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Decentralized iterative learning control for a class of large scale interconnected dynamical systems

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Abstract

The problem of decentralized iterative learning control for a class of large scale interconnected dynamical systems is considered. In this paper, it is assumed that the considered large scale dynamical systems are linear time-varying, and the interconnections between each subsystem are unknown. For such a class of uncertain large scale interconnected dynamical systems, a method is presented whereby a class of decentralized local iterative learning control schemes is constructed. It is also shown that under some given conditions, the constructed decentralized local iterative learning controllers can guarantee the asymptotic convergence of the local output error between the given desired local output and the actual local output of each subsystem through the iterative learning process. Finally, as a numerical example, the system coupled by two inverted pendulums is given to illustrate the application of the proposed decentralized iterative learning control schemes.
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1. Introduction

It is well known that in many practical control problems, there are a class of repetitive dynamical systems, such as robotic control systems, neuromuscular stimulation systems, and so on. For such repetitive dynamical systems, the so-called iterative learning control laws have been introduced in [1]. Generally speaking, by employing an iterative learning control law, one can

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gradually improve or perfect the system performance of a specified task, based on the previous performances of the identical tasks.

It seems that the main advantage of the iterative learning control strategy is to require less a priori knowledge about the system dynamics and less computational effort than many other types of control strategies. Therefore, the problem of iterative learning control for repetitive dynamical systems has received considerable attention, and many results have been obtained (see, e.g., [2–11] and references therein). In particular, there are some works in which an iterative learning control scheme has been applied to the analysis and design of time-delay systems and other types of systems. In [12], for example, the design of an iterative learning controller is considered for a class of linear systems with time delay, and an iterative learning control algorithm is proposed such that the output of the considered time-delay dynamical systems can track a given desired trajectory. In [13], a class of PID-type iterative learning control schemes is proposed for uncertain nonlinear systems with state delays, and the convergence conditions for the proposed high-order learning control are derived. However, few efforts are made to consider the problem of iterative learning control for large scale systems. It seems that for large scale systems, the similar results have not been reported yet in the control literature.

As well known, a large scale system can be characterized by a large number of variables representing system, a strong interaction between the system variables, and a complex structure. In particular, a large scale system is often considered as a set of interconnected subsystems, and referred to as large scale interconnected systems. The advantage of this aspect in controller design is to reduce complexity and this therefore allows the control implementation to be feasible. Therefore, the problem of decentralized control of large scale interconnected systems has received considerable attention, and many approaches have been developed to synthesize some types of decentralized local state (or output) feedback controllers (see, e.g., [14–20] and references therein). Thus, it is obviously meaningful to apply the iterative learning control strategy to large scale interconnected systems, and to develop some types of decentralized local iterative learning control schemes.

In this paper, we consider the problem of decentralized iterative learning control for a class of large scale interconnected systems. We assume that the considered large scale systems are linear time-varying, and the interconnections between each subsystem are unknown. For such a class of uncertain large scale interconnected systems, we want to present a method whereby a class of decentralized local iterative learning control schemes can be constructed. We also show that under some given conditions, the constructed decentralized local iterative learning controllers can guarantee the asymptotic convergence of the local output error between the given desired local output and the actual local output of each subsystem through the iterative learning process.

The paper is organized as follows. In Section 2, the decentralized iterative learning control problem to be tackled is stated and some standard assumptions are introduced. In Section 3, we propose a class of decentralized local iterative learning control schemes for large scale interconnected systems. In Section 4, a numerical example is given to illustrate the use of our results. The paper is concluded in Section 5 with a brief discussion of the results.

2. Problem formulation and assumptions

Consider a class of large scale systems S composed of N interconnected subsystems S_i, $i = 1, 2, \ldots, N$, described by the following state equations and output equations

$$\frac{dx_i(t)}{dt} = A_i(t)x_i(t) + B_i(t)u_i(t) + \sum_{j=1}^{N} A_{ij}(t)x_j(t), \tag{1a}$$

