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Note

Addendum to "The Lebesgue summability of trigonometric integrals" [J. Math. Anal. Appl. 390 (2012) 188–196]

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ABSTRACT

We observed that statement (2.9) in Theorem 2 remains valid if condition (2.7) is replaced by the weaker condition that

$$f(t) \in L^1(-T, T) \quad \text{for all } T > 0. \tag{2.7$'$}$$

We observed that statement (2.9) in Theorem 2 remains valid if condition (2.7) is replaced by the weaker condition that

$$f(t) \in L^1(-T, T) \quad \text{for all } T > 0. \tag{2.7$'$}$$

Furthermore, Theorem 3 also remains valid if condition (2.7) is replaced by (2.7′).

To be more precise, following Theorems 2′ and 3′ can be proved in the same way as Theorems 2 and 3 are proved, while using Lemmas 2′ and 3′ below instead of Lemmas 2 and 3.

Theorem 2′. *If $f : \mathbb{R} \to \mathbb{C}$ is such that conditions (2.7′) and*

$$\lim_{T\to\infty} \frac{1}{T} \int_{|t|<T} |tf(t)|dt = 0 \tag{2.8}$$

are satisfied, then we have uniformly in $x \in \mathbb{R}$ that

$$\lim_{h\downarrow 0} \left\{ \frac{\Delta\mathcal{L}(x; h)}{2h} - I_{1/h}(x) \right\} = 0. \tag{2.9}$$

Theorem 3′. *Suppose $f : \mathbb{R} \to \mathbb{C}$ is such that conditions (2.7′) and*

$$\frac{1}{T} \int_{|t|<T} |tf(t)|dt \le B \quad \textit{for all } T > T_1, \tag{2.10}$$

are satisfied, where B and T_1 are constants. If the finite limit

$$\lim_{T\to\infty} I_T(x) := \lim_{T\to\infty} \int_{|t|<T} f(t)e^{itx}dt = \ell \tag{2.3}$$

exists at some point $x \in \mathbb{R}$, then (2.9) holds at this x.

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We emphasize that in this addendum, the definition

$$\int_{\mathbb{R}} f(t)\frac{e^{itx}}{it}dx =: \mathcal{L}(x), \quad x \in \mathbb{R}, \tag{2.4}$$

is interpreted only formally; that is, the integral in (2.4) may not exist in Lebesgue's sense. However, under the conditions in Theorems 2′ and 3′, the integral in the representation

$$\frac{\Delta\mathcal{L}(x;h)}{2h} := \int_{\mathbb{R}} f(t)e^{itx}\frac{\sin th}{th}dt, \quad h > 0, \tag{2.6}$$

does exist in Lebesgue's sense.

As we have mentioned above, the proofs of Theorems 2′ and 3′ hinge on the following Lemmas 2′ and 3′. We note that we essentially use only Part (i) in our earlier Lemmas 2 and 3, while we substitute condition (2.7′) for (2.7).

Lemma 2′. *If $f : \mathbb{R} \to \mathbb{C}$ is such that condition (2.7′) and (2.8) are satisfied, then*

$$\lim_{T\to\infty} T\int_{|t|>T}\left|\frac{f(t)}{t}\right|dt = 0. \tag{3.2}$$

Lemma 3′. *If $f : \mathbb{R} \to \mathbb{C}$ is such that conditions (2.7′) and (2.10) are satisfied, then*

$$T\int_{|t|>T}\left|\frac{f(t)}{t}\right|dt \le 4B \quad \textit{for all } T > T_1. \tag{3.8}$$

We note that the converse implications (3.2) ⇒ (2.8) in Lemma 2′ and (3.8) ⇒ (2.10) in Lemma 3′ do hold under the supplementary condition (2.7). But we do not need these converse implications in the proofs of Theorems 2′ and 3′.