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Line source and point source scattering of acoustic waves by the junction of transmissive and soft–hard half planes

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ABSTRACT

Firstly, the analysis of [A. Büyükaksoy, G. Cinar, A.H. Serbest, Scattering of plane waves by the junction of transmissive and soft–hard half planes, ZAMP 55 (2004) 483–499] for the scattering of plane waves by the junction of transmissive and soft–hard half planes is extended to the case of a line source. The introduction of the line source changes the incident field and the method of solution requires a careful analysis in calculating the scattered field. The graphical results are presented using MATHEMATICA. We observe that the graphs of the plane wave situation [A. Büyükaksoy, G. Cinar, A.H. Serbest, Scattering of plane waves by the junction of transmissive and soft–hard half planes, ZAMP 55 (2004) 483–499] can be recovered by shifting the line source to a large distance. Subsequently, the problem is further extended to the case of scattering due to a point source using the results obtained for a line source excitation. The introduction of a point source (three dimensions) involves another variable which then requires the calculation of an additional integral appearing in the inverse transform.

1. Introduction

The Wiener–Hopf (WH) technique provides a significant extension of the large class of problems that can be solved by Fourier, Laplace and Mellin integral transform [1]. The WH technique provides us an approach for considering the diffraction of waves by a single half plane [1]. However there are problems in dealing with other configurations which are first attacked by using matrix version of WH equations. A comprehensive procedure for tackling the matrix version of these equations is not yet available because it is not normally easy to split the matrix into the appropriate half planes. The noncommutativity of the factor matrices and the requirement of the radiation conditions also present further problems. Nevertheless the development and improvement of this technique is progressing steadily [2]. For example the Wiener–Hopf Hilbert method introduced by Hurd [3], Rawlins [4] and Rawlins and Williams [5] is a powerful tool in the case when kernel matrix has only branch point singularities, while the Daniele–Kharapkov method proposed by Daniele [6] and Kharapkov [7] is effective for the class of matrices having only pole singularities and branch-cut singularities besides pole singularities [8–12].

Diffraction from a two part surface is an important topic in diffraction theory and constitute a canonical problem for diffraction due to abrupt changes in material properties. Recently, Büyükaksoy et al. [13] considered the scattering of plane waves by a two part surface. They developed a high frequency solution for the diffraction of plane waves by the junction of two half planes. One half plane is characterized by partially transmissive boundary conditions and the other is soft at the

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top and hard at the bottom. In [13], using the Fourier transform technique, the related boundary value problem is reduced to matrix Wiener–Hopf equation, which is solved by the usual Wiener–Hopf procedure [1].

In this paper, we have attempted two problems. In the first problem, we have extended the problem of plane wave scattering [13] to the problem of scattering due to a line source situated at (x_0, y_0). It is perhaps the first attempt to look at the line source geometry discussed by [13] involving the matrix Wiener–Hopf approach. The introduction of line source changes the incident field and the method of solution requires a careful analysis in calculating the scattered field. It is observed that when the source is shifted to a large distance our results differ from those of [13] by a multiplicative factor which agrees with, already known, facts given in the literature [2,14]. These observations can also be verified through the graphical results shown in this paper. It is observed that the graphs of plane wave situation [13] can be recovered by shifting the line source to a large distance. The problem of line source scattering is further extended to the case of point source excitation. The introduction of point source (three dimensions) introduces another variable which then requires the calculation of an additional integral appearing in the inverse transform. The analytic solution of these integrals is obtained using the method of steepest descent and the scattered field is presented.

2. The line source scattering

2.1. Mathematical formulation of the problem

We consider the problem of scattering of an acoustic wave from a line source located at (x_0, y_0) by the junction of the soft–hard half plane located at $y = 0$, $x > 0$, and the penetrable half plane located at $y = 0$, $x < 0$, respectively so that their edges lie along the z-axis. The geometry of the problem is shown in Fig. 1. Thus we can say that the field is independent of the z-axis. For the harmonic acoustic vibrations of time dependence we require the solution of the wave equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)\varphi_t(x, y) = \delta(x - x_0)\delta(y - y_0), \tag{1}$$

where φ_t is the total velocity potential, and the boundary conditions at the soft and hard surfaces are

$$\varphi_t\left(x, 0^+\right) = 0, \quad x > 0, \tag{2}$$

$$\frac{\partial \varphi_t(x, 0^-)}{\partial y} = 0, \quad x > 0, \tag{3}$$

and at the partially transmissive surface are [15]

$$\frac{\partial \varphi_t(x, 0^+)}{\partial y} + \frac{ik}{\eta}\varphi_t\left(x, 0^+\right) = 0, \quad x < 0, \tag{4}$$

$$\frac{\partial \varphi_t(x, 0^-)}{\partial y} - \frac{ik}{\eta}\varphi_t\left(x, 0^-\right) = 0, \quad x < 0, \tag{5}$$

$$\varphi_t\left(x, 0^+\right) - \varphi_t\left(x, 0^-\right) = 0, \quad x < 0. \tag{6}$$

In the above relations η is the normal specific impedance of the material relative to the impedance of the surrounding medium, k is the wave number, and a time factor $e^{-i\omega t}$ is assumed and suppressed. The boundary conditions in (4)–(6) represent the situation in which the pressure on both sides of the sheet is equal and producing the jump discontinuity in

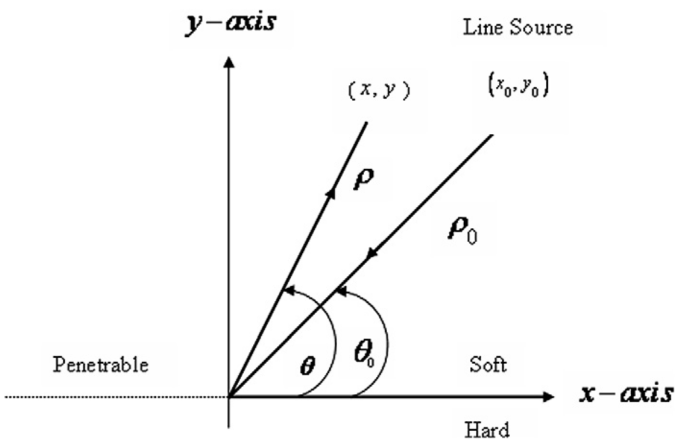

Fig. 1. Geometry of the problem.

the normal component of the fluid velocity across it. These are the valid conditions from the mathematical viewpoint and are acoustic counterpart of an electrically resistive sheet in which φ is then the tangential component of the electric field.

It is assumed that the wave number k has positive imaginary part. The lossless case can be obtained by making $\operatorname{Im} k \to 0$ in the final expressions. For the analysis purpose it is convenient to express the total field as follows [2,15,16]:

$$\varphi_t(x, y) = \varphi_0(x, y) + \varphi(x, y), \tag{7}$$

where $\varphi_0(x, y)$ is regarded as the unperturbed field that would exist if the whole plane $y = 0^+$ were a soft boundary. Hence the complementary part $\varphi(x, y)$ represents the diffracted field. In Eq. (7), we have

$$\varphi_0(x, y) = \begin{cases} \varphi^i(x, y) + \varphi^r(x, y) & \text{for } y > 0, \\ 0 & \text{for } y < 0, \end{cases} \tag{8}$$

where φ^i is the incident field satisfying the equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)\varphi^i(x, y) = \delta(x - x_0)\delta(y - y_0), \tag{9}$$

and φ^r is the corresponding reflected field. The scattered field $\varphi(x, y)$ satisfies the Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)\varphi(x, y) = 0. \tag{10}$$

For analytic convenience, we shall assume that k has small imaginary part for which $k = k_r + ik_i$, where k_r and k_i are both positive. It is appropriate to define the following Fourier transform pair as follows:

$$\bar{\varphi}(\alpha, y) = \int_{-\infty}^{\infty} \varphi(x, y)e^{i\alpha x}\,dx \tag{11a}$$

and

$$\varphi(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{\varphi}(\alpha, y)e^{-i\alpha x}\,d\alpha. \tag{11b}$$

Using Eq. (11a), Eq. (10) can be written as

$$\frac{d^2\bar{\varphi}}{dy^2} + K^2\bar{\varphi} = 0, \tag{12}$$

where $K(\alpha) = \sqrt{k^2 - \alpha^2}$. The square root function is defined in the complex α-plane cut such that $K(0) = k$.

The solution of Eq. (12) satisfying the radiation conditions can be written as

$$\bar{\varphi}(\alpha, y) = \begin{cases} A(\alpha)e^{iK(\alpha)y}, & y > 0, \\ B(\alpha)e^{-iK(\alpha)y}, & y < 0, \end{cases} \tag{13}$$

where $A(\alpha)$ and $B(\alpha)$ are the unknown coefficients to be determined.