$$y_i(t) = C_i(t)x_i(t), \tag{1b}$$

where $t \in R^+$ is the time, $x_i(t) \in R^{n_i}$ is the state vector, $u_i(t) \in R^{m_i}$ is the control (or input) vector, $y_i(t) \in R^{l_i}$ is the output vector, $A_i(\cdot)$, $B_i(\cdot)$, $C_i(\cdot)$ are continuous matrices of appropriate dimensions, and the matrix $A_{ij}(\cdot)$ accounts for the interconnection between the subsystems S_i and S_j, which may be unknown. Here, for mathematical completeness, we assume that $A_{ij}(\cdot)$ is continuous matrix of appropriate dimensions. In addition, $x(t) \in R^n$ denotes $[\,x_1^\top(t) \quad x_2^\top(t) \quad \cdots \quad x_N^\top(t)\,]^\top$, where $n = n_1 + n_2 + \cdots + n_N$, and the initial state $x_i(t_0)$ for each subsystem is assumed to be unknown.

For each subsystem S_i, $i \in \{1, 2, \ldots, N\}$, it is supposed that a desired local output trajectory $y_i^m(t) \in R^{l_i}$ is given for a finite time interval $t \in [t_0, T]$. Then, the error between the desired local output and the actual local output trajectories of each subsystem can be represented by

$$e_i(t) = y_i^m(t) - y_i(t), \quad i \in \{1, 2, \ldots, N\}, \tag{2}$$

where $t \in [t_0, T]$ and $e_i(t) \in R^{l_i}$.

Throughout this paper, we use the superscript k to denote the iteration number of learning. Therefore, $x_i^{[k]}(t)$, $u_i^{[k]}(t)$, $y_i^{[k]}(t)$ represent the corresponding vectors at the kth iteration.

Now, the main objective of this paper is to find the decentralized local iterative learning control laws for each subsystems with unknown local initial state such that the local output error $e_i(t)$ between the given desired local output $y_i^m(t)$ and the actual local output $y_i(t)$ is identical for all $t \in [t_0, T]$, through the iterative learning process. That is, for all $t \in [t_0, T]$,

$$\lim_{k\to\infty} \left\| e_i^{[k]}(t) \right\| = \lim_{k\to\infty} \left\| y_i^m(t) - y_i^{[k]}(t) \right\| = 0, \tag{3}$$

where $i \in \{1, 2, \ldots, N\}$.

Before giving our decentralized iterative learning control laws, we introduce for large scale system (1) the following standard assumptions.

Assumption 2.1. For each subsystem S_i, $i \in \{1, 2, \ldots, N\}$, the desired local output trajectory $y_i^m(t)$ is continuous differentiable vector function on $[t_0, T]$.

Assumption 2.2. For any $t \in [t_0, T]$ and each $i \in \{1, 2, \ldots, N\}$, the matrix $C_i(\cdot)B_i(\cdot)$ is full rank.

Assumption 2.3. [11] For any $t \in [t_0, T]$ and each $i \in \{1, 2, \ldots, N\}$, the control matrix $B_i(\cdot)$ is differentiable.

Remark 2.1. It is obvious that Assumption 2.1 is standard, and by this assumption we mean that one wants for each subsystem to track a continuous output trajectory. Moreover, it is possible from Assumption 2.1 that a class of D-type decentralized local iterative learning control laws is designed for each dynamical subsystem. Assumption 2.2 can guarantee the existence of decentralized local iterative learning control laws, which will be known from the conditions derived in the next sections.

Throughout this paper, $\|\cdot\|$ denotes any vector (or matrix) norm for any vector (or matrix), and $\Phi_i(t,\tau)$ stands for the state transition matrix of unforced isolated subsystem of the form

$$\frac{dx_i(t)}{dt} = A_i(t)x_i(t), \quad i \in \{1,2,\ldots,N\}, \tag{4}$$

and satisfies the following matrix differential equation

$$\frac{\partial \Phi_i(t,\tau)}{\partial t} = A_i(t)\Phi_i(t,\tau), \quad \Phi_i(\tau,\tau) = I_i, \tag{5}$$

where I_i is an identity matrix of appropriate dimension.

