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Projective invariants for equitorsion geodesic mappings of semi-symmetric affine connection spaces

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In the present paper we study invariants of equitorsion geodesic mappings of non-symmetric affine connection spaces. We generalize invariants of this mapping herein. As the main result, we examine the forms of these invariants in the case of equitorsion geodesic mappings of semi-symmetric spaces.

1. Introduction and preliminaries

A non-symmetric affine connection space in the sense of Eisenhart's definition [5] is a differentiable N-dimensional differentiable manifold equipped with an affine connection ∇ with torsion. The coefficients of this affine connection are L^i_{jk}, $L^i_{jk} \neq L^i_{kj}$.

L. P. Eisenhart started research about non-symmetric affine connection spaces [5–7]. A. Einstein based the theory of relativity on non-symmetric affine connection [2–4].

The symmetric and anti-symmetric part of the coefficient L^i_{jk} are

$$L^i_{\underline{jk}} = \frac{1}{2}(L^i_{jk} + L^i_{kj}) \text{ and } L^i_{\underset{\vee}{jk}} = \frac{1}{2}(L^i_{jk} - L^i_{kj}). \tag{1}$$

The symmetric affine connection space $\mathbb{A}_N$ equipped with the affine connection $\overset{0}{\nabla}$ whose coefficients are $L^i_{\underline{jk}}$ is called the associated space of space $\mathbb{GA}_N$. The anti-symmetric part $L^i_{\underset{\vee}{jk}}$ is called the torsion tensor.

Symmetric affine connection spaces and mappings between them have studied many authors. Some of them are N. S. Sinyukov [19], J. Mikeš [10–13], I. Hinterleitner [9,10,12,13], S. E. Stepanov [21,22] and many others.

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Special non-symmetric affine connection spaces are semi-symmetric affine connection spaces. Torsion tensor of a semi-symmetric space has a special form [1,8].

Covariant derivative with regard to affine connection of the associated space $\mathbb{A}_N$ is

$$a^i_{j|k} = a^i_{j,k} + L^i_{\underline{\alpha k}} a^\alpha_j - L^\alpha_{\underline{jk}} a^i_\alpha. \tag{2}$$

It exists only one Ricci-type identity with regard to this covariant differentiation. The curvature tensor of the associated space $\mathbb{A}_N$ obtained from this Ricci-type identity is

$$R^i_{jmn} = L^i_{\underline{jm},n} - L^i_{\underline{jn},m} + L^\alpha_{\underline{jm}} L^i_{\underline{\alpha n}} - L^\alpha_{\underline{jn}} L^i_{\underline{\alpha m}}. \tag{3}$$

Four kinds of covariant differentiation with regard to non-symmetric affine connection are defined [14–16]. These derivatives of a tensor a^i_j of the type (1,1) are:

$$\begin{aligned} a^i_{j\underset{1}{|}k} &= a^i_{j,k} + L^i_{\alpha k} a^\alpha_j - L^\alpha_{jk} a^i_\alpha, \qquad & a^i_{j\underset{2}{|}k} &= a^i_{j,k} + L^i_{k\alpha} a^\alpha_j - L^\alpha_{kj} a^i_\alpha, \\ a^i_{j\underset{3}{|}k} &= a^i_{j,k} + L^i_{\alpha k} a^\alpha_j - L^\alpha_{kj} a^i_\alpha, \qquad & a^i_{j\underset{4}{|}k} &= a^i_{j,k} + L^i_{k\alpha} a^\alpha_j - L^\alpha_{jk} a^i_\alpha. \end{aligned} \tag{4}$$

From the corresponding twelve Ricci-type identities, it is obtained the corresponding curvature tensors of the space $\mathbb{GA}_N$ (see [14–16]). These curvature tensors are elements of the family

$$K^i_{jmn} = R^i_{jmn} + u L^i_{\underset{\vee}{jm}|n} + u' L^i_{\underset{\vee}{jn}|m} + v L^\alpha_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} + v' L^\alpha_{\underset{\vee}{jn}} L^i_{\underset{\vee}{\alpha m}} + w L^\alpha_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\alpha j}}, \tag{5}$$

for real constants u, u', v, v', w. Five of the tensors in this family are linearly independent and the other ones may be expressed as linear combinations of these linearly independent tensors and the curvature tensor R^i_{jmn} of the associated space $\mathbb{A}_N$.

Geodesic mappings of symmetric affine connection space $\mathbb{A}_N$ have been studied by many authors [11–13, 19]. Weyl projective tensor of the associated space $\mathbb{A}_N$

$$W^i_{jmn} = R^i_{jmn} + \frac{1}{N+1}\delta^i_j R_{[mn]} + \frac{N}{N^2-1}\delta^i_{[m} R_{jn]} + \frac{1}{N^2-1}\delta^i_{[m} R_{n]j}, \tag{6}$$

is an invariant of geodesic mapping $f : \mathbb{A}_N \to \overline{\mathbb{A}}_N$.

Definition 1 *([16,17,20,23,24]).* A mapping $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ is the geodesic mapping if any geodesic line of the space $\mathbb{GA}_N$ turns into a geodesic line of space $\mathbb{G}\overline{\mathbb{A}}_N$. The geodesic mapping f is the equitorsion one if it preserves the torsion tensor.

Weyl projective tensor as an invariant of equitorsion geodesic mapping is generalized as (see [17,20,23,24])

$$\begin{aligned} \mathcal{E}^i_{jmn} = {} & K^i_{jmn} + \frac{1}{N+1}\delta^i_j K_{[mn]} + \frac{N}{N^2-1}\delta^i_{[m} K_{jn]} + \frac{1}{N^2-1}\delta^i_{[m} K_{n]j} - \frac{u+u'}{N+1} L^\alpha_{[\underline{m\alpha}} L^i_{\underset{\vee}{jn}]} \\ & - \frac{1}{(N+1)^2(N-1)} L^\alpha_{\underline{m\alpha}} \big[2(N-1)u\delta^i_j L^\beta_{\underset{\vee}{\beta n}} + (u - Nu' - Nu - u')\delta^i_n L^\beta_{\underset{\vee}{j\beta}}\big] \\ & + \frac{1}{(N+1)^2(N-1)} L^\alpha_{\underline{n\alpha}} \big[2(N-1)u\delta^i_j L^\beta_{\underset{\vee}{\beta m}} + (u - Nu' - Nu - u')\delta^i_m L^\beta_{\underset{\vee}{j\beta}}\big] \\ & + \frac{1}{N+1}\big((u-u') L^\alpha_{\underline{j\alpha}} L^i_{\underset{\vee}{mn}} + 2(N-1)u\delta^i_j L^\alpha_{\underline{\beta\alpha}} L^\beta_{\underset{\vee}{mn}}\big) + \frac{N(u'-u)+u+u'}{(N+1)^2(N-1)}\delta^i_{[m} L^\alpha_{\underline{j\alpha}} L^\beta_{\underset{\vee}{n]\beta}} \\ & + \frac{1}{(N+1)^2} L^\alpha_{\underline{\beta\alpha}} \big[(Nu' + u' + Nu - u)\delta^i_m L^\beta_{\underset{\vee}{jn}} + 2u\delta^i_n L^\beta_{\underset{\vee}{jm}}\big], \end{aligned} \tag{7}$$

for the above defined family K^i_{jmn} of curvature tensors.