Using Eq. (11a), from Eq. (9) we obtain the incident field and the corresponding reflected field as follows:

$$\bar{\varphi}^i(\alpha, y) = \frac{1}{2iK}e^{i\alpha x_0 + iK(\alpha)|y - y_0|} \tag{14}$$

and

$$\bar{\varphi}^r(\alpha, y) = -\frac{1}{2iK}e^{i\alpha x_0 + iK(\alpha)|y + y_0|}. \tag{15}$$

Taking Fourier transform of the boundary conditions (2)–(6), we obtain

$$\bar{\varphi}_+(\alpha, 0^+) = -\int_0^{\infty} \varphi_0(x, 0^+)e^{i\alpha x}\,dx, \tag{16}$$

$$\frac{\partial\bar{\varphi}_+(\alpha, 0^-)}{\partial y} = 0, \tag{17}$$

$$\frac{\partial\bar{\varphi}_-(\alpha, 0^+)}{\partial y} + \frac{ik}{\eta}\bar{\varphi}_-(\alpha, 0^+) = -\int_{-\infty}^{0}\left[\frac{\partial\bar{\varphi}_0(x, 0^+)}{\partial y} + \frac{ik}{\eta}\bar{\varphi}_0(x, 0^+)\right]e^{i\alpha x}\,dx, \tag{18}$$

$$\frac{\partial\bar{\varphi}_-(\alpha,0^-)}{\partial y}-\frac{ik}{\eta}\bar{\varphi}_-(\alpha,0^-)=0, \tag{19}$$

$$\bar{\varphi}_-(\alpha,0^+)-\bar{\varphi}_-(\alpha,0^-)=-\int_{-\infty}^{0}\varphi_0(x,0^+)e^{i\alpha x}\,dx. \tag{20}$$

In order to obtain the unique solution it is necessary to take into account the following edge conditions

$$\varphi(x,0)=O\left(x^{\frac{1}{4}}\right)\quad \text{as } x\to 0, \tag{21}$$

$$\frac{\partial\varphi(x,0)}{\partial y}=O\left(x^{-\frac{3}{4}}\right)\quad \text{as } x\to 0. \tag{22}$$

The substitution of Eq. (13) into boundary conditions (16)–(20) and the Fourier inversion of the resulting integral equations yield

$$A(\alpha)=\bar{\varphi}_-(\alpha,0^+)-\int_{0}^{\infty}\varphi_0(x,0^+)e^{i\alpha x}\,dx, \tag{23}$$

$$B(\alpha)=\frac{\bar{\varphi}'_-(\alpha,0^-)}{K(\alpha)}, \tag{24}$$

$$A(\alpha)-B(\alpha)=\bar{\Lambda}_+(\alpha)-\int_{-\infty}^{0}\varphi_0(x,0^+)e^{i\alpha x}\,dx, \tag{25}$$

$$\left[\frac{2k}{\eta}+K(\alpha)\right]A(\alpha)+K(\alpha)B(\alpha)=\bar{\Lambda}'_+(\alpha)-\frac{2k}{\eta}\left[\int_{-\infty}^{\infty}\varphi_0(x,0^+)e^{i\alpha x}\,dx\right]-\frac{1}{i}\int_{-\infty}^{0}\varphi'_0(x,0)e^{i\alpha x}\,dx, \tag{26}$$

where prime "′" denotes the differentiation with respect to y and $\bar{\varphi}_-(\alpha,0^+)$, $\bar{\varphi}'_-(\alpha,0^-)$, $\bar{\Lambda}_+(\alpha)$ and $\bar{\Lambda}'_+(\alpha)$ are defined by

$$\bar{\varphi}_-(\alpha,0^+)=\int_{-\infty}^{0}\varphi(x,0^+)e^{i\alpha x}\,dx, \tag{27}$$

$$\bar{\varphi}'_-(\alpha,0^-)=i\int_{-\infty}^{0}\frac{\partial\varphi(x,0^-)}{\partial y}e^{i\alpha x}\,dx, \tag{28}$$

$$\bar{\Lambda}_+(\alpha)=\int_{0}^{\infty}\left[\varphi(x,0^+)-\varphi(x,0^-)\right]e^{i\alpha x}\,dx, \tag{29}$$

$$\bar{\Lambda}'_+(\alpha)=\int_{0}^{\infty}\left[\frac{\partial\varphi(x,0^+)}{\partial y}-\frac{\partial\varphi(x,0^-)}{\partial y}\right]e^{i\alpha x}\,dx. \tag{30}$$

Due to the analytic properties of the Fourier integrals $\bar{\varphi}_-(\alpha,0^+)$, $\bar{\varphi}'_-(\alpha,0^-)$, $\bar{\Lambda}_+(\alpha)$ and $\bar{\Lambda}'_+(\alpha)$ are regular functions of α in the half planes $\mathrm{Im}(\alpha)<\mathrm{Im}(k\cos\theta_0)$ and $\mathrm{Im}(\alpha)>\mathrm{Im}(-k)$, respectively. By using the edge conditions (21)–(22) it can be easily shown that when we let $|\alpha|\to\infty$ in their respective regions of regularity we have

$$\bar{\varphi}_-(\alpha,0^+)=O\left(\alpha^{-\frac{5}{4}}\right) \tag{31}$$

and

$$\bar{\varphi}'_-(\alpha,0^-)=O\left(\alpha^{-\frac{1}{4}}\right). \tag{32}$$

The elimination of $A(\alpha)$ and $B(\alpha)$ between Eqs. (23)–(26) leads to the following matrix Wiener–Hopf equation valid in the strip $\mathrm{Im}(-k)<\mathrm{Im}(\alpha)<\mathrm{Im}(k\cos\theta_0)$

$$\begin{bmatrix}1 & -\frac{1}{K(\alpha)}\\ \frac{2k}{\eta}+K(\alpha) & 1\end{bmatrix}\begin{bmatrix}\bar{\varphi}_-(\alpha,0^+)\\ \bar{\varphi}'_-(\alpha,0^-)\end{bmatrix}=\begin{bmatrix}\bar{\Lambda}_+(\alpha)\\ \bar{\Lambda}'_+(\alpha)\end{bmatrix}+\begin{bmatrix}q\\ r\end{bmatrix}, \tag{33}$$

$$q=-\int_{-\infty}^{0}\varphi_0(x,0^+)e^{i\alpha x}\,dx+\int_{0}^{\infty}\varphi_0(x,0^+)e^{i\alpha x}\,dx, \tag{34}$$

$$r=-\frac{2k}{\eta}\left[\int_{-\infty}^{\infty}\varphi_0\big(x,0^+\big)e^{i\alpha x}\,dx\right]-\frac{1}{i}\int_{-\infty}^{0}\varphi_0'(x,0)e^{i\alpha x}\,dx+\left(\frac{2k}{\eta}+K(\alpha)\right)\int_{-\infty}^{0}\varphi_0\big(x,0^+\big)e^{i\alpha x}\,dx, \tag{35}$$

with

$$\mathbf{M}(\alpha)=\begin{bmatrix}1 & -\frac{1}{K(\alpha)}\\ \frac{2k}{\eta}+K(\alpha) & 1\end{bmatrix}. \tag{36}$$

In order to obtain the explicit solution of Eq. (33), we first need to factorize the kernel matrix $\mathbf{M}(\alpha)$ as the product of two non-singular matrices say $\mathbf{M}_+(\alpha)$ and $\mathbf{M}_-(\alpha)$ whose entries are the regular functions of α in the upper and lower half planes, respectively. The kernel matrix $\mathbf{M}(\alpha)$ is factorized by [13] using the Daniele–Kharapkov method [7]. Further details can be found in [13]. The Daniele–Kharapkov method suggests that we have to pre-multiply the matrix given in Eq. (36) with the following constant matrix

$$\mathbf{C}=\begin{bmatrix}1 & 0\\ -\frac{2k}{\eta} & 1\end{bmatrix}, \tag{37}$$

and then write it in the form necessary for the application of Kharapkov method. Thus, we have

$$\mathbf{W}(\alpha)=\mathbf{CM}(\alpha)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\frac{1}{K(\alpha)}\begin{bmatrix}0 & -1\\ k^2-\alpha^2 & \frac{2k}{\eta}\end{bmatrix}=\begin{bmatrix}F_+ & G_+\\ H_+ & J_+\end{bmatrix}\begin{bmatrix}F_- & G_-\\ H_- & J_-\end{bmatrix}. \tag{38}$$

The matrix $\mathbf{W}(\alpha)$ is a special form which can be factorized through the Kharapkov method. Omitting the details [13] we give the final expression for $\mathbf{W}_+(\alpha)$ as

$$\mathbf{W}_+(\alpha)=\left(\frac{2}{\eta}\right)^{\frac{1}{4}}\frac{1}{\sqrt{\kappa_+(\alpha)}}\begin{bmatrix}\cosh\varkappa(\alpha)-\frac{k}{\eta}\frac{\sinh\varkappa(\alpha)}{\sqrt{\alpha^2-\sigma^2}} & -\frac{\sinh\varkappa(\alpha)}{\sqrt{\alpha^2-\sigma^2}}\\ (k^2-\alpha^2)\frac{\sinh\varkappa(\alpha)}{\sqrt{\alpha^2-\sigma^2}} & \cosh\varkappa(\alpha)+\frac{k}{\eta}\frac{\sinh\varkappa(\alpha)}{\sqrt{\alpha^2-\sigma^2}}\end{bmatrix}=\begin{bmatrix}F_+ & G_+\\ H_+ & J_+\end{bmatrix}, \tag{39}$$

so that

$$\mathbf{W}_-(\alpha)=\mathbf{W}_+(-\alpha), \tag{40}$$

$$\varkappa(\alpha)=\frac{1}{4}\ln\left\{\frac{(\sigma^2+k\alpha-\sqrt{\alpha^2-\sigma^2}\sqrt{k^2-\sigma^2})(\alpha+\sqrt{k^2-\alpha^2})}{\sigma^2(k+\alpha)}\right\}, \tag{41}$$

and

$$\sigma=k\sqrt{1-\frac{1}{\eta^2}}. \tag{42}$$

In Eq. (39), $\kappa_+(\alpha)$ and $\kappa_-(\alpha)=\kappa_+(-\alpha)$ are the split functions regular and free of zeros in the upper and lower half planes, respectively, resulting from the factorization of

