



Nonautonomous discrete rogue wave solutions and interactions in an inhomogeneous lattice with varying coefficients

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ABSTRACT

In this paper we analytically investigate nonautonomous discrete rogue wave solutions and their interactions in the generalized Ablowitz–Ladik–Hirota lattice with variable coefficients, which possess complicated wave propagation in time and differ from the usual discrete rogue waves. When the amplitude of the tunnel coupling coefficient between sites decreases, these nonautonomous discrete rogue wave solutions become localized in time after they propagate over some certain large critical values. Moreover, we find that the interactions between nonautonomous discrete rogue wave solutions are elastic.

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1. Introduction

Rogue waves (alias *freak waves*, *monster waves*, *killer waves*, *giant waves* or *extreme waves*), as an important physical phenomenon, are localized both in space and time and depict a unique event that appears from nowhere and disappears without a trace [1]. Rogue waves (RWs) are also known as rogons if they reappear virtually unaffected in size or shape shortly after their interactions [2]. RWs have been found in the ocean [3,4], nonlinear optics [5–7], Bose–Einstein condensates [8], atmosphere [9], and even finance [10]. Moreover, some experimental observations have shown that optical RWs do exist and play a positive role in nonlinear optical fibers [5–7]. Such optical RWs differ from oceanic RWs that cause danger and lead to many accounts of such waves hitting passenger ships, container ships, oil tankers, fishing boats, and offshore and coastal structures, sometimes with catastrophic consequences [4]. In particular, the analytical RWs have been obtained for some physical models such as the Kadomtsev–Petviashvili equation in shallow water [11] and the nonlinear Schrödinger (NLS) equation [1,12,13], as well as some of their extensions with varying coefficients [2], higher orders [14], or higher dimensions [15].

The discrete NLS equation [16] and the Ablowitz–Ladik (AL) lattice [17,18], as two prototypical discretizations of the continuum NLS equation, have been studied extensively in the field of nonlinear science. The former is nonintegrable, but has some interesting applications on physics [19–23]. The latter is integrable and possesses an infinite number of conservation laws [17,18], as well as having been depicted as an effective lattice to study properties of the intrinsic localized modes [24]. Moreover the nonintegrable discrete NLS equation can also be regarded as a perturbation of the integrable AL lattice [25]. In addition, the Salerno model (SM) has also been presented, interpolating between the nonintegrable discrete NLS equation ($\mu = 0$) and the integrable AL lattice ($\epsilon = 0$) in the form [26–32]

$$i\psi_{n,t} + (\psi_{n+1} + \psi_{n-1})(1 + \mu|\psi_n|^2) + \epsilon|\psi_n|^2\psi_n + v\psi_n = 0 \quad (1)$$

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which can be derived on the basis of a variational principle $\delta \mathcal{L}_{\text{SM}} / \delta \psi_n^* = 0$ from the Lagrangian density

$$\mathcal{L}_{\text{SM}} = \sum_n i(\psi_n^* \psi_{n,t} - \psi_n \psi_{n,t}^*) + 4\text{Re}(\psi_n^* \psi_{n+1}) + \mu(\psi_{n+1} + \psi_{n-1})\psi_n^* |\psi_n|^2 + \epsilon |\psi_n|^4 + 2v |\psi_n|^2, \quad (2)$$

where $\psi_n \equiv \psi_n(t)$ stands for the complex field amplitude at the n th site of the lattice, the parameter μ is the intersite nonlinearity and corresponds to the nonlinear coupling between nearest neighbours, ϵ measures the intrinsic onsite nonlinearity, and v describes the inhomogeneous frequency shift. The SM has been applied in biology [26], and Bose–Einstein condensates [27]. Recently, the discrete RWs have also drawn much attention. The discrete NLS equation (i.e., for the case $\mu = 0$, $\epsilon = 1$, and $v = -2$ in Eq. (1)) has numerically been verified to support discrete RWs [33]. The SM has also been found to admit discrete RWs on the basis of statistical analysis [34]. More recently, it has also been shown that exact discrete RWs [35,36] can exist in the AL lattice (i.e., for the case $\mu = 1$, $\epsilon = 0$, and $v = -2$ in Eq. (1)) on the basis of the limit cases of their multi-soliton solutions [37]. However, there are no reports to date about nonautonomous discrete RWs except for the AL lattice and the discrete Hirota equation [35,36].

