

## Conditions for Global Attractivity of $n$ -Patches Predator–Prey Dispersion-Delay Models<sup>1</sup>

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In this paper, a nonautonomous predator–prey dispersion model with functional response and continuous time delay is studied, where all parameters are time dependent. In this system, which consists of  $n$ -patches, the prey species can disperse among  $n$ -patches, but the predator species is confined to one patch and cannot disperse. It is proved the system is uniformly persistent under any dispersion rate effect. Furthermore, the sufficient conditions are established for global attractivity of a periodic solution of the system. © 2001 Academic Press

*Key Words:* predator–prey system; dispersion; delay; persistence; global attractivity.

### 1. INTRODUCTION

For many species spatial factors are important in population dynamics, as discussed by many authors. The theoretical study of spatial distribution can be traced back at least as far as Skellam [11], and has been extensively studied in many papers (for example in [1, 4, 6, 7, 9, 10, 14] and references cited therein). Most of the previous papers focused on the coexistence of populations modelled by systems of ordinary differential equations and the stability (local and global) of equilibria. Many existing models deal with a

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single population dispersing among patches. Some of them deal with competition and predator–prey interactions in patchy environments.

On the other hand, the effect of the past history on the systems' stability is also an important problem in population biology. Recently persistence and stability of a population dynamical system involving time delays have been discussed by some authors (for example [2, 3, 8] and references cited therein). They obtained some sufficient conditions that guarantee permanence of population or stability of positive equilibria or positive periodic solution. Song and Chen [12, 13] extended the autonomous Lotka–Volterra system to a two species nonautonomous dispersion Lotka–Volterra system, and they investigated persistence of the populations and periodic behavior of the system.

In this paper, we consider a nonautonomous predator–prey dispersion model with functional response and continuous time delay. In this system, which consists of  $n$ -patches, the prey species can disperse among  $n$ -patches, but the predator species is confined to one patch and cannot disperse. Our purpose is demonstrate that the dispersion rates have no effect on uniform persistence of the solution of the system. Furthermore, we establish conditions under which the system admits a positive periodic solution which attracts all solutions.

## 2. MODEL AND BACKGROUND CONCEPT

In this paper, we consider the predator–prey dispersion-delay model

$$\begin{aligned}
 \dot{x}_1 &= x_1 \left( a_1(t) - b_1(t)x_1 - \beta_{11}(t) \int_{-\tau}^0 k_{11}(s)x_1(t+s) ds \right. \\
 &\quad \left. - \frac{c(t)y}{1 + \alpha(t)x_1} \right) \\
 &\quad + \sum_{i=2}^n D_{i1}(t)(x_i - x_1) \\
 \dot{x}_j &= x_j \left( a_j(t) - b_j(t)x_j - \beta_{1j}(t) \int_{-\tau}^0 k_{1j}(s)x_j(t+s) ds \right) \\
 &\quad + \sum_{i=1}^n D_{ij}(t)(x_i - x_j) \\
 \dot{y} &= y \left( -d(t) + \frac{e(t)x_1}{1 + \alpha(t)x_1} - q(t)y \right. \\
 &\quad \left. - \beta_{10}(t) \int_{-\tau}^0 k_{10}(s)y(t+s) ds \right),
 \end{aligned} \tag{2.1}$$

where  $j = 2, 3, \dots, n$  ( $i \neq j$ ),  $x_1$  and  $y$  are population density of prey species  $x$  and predator species  $y$  in patch 1, and  $x_j$  is density of prey species  $x$  in patch  $j$ . Predator species  $y$  is confined to patch 1, while the prey species  $x$  can disperse among  $n$ -patches.  $D_{ij}(t)$  ( $i, j = 1, 2, \dots, n$ ) are dispersion coefficients of species  $x$ .

Now we let  $f^l = \inf\{f(t): t \in R\}$  and  $f^m = \sup\{f(t): t \in R\}$ , for a continuous and bounded function  $f(t)$ .

In system (2.1), we always assume:

(H<sub>1</sub>)  $a_i(t), b_i(t), D_{ij}(t)$  ( $i, j = 1, 2, \dots, n$ ),  $c(t), d(t), e(t), q(t), \alpha(t)$ , and  $\beta_{1j}(t)$  are continuous and strictly positive functions, which satisfy

$$\min\{a_i^l, b_i^l, D_{ij}^l, c^l, e^l, d^l, q^l, \alpha^l, \beta_{1j}^l\} > 0$$

$$\max\{a_i^m, b_i^m, D_{ij}^m, c^m, d^m, e^m, q^m, \alpha^m, \beta_{1j}^m\} < \infty.$$

(H<sub>2</sub>)  $k_{1j}(s) \geq 0$  on  $[-\tau, 0]$ , ( $0 \leq \tau < \infty$ ), and  $k_{1j}(s)$  is a piecewise continuous and normalized function such that  $\int_{-\tau}^0 k_{1j}(s) ds = 1$  ( $j = 0, 1, 2, \dots, n$ ).