$$\kappa(\alpha)=\frac{K(\alpha)}{k+\eta K(\alpha)} \tag{43}$$

as

$$\kappa(\alpha)=\kappa_-(\alpha)\kappa_+(\alpha). \tag{44}$$

Noticing that $\kappa_+(\alpha)$ and $\kappa_-(\alpha)$ can be expressed in terms of Maliuzhinetz function [17] as follows

$$\begin{aligned}\kappa_-(k\cos\theta)=2^{\frac{3}{2}}\sqrt{\frac{2}{\eta}}\sin\frac{\theta}{2}\left\{\frac{M_\pi(\frac{3\pi}{2}-\theta-\psi)M_\pi(\frac{\pi}{2}-\theta+\psi)}{M_\pi^2(\frac{\pi}{2})}\right\}\Bigg\{\left[1+\sqrt{2}\cos\left(\frac{\frac{\pi}{2}-\theta+\psi}{2}\right)\right]\\ \times\left[1+\sqrt{2}\cos\left(\frac{\frac{3\pi}{2}-\theta-\psi}{2}\right)\right]\Bigg\}^{-1}\end{aligned} \tag{45}$$

with

$$\sin\psi=\frac{1}{\eta} \tag{46}$$

and

$$M_\pi(z)=\exp\left\{-\frac{1}{8\pi}\int_0^z\frac{\pi\sin u-2\sqrt{2}\sin\frac{u}{2}+2u}{\cos u}\,du\right\}, \tag{47}$$

and when we let $|\alpha| \to \infty$ in the upper half plane, we obtain

$$\mathbf{W}_{+}(\alpha) \approx \frac{1}{\sqrt{2}}\left\{\frac{k-\sqrt{k^2-\sigma^2}}{\sigma^2}\right\}^{\frac{1}{4}}\begin{bmatrix}\alpha^{\frac{1}{4}} & \alpha^{-\frac{3}{4}}\\ \alpha^{\frac{5}{4}} & \alpha^{\frac{1}{4}}\end{bmatrix}, \tag{48}$$

$$\mathbf{W}_{-}(\alpha) = \mathbf{W}_{+}(-\alpha). \tag{49}$$

With this factorization of the kernel matrix, Eq. (33) can be rearranged as

$$\mathbf{W}_{+}(\alpha)\mathbf{W}_{-}(\alpha)\begin{bmatrix}\bar{\varphi}_{-}(\alpha,0^{+})\\ \bar{\varphi}'_{-}(\alpha,0^{-})\end{bmatrix} = \mathbf{C}\begin{bmatrix}\bar{\Lambda}_{+}(\alpha)\\ \bar{\Lambda}'_{+}(\alpha)\end{bmatrix} + \mathbf{C}\begin{bmatrix}q\\ r\end{bmatrix} \tag{50}$$

or

$$\mathbf{W}_{-}(\alpha)\begin{bmatrix}\bar{\varphi}_{-}(\alpha,0^{+})\\ \bar{\varphi}'_{-}(\alpha,0^{-})\end{bmatrix} = \mathbf{W}_{+}^{-1}(\alpha)\mathbf{C}\begin{bmatrix}\bar{\Lambda}_{+}(\alpha)\\ \bar{\Lambda}'_{+}(\alpha)\end{bmatrix} + \mathbf{W}_{+}^{-1}(\alpha)\mathbf{C}\begin{bmatrix}q\\ r\end{bmatrix}. \tag{51}$$

Eq. (51) is the matrix Wiener–Hopf equation. To make it regular in the upper and lower half planes we need to split the term

$$\mathbf{W}_{+}^{-1}(\alpha)\mathbf{C}\begin{bmatrix}q\\ r\end{bmatrix}.$$

This can be achieved by using the additive decomposition theorem [1, p. 14]. This term can be decomposed as follows:

$$\mathbf{W}_{+}^{-1}(\alpha)\mathbf{C}\begin{bmatrix}q\\ r\end{bmatrix} = \begin{bmatrix}T\\ S\end{bmatrix} = \begin{bmatrix}T_{+}+T_{-}\\ S_{+}+S_{-}\end{bmatrix}. \tag{52}$$

Using Eqs. (37) and (39), we arrive at

$$\mathbf{W}_{+}^{-1}(\alpha)\mathbf{C}\begin{bmatrix}q\\ r\end{bmatrix} = \begin{bmatrix}J_{+}q - G_{+}(-\frac{2k}{\eta}+r)\\ -H_{+}q + F_{+}(-\frac{2k}{\eta}+r)\end{bmatrix} = \begin{bmatrix}T\\ S\end{bmatrix} = \begin{bmatrix}T_{+}+T_{-}\\ S_{+}+S_{-}\end{bmatrix}, \tag{53}$$

where

$$T_{\pm}(\alpha) = \pm\frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic}\frac{T(\xi)}{\xi-\alpha}\,d\xi = \pm\frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic}\left[\frac{J_{+}(\xi)q(\xi) - G_{+}(\xi)(-\frac{2k}{\eta}+r(\xi))}{\xi-\alpha}\right]d\xi, \tag{54}$$

$$S_{\pm}(\alpha) = \pm\frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id}\frac{S(\xi)}{\xi-\alpha}\,d\xi = \pm\frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id}\left[\frac{-H_{+}(\xi)q(\xi) + F_{+}(\xi)(-\frac{2k}{\eta}+r(\xi))}{\xi-\alpha}\right]d\xi. \tag{55}$$

Using Eqs. (34), (35) and (39) in Eqs. (54) and (55), the explicit expressions for $T_{-}(\alpha)$ and $S_{-}(\alpha)$ are given as follows:

$$T_{-}(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\left[\frac{(\frac{\eta}{2})^{\frac{1}{4}}\sqrt{\kappa_{+}(\xi)}\exp(i\xi x_0 + iK(\xi)|y_0|)\frac{\sinh\varkappa(\xi)}{\sqrt{\xi^2-\sigma^2}}}{\xi-\alpha}\right]d\xi \tag{56}$$

and

$$S_{-}(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\left[\frac{(\frac{\eta}{2})^{\frac{1}{4}}\sqrt{\kappa_{+}(\xi)}\exp(i\xi x_0 + iK(\xi)|y_0|)\{\cosh\varkappa(\xi) - \frac{k}{\eta}\frac{\sinh\varkappa(\xi)}{\sqrt{\xi^2-\sigma^2}}\}}{\xi-\alpha}\right]d\xi. \tag{57}$$

Using Eq. (52) in Eq. (51) and separating into positive and negative portions, we arrive at

$$\mathbf{W}_{-}(\alpha)\begin{bmatrix}\bar{\varphi}_{-}(\alpha,0^{+})\\ \bar{\varphi}'_{-}(\alpha,0^{-})\end{bmatrix} - \begin{bmatrix}T_{-}\\ S_{-}\end{bmatrix} = \mathbf{W}_{+}^{-1}(\alpha)\mathbf{C}\begin{bmatrix}\bar{\Lambda}_{+}(\alpha)\\ \bar{\Lambda}'_{+}(\alpha)\end{bmatrix} + \begin{bmatrix}T_{+}\\ S_{+}\end{bmatrix}. \tag{58}$$

The left-hand side of Eq. (58) is regular in the lower half plane $\mathrm{Im}(\alpha) < \mathrm{Im}(k\cos\theta_0)$ and the right-hand side is regular in the upper half plane $\mathrm{Im}(\alpha) > \mathrm{Im}(-k)$. Hence by analytic continuation principle both sides define an entire matrix-valued function $\mathbf{P}(\alpha)$. To find the exact value of $\mathbf{P}(\alpha)$, we take into account the order relations in Eqs. (31), (32), (48) and (49) which help us to conclude from the extended Liouville's theorem that the $\mathbf{P}(\alpha)$ is a constant matrix of the form

$$\mathbf{P}(\alpha) = \begin{bmatrix}0\\ p^{*}\end{bmatrix}, \tag{59}$$

where p^{*} can be evaluated as follows.

From Eq. (58), we obtain

$$\begin{bmatrix} \bar{\varphi}_-(\alpha, 0^+) \\ \bar{\varphi}'_-(\alpha, 0^-) \end{bmatrix} = \begin{bmatrix} J_- & -G_- \\ -H_- & F_- \end{bmatrix} \begin{bmatrix} T_- \\ S_- + p^* \end{bmatrix}. \tag{60}$$

The above equation can further be simplified to get

$$\begin{bmatrix} \bar{\varphi}_-(\alpha, 0^+) \\ \bar{\varphi}'_-(\alpha, 0^-) \end{bmatrix} = \begin{bmatrix} J_-T_- - G_-(S_- + p^*) \\ -H_-T_- + F_-(S_- + p^*) \end{bmatrix}. \tag{61}$$