In the present paper, we will explore exact nonautonomous discrete RW solutions and interaction of the generalized Ablowitz–Ladik–Hirota (ALH) lattice with variable coefficients given by Eq. (3), i.e., the generalized case of Eq. (1) without the intrinsic onsite nonlinearity, where the tunnel coupling coefficients and the intersite nonlinearity are the time-modulated, complex-valued and real-valued functions, respectively, the inhomogeneous frequency shifts are space- and time-modulated, real-valued functions, and the time-dependent gain or loss term is added. To do so, we will make use of the differential-difference symmetry analysis that can connect this equation with variable coefficients with the simpler ones. We show that, for the attractive intersite nonlinearity, the generalized ALH lattice with variable coefficients can support nonautonomous discrete Hirota RWs in terms of rogue wave solutions of the discrete Hirota equation. Moreover, we also exhibit wave propagation of nonautonomous discrete RW solutions and their interaction for some chosen parameters and functions. For the given periodic gain or loss term, when the amplitude of the tunnel coupling coefficient between sites decreases, these nonautonomous discrete rogue wave solutions are localized in space and keep the localization longer in time, which differ from usual discrete rogue waves.

2. Differential-difference similarity reductions and constraints

We here address the generalized Ablowitz–Ladik–Hirota (ALH) lattice with variable coefficients modeled by the following lattice

$$i\Phi_{n,t} + [\Lambda(t)\Phi_{n+1} + \Lambda^*(t)\Phi_{n-1}](1 + g(t)|\Phi_n|^2) - 2v_n(t)\Phi_n + i\gamma(t)\Phi_n = 0, \quad (3)$$

which can be derived in terms of a variational principle $\delta \mathcal{L}_{\text{SM}} / \delta \psi_n^* = 0$ from the following Lagrangian density

$$\begin{aligned} \mathcal{L}_{\text{ALH}} = \sum_n i(\Phi_n^* \Phi_{n,t} - \Phi_n \Phi_{n,t}^*) + 4\text{Re}[\Lambda(t)\psi_n^* \Phi_{n+1}] \\ + g(t)(\Lambda(t)\Phi_{n+1} + \Lambda^*(t)\Phi_{n-1})\Phi_n^* |\Phi_n|^2 - 2[2v_n(t) - i\gamma(t)]|\psi_n|^2, \end{aligned} \quad (4)$$

where $\Phi_n \equiv \Phi_n(t)$ stands for the complex field amplitude at the n th site of the lattice, the complex-valued function $\Lambda(t)$ is the coefficient of tunnel coupling between sites and can be rewritten as $\Lambda(t) = \alpha(t) + i\beta(t)$ with $\alpha(t)$ and $\beta(t)$ being differentiable, real-valued functions, $g(t)$ stands for the time-modulated intersite nonlinearity, $v_n(t)$ is the space- and time-modulated inhomogeneous frequency shift, and $\gamma(t)$ denotes the time-modulated effective gain or loss term.

In fact, this nonlinear lattice model (3) contains many special lattice models, such as the AL lattice for the case $\alpha(t) = \text{const}$, $\beta(t) = v_n(t) = \gamma(t) = 0$, and $g(t) = \text{const}$ [16,17], the AL equation with an additional term accounting for dissipation for the case $\alpha(t) = \text{const}$, $\beta(t) = v_n(t) = 0$, $\gamma(t) = \text{const}$, and $g(t) = \text{const}$ [27], the discrete Hirota equation for the case $\alpha(t) = \text{const}$, $\beta(t) = \text{const}$, $v_n(t) = \gamma(t) = 0$, and $g(t) = \text{const}$ [37], the generalized AL lattice given by Eq. (1) for the case $\alpha(t) = \text{const}$, $\beta(t) = \gamma(t) = 0$, and $g(t) = \text{const}$ [38], and the discrete modified KdV equation for the case $\alpha(t) = v_n(t) = \gamma(t) = 0$, $\beta(t) = \text{const}$, and $g(t) = \text{const}$ [39].

We consider spatially localized solutions of Eq. (3), i.e., $\lim_{|n| \rightarrow \infty} \Psi_n(t) = 0$. We search for a proper similarity transformation connecting solutions of Eq. (3) with those of the following discrete Hirota equation with constant coefficients [37]

$$i\Phi_{n,t} + [\lambda\Phi_{n+1} + \lambda^*\Phi_{n-1}](1 + |\Phi_n|^2) - 2\text{Re}(\lambda)\Phi_n = 0, \quad (5)$$

where $\Phi_n \equiv \Phi_n(\tau)$ is a physical field of space n and time $\tau \equiv \tau(t)$, which is a real-valued function of time to be determined, and the complex-valued parameter λ can be rewritten as $\lambda = a + ib$ with a and b being real-valued parameters. The discrete Hirota model (5) contains some special physical models, such as the AL lattice for the case $a = 1$ and $b = 0$ [17,18] and the discrete mKdV equation for the case $a = 0$ and $b = 1$ [39]. It has been shown in Ref. [40] that the discrete Hirota equation (5) is in fact an integrable discretization of the three-order NLS equation (also known as the Hirota equation) [41]

$$iq_t + a(q_{xx} + |q|^2 q) - ib(q_{xxx} + 6|q|^2 q_x) = 0, \quad (6)$$

which plays an important role in nonlinear optics [42,43].