We adopt the following notations and concepts throughout this paper.

Let  $x = (x_1, x_2, \dots, x_n, y) \in R_+^{n+1} = \{x \in R_+^{n+1}: x_i > 0 \text{ } (i = 1, 2, \dots, n), y > 0\}$ . Denote  $x > 0$  if  $x \in \text{Int } R_+^{n+1}$ . For ecological reasons, we consider system (2.1) only in  $\text{Int } R_+^{n+1}$ .

Let  $C^+ = C([-\tau, 0]; R_+^{n+1})$  denote the Banach space of all nonnegative continuous functions with

$$\|\Phi\| = \sup_{s \in [-\tau, 0]} |\Phi(s)|, \quad \text{for } \Phi \in C^+.$$

Then, if we choose the initial function space of system (2.1) to be  $C^+$ , it is easy to see that, for any  $\Phi = (\Phi_1, \Phi_2, \dots, \Phi_{n+1}) \in C^+$  and  $\Phi(0) > 0$ , there exists  $\alpha \in (0, \infty)$  and a unique solution  $x(t, \Phi)$  of system (2.1) on  $[-\tau, \alpha]$ , which remains positive for all  $t \in [0, \alpha]$ ; such solutions of system (2.1) are called positive solutions. Hence, in the rest of this paper, we always assume that

$$\Phi \in C^+, \quad \Phi(0) > 0. \quad (2.2)$$

**DEFINITION.** The system (2.1) is said to be uniformly persistent if there exists a compact region  $D \subset \text{Int}(R_+^{n+1})$  such that every solution  $Z(t) = (x_1(t), x_2(t), \dots, x_n(t), y(t))$  of system (2.1) with initial condition (2.2) eventually enters and remains in region  $D$ .

## 3. UNIFORM PERSISTENCE

LEMMA 3.1. *Every solution  $Z(t)$  of system (2.1) with initial conditions (2.2) exists in the interval  $[0, +\infty)$  and remains positive for all  $t \geq 0$ .*

*Proof.* It is true because

$$\dot{x}_1|_{x_1=0} = \sum_{i=2}^n D_{i1}(t)x_i > 0 \quad \text{for } x_i > 0,$$

$$\dot{x}_j|_{x_j=0} = \sum_{i=1}^n D_{ij}(t)x_i > 0 \quad \text{for } x_i > 0 \ (i \neq j),$$

$$y(t) = y(0) \exp \left\{ \int_0^t \left[ -d(s) + \frac{e(s)x_1}{1 + \alpha(s)x_1} - q(s)y - \beta_{10}(s) \int_{-\tau}^0 k_{10}(v)y(s+v) dv \right] ds \right\},$$

for  $y(0) > 0$ .

LEMMA 3.2. *Let  $Z(t) = \{x_1(t), x_2(t), \dots, x_n(t), y(t)\}$  denote any positive solution of system (2.1) with the initial conditions (2.2). Then there exists a  $T > 0$  such that*

$$x_i(t) \leq M_1 \quad (i = 1, 2, \dots, n), \quad y(t) \leq M_2, \quad \text{for } t \geq T \quad (3.1)$$

where

$$M_1 > M_1^*, \quad M_2 > M_2^*$$

$$M_1^* = \max \left\{ \frac{a_1^m}{b_1^l}, \frac{a_2^m}{b_2^l}, \dots, \frac{a_n^m}{b_n^l} \right\}, \quad M_2^* = \frac{e^m M_1^*}{q^l}. \quad (3.2)$$

*Proof.* We define

$$V(t) = \max\{x_1(t), x_2(t), \dots, x_n(t)\}.$$

Calculating the upper right derivative of  $V$  along the positive solution of system (2.1), we have

(P<sub>1</sub>) If  $V(t) = x_1(t)$ , then

$$\begin{aligned} D^+V(t) &= \dot{x}_1 \\ &\leq x_1(t)[a_1(t) - b_1(t)x_1(t)] \\ &\leq V(t)[a_1^m - b_1^l V(t)]. \end{aligned}$$