The unknown constant p^* can be specified by taking into account the order relations in Eqs. (31) and (32). By using (40) one obtains

$$\begin{bmatrix} \bar{\varphi}_-(\alpha, 0^+) \\ \bar{\varphi}'_-(\alpha, 0^-) \end{bmatrix} \approx \sqrt{2}\left\{\frac{k - \sqrt{k^2 - \sigma^2}}{\sigma^2}\right\}^{-\frac{1}{4}} [p^* - \widetilde{T}_-] \begin{bmatrix} (-\alpha)^{-\frac{3}{4}} \\ (-\alpha)^{-\frac{1}{4}} \end{bmatrix} + O\begin{bmatrix} (-\alpha)^{-\frac{5}{4}} \\ (-\alpha)^{-\frac{1}{4}} \end{bmatrix}, \tag{62}$$

with

$$\widetilde{T}_- = \lim_{\alpha \to \infty} \alpha T_-. \tag{63}$$

The correct behavior of $\bar{\varphi}_-(\alpha, 0^+)$ and $\bar{\varphi}'_-(\alpha, 0^-)$ is recovered if we choose

$$p^* = \widetilde{T}_-. \tag{64}$$

Hence the explicit expressions for $\bar{\varphi}_-(\alpha, 0^+)$ and $\bar{\varphi}'_-(\alpha, 0^-)$ are given as

$$\begin{aligned} \bar{\varphi}_-\big(\alpha, 0^+\big) = {} & \left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_-(\alpha)} \Bigg[\Bigg\{\frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} \left(\frac{1}{\xi - \alpha}\right)\left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_+(\xi)} \exp i\xi x_0 + iK(\xi)|y_0| \left(\frac{\sinh \varkappa(\xi)}{\sqrt{\xi^2 - \sigma^2}}\right) d\xi \Bigg\} \\ & \times \left\{\cosh \varkappa(-\alpha) + \frac{k}{\eta} \frac{\sinh \varkappa(-\alpha)}{\sqrt{\alpha^2 - \sigma^2}}\right\} + \left\{\frac{\sinh \varkappa(-\alpha)}{\sqrt{\alpha^2 - \sigma^2}}\right\} \Bigg\{\frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} \left(\frac{1}{\xi - \alpha}\right)\left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_+(\xi)} \\ & \times \exp\big(i\xi x_0 + iK(\xi)|y_0|\big)\left(\cosh \varkappa(\xi) - \frac{k}{\eta} \frac{\sinh \varkappa(\xi)}{\sqrt{\xi^2 - \sigma^2}}\right) d\xi + p^* \Bigg\}\Bigg], \end{aligned} \tag{65}$$

and

$$\begin{aligned} \bar{\varphi}'_-\big(\alpha, 0^-\big) = {} & \left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_-(\alpha)} \Bigg[\Bigg\{\frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} \left(\frac{1}{\xi - \alpha}\right)\left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_+(\xi)} \exp\big(i\xi x_0 + iK(\xi)|y_0|\big) \left(\frac{\sinh \varkappa(\xi)}{\sqrt{\xi^2 - \sigma^2}}\right) d\xi \Bigg\} \\ & \times \left\{-\big(k^2 - \alpha^2\big)\frac{\sinh \varkappa(-\alpha)}{\sqrt{\alpha^2 - \sigma^2}}\right\} + \left\{\cosh \varkappa(-\alpha) - \frac{k}{\eta} \frac{\sinh \varkappa(-\alpha)}{\sqrt{\alpha^2 - \sigma^2}}\right\} \Bigg\{\frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} \left(\frac{1}{\xi - \alpha}\right)\left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_+(\xi)} \\ & \times \exp\big(i\xi x_0 + iK(\xi)|y_0|\big)\left(\cosh \varkappa(\xi) - \frac{k}{\eta} \frac{\sinh \varkappa(\xi)}{\sqrt{\xi^2 - \sigma^2}}\right) d\xi + p^* \Bigg\}\Bigg]. \end{aligned} \tag{66}$$

2.2. The far field solution

Now by substituting Eqs. (65) and (66) into Eqs. (23) and (24) and then resulting equations in Eq. (13) and taking the inverse Fourier transform, we obtain for $y > 0$,

$$\begin{aligned} \varphi(x, y) = {} & \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_-(\alpha)} \Bigg[\Bigg\{\frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} \left(\frac{1}{\xi - \alpha}\right)\left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_+(\xi)} \exp\big(i\xi x_0 + iK(\xi)|y_0|\big) \left(\frac{\sinh \varkappa(\xi)}{\sqrt{\xi^2 - \sigma^2}}\right) d\xi \Bigg\} \\ & \times \left\{\cosh \varkappa(-\alpha) + \frac{k}{\eta} \frac{\sinh \varkappa(-\alpha)}{\sqrt{\alpha^2 - \sigma^2}}\right\} + \left\{\frac{\sinh \varkappa(-\alpha)}{\sqrt{\alpha^2 - \sigma^2}}\right\} \Bigg\{\frac{1}{2\pi i} \int_{-\infty+ia}^{\infty+ia} \left(\frac{1}{\xi - \alpha}\right)\left(\frac{\eta}{2}\right)^{\frac{1}{4}} \sqrt{\kappa_+(\xi)} \\ & \times \exp\big(i\xi x_0 + iK(\xi)|y_0|\big)\left(\cosh \varkappa(\xi) - \frac{k}{\eta} \frac{\sinh \varkappa(\xi)}{\sqrt{\xi^2 - \sigma^2}}\right) d\xi + p^* \Bigg\}\Bigg] e^{iK(\alpha)y - i\alpha x}\, d\alpha, \end{aligned} \tag{67}$$

and for $y < 0$

$$\varphi(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_-(\alpha)}\Bigg[\Bigg\{\frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\left(\frac{1}{\xi-\alpha}\right)\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_+(\xi)}\exp\bigl(i\xi x_0 + iK(\xi)|y_0|\bigr)\left(\frac{\sinh\varkappa(\xi)}{\sqrt{\xi^2-\sigma^2}}\right)d\xi\Bigg\}$$
$$\times\left\{-\bigl(k^2-\alpha^2\bigr)\frac{\sinh\varkappa(-\alpha)}{\sqrt{\alpha^2-\sigma^2}}\right\} + \left\{\cosh\varkappa(-\alpha) - \frac{k}{\eta}\frac{\sinh\varkappa(-\alpha)}{\sqrt{\alpha^2-\sigma^2}}\right\}\Bigg\{\frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\left(\frac{1}{\xi-\alpha}\right)\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_+(\xi)}$$
$$\times\exp\bigl(i\xi x_0 + iK(\xi)|y_0|\bigr)\left(\cosh\varkappa(\xi) - \frac{k}{\eta}\frac{\sinh\varkappa(\xi)}{\sqrt{\xi^2-\sigma^2}}\right)d\xi + p^*\Bigg\}\Bigg]e^{-iK(\alpha)y - i\alpha x}\,d\alpha. \tag{68}$$

To determine the far field behavior of the scattered field we introduce the following substitutions

$$x = \rho\cos\theta, \quad y = \rho\sin\theta \quad (0 < \theta < \pi), \tag{69}$$

$$x_0 = \rho_0\cos\theta_0, \quad y_0 = \rho_0\sin\theta_0 \quad (\pi < \theta_0 < 0) \tag{70}$$

and the transformation

$$\alpha = -k\cos(\theta + it_1). \tag{71}$$

The explicit expression for the constant p^* is determined from Eqs. (56), (63) and (64) which give it as

$$p^* = \frac{1}{2\pi i}\sqrt{\frac{2\pi}{k\rho_0}}\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\frac{k}{\pi}\sin\theta_0\frac{\sqrt{\kappa_+(k\cos\theta_0)}}{\sqrt{k^2\cos^2\theta-\sigma^2}}\sinh\varkappa(k\cos\theta_0)\exp\left(ik\rho_0 + i\frac{\pi}{4}\right), \tag{72}$$

where t_1, given in Eq. (71) is real. The contour of integration over α in Eqs. (67) and (68) goes into the branch of hyperbola around $-ik$ if $\frac{\pi}{2} < \theta < \pi$. We further observe that, in deforming the contour into a hyperbola the pole $\alpha = \xi$ may be crossed. If we also make the transformation $\xi = k\cos(\theta_0 + i\tau_1)$ the contour over ξ also goes into a hyperbola. The two hyperbolae will not cross each other if $\theta < \theta_0$. However, if the inequality is reversed there will be a contribution from the pole which, in fact, cancels the incident wave in the shadow region. Omitting the details of calculations, using the method of steepest descent, the field due to a line source at a large distance from the edge is given for both cases $y > 0$ and $y < 0$, respectively. For $y > 0$, we have

$$\varphi(\rho,\theta) \approx \frac{e^{\frac{-i\pi}{2}}}{2\pi}\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_+(k\cos\theta)}\frac{\exp(ik\rho + ik\rho_0)}{\sqrt{\rho\rho_0}}\Bigg[\Bigg[\left\{\cosh\varkappa(k\cos\theta) + \frac{k}{\eta}\frac{\sinh\varkappa(k\cos\theta)}{\sqrt{k^2\cos^2\theta-\sigma^2}}\right\}$$
$$\times\left\{i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\frac{\sqrt{\kappa_+(k\cos\theta_0)}}{\sqrt{k^2\cos^2\theta_0-\sigma^2}}\sinh\varkappa(k\cos\theta_0)\right\} + \frac{\sinh\varkappa(k\cos\theta)}{\sqrt{k^2\cos^2\theta-\sigma^2}}$$
$$\times\Bigg\{\left(i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\sqrt{\kappa_+(k\cos\theta_0)}\right)\left(\cosh\varkappa(k\cos\theta_0) - \frac{k}{\eta}\frac{\sinh\varkappa(k\cos\theta_0)}{\sqrt{k^2\cos^2\theta_0-\sigma^2}}\right)$$
$$+\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}(\cos\theta_0+\cos\theta_0)p\Bigg\}\Bigg]\Bigg[F\left(\sqrt{2k\rho}\cos\frac{\theta-\theta_0}{2}\right) + F\left(\sqrt{2k\rho}\cos\frac{\theta+\theta_0}{2}\right)\Bigg]. \tag{73}$$