In order to reduce Eq. (3) to (5), we here apply the similarity transformation in the form

$$\Psi_n(t) = \rho(t)e^{i\varphi_n(t)}\Phi_n[\tau(t)], \quad (7)$$

to Eq. (3), where the function $\rho(t)$ and phase $\varphi_n(t)$ are both real-valued functions of indicated variables to be determined.

To conveniently substitute ansatz (7) into Eq. (3) and to further balance the phases in every term, i.e., $\Psi_n(t)$, $\Psi_{n+1}(t)$, and $\Psi_{n-1}(t)$, in Eq. (3), we should firstly know the explicit expression of the phase in space. Similar to the phases in the continuous GP (NLS) equations with variable coefficients [44], we here consider the case that the phase is expressed as a quadratic polynomial in space with coefficients being functions of time in the form

$$\varphi_n(t) = p_2(t)n^2 + p_1(t)n + p_0(t). \quad (8)$$

Based on symmetry analysis, we simply balance the coefficients of these terms $\Psi_n(t)$, $\Psi_{n+1}(t)$, and $\Psi_{n-1}(t)$ such that we find that the phase in transformation (7) should be a first degree polynomial in space with coefficients being functions of time, namely

$$\varphi_n(t) = p_1(t)n + p_0(t), \quad (9)$$

where $p_{0,1}(t)$ are functions of time to be determined.

Eq. (7) allows us to reduce Eq. (3) to (5), variables in this reduction can be determined from the requirement for the new physical field $\Phi_n(\tau(t))$ to satisfy Eq. (5). Thus, we substitute transformation (7) into Eq. (3) and after relatively simple algebra obtain the system of nonlinear partial differential equations

$$\dot{\rho}(t) + \gamma(t)\rho(t) = 0, \quad (10a)$$

$$a\dot{\tau}(t) - \alpha(t)\cos p_1(t) + \beta(t)\sin p_1(t) = 0, \quad (10b)$$

$$[b\beta(t) + a\alpha(t)]\sin p_1(t) - [b\alpha(t) - a\beta(t)]\cos p_1(t) = 0, \quad (10c)$$

$$2v_n(t) + \dot{p}_1(t)n + \dot{p}_0(t) - 2a\dot{\tau}(t) = 0, \quad (10d)$$

$$g(t)\rho^2(t) = 1. \quad (10e)$$

where the dot denotes the derivative with respect to time.

Therefore, if system (10) is consistent, then we have constructed an algorithm generating nonautonomous discrete rogue wave solutions of Eq. (3) based on transformation (7) and Eq. (5):

First, we solve Eqs. (10a)–(10c) to obtain the functions $\rho(t)$, $\tau(t)$ and $p_1(t)$ in transformation (7).

And then we consider Eqs. (10d) and (10e) to determine the external potential $v_n(t)$ and nonlinearity $g(t)$ in Eq. (3) in terms of the above-obtained functions $\rho(t)$, $\tau(t)$ and $p_1(t)$.

Thus, we have established a transformation (7) connecting solutions of Eq. (5) and those of Eq. (3). In particular, we here exhibit our approach in terms of two lowest-order discrete rogue wave solutions of Eq. (5) as seeding solutions to find nonautonomous discrete rogue wave solutions of Eq. (3).

3. Determining the similarity transformation and coefficients

It follows from Eqs. (10a)–(10c) that we can write the variables $\rho(t)$, $p_1(t)$ and $\tau(t)$ in transformation (7) in the form

$$\rho(t) = \rho_0 \exp \left[- \int_{c_1}^t \gamma(s) ds \right], \quad (11a)$$

$$p_1(t) = \tan^{-1} \left[\frac{b\alpha(t) - a\beta(t)}{a\alpha(t) + b\beta(t)} \right], \quad (11b)$$

$$\tau(t) = (a^2 + b^2)^{-1/2} \int_{c_2}^t [\alpha^2(s) + \beta^2(s)]^{1/2} ds, \quad (11c)$$

where $c_{1,2}$ are constants and chosen as zero in the following context, ρ_0 is an integration constant. Now it follows from Eqs. (10d) and (10e) that we further find the external potential $v_n(t)$ and nonlinearity $g(t)$ in the form

$$v_n(t) = v_1(t)n + v_0(t), \quad (12a)$$

$$g(t) = \frac{1}{\rho_0^2} \exp \left[2 \int_0^t \gamma(s) ds \right], \quad (12b)$$

where we have introduced two functions in external potential $v_n(t)$ in the form

$$v_1(t) = -\frac{1}{2}\dot{p}_1(t), \quad (13a)$$

$$v_0(t) = -\frac{\dot{p}_0(t)}{2} + \left[\frac{\alpha^2(t) + \beta^2(t)}{a^2 + b^2} \right]^{1/2}. \quad (13b)$$