(P<sub>2</sub>) If  $V(t) = x_j(t)$  ( $j = 2, 3, \dots, n$ ), then

$$\begin{aligned} D^+V(t) &= \dot{x}_j(t) \\ &\leq x_j(t)[a_j(t) - b_j(t)x_j(t)] \\ &\leq V(t)[a_j^m - b_j^l V(t)] \quad (j = 2, 3, \dots, n). \end{aligned}$$

From (P<sub>1</sub>) and (P<sub>2</sub>), we derive

$$D^+V(t) \leq V(t)[a_i^m - b_i^l V(t)]. \quad (3.3)$$

From (3.3), we can obtain

(I) If  $\max\{x_1(0), x_2(0), \dots, x_n(0)\} \leq M_1$ , then  $\max\{x_1(t), x_2(t), \dots, x_n(t)\} \leq M_1$ ,  $t \geq 0$ .

(II) If  $\max\{x_1(0), x_2(0), \dots, x_n(0)\} > M_1$ , then let  $-\alpha = \max\{M_1(a_i^m - b_i^l M_1)\}$  ( $\alpha > 0$ ); if  $V(0) = x_j(0) > M_1$  holds, then there exists  $\epsilon > 0$ , such that if  $t \in [0, \epsilon)$ ,  $V(t) = x_j(t) > M_1$  and we have

$$D^+V(x_1(t), x_2(t), \dots, x_n(t)) = \dot{x}_j(t) < -\alpha < 0 \quad (j = 1, 2, \dots, n).$$

So there exists  $T_1 > 0$  if  $t \geq T_1$ , and we have

$$V(t) = \max\{x_1(t), x_2(t), \dots, x_n(t)\} \leq M_1.$$

In addition, from the system (2.1), we obtain

$$\dot{y}(t) \leq y(t)(e^m M_1 - q^l y(t)) \quad \text{for } t \geq T_1. \quad (3.4)$$

Suppose  $y(t)$  is not oscillatory about  $e^m M_1 / q^l$ ; that is, there exists a  $T_2 > 0$  ( $T_2 > T_1$ ), such that

$$y(t) > \frac{e^m M_1}{q^l}, \quad \text{for } t \geq T_2 \quad (3.5)$$

or

$$y(t) < \frac{e^m M_1}{q^l}, \quad \text{for } t \geq T_2. \quad (3.6)$$

Suppose (3.6) holds; then (3.1) follows.

Suppose (3.5) holds. We can choose  $M_2$  such that  $y(t) > M_2 > e^m M_1 / q^l > M_2^*$  and  $e^m M_1 - q^l M_2 < 0$ ; if we let  $-\beta = M_2(e^m M_1 - q^l M_2)$  ( $\beta > 0$ ), then  $\dot{y}(t) < -\beta < 0$ . Hence, in this case  $y(t)$  is strictly monotone decreasing with speed at least  $\beta$ . So there exists  $T_2^0 > 0$  ( $T_2^0 > T_2$ ), such that  $y(t) \leq M_2$ , if  $t \geq T_2^0$ .

Suppose that  $y(t)$  is oscillatory about  $e^m M_1 / q^l$ . Let  $y(\tilde{t})$  denote an arbitrary local maximum of  $y(t)$ . It is easy to see from (3.4) that

$$0 = \frac{dy(\tilde{t})}{dt} \leq y(\tilde{t})(e^m M_1 - q^l y(\tilde{t})), \quad (3.7)$$

and this implies

$$y(\tilde{t}) \leq \frac{e^m M_1}{q^l}. \quad (3.8)$$

Since  $y(\tilde{t})$  is an arbitrary local maximum of  $y(t)$ , we can conclude that  $y(t) < M_2$  holds eventually.

Let  $T = T_2^0$ . Then

$$x_i(t) \leq M_1 \quad (i = 1, 2, \dots, n), \quad y(t) \leq M_2, \quad \text{for } t \geq T.$$

The proof is complete.

We let

$$m_1^* = \min \left\{ \frac{a_1^l - c^m M_2^* - \beta_{11}^m M_1^*}{b_1^m}, \frac{a_2^l - \beta_{12}^m M_1^*}{b_2^m}, \dots, \frac{a_n^l - \beta_{1n}^m M_1^*}{b_n^m} \right\}.$$

**THEOREM 3.1.** *Suppose the system (2.1) satisfies*

$$(H_3) \quad m_1^* > 0,$$

$$(H_4) \quad (1 + \alpha^m M_1^*)^{-1} e^l m_1^* > d^m + \beta_{10}^m M_2^*;$$

*then system (2.1) is uniformly persistent*

*Proof.* Suppose  $Z(t) = (x_1(t), x_2(t), \dots, x_n(t), y(t))$  is a solution of system (2.1) which satisfies (2.2).