U. C. De, Y. L. Han, P. B. Zhao [1], A. K. Mondal, U. C. De [18], Y. Han, H. T. Yun, P. Zhao [8] have studied the semi-symmetric affine connection spaces $\mathbb{GA}_N$.

An affine connection space $\mathbb{GA}_N$ is called the semi-symmetric affine connection space if the torsion tensor $L^i_{\underset{\vee}{jk}}$ of the affine connection of this space has the form

$$L^i_{\underset{\vee}{jk}} = u_k\delta^i_j - u_j\delta^i_k, \tag{8}$$

for a 1-form u_j. After contract this equation by indices i and k, we obtain that is

$$u_j = -\frac{1}{N-1}L^\alpha_{\underset{\vee}{j\alpha}}. \tag{9}$$

Our aim in this paper is to obtain invariants of equitorsion geodesic mapping of the space $\mathbb{GA}_N$. The main purpose of this paper is to generalize Weyl projective tensor as an invariant of an equitorsion geodesic mapping of a semi-symmetric affine connection space.

2. Invariants of equitorsion geodesic mappings

Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an equitorsion geodesic mapping. The basic equation of this mapping is

$$\overline{L}^i_{jk} = L^i_{jk} + \psi_j\delta^i_k + \psi_k\delta^i_j, \tag{10}$$

for a 1-form ψ_j. From this equation, we obtain that the symmetric parts $L^i_{\underline{jk}}$ and $\overline{L}^i_{\underline{jk}}$ satisfy the equation

$$\overline{L}^i_{\underline{jk}} = L^i_{\underline{jk}} + \psi_j\delta^i_k + \psi_k\delta^i_j. \tag{11}$$

After contract the last equation by indices i and k, we obtain that is

$$\psi_j = \frac{1}{N+1}\big(\overline{L}^\alpha_{\underline{j\alpha}} - L^\alpha_{\underline{j\alpha}}\big).$$

For this reason, the symmetric part of the deformation tensor $P^i_{\underline{jk}}$ of the mapping f is

$$P^i_{\underline{jk}} = \overline{L}^i_{\underline{jk}} - L^i_{\underline{jk}} = \frac{1}{N+1}\big(\overline{L}^\alpha_{\underline{j\alpha}}\delta^i_k + \overline{L}^\alpha_{\underline{k\alpha}}\delta^i_j\big) - \frac{1}{N+1}\big(L^\alpha_{\underline{j\alpha}}\delta^i_k + L^\alpha_{\underline{k\alpha}}\delta^i_j\big). \tag{12}$$

From the equations (11), (12), it is obtained in [16] that a mapping $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ is an equitorsion geodesic mapping if and only if the generalized Thomas projective parameter

$$T^i_{jk} = L^i_{\underline{jk}} - \frac{1}{N+1}\big(L^\alpha_{\underline{j\alpha}}\delta^i_k + L^\alpha_{\underline{k\alpha}}\delta^i_j\big) \tag{13}$$

is an invariant of this mapping.

Based on the equation (12) and the invariance $\overline{L}^i_{\underset{\vee}{jk}} = L^i_{\underset{\vee}{jk}}$, we obtain that the following equations are satisfied equivalently:

$$\overline{L}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = \overline{L}^i_{\underline{\alpha n}}\overline{L}^\alpha_{\underset{\vee}{jm}} - \overline{L}^\alpha_{\underline{jn}}\overline{L}^i_{\underset{\vee}{\alpha m}} - \overline{L}^\alpha_{\underline{mn}}\overline{L}^i_{\underset{\vee}{j\alpha}} - L^i_{\underline{\alpha n}}L^\alpha_{\underset{\vee}{jm}} + L^\alpha_{\underline{jn}}L^i_{\underset{\vee}{\alpha m}} + L^\alpha_{\underline{mn}}L^i_{\underset{\vee}{j\alpha}}, \tag{14}$$

$$\begin{aligned}\overline{L}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = {} & \frac{1}{N+1}\delta^i_n\overline{L}^\beta_{\underline{\alpha\beta}}\overline{L}^\alpha_{\underset{\vee}{jm}} + \frac{1}{N+1}\overline{L}^\alpha_{\underline{n\alpha}}\overline{L}^i_{\underset{\vee}{jm}} - \overline{L}^\alpha_{\underline{jn}}\overline{L}^i_{\underset{\vee}{\alpha m}} - \overline{L}^\alpha_{\underline{mn}}\overline{L}^i_{\underset{\vee}{j\alpha}}\\ & - \frac{1}{N+1}\delta^i_n L^\beta_{\underline{\alpha\beta}}L^\alpha_{\underset{\vee}{jm}} - \frac{1}{N+1}L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}} + L^\alpha_{\underline{jn}}L^i_{\underset{\vee}{\alpha m}} + L^\alpha_{\underline{mn}}L^i_{\underset{\vee}{j\alpha}},\end{aligned} \tag{15}$$