It follows from Eqs. (12a)–(13b) that, in these coefficients of Eq. (3), the intersite nonlinearity $g(t)$ (the inhomogeneous frequency shift $v_n(t)$) is related to the gain or loss term $\gamma(t)$ (the tunnel coupling $\Lambda(t) = \alpha(t) + i\beta(t)$). This means that only two varying coefficients (e.g., $\gamma(t)$ and $\Lambda(t) = \alpha(t) + i\beta(t)$) are left free. Moreover, it follows from Eq. (12a) that the intersite nonlinearity $g(t)$ is always positive (i.e., the attractive intersite nonlinearity).

For the inhomogeneous frequency shift $v_n(t)$ given by Eq. (13b), when $\alpha(t) \neq c\beta(t)$ with c being a constant, the frequency shift $v_n(t)$ is a linear function of space n with coefficients being functions of time. When the inhomogeneous frequency shift depends on only time, i.e., $v_n(t) \equiv v(t)$, which means that $\dot{p}_1(t) = 0$ on the basis of Eq. (10d), there exist two cases to be discussed:

(i) If $p_1(t) \neq 0$ in which we have $p_1(t) = p_1 = \text{const.} = 0$, then this means that $\varphi_n(t)$ is still a linear function of the discrete space n , i.e., $\varphi_n(t) = p_1 n + p_0(t)$. In this case, the variable function $\tau(t)$ and the inhomogeneous frequency shift $v_n(t)$ are given by

$$\tau(t) = \frac{\int_0^t \alpha(s) ds}{a \cos(p_1) + b \sin(p_1)}, \quad (14a)$$

$$v_n(t) = \frac{a\alpha(t)}{a \cos(p_1) + b \sin(p_1)} - \frac{\dot{p}_1(t)}{2}. \quad (14b)$$

and $\rho(t)$, $g(t)$ are same as Eqs. (11a) and (12a).

(ii) If $p_1(t) = 0$, which means that the relation for the coefficients in Eq. (3), $\alpha(t) = (a/b)\beta(t)$, is required and $\varphi_n(t)$ is only a function of time, i.e., $\varphi_n(t) \equiv p_0(t)$, then the variable function $\tau(t)$ and the inhomogeneous frequency shift $v_n(t)$ are given by

$$\tau(t) = \frac{1}{a} \int_0^t \alpha(s) ds, \quad v(t) = \alpha(t) - \frac{\dot{p}_0(t)}{2}, \quad (15)$$

and $\rho(t)$, $g(t)$ are the same as Eqs. (11a) and (12a), where $\gamma(t)$, $\alpha(t)$ and $p_0(t)$ are free functions of time, and a , b , ρ_0 are all free parameters.

4. Nonautonomous discrete rogue wave solutions and interaction

In general, we have a large degree of freedom in choosing the coefficients of similarity transformation (8) and Eq. (3). As a consequence, we can obtain an infinitely large family of exact solutions of the generalized ALH lattice with variable coefficients given by Eq. (3) in terms of exact solutions of the discrete Hirota equation (5) and transformation (8). In particular, if we consider discrete rogue wave solutions of Eq. (5) as seeding solutions, then we can obtain many types of nonautonomous (including arbitrary time-dependent functions) discrete rogue wave (rogue wave) solutions of Eq. (3). As two representative examples, we consider the lowest-order discrete rogue wave solutions of Eq. (5) as two examples [35] to study the dynamics of nonautonomous discrete rogue wave solutions of Eq. (3).

4.1. Nonautonomous discrete one-rogue-wave solutions

Based on the similarity transformation (7) and one-rogue-wave solution [34] of the Hirota equation (5), we present the nonautonomous discrete one-rogue-wave solution (also known as the first-order rational solution) of Eq. (3) in the form

$$\Psi_n^{(1)}(t) = \rho_0 \sqrt{\mu} \exp \left\{ - \int_0^t \gamma(s) ds + i[\varphi_n(t) + \hat{\varphi}_n(\tau)] \right\} \left[1 - \frac{4(1+\mu) \left[1 + 4i\mu \sqrt{a^2 + b^2} \tau(t) \right]}{1 + 4\mu n^2 + 16\mu^2(1+\mu)(a^2 + b^2)\tau^2(t)} \right], \quad (16)$$

where the phase $\hat{\varphi}_n(\tau)$ is defined by

$$\hat{\varphi}_n(\tau) = 2\tau(t) \left[(1+\mu)\sqrt{a^2 + b^2} - a \right] - n \tan^{-1} \left(\frac{b}{a} \right), \quad (17)$$

μ is a positive parameter, the variable $\tau(t)$ is given by Eq. (11c), and the phase $\varphi_n(t) = p_1(t)n + p_0(t)$ with $p_1(t)$ given by Eq. (11b) and $p_0(t)$ being an arbitrary differentiable function of time.