According to the system (2.1) and Lemma 3.1 if  $t > T$ , we can obtain

$$\begin{aligned} \dot{x}_1 &\geq x_1(a_1^l - c^m M_2 - \beta_{11}^m M_1 - b_1^m x_1) \\ &\quad + \sum_{i=2}^n D_{i1}(t)(x_i - x_1), \\ \dot{x}_j &\geq x_j(a_j^l - \beta_{1j}^m M_1 - b_j^m x_j) \\ &\quad + \sum_{i=1}^n D_{ij}(t)(x_i - x_j) \quad (j = 2, 3, \dots, n). \end{aligned} \quad (3.9)$$

From (3.2) and  $(H_3)$ , we know  $a_1^l - c^m M_2^* - \beta_{11}^m M_1^* > 0$ ,  $a_j^l - \beta_{1j}^m M_1^* > 0$  hold. Also from the proof of Lemma 3.1, we obtain that  $M_i$  can be chosen

close enough to  $M_i^*$  ( $i = 1, 2$ ) to make  $a_1^l - c^m M_2 - \beta_{11}^m M_1 > 0$ ,  $a_j^l - \beta_{1j}^m M_1 > 0$  hold.

We choose  $m_1$  as  $0 < m_1 < m_1^*$ .

Define  $V_1(t) = \min\{x_1(t), x_2(t), \dots, x_n(t)\}$ . Then by calculating the lower right derivative of  $V_1(t)$  along the positive solution of system (2.1), similar to the discussion for the inequality (3.3), it is easy to obtain

$$D_+ V_1(t) = \dot{x}_i(t) \geq \begin{cases} x_1(t) [a_1^l - c^m M_2 - \beta_{11}^m M_1 - b_1^m x_1(t)] \\ \quad (i = 1) \\ x_j(t) [a_j^l - \beta_{1j}^m M_1 - b_j^m x_j(t)] \\ \quad (j = 2, 3, \dots, n) \end{cases} \quad \text{for } t \geq T. \quad (3.10)$$

From (3.10), we can derive

(I) If  $V_1(0) = \min\{x_1(0), x_2(0), \dots, x_n(0)\} \geq m_1$ , then  $\min\{x_1(t), x_2(t), \dots, x_n(t)\} \geq m_1$ ,  $t \geq 0$ .

(II) If  $V_1(0) = \min\{x_1(0), x_2(0), \dots, x_n(0)\} < m_1$ , then let

$$\mu = \min\{x_1(0)(a_1^l - c^m M_2 - \beta_{11}^m M_1 - b_1^m m_1), \\ x_2(0)(a_2^l - \beta_{12}^m M_1 - b_2^m m_1), \dots, x_n(0)(a_n^l - \beta_{1n}^m M_1 - b_n^m m_1)\};$$

if  $V_1(0) = x_j(0) < m_1$  holds, then there exists  $\epsilon > 0$  such that if  $t \in [0, \epsilon)$ , we have  $V_1(x_1(t), x_2(t), \dots, x_n(t)) = x_j(t)$  and  $D_+ V_1(t) = \dot{x}_j(t) > \mu > 0$  ( $j = 1, 2, \dots, n$ ). So there exists  $\tilde{T}_1 > T > 0$  such that, if  $t > \tilde{T}_1$ , we have

$$V_1(x_1(t), x_2(t), \dots, x_n(t)) \geq m_1.$$

From the system (2.1) and Lemma 3.1, we know that there exists  $\tilde{T}_2 \geq \tilde{T}_1 + \tau$  such that

$$\begin{aligned} \dot{y}(t) &\geq y(t) \left[ -d^m + (1 + \alpha^m M_1)^{-1} e^l m_1 - q^m y(t) - \beta_{10}^m M_2 \right] \\ &= y(t) \left[ (1 + \alpha^m M_1)^{-1} e^l m_1 - (d^m + \beta_{10}^m M_2) - q^m y(t) \right]. \end{aligned}$$

From (3.2) and (H<sub>4</sub>), the inequality  $(1 + \alpha^m M_1^*)^{-1} e^l m_1^* - (d^m + \beta_{10}^m M_2^*) > 0$  holds; we know that  $m_1$  can be close to  $m_1^*$  and  $M_i$  can be sufficiently close to  $M_i^*$  ( $i = 1, 2$ ) to make the inequality  $(1 + \alpha^m M_1)^{-1} e^l m_1 - (d^m + \beta_{10}^m M_2) > 0$  hold.