For $y < 0$, the far field is given as follows:

$$\varphi(\rho,\theta) \approx \frac{e^{\frac{-i\pi}{2}}}{2\pi}\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_+(k\cos\theta)}\frac{\exp(ik\rho + ik\rho_0)}{k\sqrt{\rho\rho_0}}\Bigg[\Bigg[\left\{-k^2\sin^2\theta\frac{\sinh\varkappa(k\cos\theta)}{\sqrt{k^2\cos^2\theta-\sigma^2}}\right\}$$
$$\times\left\{i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}\frac{\sqrt{\kappa_+(k\cos\theta_0)}}{\sqrt{k^2\cos^2\theta_0-\sigma^2}}\sinh\varkappa(k\cos\theta_0)\right\} + \left\{\cosh\varkappa(k\cos\theta) - \frac{k}{\eta}\frac{\sinh\varkappa(k\cos\theta)}{\sqrt{k^2\cos^2\theta-\sigma^2}}\right\}$$
$$\times\Bigg\{\left(i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}\sqrt{\kappa_+(k\cos\theta_0)}\right)\left(\cosh\varkappa(k\cos\theta_0) - \frac{k}{\eta}\frac{\sinh\varkappa(k\cos\theta_0)}{\sqrt{k^2\cos^2\theta_0-\sigma^2}}\right)$$
$$+\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}(\cos\theta_0+\cos\theta_0)p\Bigg\}\Bigg]\Bigg[F\left(\sqrt{2k\rho}\cos\frac{\theta-\theta_0}{2}\right) + F\left(\sqrt{2k\rho}\cos\frac{\theta+\theta_0}{2}\right)\Bigg]. \tag{74}$$

We observe that the unknown constant p^* goes into p as determined by [13] at this stage and $F(z)$ stands for the Fresnel function as defined in [1,16]

$$F(z) = e^{-iz^2}\int_z^{\infty}e^{it^2}\,dt.$$

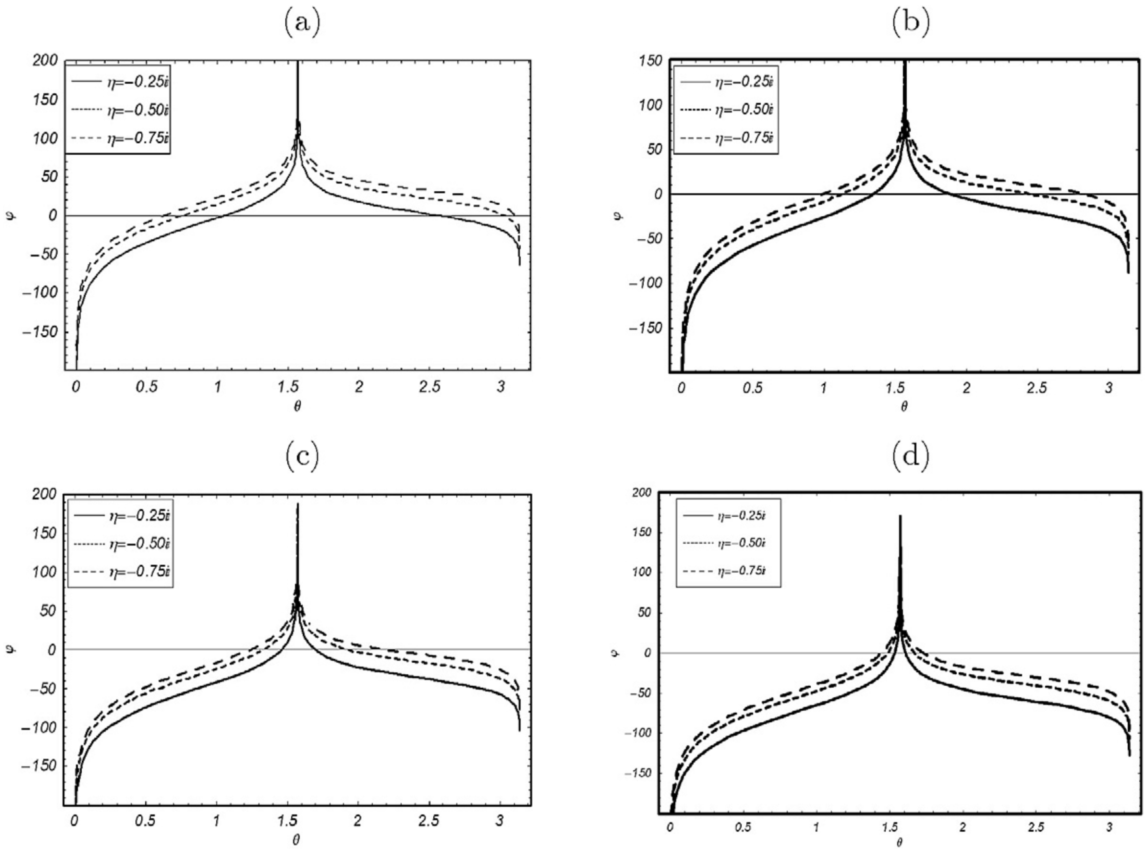

Fig. 2. The amplitude of the diffracted field ($y > 0$) versus the observation angle for different values of η (imaginary) for (panel (a)) $\rho_0 = 0.001$, (panel (b)) $\rho_0 = 0.01$, (panel (c)) $\rho_0 = 0.05$ and (panel (d)) $\rho_0 = 0.5$. The other parameters are $\theta_0 = \pi/2$, $\rho = 1$ and $k = 1$.

2.3. Computational results

In this section we will present some graphical results (Figs. 2–6) showing the effects of resistivity η and the line source parameter ρ_0 on the diffraction phenomenon. By increasing the parameter η, whether it is real or pure imaginary, and fixing the parameter ρ_0 the diffracted field increases for both the cases $y > 0$ and $y < 0$. Also by increasing the parameter ρ_0 and fixing the parameter η the diffracted field decreases for both the cases $y > 0$ and $y < 0$, respectively.

3. The point source scattering

3.1. Mathematical formulation

For the case of point source scattering we suppose that a point source is occupying the position (x_0, y_0, z_0). Thus we require the solution of the equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right)\Phi_t(x, y, z) = \delta(x - x_0)\delta(y - y_0)\delta(z - z_0), \tag{75}$$

subject to the following boundary conditions, for $x > 0$

$$\Phi_t\big(x, 0^+, z\big) = 0, \tag{76}$$

$$\frac{\partial \Phi_t(x, 0^-, z)}{\partial y} = 0, \tag{77}$$

and for $x < 0$

$$\frac{\partial \Phi_t(x, 0^+, z)}{\partial y} + \frac{ik}{\eta}\Phi_t\big(x, 0^+, z\big) = 0, \tag{78}$$

$$\frac{\partial \Phi_t(x, 0^-, z)}{\partial y} - \frac{ik}{\eta}\Phi_t\big(x, 0^-, z\big) = 0, \tag{79}$$

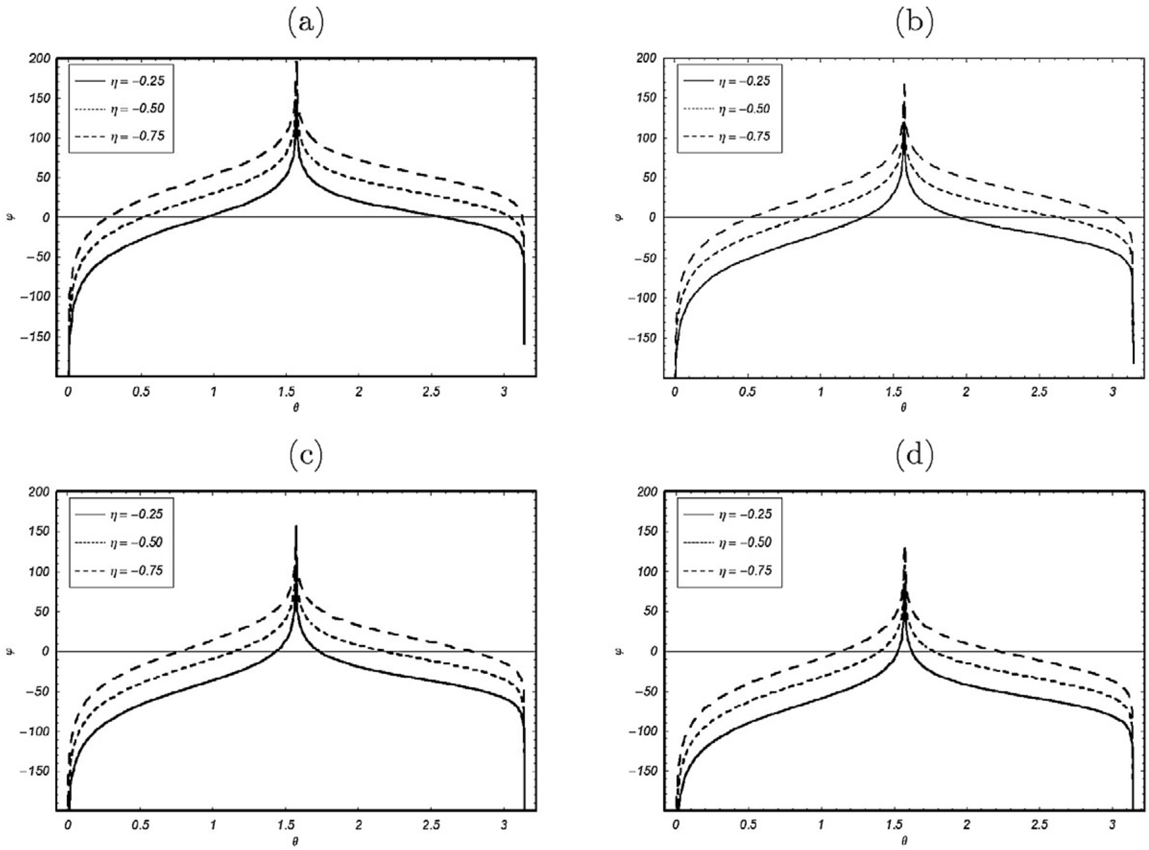

Fig. 3. The amplitude of the diffracted field ($y > 0$) versus the observation angle for different values of η (real) for (panel (a)) $\rho_0 = 0.001$, (panel (b)) $\rho_0 = 0.01$, (panel (c)) $\rho_0 = 0.05$ and (panel (d)) $\rho_0 = 0.5$. The other parameters are $\theta_0 = \pi/2$, $\rho = 1$ and $k = 1$.