To illustrate the wave propagation of the obtained nonautonomous discrete one-rogue-wave solution, we can choose these free parameters in the form

$$\begin{aligned} \alpha(t) &= c_1 \sin(2t), & \beta(t) &= c_2 \cos(t), \\ \gamma(t) &= \gamma_0 \sin(t) \cos^2(t), & a = b = \mu = \rho_0 &= 1, \end{aligned} \quad (18)$$

where γ_0 , $c_{1,2}$ are constants.

Fig. 1 depicts the profiles of the coefficient $v_1(t) = -(1/2)\dot{p}_1(t)$ of the first degree term of the linear potential $v_n(t)$ given by Eq. (13a), the nonlinearity $g(t)$ given by Eq. (12b), and the gain or loss term $\gamma(t)$ vs time for the parameters given

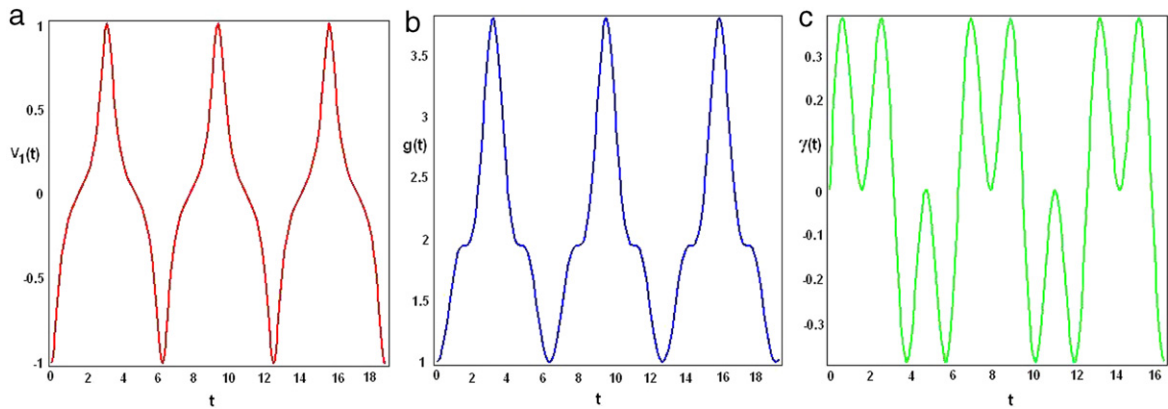


Fig. 1. Profiles of (a) the coefficient $v_1(t) = -(1/2)\dot{p}_1(t)$ of the first degree term of the external potential $v_n(t)$ given by Eq. (12a), (b) nonlinearity $g(t)$ given by Eq. (12b), and (c) the gain or loss term $\gamma(t)$ vs time for the parameters given by Eq. (18) with $\gamma_0 = c_1 = c_2 = 1$.

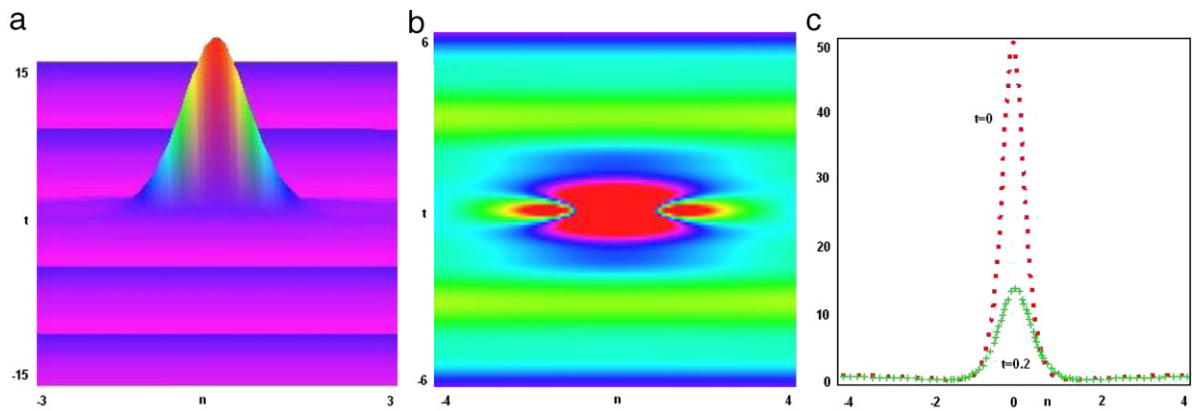


Fig. 2. Profiles of (a) the intensity distribution $|\Psi_n^{(1)}(t)|^2$ with $\max_{(n,t)} |\Psi_n^{(1)}(t)|^2 = 4.7$, (b) the density distribution $|\Psi_n^{(1)}(t)|^2$; (c) the intensity distributions $|\Psi_n^{(1)}(t)|^2$ for $t = 0, 0.2$. Here we choose the parameters as $\gamma_0 = c_1 = c_2 = 1$ in Eq. (18).