Suppose  $y(t)$  is not oscillatory about

$$\frac{(1 + \alpha^m M_1)^{-1} e^l m_1 - (d^m + \beta_{10}^m M_2)}{q^m} (> 0);$$

then either

$$y(t) < \frac{(1 + \alpha^m M_1)^{-1} e^l m_1 - (d^m + \beta_{10}^m M_2)}{q^m}, \quad \text{for } t \geq \tilde{T}_2 \quad (3.11)$$

or

$$y(t) > \frac{(1 + \alpha^m M_1)^{-1} e^l m_1 - (d^m + \beta_{10}^m M_2)}{q^m}, \quad \text{for } t \geq \tilde{T}_2. \quad (3.12)$$

If (3.11) holds, then there exists a constant  $m_2$ ,

$$0 < m_2 < M_3^* = \frac{(1 + \alpha^m M_1)^{-1} e^l m_1 - (d^m + \beta_{10}^m M_2)}{q^m},$$

such that  $y(t) \leq m_2$  and  $(1 + \alpha^m M_1)^{-1} e^l m_1 - (d^m + \beta_{10}^m M_2) - m_2 q^m > 0$ ; thus, letting  $\lambda = (1 + \alpha^m M_1)^{-1} e^l m_1 - (d^m + \beta_{10}^m M_2) - m_2 q^m$ , we obtain

$$\dot{y}(t) > \lambda y(t) > 0.$$

This implies that  $y(t)$  is strictly monotone increasing with speed  $\lambda$ . Hence there exists  $\tilde{T}_3 > \tilde{T}_2$  such that

$$y(t) \geq m_2 \quad \text{for } t \geq \tilde{T}_3.$$

If (3.12) holds, then

$$y(t) > M_3^* > m_2 \quad \text{for } t \geq \tilde{T}_2.$$

Suppose now that  $y(t)$  is oscillatory about  $M_3^*$ . Let  $y(t^*)$  ( $t^* \geq \tilde{T}_2$ ) denote an arbitrary local minimum of  $y(t)$ ; it is easy to see from system (2.1) that

$$0 = \frac{dy(t^*)}{dt} \geq y(t^*) \left[ -d^m + (1 + \alpha^m M_1)^{-1} e^l m_1 - \beta_{10}^m M_2 - q^m y(t^*) \right],$$

and this implies

$$m_2 < M_3^* \leq y(t^*) \quad \text{for } t^* \geq \tilde{T}_2.$$

Since  $y(t^*)$  is an arbitrary local minimum of  $y(t)$ , we conclude that  $0 < m_2 < M_3^* \leq y(t)$  eventually.

Finally we let

$$D = \{(x_1(t), x_2(t), \dots, x_n(t), y(t)) : \\ m_1 \leq x_i(t) \leq M_1 \ (i = 1, 2, \dots, n), m_2 \leq y(t) \leq M_2\}.$$



Then  $D$  is a bounded compact region in  $R_+^{n+1}$  which has positive distance from coordinate hyperplanes. Let  $\tilde{T} = \tilde{T}_3$ ; then from the proof above, we obtain that if  $t \geq \tilde{T}$ , then every positive solution of system (2.1) with the initial condition (2.2) eventually enters and remains in region  $D$ . The proof is complete.

#### 4. GLOBAL ATTRACTIVITY OF PERIODIC SOLUTION

In this section, we suppose that all the coefficients in system (2.1) are continuous and  $\omega$ -periodic positive functions; then system (2.1) is a  $\omega$ -periodic system. Naturally, assumption  $(H_1)$  holds.

We let  $Z(t, Z^0) = \{x_1(t, Z^0), x_2(t, Z^0), \dots, x_n(t, Z^0), y(t, Z^0)\}$  denote the unique solution of periodic system (2.1) for initial value  $Z^0 = \{x_1^0, x_2^0, \dots, x_n^0, y^0\}$ .