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$$\begin{aligned}\overline{L}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = {} & -\frac{1}{N+1}\big(\overline{L}^\alpha_{\underline{j\alpha}}\overline{L}^i_{\underset{\vee}{nm}} + \overline{L}^\alpha_{\underline{n\alpha}}\overline{L}^i_{\underset{\vee}{jm}}\big) + \overline{L}^i_{\underline{\alpha n}}\overline{L}^\alpha_{\underset{\vee}{jm}} - \overline{L}^\alpha_{\underline{mn}}\overline{L}^i_{\underset{\vee}{j\alpha}}\\ & + \frac{1}{N+1}\big(L^\alpha_{\underline{j\alpha}}L^i_{\underset{\vee}{nm}} + L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}}\big) - L^i_{\underline{\alpha n}}L^\alpha_{\underset{\vee}{jm}} + L^\alpha_{\underline{mn}}L^i_{\underset{\vee}{j\alpha}},\end{aligned} \tag{16}$$

$$\begin{aligned}\overline{L}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = {} & -\frac{1}{N+1}\big(\overline{L}^\alpha_{\underline{n\alpha}}\overline{L}^i_{\underset{\vee}{jm}} + \overline{L}^\alpha_{\underline{m\alpha}}\overline{L}^i_{\underset{\vee}{jn}}\big) + \overline{L}^i_{\underline{\alpha n}}\overline{L}^\alpha_{\underset{\vee}{jm}} - \overline{L}^\alpha_{\underline{jn}}\overline{L}^i_{\underset{\vee}{\alpha m}}\\ & + \frac{1}{N+1}\big(L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}} + L^\alpha_{\underline{m\alpha}}L^i_{\underset{\vee}{jn}}\big) - L^i_{\underline{\alpha n}}L^\alpha_{\underset{\vee}{jm}} + L^\alpha_{\underline{jn}}L^i_{\underset{\vee}{\alpha m}},\end{aligned} \tag{17}$$

$$\begin{aligned}\overline{L}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = {} & \frac{1}{N+1}\delta^i_n\overline{L}^\beta_{\underline{\alpha\beta}}\overline{L}^\alpha_{\underset{\vee}{jm}} - \frac{1}{N+1}\overline{L}^\alpha_{\underline{j\alpha}}\overline{L}^i_{\underset{\vee}{nm}} - \overline{L}^\alpha_{\underline{mn}}\overline{L}^i_{\underset{\vee}{j\alpha}}\\ & - \frac{1}{N+1}\delta^i_n L^\beta_{\underline{\alpha\beta}}L^\alpha_{\underset{\vee}{jm}} + \frac{1}{N+1}L^\alpha_{\underline{j\alpha}}L^i_{\underset{\vee}{nm}} + L^\alpha_{\underline{mn}}L^i_{\underset{\vee}{j\alpha}},\end{aligned} \tag{18}$$

$$\begin{aligned}\overline{L}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = {} & \frac{1}{N+1}\delta^i_n\overline{L}^\beta_{\underline{\alpha\beta}}\overline{L}^\alpha_{\underset{\vee}{jm}} - \frac{1}{N+1}\overline{L}^\alpha_{\underline{m\alpha}}\overline{L}^i_{\underset{\vee}{jn}} - \overline{L}^\alpha_{\underline{jn}}\overline{L}^i_{\underset{\vee}{\alpha m}}\\ & - \frac{1}{N+1}\delta^i_n L^\beta_{\underline{\alpha\beta}}L^\alpha_{\underset{\vee}{jm}} + \frac{1}{N+1}L^\alpha_{\underline{m\alpha}}L^i_{\underset{\vee}{jn}} + L^\alpha_{\underline{jn}}L^i_{\underset{\vee}{\alpha m}},\end{aligned} \tag{19}$$

$$\begin{aligned}\overline{L}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = {} & -\frac{1}{N+1}\big(\overline{L}^\alpha_{\underline{j\alpha}}\overline{L}^i_{\underset{\vee}{nm}} + \overline{L}^\alpha_{\underline{m\alpha}}\overline{L}^i_{\underset{\vee}{jn}} + 2\overline{L}^\alpha_{\underline{n\alpha}}\overline{L}^i_{\underset{\vee}{jm}}\big) + \overline{L}^i_{\underline{\alpha n}}\overline{L}^\alpha_{\underset{\vee}{jm}}\\ & + \frac{1}{N+1}\big(L^\alpha_{\underline{j\alpha}}L^i_{\underset{\vee}{nm}} + L^\alpha_{\underline{m\alpha}}L^i_{\underset{\vee}{jn}} + 2L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}}\big) - L^i_{\underline{\alpha n}}L^\alpha_{\underset{\vee}{jm}},\end{aligned} \tag{20}$$

$$\begin{aligned}\overline{L}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = {} & \frac{1}{N+1}\delta^i_n\overline{L}^\beta_{\underline{\alpha\beta}}\overline{L}^\alpha_{\underset{\vee}{jm}} - \frac{1}{N+1}\big(\overline{L}^\alpha_{\underline{j\alpha}}\overline{L}^i_{\underset{\vee}{nm}} + \overline{L}^\alpha_{\underline{m\alpha}}\overline{L}^i_{\underset{\vee}{jn}} + \overline{L}^\alpha_{\underline{n\alpha}}\overline{L}^i_{\underset{\vee}{jm}}\big)\\ & - \frac{1}{N+1}\delta^i_n L^\beta_{\underline{\alpha\beta}}L^\alpha_{\underset{\vee}{jm}} + \frac{1}{N+1}\big(L^\alpha_{\underline{j\alpha}}L^i_{\underset{\vee}{nm}} + L^\alpha_{\underline{m\alpha}}L^i_{\underset{\vee}{jn}} + L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}}\big).\end{aligned} \tag{21}$$

In this way, we proved that is

$$\overline{L}^i_{\underset{\vee}{jm}||n} = L^i_{\underset{\vee}{jm}|n} + \underset{(p)}{\overline{\eta}}{}^i_{jmn} - \underset{(p)}{\eta}{}^i_{jmn}, \tag{22}$$

$p = 1, \dots, 8$, for

$$\underset{(1)}{\eta}{}^i_{jmn} = L^i_{\underline{\alpha n}}L^\alpha_{\underset{\vee}{jm}} - L^\alpha_{\underline{jn}}L^i_{\underset{\vee}{\alpha m}} - L^\alpha_{\underline{mn}}L^i_{\underset{\vee}{j\alpha}}, \tag{23}$$

$$\underset{(2)}{\eta}{}^i_{jmn} = \frac{1}{N+1}\delta^i_n L^\beta_{\underline{\alpha\beta}}L^\alpha_{\underset{\vee}{jm}} + \frac{1}{N+1}L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}} - L^\alpha_{\underline{jn}}L^i_{\underset{\vee}{\alpha m}} - L^\alpha_{\underline{mn}}L^i_{\underset{\vee}{j\alpha}}, \tag{24}$$

$$\underset{(3)}{\eta}{}^i_{jmn} = -\frac{1}{N+1}\big(L^\alpha_{\underline{j\alpha}}L^i_{\underset{\vee}{nm}} + L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}}\big) + L^i_{\underline{\alpha n}}L^\alpha_{\underset{\vee}{jm}} - L^\alpha_{\underline{mn}}L^i_{\underset{\vee}{j\alpha}}, \tag{25}$$