by Eq. (18). The evolution of the intensity distribution for the one-rogue-wave solution given by Eq. (16) is illustrated in Fig. 2 for parameters $\gamma_0 = c_1 = c_2 = 1$. The discrete rogue wave solution is localized both in space and in time, thus revealing the usual discrete rogue wave features. However if we fix the coefficient $\gamma_0 = 1$ of the gain or loss term and adjust the coefficients $c_1 = 0.01$, $c_2 = 0.02$ of the tunnel couplings $\alpha(t)$ and $\beta(t)$ given by Eq. (17), then the evolution of the intensity distribution for the one-rogue-wave solution is changed (see Fig. 3), and it follows from Fig. 3 that the discrete one-rogue-wave solution in this case is localized in space and keeps the localization longer in time than usual rogue waves (see [34]). Moreover, it follows from Figs. 2(c) and 3(c) that the amplitude of the discrete one-rogue-wave solution decreases as time increases, and the amplitude in Fig. 2(c) is decreased faster than that in Fig. 3(c) as time increases.

4.2. Nonautonomous discrete two-rogue-wave solutions

Here we consider the interaction between nonautonomous discrete rogue wave solutions. To do so, we apply a second-order rational solution [35] of the discrete Hirota equation (5) to the similarity transformation (7). As a result, we can obtain the nonautonomous discrete two-rogue-wave solution of Eq. (3) in the form

$$\Psi_n^{(2)}(t) = \rho_0 \sqrt{\mu} \exp \left\{ - \int_0^t \gamma(s) ds + i[\varphi_n(t) + \hat{\varphi}_n(\tau)] \right\} \left[1 - 12(1 + \mu) \frac{\mathcal{P}_n^{(2)}(\tau) + i\sqrt{\frac{\mathcal{T}(\tau)}{1+\mu}} \mathcal{Q}_n^{(2)}(\tau)}{\mathcal{H}_n^{(2)}(\tau)} \right], \quad (19)$$

where μ is a positive parameter, the functions $\mathcal{P}_n^{(2)}(\tau)$, $\mathcal{Q}_n^{(2)}(\tau)$ and $\mathcal{H}_n^{(2)}(\tau)$ are given by

$$\mathcal{P}_n^{(2)}(\tau) = 5\mathcal{T}^2 + 6(\mathcal{N} + 2\mu + 3)\mathcal{T} + \mathcal{N}^2 + (6 - 4\mu)\mathcal{N} - 3(4\mu + 1), \quad (20a)$$

$$\mathcal{Q}_n^{(2)}(\tau) = \mathcal{T}^2 + 2(\mathcal{N} + 1)\mathcal{T} + \mathcal{N}^2 - (16\mu + 6)\mathcal{N} - 3(8\mu + 5), \quad (20b)$$