Define Poincaré mapping  $A: R_+^{n+1} \rightarrow R_+^{n+1}$  as follows:

$$A(Z^0) = Z(\omega, Z^0), \quad Z^0 \in R_+^{n+1}.$$

$D$  is a bound, closed, and convex set in  $R_+^{n+1}$ . The mapping  $A$  is continuous because the solution of system (2.1) is continuous about the initial value, and from the proof of Theorem 3.1, we know that mapping  $A$  maps  $D$  into itself. Under assumptions  $(H_3)$  and  $(H_4)$ , from the Brouwer fixed point theorem, we obtain the following:

**THEOREM 4.1.** *If  $\omega$ -periodic system (2.1) satisfies assumptions  $(H_3)$  and  $(H_4)$ , then there is at least one strictly positive periodic solution of system (2.1).*

**THEOREM 4.2.** *In addition to  $(H_3)$  and  $(H_4)$ , assume further that system (2.1) satisfies*

$(H_5)$

$$\begin{aligned} \beta_{11}^m + \frac{c^m \alpha^m M_2^* + e^m}{(1 + \alpha^l m_1^*)^2} + \sum_{i=2}^n \frac{D_{1i}^m}{m_1^*} &< b_1^l, \\ \beta_{1j}^m + \frac{D_{j1}^m}{m_1^*} + \sum_{i=2}^n \frac{D_{ij}^m}{m_1^*} &< b_j^l \quad (j = 2, 3, \dots, n), \\ \beta_{10}^m + \frac{c^m}{1 + \alpha^l m_1^*} &< q^l; \end{aligned}$$

then system (2.1) has a globally attractive positive periodic solution.

*Proof.* Suppose  $Z(t) = (x_1(t), x_2(t), \dots, x_n(t), y(t))$  is a solution of system (2.1) with  $x_i(0) > 0$  ( $i = 1, 2, \dots, n$ ),  $y(0) > 0$ ,  $U(t) =$

$(u_1(t), u_2(t), \dots, u_n(t), v(t))$  is a strictly positive periodic solution of system (2.1). We have from uniform persistence of system (2.1) that there exist positive constants  $m_i$  and  $M_i$ , ( $i = 1, 2, \dots, n$ ) such that for all  $t \geq t^*$  ( $t^*$  sufficiently large),

$$\begin{aligned} 0 < m_1 \leq x_i(t) \leq M_1 & \quad (i = 1, 2, \dots, n), & 0 < m_2 \leq y(t) \leq M_2 \\ 0 < m_1 \leq u_i(t) \leq M_1 & \quad (i = 1, 2, \dots, n), & 0 < m_2 \leq v(t) \leq M_2. \end{aligned}$$

We define

$$\begin{aligned} \tilde{x}_i(t) &= \ln x_i(t), & \tilde{y}(t) &= \ln y(t), & \tilde{u}_i(t) &= \ln u_i(t), \\ \tilde{v}(t) &= \ln v(t) & (i = 1, 2, \dots, n). \end{aligned}$$

Consider the following Lyapunov functional:

$$\begin{aligned} V_2(t) &= \sum_{i=1}^n \left( |\tilde{x}_i(t) - \tilde{u}_i(t)| \right. \\ &\quad \left. + \beta_{1i}^m \int_{-\tau}^0 k_{1i}(s) \int_{t+s}^t |x_i(\theta) - u_i(\theta)| d\theta ds \right) \\ &\quad + |\tilde{y}(t) - \tilde{v}(t)| + \beta_{10}^m \int_{-\tau}^0 k_{10}(s) \int_{t+s}^t |y(\theta) - v(\theta)| d\theta ds. \end{aligned}$$

Now we calculate and estimate the upper right derivative of  $V_2(t)$  along the solutions of system (2.1):