$$\underset{(4)}{\eta}{}^i_{jmn} = -\frac{1}{N+1}\big(L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}} + L^\alpha_{\underline{m\alpha}}L^i_{\underset{\vee}{jn}}\big) + L^i_{\underline{\alpha n}}L^\alpha_{\underset{\vee}{jm}} - L^\alpha_{\underline{jn}}L^i_{\underset{\vee}{\alpha m}}, \tag{26}$$

$$\underset{(5)}{\eta}{}^i_{jmn} = \frac{1}{N+1}\delta^i_n L^\beta_{\underline{\alpha\beta}}L^\alpha_{\underset{\vee}{jm}} - \frac{1}{N+1}L^\alpha_{\underline{j\alpha}}L^i_{\underset{\vee}{nm}} - L^\alpha_{\underline{mn}}L^i_{\underset{\vee}{j\alpha}}, \tag{27}$$

$$\underset{(6)}{\eta}{}^i_{jmn} = \frac{1}{N+1}\delta^i_n L^\beta_{\underline{\alpha\beta}}L^\alpha_{\underset{\vee}{jm}} - \frac{1}{N+1}L^\alpha_{\underline{m\alpha}}L^i_{\underset{\vee}{jn}} - L^\alpha_{\underline{jn}}L^i_{\underset{\vee}{\alpha m}}, \tag{28}$$

$$\underset{(7)}{\eta}{}^i_{jmn} = -\frac{1}{N+1}\big(L^\alpha_{\underline{j\alpha}}L^i_{\underset{\vee}{nm}} + L^\alpha_{\underline{m\alpha}}L^i_{\underset{\vee}{jn}} + 2L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}}\big) + L^i_{\underline{\alpha n}}L^\alpha_{\underset{\vee}{jm}}, \tag{29}$$

$$\underset{(8)}{\eta}{}^i_{jmn} = \frac{1}{N+1}\delta^i_n L^\beta_{\underline{\alpha\beta}}L^\alpha_{\underset{\vee}{jm}} - \frac{1}{N+1}\big(L^\alpha_{\underline{j\alpha}}L^i_{\underset{\vee}{nm}} + L^\alpha_{\underline{m\alpha}}L^i_{\underset{\vee}{jn}} + L^\alpha_{\underline{n\alpha}}L^i_{\underset{\vee}{jm}}\big), \tag{30}$$

and the corresponding $\underset{(p)}{\overline{\eta}}{}^i_{jmn}$.

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Let us denote by

$$\begin{aligned}
&\underset{1}{X}{}^i_{jmn} = \big(\overline{L}{}^i_{\underline{\alpha n}} - L^i_{\underline{\alpha n}}\big) L^\alpha_{\underset{\vee}{jm}}, \qquad \underset{2}{X}{}^i_{jmn} = \big(\overline{L}{}^\alpha_{\underline{jn}} - L^\alpha_{\underline{jn}}\big) L^i_{\underset{\vee}{\alpha m}}, \qquad \underset{3}{X}{}^i_{jmn} = \big(\overline{L}{}^\alpha_{\underline{mn}} - L^\alpha_{\underline{mn}}\big) L^i_{\underset{\vee}{j\alpha}},\\
&\underset{4}{X}{}^i_{jmn} = \delta^i_n \big(\overline{L}{}^\beta_{\underline{\alpha\beta}} - L^\beta_{\underline{\alpha\beta}}\big) L^\alpha_{\underset{\vee}{jm}}, \quad \underset{5}{X}{}^i_{jmn} = \big(\overline{L}{}^\alpha_{\underline{n\alpha}} - L^\alpha_{\underline{n\alpha}}\big) L^i_{\underset{\vee}{jm}}, \quad \underset{6}{X}{}^i_{jmn} = \big(\overline{L}{}^\alpha_{\underline{j\alpha}} - L^\alpha_{\underline{j\alpha}}\big) L^i_{\underset{\vee}{nm}},\\
&\underset{7}{X}{}^i_{jmn} = \big(\overline{L}{}^\alpha_{\underline{m\alpha}} - L^\alpha_{\underline{m\alpha}}\big) L^i_{\underset{\vee}{jn}}.
\end{aligned} \tag{31}$$

The equations (14)–(21) may be expressed in the forms:

$$(14): \quad \overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = \underset{1}{X}{}^i_{jmn} - \underset{2}{X}{}^i_{jmn} - \underset{3}{X}{}^i_{jmn}, \tag{32}$$

$$(15): \quad \overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = -\underset{2}{X}{}^i_{jmn} - \underset{3}{X}{}^i_{jmn} + \tfrac{1}{N+1}\underset{4}{X}{}^i_{jmn} + \tfrac{1}{N+1}\underset{5}{X}{}^i_{jmn}, \tag{33}$$

$$(16): \quad \overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = \underset{1}{X}{}^i_{jmn} - \underset{3}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{5}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{6}{X}{}^i_{jmn}, \tag{34}$$

$$(17): \quad \overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = \underset{1}{X}{}^i_{jmn} - \underset{2}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{5}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{7}{X}{}^i_{jmn}, \tag{35}$$

$$(18): \quad \overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = -\underset{3}{X}{}^i_{jmn} + \tfrac{1}{N+1}\underset{4}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{6}{X}{}^i_{jmn}, \tag{36}$$

$$(19): \quad \overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = \underset{2}{X}{}^i_{jmn} + \tfrac{1}{N+1}\underset{4}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{7}{X}{}^i_{jmn}, \tag{37}$$

$$(20): \quad \overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = \underset{2}{X}{}^i_{jmn} - \tfrac{2}{N+1}\underset{5}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{6}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{7}{X}{}^i_{jmn}, \tag{38}$$

$$(21): \quad \overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = \tfrac{1}{N+1}\underset{4}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{5}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{6}{X}{}^i_{jmn} - \tfrac{1}{N+1}\underset{7}{X}{}^i_{jmn}. \tag{39}$$

The p-th, $p = 1, \dots, 8$, of the equations (32)–(39) has the form

$$\overline{L}{}^i_{\underset{\vee}{jm}||n} - L^i_{\underset{\vee}{jm}|n} = \sum_{q=1}^{7} \underset{p}{x}{}_q \underset{q}{X}{}^i_{jmn}, \tag{40}$$

for the corresponding real constants $\underset{p}{x}{}_q$.