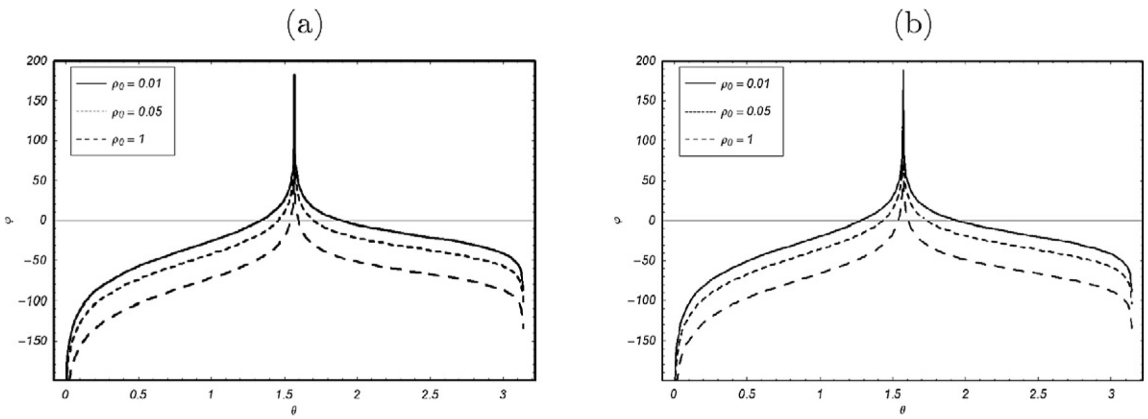

Fig. 4. The amplitude of the diffracted field ($y > 0$) versus the observation angle for different values of ρ_0 for (panel (a)) $\eta = -0.25i$, (panel (b)) $\eta = -0.25$. The other parameters are $\theta_0 = \pi/2$, $\rho = 1$ and $k = 1$.

$$\Phi_t(x, 0^+, z) - \Phi_t(x, 0^-, z) = 0, \tag{80}$$

where Φ_t is the total acoustic field, defined as

$$\Phi_t(x, y, z) = \Phi_0(x, y, z) + \Phi(x, y, z), \tag{81}$$

where Φ is the scattered field and Φ_0 represents the effect due to a point source.

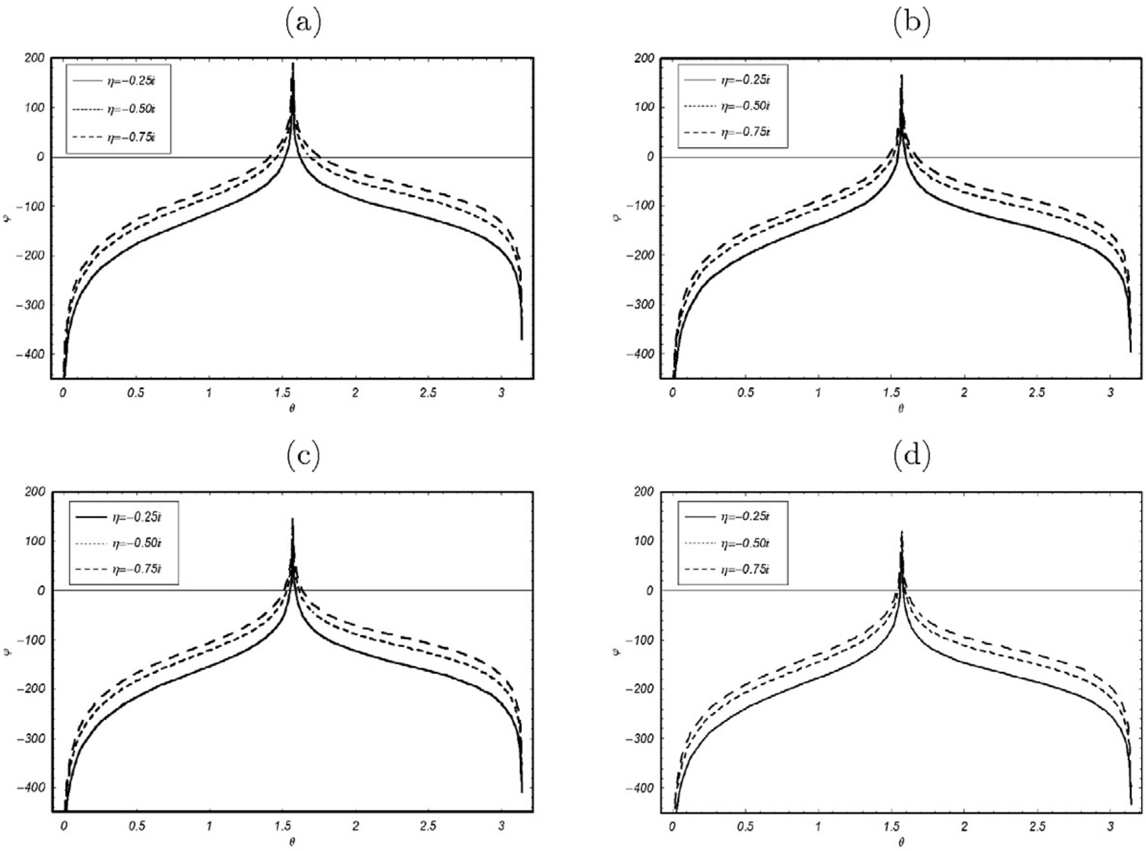

Fig. 5. The amplitude of the diffracted field ($y < 0$) versus the observation angle for different values of η (imaginary) for (panel (a)) $\rho_0 = 0.001$, (panel (b)) $\rho_0 = 0.01$, (panel (c)) $\rho_0 = 0.05$ and (panel (d)) $\rho_0 = 0.5$. The other parameters are $\theta_0 = \pi/2$, $\rho = 1$ and $k = 1$.

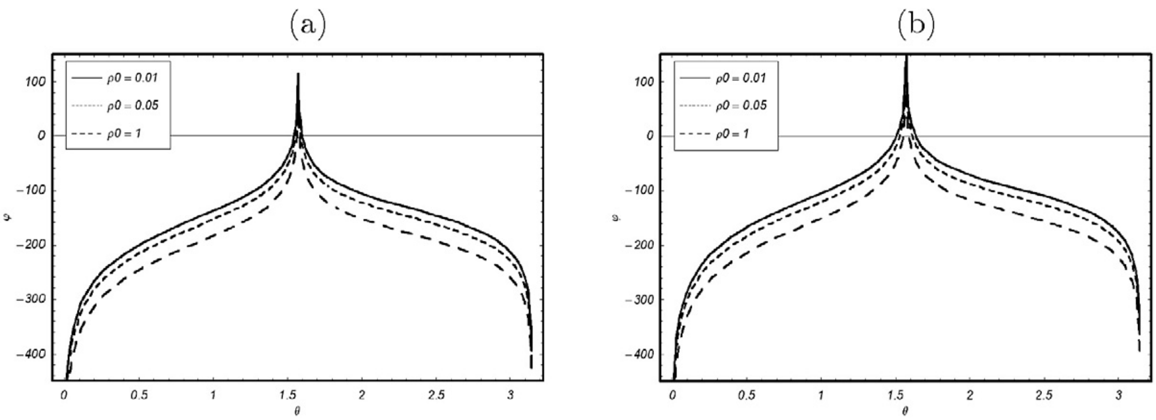

Fig. 6. The amplitude of the diffracted field ($y < 0$) versus the observation angle for different values of ρ_0 for (panel (a)) $\eta = -0.25i$, (panel (b)) $\eta = -0.5i$. The other parameters are $\theta_0 = \pi/2$, $\rho = 1$ and $k = 1$.

Let us define the Fourier transform and the inverse Fourier transform with respect to the variable z as follows:

$$\bar{\Phi}(x, y, \mu) = \int_{-\infty}^{\infty} \Phi(x, y, z) e^{ik\mu z}\, dz, \tag{82a}$$

$$\Phi(x, y, z) = \frac{k}{2\pi} \int_{-\infty}^{\infty} \bar{\Phi}(x, y, \mu) e^{-ik\mu z}\, d\mu. \tag{82b}$$

Taking Fourier transform of Eqs. (75) to (79), the problem with boundary conditions in the transformed domain μ takes the following form

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\gamma^2\right)\bar{\Phi}_t = a\delta(x-x_0)\delta(y-y_0), \tag{83}$$

with $\gamma=\sqrt{1-\mu^2}$, and $a=e^{ik\mu z_0}$.

The transformed boundary conditions take the form

$$\bar{\Phi}_t\big(x,0^+,\mu\big)=0, \tag{84}$$

$$\frac{\partial\bar{\Phi}_t(x,0^-,\mu)}{\partial y}=0, \tag{85}$$

$$\frac{\partial\bar{\Phi}_t(x,0^+,\mu)}{\partial y}+\frac{ik}{\eta}\bar{\Phi}_t\big(x,0^+,\mu\big)=0, \tag{86}$$

$$\frac{\partial\bar{\Phi}_t(x,0^-,\mu)}{\partial y}-\frac{ik}{\eta}\bar{\Phi}_t\big(x,0^-,\mu\big)=0, \tag{87}$$

$$\bar{\Phi}_t\big(x,0^+,\mu\big)-\bar{\Phi}_t\big(x,0^-,\mu\big)=0. \tag{88}$$

Thus we see that the problem (83) together with the boundary conditions (84)–(88) in the transformed domain μ is the same as in the case of two dimensions formulated in Section 2 except that $k^2\gamma^2$ replaces k^2.