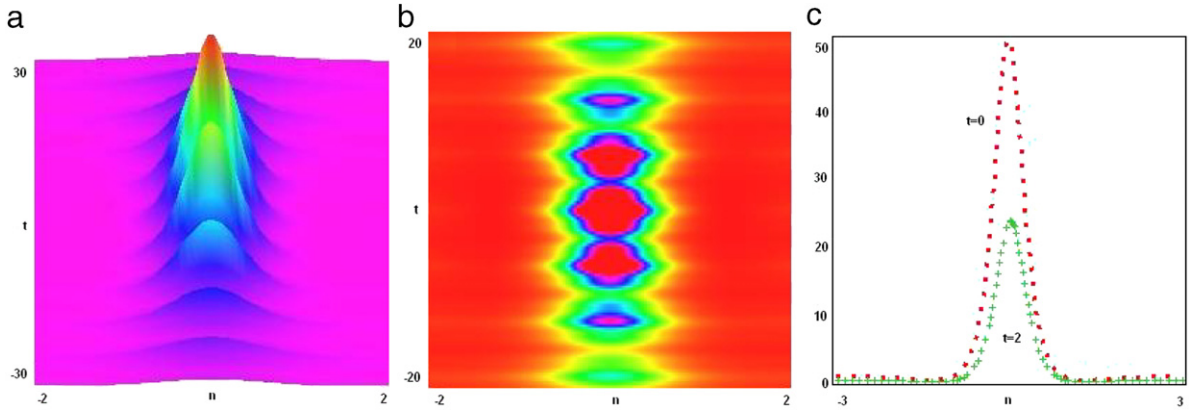


Fig. 3. Profiles of (a) the intensity distribution $|\psi_n^{(1)}(t)|^2$ with $\max_{(n,t)} |\psi_n^{(1)}(t)|^2 = 38$, (b) the density distribution $|\psi_n^{(1)}(t)|^2$; (c) the intensity distributions $|\psi_n^{(1)}(t)|^2$ for $t = 0, 2$. Here we choose the parameters as $\gamma_0 = 2$, $c_1 = 0.01$, $c_2 = 0.02$ in Eq. (18).

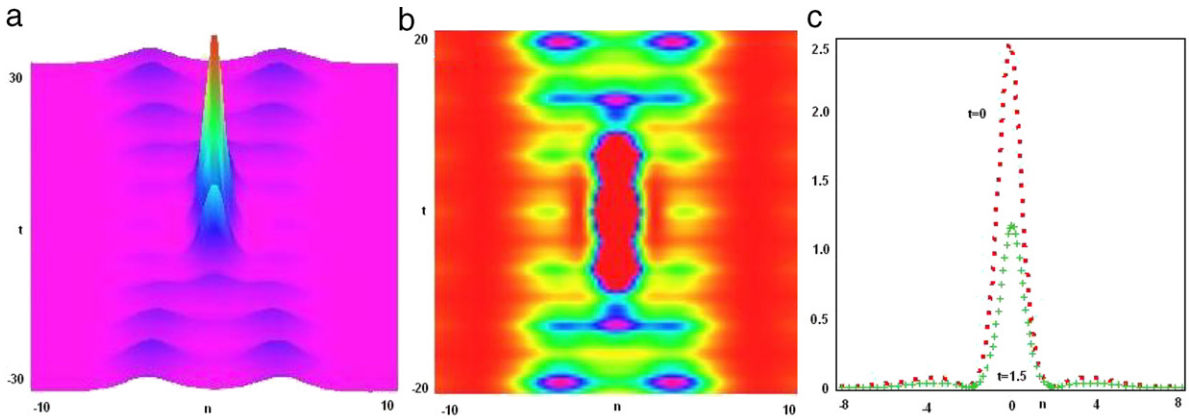


Fig. 4. Profiles of (a) the intensity distribution $|\psi_n^{(2)}(t)|^2$ with $\max_{(n,t)} |\psi_n^{(2)}(t)|^2 = 2.11$, (b) the density distribution $|\psi_n^{(2)}(t)|^2$; (c) the intensity distributions $|\psi_n^{(2)}(t)|^2$ for $t = 0, 1.5$. Here we choose the parameters as $\gamma_0 = 2$, $c_1 = 0.2$, $c_2 = 0.1$ in Eq. (18).

$$\mathcal{H}_n^{(2)}(\tau) = 9 + \mathcal{T}^3 + 3(\mathcal{N} + 8\mu + 9)\mathcal{T}^2 + 3(\mathcal{N}^2 - 6\mathcal{N} - 16\mu\mathcal{N} + 48\mu^2 + 72\mu + 33)\mathcal{T} + \mathcal{N}^3 + (3 - 8\mu)\mathcal{N}^2 + (27 + 24\mu + 16\mu^2)\mathcal{N}, \quad (20c)$$

where we have introduced $\mathcal{N}(n) = 4\mu n^2$ and $\mathcal{T}(\tau) = 16\mu^2(1 + \mu)(a^2 + b^2)\tau^2$, $\tau \equiv \tau(t)$ is given by Eq. (11c), the part phase $\varphi_n(t) = p_1(t)n + p_0(t)$ with $P_1(t)$ given by Eq. (11b) and $p_0(t)$ being an arbitrary differentiable function of time, and the phase $\hat{\varphi}_n(\tau)$ is given by Eq. (17).