The above defined geometrical objects $\underset{p}{X}{}^i_{jmn}$ are linearly independent. The rank of matrix

$$\mathcal{X} = \begin{bmatrix} \underset{1}{x}{}_1 & \cdots & \underset{1}{x}{}_7 \\ \vdots & \ddots & \vdots \\ \underset{8}{x}{}_1 & \cdots & \underset{8}{x}{}_7 \end{bmatrix}$$

of the type 8×7 is 6. The following lemma is proved in this way.

Lemma 2.1. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an equitorsion geodesic mapping. Six of equivalent rules of transformations (14)–(21) are linearly independent.* $\square$

2.1. Weyl projective tensor and other invariants of geodesic mappings

Based on the invariance

$$\overline{T}{}^i_{jm,n} - \overline{T}{}^i_{jn,m} + \overline{T}{}^\alpha_{jm}\overline{T}{}^i_{\alpha n} - \overline{T}{}^\alpha_{jn}\overline{T}{}^i_{\alpha m} = T^i_{jm,n} - T^i_{jn,m} + T^\alpha_{jm}T^i_{\alpha n} - T^\alpha_{jn}T^i_{\alpha m},$$

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for the above defined generalized Thomas projective parameter, we obtain that is

$$\overline{\underset{(1)}{W}}{}^i_{jmn} = \underset{(1)}{W}{}^i_{jmn},$$

for

$$\begin{aligned}\underset{(1)}{W}{}^i_{jmn} = R^i_{jmn} + \frac{1}{N+1}\delta^i_j R_{[mn]} - \frac{1}{(N+1)^2}\big((N+1)L^{\alpha}_{\underline{j\alpha}|n} + L^{\alpha}_{\underline{j\alpha}}L^{\beta}_{\underline{n\beta}}\big)\delta^i_m \\ + \frac{1}{(N+1)^2}\big((N+1)L^{\alpha}_{\underline{j\alpha}|m} + L^{\alpha}_{\underline{j\alpha}}L^{\beta}_{\underline{m\beta}}\big)\delta^i_n,\end{aligned} \tag{41}$$

and the corresponding $\overline{\underset{(1)}{W}}{}^i_{jmn}$. After contract the equality

$$\overline{\underset{(1)}{W}}{}^i_{jmn} - \underset{(1)}{W}{}^i_{jmn} = 0$$

by indices i and n, we obtain that Weyl projective tensor $\underset{(2)}{W}{}^i_{jmn} = W^i_{jmn}$ from the equation (6) is an invariant of the equitorsion geodesic mapping $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$.

Definition 2. The invariants $\underset{(1)}{W}{}^i_{jmn}$ and $\underset{(2)}{W}{}^i_{jmn}$ of geodesic mapping $f : \mathbb{GA}_N \to \mathbb{GA}_N$ are **the first** and **the second Weyl projective tensor**.

From the invariance of the geometrical object $\underset{(1)}{W}{}^i_{jmn}$ and Weyl projective tensor (6), we establish that is

$$\overline{R}^i_{jmn} = R^i_{jmn} + \overline{\underset{(\rho)}{\mathcal{D}}}{}^i_{jmn} - \underset{(\rho)}{\mathcal{D}}{}^i_{jmn}, \tag{42}$$

$\rho = 1, 2$, for

$$\begin{aligned}\underset{(1)}{\mathcal{D}}{}^i_{jmn} = -\frac{1}{N+1}\delta^i_j R_{[mn]} + \frac{1}{(N+1)^2}\big((N+1)L^{\alpha}_{\underline{j\alpha}|n} + L^{\alpha}_{\underline{j\alpha}}L^{\beta}_{\underline{n\beta}}\big)\delta^i_m \\ - \frac{1}{(N+1)^2}\big((N+1)L^{\alpha}_{\underline{j\alpha}|m} + L^{\alpha}_{\underline{j\alpha}}L^{\beta}_{\underline{m\beta}}\big)\delta^i_n,\end{aligned} \tag{43}$$

$$\underset{(2)}{\mathcal{D}}{}^i_{jmn} = -\frac{1}{N+1}\delta^i_j R_{[mn]} - \frac{1}{N^2-1}\delta^i_m\big(NR_{jn} + R_{nj}\big) + \frac{1}{N^2-1}\delta^i_n\big(NR_{jm} + R_{mj}\big), \tag{44}$$

and the corresponding $\overline{\underset{(\rho)}{\mathcal{D}}}{}^i_{jmn}$.

The following lemma holds:

Lemma 2.2. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an equitorsion geodesic mapping. The geometrical objects $\underset{(1)}{W}{}^i_{jmn}$ and $\underset{(2)}{W}{}^i_{jmn}$ given by the equations (6, 41) are invariants of this mapping.* □

Corollary 2.1. *The invariants $\underset{(1)}{W}{}^i_{jmn}$ and $\underset{(2)}{W}{}^i_{jmn}$ of an equitorsion geodesic mapping $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ satisfy the equation*

$$\underset{(2)}{W}{}^i_{jmn} = \underset{(1)}{W}{}^i_{jmn} + \underset{(1)}{\mathcal{D}}{}^i_{jmn} - \underset{(2)}{\mathcal{D}}{}^i_{jmn}, \tag{45}$$

for the above defined $\underset{(1)}{\mathcal{D}}{}^i_{jmn}$ and $\underset{(2)}{\mathcal{D}}{}^i_{jmn}$. □

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From the equations (22), (42) we obtain that the family (5) of curvature tensors of the space $\mathbb{GA}_N$ satisfies the equation

$$\overline{K}{}^i_{jmn} = K^i_{jmn} + \underset{(\rho)}{\overline{\mathcal{D}}}{}^i_{jmn} + u\,\underset{(p)}{\overline{\eta}}{}^i_{jmn} + u'\,\underset{(q)}{\overline{\eta}}{}^i_{jnm} - \underset{(\rho)}{\mathcal{D}}{}^i_{jmn} - u\,\underset{(p)}{\eta}{}^i_{jmn} - u'\,\underset{(q)}{\eta}{}^i_{jmn}, \tag{46}$$