3.2. Solution of the problem

As mentioned before, the mathematical problem (83) together with the boundary conditions (84)–(88) in the transformed domain μ is the same as in the case of two dimensions formulated in Section 2 except that $k^2\gamma^2$ replaces k^2 [18,19]. Thus making use of Eqs. (73) and (74) we can calculate the scattered field due to a point source as follows.

For $y>0$

$$\begin{aligned}
\bar{\Phi}(\rho,\theta,\mu) \approx{}& \left(\frac{e^{\frac{-i\pi}{2}}}{2\pi}\left(\frac{\eta}{2}\right)^{\frac14}\sqrt{\kappa_+(k\gamma\cos\theta)}\right)\left[\left\{\cosh\varkappa(k\gamma\cos\theta)+\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\right\}\right.\\
&\times\left\{i\left(\frac{\eta}{2}\right)^{\frac14}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\sinh\varkappa(k\gamma\cos\theta_0)\right\}+\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\\
&\times\left\{\left(i\left(\frac{\eta}{2}\right)^{\frac14}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\sqrt{\kappa_+(k\gamma\cos\theta_0)}\right)\right.\\
&\left.\left.\times\begin{pmatrix}\cosh\varkappa(k\gamma\cos\theta_0)\\ -\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta_0)}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\end{pmatrix}+\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}(\cos\theta+\cos\theta_0)p^{**}\right\}\right]\\
&\times\left[\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta-\theta_0}{2}\right)+\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta+\theta_0}{2}\right)\right]\frac{\exp[ik\gamma(\rho+\rho_0)+ik\mu z_0]}{\sqrt{\rho\rho_0}}.
\end{aligned}\tag{89}$$

For $y<0$

$$\begin{aligned}
\bar{\Phi}(\rho,\theta,\mu) \approx{}& \left(\frac{e^{\frac{-i\pi}{2}}}{2\pi}\left(\frac{\eta}{2}\right)^{\frac14}\sqrt{\kappa_+(k\gamma\cos\theta)}\right)\left[\left\{-k^2\gamma^2\sin^2\theta\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\right\}\left\{i\left(\frac{\eta}{2}\right)^{\frac14}\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}\right.\right.\\
&\left.\times\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\sinh\varkappa(k\gamma\cos\theta_0)\right\}+\left(\cosh\varkappa(k\gamma\cos\theta)-\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\right)\\
&\times\left\{\left(i\left(\frac{\eta}{2}\right)^{\frac14}\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\right)\begin{pmatrix}\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}\cosh\varkappa(k\gamma\cos\theta_0)\\ -\frac{k\gamma}{\eta}\sinh\varkappa(k\gamma\cos\theta_0)\end{pmatrix}\right.\\
&\left.\left.+\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}(\cos\theta+\cos\theta_0)p^{**}\right\}\right]\left[\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta-\theta_0}{2}\right)+\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta+\theta_0}{2}\right)\right]\\
&\times\frac{\exp[ik\gamma(\rho+\rho_0)+ik\mu z_0]}{k\gamma\sqrt{\rho\rho_0}}.
\end{aligned}\tag{90}$$

The scattered field in the spatial domain can now be obtained by taking the inverse Fourier transform of Eqs. (89) and (90). Thus, for $y>0$,

$$\begin{aligned}\Phi(\rho,\theta,z)\approx{}&\frac{k}{2\pi}\int_{-\infty}^{\infty}\left(\frac{e^{\frac{-i\pi}{2}}}{2\pi}\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_+(k\gamma\cos\theta)}\right)\left[\left\{\cosh\varkappa(k\gamma\cos\theta)+\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\right\}\right.\\&\times\left\{i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\sinh\varkappa(k\gamma\cos\theta_0)\right\}+\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\\&\times\left\{\left(i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\sqrt{\kappa_+(k\gamma\cos\theta_0)}\right)\left(\cosh\varkappa(k\gamma\cos\theta_0)-\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta_0)}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\right)\right.\\&\left.\left.+\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}(\cos\theta+\cos\theta_0)p^{**}\right\}\right]\left[\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta-\theta_0}{2}\right)+\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta+\theta_0}{2}\right)\right]\\&\times\frac{\exp[ik\gamma(\rho+\rho_0)+ik\mu z_0-ik\mu z]}{k\gamma\sqrt{\rho\rho_0}}\,d\mu,\end{aligned}\tag{91}$$

and for $y<0$

$$\begin{aligned}\Phi(\rho,\theta,\mu)\approx{}&\frac{k}{2\pi}\int_{-\infty}^{\infty}\left(\frac{e^{\frac{-i\pi}{2}}}{2\pi}\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_+(k\gamma\cos\theta)}\right)\left[\left\{-k^2\gamma^2\sin^2\theta\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\right\}\left\{i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}\right.\right.\\&\left.\times\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\sinh\varkappa(k\gamma\cos\theta_0)\right\}+\left(\cosh\varkappa(k\gamma\cos\theta)-\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\right)\\&\times\left\{i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\begin{pmatrix}\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}\cosh\varkappa(k\gamma\cos\theta_0)\\-\frac{k\gamma}{\eta}\sinh\varkappa(k\gamma\cos\theta_0)\end{pmatrix}\right.\\&\left.\left.+\frac{\sin\frac{\theta_0}{2}}{2\cos\frac{\theta}{2}}(\cos\theta+\cos\theta_0)\ p^{**}\right\}\right]\left[\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta-\theta_0}{2}\right)+\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta+\theta_0}{2}\right)\right]\\&\times\frac{\exp[ik\gamma(\rho+\rho_0)+ik\mu z_0-ik\mu z]}{k\gamma\sqrt{\rho\rho_0}}\,d\mu.\end{aligned}\tag{92}$$

The integrals appearing in expressions (91) and (92) can be evaluated asymptotically by the method of steepest descent (see Appendix A), and the far field for $y>0$ and $y<0$ are finally given as follows.

For $y>0$

$$\begin{aligned}\Phi(\rho,\theta,z)\approx{}&\exp\left[-i\frac{\pi}{4}+ikR_1\right]\left[g_1(s_1)\epsilon_1\sqrt{\frac{a_{1+}\rho+\rho_0}{R_1(R_{11}+R_1)}}\widetilde{F}(\tau_{R_1})+g_1(s_2)\epsilon_2\sqrt{\frac{a_{2+}\rho+\rho_0}{R_1(R_{12}+R_1)}}\widetilde{F}(\tau_{R_2})\right]\\&+\sqrt{\frac{a_1}{2\pi}}\exp\left[-i\frac{\pi}{4}+ikR_{11}\right]g_1\left(\frac{a_1}{R_{11}}\right)\frac{1}{R_{11}}H(-\epsilon_1)+\sqrt{\frac{a_2}{2\pi}}\exp\left[-i\frac{\pi}{4}+ikR_{12}\right]g_1\left(\frac{a_2}{R_2}\right)\frac{1}{R_{12}}H(-\epsilon_2),\end{aligned}\tag{93}$$

for $y<0$

$$\begin{aligned}\Phi(\rho,\theta,z)\approx{}&\exp\left[-i\frac{\pi}{4}+ikR_1\right]\left[g_2(s_1)\epsilon_1\sqrt{\frac{a_{1+}\rho+\rho_0}{R_1(R_{11}+R_1)}}\widetilde{F}(\tau_{R_1})+g_2(s_2)\epsilon_2\sqrt{\frac{a_{2+}\rho+\rho_0}{R_1(R_{12}+R_1)}}\widetilde{F}(\tau_{R_2})\right]\\&+\sqrt{\frac{a_1}{2\pi}}\exp\left[-i\frac{\pi}{4}+ikR_{11}\right]g_2\left(\frac{a_1}{R_{11}}\right)\frac{1}{R_{11}}H(-\epsilon_1)+\sqrt{\frac{a_2}{2\pi}}\exp\left[-i\frac{\pi}{4}+ikR_{12}\right]g_2\left(\frac{a_2}{R_2}\right)\frac{1}{R_{12}}H(-\epsilon_2),\end{aligned}\tag{94}$$

where

$$s_1=\frac{\sqrt{\tau_{R_1}^2(2kR_{11}+\tau_{R_1}^2)+k^2a_1^2}}{\tau_{R_1}^2+kR_{11}}\quad\text{and}\quad s_2=\frac{\sqrt{\tau_{R_2}^2(2kR_{12}+\tau_{R_2}^2)+k^2a_2^2}}{\tau_{R_1}^2+kR_{12}}.\tag{95}$$

In expressions (93) and (94) $H(.)$ is the usual Heaviside function and the quantities μ_1, $f_1(\mu)$, $g_1(\mu)$, a_1, R_{11}, τ_{R_1}, ϵ_1 and R_1 have already been explained in Appendix A. We wish to remark here that the other quantities for e.g., R_{12}, τ_{R_2}, ϵ_2 etc. may be seen from [18,19].