$$\begin{aligned} D^+ V_2(t) &= \frac{\tilde{x}_1(t) - \tilde{u}_1(t)}{|\tilde{x}_1(t) - \tilde{u}_1(t)|} \left[ -b_1(t)(x_1(t) - u_1(t)) \right. \\ &\quad \left. - \beta_{11}(t) \int_{-\tau}^0 k_{11}(s)(x_1(t+s) - u_1(t+s)) ds \right. \\ &\quad \left. - c(t) \left( \frac{y(t)}{1 + \alpha(t)x_1(t)} - \frac{v(t)}{1 + \alpha(t)u_1(t)} \right) \right. \\ &\quad \left. + \sum_{i=2}^n D_{i1}(t) \left( \frac{x_i(t)}{x_1(t)} - \frac{u_i(t)}{u_1(t)} \right) \right] \\ &\quad + \sum_{j=2}^n \frac{\tilde{x}_j(t) - \tilde{u}_j(t)}{|\tilde{x}_j(t) - \tilde{u}_j(t)|} \left[ -b_j(t)(x_j(t) - u_j(t)) \right. \\ &\quad \left. - \beta_{1j}(t) \int_{-\tau}^0 k_{1j}(s)(x_j(t+s) - u_j(t+s)) ds \right] \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^n D_{ij}(t) \left( \frac{x_i(t)}{x_j(t)} - \frac{u_i(t)}{u_j(t)} \right) \Bigg] \\
& + \frac{\tilde{y}(t) - \tilde{v}(t)}{|\tilde{y}(t) - \tilde{v}(t)|} \left[ e(t) \left( \frac{x_1(t)}{1 + \alpha(t)x_1(t)} - \frac{u_1(t)}{1 + \alpha(t)u_1(t)} \right) \right. \\
& - q(t)(y(t) - v(t)) \\
& \left. - \beta_{10}(t) \int_{-\tau}^0 k_{10}(s)(y(t+s) - v(t+s)) ds \right] \\
& + \sum_{i=1}^n \beta_{1i}^m \left[ \int_{-\tau}^0 k_{1i}(s) |x_i(t) - u_i(t)| ds \right. \\
& \left. - \int_{-\tau}^0 k_{1i}(s) |x_i(t+s) - u_i(t+s)| ds \right] \\
& + \beta_{10}^m \int_{-\tau}^0 k_{10}(s) |y(t) - v(t)| ds \\
& - \beta_{10}^m \int_{-\tau}^0 k_{10}(s) |y(t+s) - v(t+s)| ds, \\
& - c(t) \frac{\tilde{x}_1(t) - \tilde{u}_1(t)}{|\tilde{x}_1(t) - \tilde{u}_1(t)|} \left( \frac{y(t)}{1 + \alpha(t)x_1(t)} - \frac{v(t)}{1 + \alpha(t)u_1(t)} \right) \\
& \leq \frac{c(t)}{1 + \alpha(t)x_1(t)} |y(t) - v(t)| \\
& + \frac{c(t)\alpha(t)v(t)}{(1 + \alpha(t)x_1(t))(1 + \alpha(t)u_1(t))} |x_1(t) - u_1(t)| \\
& \leq \frac{c^m}{1 + \alpha^l m_1} |y(t) - v(t)| + \frac{c^m \alpha^m M_2}{(1 + \alpha^l m_1)^2} |x_1(t) - u_1(t)|, \\
& \frac{\tilde{y}(t) - \tilde{v}(t)}{|\tilde{y}(t) - \tilde{v}(t)|} \left[ e(t) \left( \frac{x_1(t)}{1 + \alpha(t)x_1(t)} - \frac{u_1(t)}{1 + \alpha(t)u_1(t)} \right) \right] \\
& \leq e(t) \frac{|x_1(t) - u_1(t)|}{(1 + \alpha(t)x_1(t))(1 + \alpha(t)u_1(t))} \\
& \leq \frac{e^m}{(1 + \alpha^l m_1)^2} |x_1(t) - u_1(t)|.
\end{aligned}$$

We let

$$\begin{aligned}\tilde{D}_1(t) &= \frac{\tilde{x}_1(t) - \tilde{u}_1(t)}{|\tilde{x}_1(t) - \tilde{u}_1(t)|} \sum_{i=2}^n D_{i1}(t) \left( \frac{x_i(t)}{x_1(t)} - \frac{u_i(t)}{u_1(t)} \right), \\ \tilde{D}_j(t) &= \frac{\tilde{x}_j(t) - \tilde{u}_j(t)}{|\tilde{x}_j(t) - \tilde{u}_j(t)|} \sum_{i=1}^n D_{ij}(t) \left( \frac{x_i(t)}{x_j(t)} - \frac{u_i(t)}{u_j(t)} \right) \\ &\quad (j = 2, 3, \dots, n).\end{aligned}$$

There are the following three cases to consider for  $\tilde{D}_1(t)$ :

(i) If  $x_1(t) > u_1(t)$  and  $t \geq t^*$ , then

$$\tilde{D}_1(t) \leq \sum_{i=2}^n \frac{D_{i1}(t)}{u_1(t)} (x_i(t) - u_i(t)) \leq \sum_{i=2}^n \frac{D_{i1}^m}{m_1} |x_i(t) - u_i(t)|.$$

(ii) If  $x_1(t) < u_1(t)$  and  $t \geq t^*$ , then

$$\tilde{D}_1(t) \leq \sum_{i=2}^n \frac{D_{i1}(t)}{x_1(t)} (u_i(t) - x_i(t)) \leq \sum_{i=2}^n \frac{D_{i1}^m}{m_1} |x_i(t) - u_i(t)|.$$

(iii) If  $x_1(t) = u_1(t)$ , similar to the argument above, we can derive the same conclusion as (i) and (ii).