$\rho = 1, 2$, for $\underset{(p)}{\eta}{}^i_{jmn}, \underset{(\rho)}{\mathcal{D}}{}^i_{jmn}$ defined by the equations (23)–(30), (43), (44) and the corresponding $\underset{(p)}{\overline{\eta}}{}^i_{jmn}, \underset{(\rho)}{\overline{\mathcal{D}}}{}^i_{jmn}$. In this way, we proved that

$$\underset{(1)}{\underset{(p,q)}{\overline{W}}}{}^i_{jmn} = \underset{(1)}{\underset{(p,q)}{W}}{}^i_{jmn} \text{ and } \underset{(2)}{\underset{(p,q)}{\overline{W}}}{}^i_{jmn} = \underset{(2)}{\underset{(p,q)}{W}}{}^i_{jmn},$$

for

$$\underset{(1)}{\underset{(p,q)}{W}}{}^i_{jmn} = K^i_{jmn} - \underset{(1)}{\mathcal{D}}{}^i_{jmn} - u\,\underset{(p)}{\eta}{}^i_{jmn} - u'\,\underset{(q)}{\eta}{}^i_{jnm},\ \underset{(2)}{\underset{(p,q)}{W}}{}^i_{jmn} = K^i_{jmn} - \underset{(2)}{\mathcal{D}}{}^i_{jmn} - u\,\underset{(p)}{\eta}{}^i_{jmn} - u'\,\underset{(q)}{\eta}{}^i_{jnm}, \tag{47}$$

$p, q = 1, \dots, 4$, and the corresponding $\underset{(1)}{\underset{(p,q)}{\overline{W}}}{}^i_{jmn}, \underset{(2)}{\underset{(p,q)}{\overline{W}}}{}^i_{jmn}$. The following theorem holds:

Theorem 2.1. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an equitorsion geodesic mapping. The geometrical objects $\underset{(\rho)}{\underset{(p,q)}{W}}{}^i_{jmn}, \rho = 1, 2; p, q = 1, \dots, 8$, defined by the equation (47) are families of invariants of the mapping f.* □

Corollary 2.2. *A family of invariants $\underset{(\rho)}{\underset{(p,q)}{W}}{}^i_{jmn}, \rho = 1, 2$, and the invariant $\underset{(\rho)}{W}{}^i_{jmn}$ satisfy the equation*

$$\underset{(\rho)}{\underset{(p,q)}{W}}{}^i_{jmn} = \underset{(\rho)}{W}{}^i_{jmn} - u\,\underset{(p)}{\eta}{}^i_{jmn} - u'\,\underset{(q)}{\eta}{}^i_{jnm} + vL^\alpha_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} + v' L^\alpha_{\underset{\vee}{jn}} L^i_{\underset{\vee}{\alpha m}} + w L^\alpha_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\alpha j}}. \tag{48}$$

The families of invariants $\underset{(1)}{\underset{(p,q)}{W}}{}^i_{jmn}$ and $\underset{(2)}{\underset{(p,q)}{W}}{}^i_{jmn}$ satisfy the equation

$$\underset{(2)}{\underset{(p,q)}{W}}{}^i_{jmn} = \underset{(1)}{\underset{(p,q)}{W}}{}^i_{jmn} + \underset{(1)}{\mathcal{D}}{}^i_{jmn} - \underset{(2)}{\mathcal{D}}{}^i_{jmn}, \tag{49}$$

for the above defined $\underset{(1)}{\mathcal{D}}{}^i_{jmn}$ and $\underset{(2)}{\mathcal{D}}{}^i_{jmn}$. □

To examine how many of the families of invariants $\underset{(\rho_0)}{\underset{(p,q)}{W}}{}^i_{jmn}$ given by the equation (47) are linearly independent, we need to define the following geometrical objects

$$\begin{aligned}
&\underset{1}{Y}{}^i_{jmn} = L^i_{\underline{\alpha n}} L^\alpha_{\underset{\vee}{jm}}, \quad \underset{2}{Y}{}^i_{jmn} = L^\alpha_{\underline{jn}} L^i_{\underset{\vee}{\alpha m}}, \quad \underset{3}{Y}{}^i_{jmn} = L^\alpha_{\underline{mn}} L^i_{\underset{\vee}{j\alpha}}, \quad \underset{4}{Y}{}^i_{jmn} = \delta^i_n L^\beta_{\underline{\alpha\beta}} L^\alpha_{\underset{\vee}{jm}},\\
&\underset{5}{Y}{}^i_{jmn} = L^\alpha_{\underline{n\alpha}} L^i_{\underset{\vee}{jm}}, \quad \underset{6}{Y}{}^i_{jmn} = L^\alpha_{\underline{j\alpha}} L^i_{\underset{\vee}{nm}}, \quad \underset{7}{Y}{}^i_{jmn} = L^\alpha_{\underline{m\alpha}} L^i_{\underset{\vee}{jn}}, \quad \underset{8}{Y}{}^i_{jmn} = L^i_{\underline{\alpha m}} L^\alpha_{\underset{\vee}{jn}},\\
&\underset{9}{Y}{}^i_{jmn} = L^\alpha_{\underline{jm}} L^i_{\underset{\vee}{\alpha n}}, \quad \underset{10}{Y}{}^i_{jmn} = \delta^i_m L^\beta_{\underline{\alpha\beta}} L^\alpha_{\underset{\vee}{jn}}.
\end{aligned} \tag{50}$$

The (p, q)-th invariant (47) has the form

$$\underset{(\rho_0)}{\underset{(p,q)}{W}}{}^{i}_{jmn} = \underset{0}{Y}{}^{i}_{jmn} + \sum_{r=1}^{10} \overset{p}{\underset{q}{y}}_r \underset{r}{Y}{}^{i}_{jmn}, \tag{51}$$

for $\underset{0}{Y}{}^{i}_{jmn} = \underset{(\rho_0)}{W}{}^{i}_{jmn} + vL^{\alpha}_{\underset{\vee}{jm}} L^{i}_{\underset{\vee}{\alpha n}} + v' L^{\alpha}_{\underset{\vee}{jn}} L^{i}_{\underset{\vee}{\alpha m}} + wL^{\alpha}_{\underset{\vee}{mn}} L^{i}_{\underset{\vee}{\alpha j}}$, the above defined $\underset{r}{Y}{}^{i}_{jmn}$ and the corresponding real constants $\overset{p}{\underset{q}{y}}_r$, $r = 1, \dots, 10$.