The unknown constant p^{**} for the case of point source scattering is given as follows:

$$p^{**}=\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\frac{k\gamma}{\pi i}\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\sinh\varkappa(k\gamma\cos\theta_0).\tag{96}$$

4. Concluding remarks

In this article, the line source and the point source scattering of acoustic waves by the junction of partially transmissive and soft–hard half planes are studied. The boundary value problem is reduced to a matrix Wiener–Hopf equation by using the Fourier transform technique. Then solution of the problem requires the Wiener–Hopf factorization of the kernel matrix involved in the equation. This factorization is performed by Büyükaksoy et al. [13] which can be used for our analysis. The problem is then solved completely. It is observed that our analysis differs from [13] by a multiplicative factor which agrees well with the literature [2,14]. Finally the graphs (Figs. 2–6) for the line source situation are presented. It is also observed that the graphs of [13] can be recovered, by shifting the line source at large distance.

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Appendix A

In this appendix, we evaluate the integral

$$I_1 \approx \frac{k}{2\pi}\int_{-\infty}^{\infty}\left(\frac{e^{\frac{-i\pi}{2}}}{2\pi}\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_+(k\gamma\cos\theta)}\right)\left[\left[\left\{\cosh\varkappa(k\gamma\cos\theta)+\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\right\}\left\{i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\right.\right.\right.$$
$$\left.\times\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\sinh\varkappa(k\gamma\cos\theta_0)\right\}+\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\left\{\left(i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\sqrt{\kappa_+(k\gamma\cos\theta_0)}\right)\right.$$
$$\left.\left.\times\begin{pmatrix}\cosh\varkappa(k\gamma\cos\theta_0)\\ -\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta_0)}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\end{pmatrix}+\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}(\cos\theta+\cos\theta_0)p^{**}\right\}\right]\left[\widetilde{F}\left(\sqrt{2k\gamma\rho}\cos\frac{\theta-\theta_0}{2}\right)\right]$$
$$\times\frac{\exp[ik\gamma\rho+ik\gamma\rho_0+ik\mu z_0-ik\mu z]}{\sqrt{\rho\rho_0}}\,d\mu. \tag{A.1}$$

Substitute

$$\mu_1=\sqrt{2\rho}\cos\frac{\theta-\theta_0}{2}$$

and

$$f_1(\mu)=\left(\frac{e^{\frac{-i\pi}{2}}}{2\pi}\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sqrt{\kappa_+(k\gamma\cos\theta)}\right)\left[\left\{\cosh\varkappa(k\gamma\cos\theta)+\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\right\}\left\{i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\right.\right.$$
$$\left.\times\frac{\sqrt{\kappa_+(k\gamma\cos\theta_0)}}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\sinh\varkappa(k\gamma\cos\theta_0)\right\}+\frac{\sinh\varkappa(k\gamma\cos\theta)}{\sqrt{k^2\gamma^2\cos^2\theta-\sigma^2}}\left\{\left(i\left(\frac{\eta}{2}\right)^{\frac{1}{4}}\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}\sqrt{\kappa_+(k\gamma\cos\theta_0)}\right)\right.$$
$$\left.\left.\times\begin{pmatrix}\cosh\varkappa(k\gamma\cos\theta_0)\\ -\frac{k\gamma}{\eta}\frac{\sinh\varkappa(k\gamma\cos\theta_0)}{\sqrt{k^2\gamma^2\cos^2\theta_0-\sigma^2}}\end{pmatrix}+\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}(\cos\theta+\cos\theta_0)p^{**}\right\}\right].$$

Eq. (A.1) will take the form

$$I_1=\frac{k}{2\pi}\int_{-\infty}^{\infty}f_1(\mu)\widetilde{F}\left(\mu_1\left(k\sqrt{1-\mu^2}\right)^{\frac{1}{2}}\right)\frac{\exp[ik\gamma(\rho+\rho_0)-ik\mu(z-z_0)]}{\sqrt{\rho\rho_0}}\,d\mu. \tag{A.2}$$

Making use of the result

$$\int_z^{\infty}e^{i\lambda t^2}\,dt=e^{i\lambda z^2}\frac{F(\lambda^{\frac{1}{2}}z)}{\lambda^{\frac{1}{2}}}, \tag{A.3}$$

expression (A.2) will take the form [18,19]

$$I_1=\frac{k}{2\pi}\int_{\mu_1}^{\infty}\int_{-\infty}^{\infty}\frac{f_1(\mu)(k\sqrt{1-\mu^2})^{\frac{1}{2}}}{\sqrt{\rho\rho_0}}e^{-ik[\mu(z-z_0)-\sqrt{1-\mu^2}(\rho+\rho_0+t^2-\mu^2)]}\,d\mu\,dt. \tag{A.4}$$

Let $g_1(\mu) = \frac{f_1(\mu)(k\sqrt{1-\mu^2})^{\frac{1}{2}}}{\sqrt{\rho\rho_0}}$ and consider the integral

$$I_1' = \int_{-\infty}^{\infty} g_1(\mu) e^{-ik[\mu(z-z_0)-\sqrt{1-\mu^2}(\rho+\rho_{0+}t^2-\mu^2)]}\, d\mu. \tag{A.5}$$

In order to solve the integral (A.5) we introduce the following substitutions:

$$\mu = \cos\beta, \qquad \gamma = \sqrt{1-\mu^2} = \sin\beta,$$
$$z - z_0 = R_1\cos\nu, \qquad P = R_1\sin\nu. \tag{A.6}$$

I_1' takes the form

$$I_1' = \int_{-\infty}^{\infty} g_1(\beta) e^{-ikR_1\cos(\beta+\nu)}(-\sin\beta)\, d\beta. \tag{A.7}$$

We apply the method of steepest descent to solve the integral I_1'. For this, we deform the contour of integration so as to pass through the point of steepest descent $\beta = -\nu$, so that the major part of integrand is given by integration over the part of deformed contour near $-\nu$ with $g_1(\beta)$ slowly varying around it. Therefore,

$$I_1' \approx \pi g_1(-\nu)\sin\nu H_0^{(1)}(kR_1) \approx \pi g_1(\Omega) H_0^{(1)}\big[k\big\{(z-z_0)^2 + P^2\big\}^{\frac{1}{2}}\big]\Omega, \tag{A.8}$$

where $\Omega = \frac{P}{[(z-z_0)^2+P^2]^{\frac{1}{2}}}$.

Using (A.8), expression (A.4) will take the form

$$I_1 = \frac{k}{2}\int_{\mu_1}^{\infty} g_1(\Omega) H_0^{(1)}\big[k\big\{(z-z_0)^2 + \big(\rho+\rho_0+t^2-\mu_1^2\big)^2\big\}^{\frac{1}{2}}\big] \frac{\rho+\rho_0+t^2-\mu_1^2}{\{(z-z_0)^2+(\rho+\rho_0+t^2-\mu_1^2)^2\}^{\frac{1}{2}}}\, dt. \tag{A.9}$$

If we make the substitutions

$$t^2 = -a_1 + \sqrt{a_1^2 + R_{11}^2\sinh^2 u}, \quad a_1 = \rho+\rho_0-\mu_1^2 \quad \text{and} \quad R_{11}^2 = (z-z_0)^2 + a_1^2, \tag{A.10}$$

in (A.9), it will yield

$$I_1 = \frac{k}{4}\int_{\varepsilon_1}^{\infty} \big[g_1(\widetilde{\Omega}) H_0^{(1)}(kR_{11}\cosh u)\big(\sqrt{a_1^2+R_{11}^2\sinh^2 u} + a_1\big)^{\frac{1}{2}}\big]\, du, \tag{A.11}$$

where

$$\widetilde{\Omega} = \frac{\sqrt{a_1^2+R_{11}^2\sinh^2 u}}{R_{11}\cosh u} \quad \text{and} \quad \varepsilon_1 = \sinh^{-1}\left\{\frac{\mu_1\sqrt{\mu_1^2+2a_1}}{R_{11}}\right\}.$$

The integral in expression (A.11) can be solved asymptotically for $kR_{11}\cosh u \gg 1$. Therefore the Hankel function can be replaced by the first term of its asymptotic expansion to give

$$I_1 = \frac{k}{4}\int_{\varepsilon_1}^{\infty} \left[g_1(\widetilde{\Omega})\left\{\sqrt{\frac{2}{\pi kR_{11}\cosh u}}\right\}^{\frac{1}{2}} e^{i(kR_{11}\cosh u-\frac{\pi}{4})}\big(\sqrt{a_1^2+R_{11}^2\sinh^2 u} + a_1\big)^{\frac{1}{2}}\right] du. \tag{A.12}$$

If we let $\tau = \sqrt{2kR_{11}}\sinh u$ in the integral appearing in expression (A.12), then

$$I_1 = \sqrt{\frac{2k}{\pi}}\frac{e^{-i\frac{\pi}{4}+ikR_{11}}}{2}\int_{\tau_{R_1}}^{\infty} g_1(\tau)e^{-i\tau^2}\, d\tau, \tag{A.13}$$

where

$$g_1(\tau) = \left\{\frac{\sqrt{\tau^2(\tau^2+2kR_{11})+k^2a_1^2} + ka_1}{(\tau^2+kR_{11})(\tau^2+2kR_{11})}\right\}^{\frac{1}{2}} g_1(\widehat{\Omega}), \qquad \widehat{\Omega} = \frac{\sqrt{\tau^2(\tau^2+2kR_{11})+k^2a_1^2}}{\tau^2+kR_{11}},$$
$$\tau_{R_1} = \sqrt{k(R_1-R_{11})} \quad \text{and} \quad \epsilon_1 = \operatorname{sgn}(\tau_{R_1}). \tag{A.14}$$

An asymptotic expansion of I_1 then follows by putting τ equal to its lower limit value in the nonexponential part of the integrand plus the contribution from $\tau = 0$ if zero lies in the interval of integration. Hence in our case, for I_1 it is given in Eq. (93).

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