From (i), (ii), and (iii), we have

$$\tilde{D}_1(t) \leq \sum_{i=2}^n \frac{D_{i1}^m}{m_1} |x_i(t) - u_i(t)|, \quad \text{for } t \geq t^*.$$

Consider that for  $\tilde{D}_j(t)$  in the same way we can obtain

$$\tilde{D}_j(t) \leq \sum_{i=1}^n \frac{D_{ij}^m}{m_1} |x_i(t) - u_i(t)| \quad (j = 2, 3, \dots, n).$$

Hence we have

$$\begin{aligned}D^+ V_2(t) &\leq - \left( b_1^l - \beta_{11}^m - \frac{c^m \alpha^m M_2 + e^m}{(1 + \alpha^l m_1)^2} - \sum_{j=2}^n \frac{D_{1j}^m}{m_1} \right) |x_1(t) - u_1(t)| \\ &\quad - \sum_{j=2}^n \left( b_j^l - \frac{D_{j1}^m}{m_1} - \sum_{i=2}^n \frac{D_{ij}^m}{m_1} - \beta_{1j}^m \right) |x_j(t) - u_j(t)| \\ &\quad - \left( q^l - \frac{c^m}{1 + \alpha^l m_1} - \beta_{10}^m \right) |y(t) - v(t)|.\end{aligned}$$

From the proof of Theorem 3.1 and assumption  $(H_5)$ , we can select  $M_2$  close to  $M_2^*$  sufficiently and get  $m_1$  close to  $m_1^*$  enough too, such that

$$\begin{aligned}\beta_{11}^m + \frac{c^m \alpha^m M_2 + e^m}{(1 + \alpha^l m_1)^2} + \sum_{i=2}^n \frac{D_{1i}^m}{m_1} &< b_1^l, \\ \beta_{1j}^m + \frac{D_{j1}^m}{m_1} + \sum_{i=2}^n \frac{D_{ij}^m}{m_1} &< b_j^l \quad (j = 2, 3, \dots, n), \\ \beta_{10}^m + \frac{c^m}{1 + \alpha^l m_1} &< q^l.\end{aligned}$$

So there exists  $\alpha_1 > 0$  such that

$$D^+ V_2(t) \leq -\alpha_1 \left( \sum_{i=1}^n |x_i(t) - u_i(t)| + |y(t) - v(t)| \right). \quad (4.1)$$

Integrating both sides of (4.1) leads to

$$\begin{aligned}V_2(t) + \alpha_1 \int_{t^*}^t \left( \sum_{i=1}^n |x_i(s) - u_i(s)| + |y(s) - v(s)| \right) ds \\ \leq V_2(t^*) < +\infty \quad \text{for } t > t^*,\end{aligned}$$

which leads to

$$\sum_{i=1}^n |x_i(t) - u_i(t)| + |y(t) - v(t)| \in L^1(t^*, +\infty).$$

From the persistence hypothesis of (2.1) and the boundedness of the solutions of (2.1), we can obtain that  $[x_i(t) - u_i(t)]$  ( $i = 1, 2, \dots, n$ ),  $[y(t) - v(t)]$ , and their derivatives remain bounded on  $[0, \infty)$ . As a consequence

$$\sum_{i=1}^n |x_i(t) - u_i(t)| + |y(t) - v(t)|$$

is uniformly continuous. By Barbalat's lemma [5, P<sub>4</sub>, Lemma 1.2.2], it follows that

$$\lim_{t \rightarrow \infty} \left( \sum_{i=1}^n |x_i(s) - u_i(s)| + |y(s) - v(s)| \right) = 0.$$

Hence

$$\lim_{t \rightarrow \infty} |x_i(t) - u_i(t)| = 0 \quad (i = 1, 2, \dots, n),$$

$$\lim_{t \rightarrow \infty} |y(t) - v(t)| = 0.$$

This result implies that the system (2.1) has a globally attractive positive periodic solution. The proof is complete.

## 5. CONCLUSIONS

In this paper, we consider a predator–prey system with functional response and time delay in which the prey population can disperse among  $n$ -patches. Moreover, all coefficients in system (2.1) are time dependent. We first show that the system is persistent independent of the dispersion rates. In the second part we assume that all the coefficients are indeed periodic and prove that all solutions converge to a periodic solution of the system.

From this paper, we can find that the dispersion rates have also no effect on the existence of the positive periodic solution, but they have an effect on the global attractivity of the periodic solution.

We expect a similar technique to work in higher-dimensional systems with discrete time delays and dispersion. We leave this investigation for future work.

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