Similarly, we can choose these free parameters given by Eq. (17) for the nonautonomous discrete two-rogue-wave solution given by Eq. (18) except for $\mu = 1/16$. Figs. 4 and 5 depict the evolution of intensity distribution for the interaction between nonautonomous discrete rogue wave solutions (discrete two-rogue-wave solution) given by Eq. (18) for different parameters γ_0 , c_1 , and c_2 , respectively. Moreover, we find that the interaction between nonautonomous discrete rogue waves is elastic, which may denote that the ALH equation (3) is integrable for some coefficients. Fig. 4 shows that the nonautonomous discrete two-rogue-wave solution is localized both in space and in time, thus revealing the usual discrete rogue wave features for the chosen parameters $\gamma_0 = 1$, $c_1 = 2$, $c_2 = 1$, but if we fix the coefficient $\gamma_0 = 1$ of the gain or loss term and adjust the coefficients $c_1 = 0.2$, $c_2 = 0.1$ of the tunnel couplings $\alpha(t)$ and $\beta(t)$ given by Eq. (17), then the evolution of the intensity distribution for the discrete two-rogue-wave solution is changed (see Fig. 5), and it follows from Fig. 5 that the nonautonomous discrete two-rogue wave solution is localized in space and keeps the localization longer in time than usual rogue waves. Moreover, it follows from Figs. 4(c) and 5(c) that the amplitude of the discrete two-rogue-wave solution decreases as time increases, and the amplitude in Fig. 4(c) decreases faster than that in Fig. 5(c) as time increases.

5. Conclusions and discussion

In conclusion, we have studied nonautonomous discrete rogue wave solutions and their interaction in the generalized Ablowitz–Ladik–Hirota lattice with the varying tunnel coupling, intersite nonlinearity, inhomogeneous frequency shift, and gain or loss term given by Eq. (3). Based on the differential-difference similarity reduction (8) and the discrete rogue

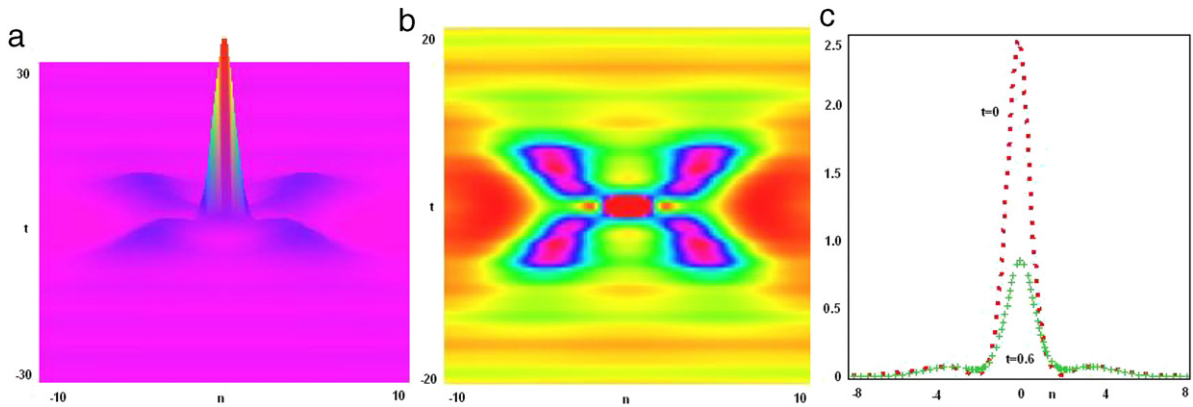


Fig. 5. Profiles of (a) the intensity distribution $|\Psi_n^{(2)}(t)|^2$ with $\max_{(n,t)} |\Psi_n^{(2)}(t)|^2 = 1.65$, (b) the density distribution $|\Psi_n^{(2)}(t)|^2$; (c) the intensity distributions $|\Psi_n^{(2)}(t)|^2$ for $t = 0, 0.6$. Here we choose the parameters as $\gamma_0 = 1$, $c_1 = 2$, $c_2 = 1$ in Eq. (18).

waves of the AL lattice and the discrete Hirota equation [34], we obtain nonautonomous discrete rogue wave solutions when the intersite nonlinearity $g(t)$ (the inhomogeneous frequency shift $v_n(t)$) is related to the gain or loss term $\gamma(t)$ (the tunnel coupling $\Lambda(t) = \alpha(t) + i\beta(t)$) (see Eqs. (12a) and (12b)). This denotes that only two coefficients (i.e., $\gamma(t)$ and $\Lambda(t) = \alpha(t) + i\beta(t)$) are left free.

In particular, we have studied wave propagation of nonautonomous discrete rogue wave solutions and their interaction for some chosen parameters, which exhibit complicated rogue wave constructs. For the given periodic gain or loss term, when the amplitude of the tunnel coupling coefficient between sites decreases, these nonautonomous discrete rogue wave solutions are localized in space and keep the localization longer in time, which differ from the usual discrete rogue waves of nonlinear discrete equations (e.g., the AL lattice and the discrete Hirota equation) [34].

Moreover, nonautonomous discrete rogue wave solutions and interaction may provide more evidence to further understand the physical mechanism of discrete rogue wave phenomena. The approach may also be extended to other discrete nonlinear lattices with variable coefficients for studying their discrete rogue wave solutions and wave propagation.

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