Geometrical objects $\underset{0}{Y}{}^{i}_{jmn}, \underset{1}{Y}{}^{i}_{jmn}, \dots, \underset{10}{Y}{}^{i}_{jmn}$ are linearly independent. The rank of matrix

$$\mathcal{Y} = \begin{bmatrix} 1 & \overset{1}{\underset{1}{y}}_1 & \dots & \overset{1}{\underset{1}{y}}_{10} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \overset{1}{\underset{8}{y}}_1 & \dots & \overset{1}{\underset{8}{y}}_{10} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \overset{8}{\underset{1}{y}}_1 & \dots & \overset{8}{\underset{1}{y}}_{10} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \overset{8}{\underset{8}{y}}_1 & \dots & \overset{8}{\underset{8}{y}}_{10} \end{bmatrix}$$

is 8. The following theorem is proved in this way:

Theorem 2.2. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an equitorsion geodesic mappings. There are eight linearly independent families (47) of invariants of this mapping which are generalizations of Weyl projective tensor.* □

Corollary 2.3. *There are nine linearly independent families of invariants of an equitorsion geodesic mapping $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ which are elements of the set $\{\underset{(1)}{\underset{(p,q)}{W}}{}^{i}_{jmn}, \underset{(2)}{\underset{(p,q)}{W}}{}^{i}_{jmn}\}$.* □

2.2. Geodesic mappings of semi-symmetric spaces

Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an equitorsion geodesic mapping of a semi-symmetric space $\mathbb{GA}_N$. The geometrical objects $\underset{(1)}{W}{}^{i}_{jmn}$ and $\underset{(2)}{W}{}^{i}_{jmn} = W^{i}_{jmn}$ given by the equations (6), (41) are invariants of the mapping f.

In the case of semi-symmetric affine connection space $\mathbb{GA}_N$, the geometrical objects (23)–(30) are

$$\underset{(1)}{\eta}{}^{i}_{jmn} = L^{\alpha}_{\underline{jn}} \delta^{i}_{m} u_{\alpha} - L^{\alpha}_{\underline{mn}} \delta^{i}_{j} u_{\alpha}, \tag{52}$$

$$\begin{aligned} \underset{(2)}{\eta}{}^{i}_{jmn} = {} & \frac{1}{N+1} \delta^{i}_{n} (L^{\alpha}_{\underline{j\alpha}} u_m - L^{\alpha}_{\underline{m\alpha}} u_j) + \frac{1}{N+1} \delta^{i}_{j} \big(L^{\alpha}_{\underline{n\alpha}} u_m - L^{\alpha}_{\underline{mn}} u_{\alpha} \big) \\ & - \frac{1}{N+1} \delta^{i}_{m} \big(L^{\alpha}_{\underline{n\alpha}} u_j - L^{\alpha}_{\underline{jn}} u_{\alpha} \big) - L^{i}_{\underline{jn}} u_m + L^{i}_{\underline{mn}} u_j, \end{aligned} \tag{53}$$

$$\begin{aligned} \underset{(3)}{\eta}{}^{i}_{jmn} = {} & -\frac{1}{N+1} \delta^{i}_{j} \big(L^{\alpha}_{\underline{n\alpha}} u_m + (N+1) L^{\alpha}_{\underline{mn}} u_{\alpha} \big) + \frac{1}{N+1} \delta^{i}_{m} \big(L^{\alpha}_{\underline{j\alpha}} u_n + L^{\alpha}_{\underline{n\alpha}} u_j \big) \\ & - \frac{1}{N+1} \delta^{i}_{n} L^{\alpha}_{\underline{j\alpha}} u_m + L^{i}_{\underline{jn}} u_m, \end{aligned} \tag{54}$$

$$\begin{aligned}\underset{(4)}{\eta}{}^i_{jmn} &= -\frac{1}{N+1}\delta^i_j\big(L^\alpha_{\underline{n\alpha}}u_m + L^\alpha_{\underline{m\alpha}}u_n\big) + \frac{1}{N+1}\delta^i_m\big(L^\alpha_{\underline{n\alpha}}u_m + (N+1)L^\alpha_{\underline{jn}}u_\alpha\big)\\ &+ \frac{1}{N+1}\delta^i_n L^\alpha_{\underline{m\alpha}}u_j - L^i_{\underline{mn}}u_j,\end{aligned} \tag{55}$$

$$\underset{(5)}{\eta}{}^i_{jmn} = -\delta^i_j L^\alpha_{\underline{mn}}u_\alpha + \frac{1}{N+1}\delta^i_m L^\alpha_{\underline{j\alpha}}u_n - \frac{1}{N+1}\delta^i_n L^\alpha_{\underline{m\alpha}}u_j + L^i_{\underline{mn}}u_j, \tag{56}$$

$$\underset{(6)}{\eta}{}^i_{jmn} = -\frac{1}{N+1}\delta^i_j L^\alpha_{\underline{m\alpha}}u_n + \delta^i_m L^\alpha_{\underline{jn}}u_\alpha + \frac{1}{N+1}\delta^i_n L^\alpha_{\underline{j\alpha}}u_m - L^i_{\underline{jn}}u_m, \tag{57}$$

$$\begin{aligned}\underset{(7)}{\eta}{}^i_{jmn} &= -\frac{1}{N+1}\delta^i_j\big(L^\alpha_{\underline{m\alpha}}u_n + 2L^\alpha_{\underline{n\alpha}}u_m\big) + \frac{1}{N+1}\delta^i_m\big(L^\alpha_{\underline{j\alpha}}u_n + 2L^\alpha_{\underline{n\alpha}}u_j\big)\\ &- \frac{1}{N+1}\delta^i_n L^\alpha_{\underline{j\alpha}}u_m + L^i_{\underline{jn}}u_m - L^i_{\underline{mn}}u_j,\end{aligned} \tag{58}$$

$$\underset{(8)}{\eta}{}^i_{jmn} = \frac{1}{N+1}\delta^i_j\big(L^\alpha_{\underline{m\alpha}}u_n + L^\alpha_{\underline{m\alpha}}u_n\big) + \frac{1}{N+1}\delta^i_m\big(L^\alpha_{\underline{j\alpha}}u_n - L^\alpha_{\underline{n\alpha}}u_j\big) - \frac{2}{N+1}\delta^i_n L^\alpha_{\underline{m\alpha}}u_j. \tag{59}$$

With regard to semi-symmetric affine connection, we have that is

$$L^\alpha_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} = \delta^i_j u_m u_n - \delta^i_m u_j u_n. \tag{60}$$

The family of curvature tensors (5) of the semi-symmetric space $\mathbb{GA}_N$ is

$$\begin{aligned}K^i_{jmn} &= R^i_{jmn} + u(u_{m|n}\delta^i_j - u_{j|n}\delta^i_m) + u'(u_{n|m}\delta^i_j - u_{j|m}\delta^i_n)\\ &+ (v+v')u_m u_n\delta^i_j - (v-w)u_j u_n\delta^i_m - (v+w)u_j u_m\delta^i_n.\end{aligned} \tag{61}$$

With regard to the equations (48), (52)–(59), (60), we conclude that the following theorem holds:

Theorem 2.3. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an equitorsion geodesic mapping of semi-symmetric space $\mathbb{GA}_N$. The set of families of geometrical objects*

$$\begin{aligned}\underset{\substack{(p,q)\\(\rho)}}{\widetilde{W}}{}^i_{jmn} &= \underset{(\rho)}{W}{}^i_{jmn} - u\,\underset{(p)}{\eta}{}^i_{jmn} - u'\,\underset{(q)}{\eta}{}^i_{jmn}\\ &+ \frac{u+u'}{(N-1)^2}\delta^i_j L^\alpha_{\underset{\vee}{m\alpha}} L^\beta_{\underset{\vee}{n\beta}} - \frac{u-w}{(N-1)^2}\delta^i_m L^\alpha_{\underset{\vee}{j\alpha}} L^\beta_{\underset{\vee}{n\beta}} - \frac{u'+w}{(N-1)^2}\delta^i_n L^\alpha_{\underset{\vee}{j\alpha}} L^\beta_{\underset{\vee}{m\beta}},\end{aligned} \tag{62}$$

is the family of invariants of the mapping f. □

Corollary 2.4. *The invariant (62) of an equitorsion geodesic mapping $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ is*

$$\underset{\substack{(p,q)\\(\rho)}}{\widetilde{W}}{}^i_{jmn} = K^i_{jmn} - \underset{(\rho)}{\mathcal{D}}{}^i_{jmn} - u\,\underset{(p)}{\eta}{}^i_{jmn} - u'\,\underset{(q)}{\eta}{}^i_{jnm}, \tag{63}$$

for the above defined $\underset{(p)}{\eta}{}^i_{jmn}$, curvature tensor K^i_{jmn} of semi-symmetric space $\mathbb{GA}_N$ and real constants u, u'. □

3. Conclusion

In this paper, we studied and developed the theory of invariants of equitorsion geodesic mappings of symmetric and non-symmetric affine connection spaces. At the start, we obtained the first and the second

Weyl projective tensor. Motivated by these findings, we found the invariants of equitorsion geodesic mappings of non-symmetric affine connection space $\mathbb{GA}_N$. Specially, we obtained the invariants of equitorsion geodesic mappings of a semi-symmetric affine connection space.

References

[1] U.C. De, Y.L. Han, P.B. Zhao, A special type of semi-symmetric non-metric connection on a Riemannian manifold, Facta Univ. Ser. Math. Inform. 31 (2) (2016) 529–541.

[2] A. Einstein, A generalization of the relativistic theory of gravitation, Ann. of Math. 45 (2) (1945) 576–584.

[3] A. Einstein, Bianchi identities in the generalized theory of gravitation, Canad. J. Math. (2) (1950) 120–128.

[4] A. Einstein, Relativistic Theory of the Non-symmetric Field, 5th edition, Princeton University Press, New Jersey, 1954.

[5] L.P. Eisenhart, Non-Riemannian Geometry, 1927, New York.

[6] L.P. Eisenhart, Generalized Riemannian spaces, Proc. Natl. Acad. Sci. USA 37 (1951) 311–315.

[7] L.P. Eisenhart, Generalized Riemannian spaces, II, Proc. Natl. Acad. Sci. USA 38 (1952) 505–508.

[8] Y. Han, H.T. Yun, P. Zhao, Some invariants of quarter-symmetric metric connections under the projective transformation, Filomat 27 (4) (2013) 679–691.

[9] I. Hinterleitner, Geodesic mappings on compact Riemannian manifolds with conditions on sectional curvature, Publ. Inst. Math. (Beograd) (N.S.) 95 (108) (2013) 125–130.

[10] I. Hinterleitner, J. Mikeš, Fundamental equations of geodesic mappings and their generalizations, in: Geometry, in: Itogi Nauki Tekh. Ser. Sovrem. Mat. Prilozh. Temat. Obz., vol. 124, Cseross. Inst. Nauchn. i Tekhn. Inform. (VINITI), Moscow, 2010, pp. 7–34.

[11] J. Mikeš, V. Kiosak, A. Vanžurová, Geodesic Mappings of Manifolds with Affine Connection, Palacký University, Olomouc, 2008.

[12] J. Mikeš, E. Stepanova, A. Vanžurová, et al., Differential Geometry of Special Mappings, first edition, Palacký University, Olomouc, 2015.

[13] J. Mikeš, A. Vanžurová, I. Hinterleitner, Geodesic Mappings and Some Generalizations, Palacky University, Olomouc, 2009.

[14] S.M. Minčić, On curvature tensors and pseudotensors of the spaces with non-symmetric affine connection, Math. Balkanica 4 (76) (1974) 427–430.

[15] S.M. Minčić, Independent curvature tensors and pseudotensors of spaces with non-symmetric affine connexion, in: Differential Geometry, in: Colloquia Mathematica Societatis Janos Bolayai, vol. 31, 1979, pp. 445–460, Budapest, Hungary.

[16] S.M. Minčić, M.S. Stanković, On geodesic mappings of general affine connexion spaces and of generalized Riemannian spaces, Mat. Vesnik 49 (1997) 27–33.

[17] S.M. Minčić, M.S. Stanković, Equitorsion geodesic mappings of generalized Riemannian spaces, Publ. Inst. Math. (N. S) 61 (75) (1997) 97–104.

[18] A.K. Mondal, U.C. De, Some properties of a quarter-symmetric metric connection on a Sasakian manifold, Bull. Math. Anal. Appl. 1 (3) (2009) 99–108.

[19] N.S. Sinyukov, Geodesic Mappings of Riemannian Spaces, Nauka, Moscow, 1979 (in Russian).

[20] M.S. Stanković, S.M. Minčić, Lj.S. Velimirović, M.Lj. Zlatanović, On equitorsion geodesic mappings of general affine connection spaces, Rend. Semin. Mat. Univ. Padova 124 (2010) 77–90.

[21] S.E. Stepanov, On the global theory of some classes of mappings, Ann. Global Anal. Geom. 13 (1995) 239, https://doi.org/10.1007/BF00773658.

[22] S.E. Stepanov, Curvature and Tachibana numbers, J. Math. Sci. 172 (2011) 901, https://doi.org/10.1007/s10958-011-0232-y.

[23] N.O. Vesić, Weyl projective objects $\underset{1}{\mathcal{W}}, \underset{2}{\mathcal{W}}, \underset{3}{\mathcal{W}}$ for equitorsion geodesic mappings, Miskolc Math. Notes 19 (1) (2018) 677–689.

[24] M.Lj. Zlatanović, New projective tensors for equitorsion geodesic mappings, Appl. Math. Lett. 25 (2012) 890–897.

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