3. Decentralized iterative learning control laws

In this section, for the problem stated in Section 2, we employ a local input updating law for decentralized iterative learning control as follows:

$$u_i^{[k+1]}(t) = u_i^{[k]}(t) + \Gamma_i^{[k]}(t)\dot{e}_i^{[k]}(t), \quad i \in \{1,2,\ldots,N\}, \tag{6}$$

where $\Gamma_i(t) \in R^{m_i \times l_i}$ is a continuous learning gain matrix which will be determined later, together with an initial state learning algorithm described by

$$x_i^{[k+1]}(t_0) = x_i^{[k]}(t_0) + B_i(t_0)\Gamma_i^{[k]}(t_0)e_i^{[k]}(t_0), \quad i \in \{1,2,\ldots,N\}, \tag{7}$$

where $t \in [t_0, T]$, $u_i^{[0]}(t)$ is an arbitrary continuous initial control input, and $x_i^{[0]}(t_0)$ is an arbitrary initial state, which may be different from the unknown desired initial state $x_i(t_0)$ for each subsystem. Furthermore, it is assumed that for each $i \in \{1,2,\ldots,N\}$, the iterative learning control gain matrix $\Gamma_i(t)$ is continuous differentiable over $[t_0, T]$, and is chosen such that $\Gamma_i(t_0) \neq 0$.

Then the following theorem can be obtained which shows that the decentralized iterative learning control laws given in (6) and (7) can guarantee the asymptotic convergence of the local output error of each subsystem S_i.

Theorem 3.1. *Consider the large scale interconnected dynamical systems described by* (1) *which satisfies Assumptions* 2.2 *and* 2.3. *Given the desired local output trajectory* $y_i^m(t)$, *which satisfies Assumption* 2.1, *over the finite time interval* $[t_0, T]$, *by employing the decentralized local iterative learning control law described by* (6) *and initial state learning algorithm described by* (7), *the local output error* $e_i(t)$ *of each subsystem* S_i, $i \in \{1,2,\ldots,N\}$, *can be guaranteed to asymptotically converge to zero, i.e., for any* $t \in [t_0, T]$,

$$\lim_{k\to\infty} e_i^{[k]}(t) = \lim_{k\to\infty} \left(y_i^m(t) - y_i^{[k]}(t)\right) = 0, \quad i \in \{1,2,\ldots,N\}, \tag{8}$$

if there exists a learning gain matrix $\Gamma_i(t)$ *such that for any* $t \in [t_0, T]$,

$$\rho_i(t) < 1, \quad i \in \{1,2,\ldots,N\}, \tag{9}$$

where

$$\rho_i(t) = \sup_{\tau\in[t_0,t]} \left\| I_i - C_i(\tau)B_i(\tau)\Gamma_i(\tau) \right\| \tag{10}$$

and where $I_i \in R^{l_i \times l_i}$ *is an identity matrix.*

Proof. It is obvious that for any given local control input $u_i(t)$, $t \in [t_0, T]$, the general solution $x_i(t)$ to each subsystem S_i, described by (1), can be written in the following form:

$$x_i(t) = \Phi_i(t, t_0)x_i(t_0) + \int_{t_0}^{t} \Phi_i(t, \tau)B_i(\tau)u_i(\tau)\,d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \Phi_i(t, \tau)A_{ij}(\tau)x_j(\tau)\,d\tau, \tag{11}$$

where $\Phi_i(t, \tau)$ is the state transition matrix of the ith unforced isolated subsystem and satisfied the matrix differential equation described by (5).

Thus, for the kth iteration, it can be known from (11) that for any $t \in [t_0, T]$,

$$x_i^{[k]}(t) = \Phi_i(t, t_0)x_i^{[k]}(t_0) + \int_{t_0}^{t} \Phi_i(t, \tau)B_i(\tau)u_i^{[k]}(\tau)\,d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \Phi_i(t, \tau)A_{ij}(\tau)x_j^{[k]}(\tau)\,d\tau. \tag{12}$$

Furthermore, by making use of (12) together with (6) and (7), the state error between the $(k+1)$th and kth iterations can be expressed as

$$x_i^{[k+1]}(t) - x_i^{[k]}(t) = \Phi_i(t, t_0)B_i(t_0)\Gamma_i(t_0)e_i^{[k]}(t_0) + \int_{t_0}^{t} \Phi_i(t, \tau)B_i(\tau)\Gamma_i(\tau)\dot{e}_i^{[k]}(\tau)\,d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \Phi_i(t, \tau)A_{ij}(\tau)\big(x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\big)\,d\tau. \tag{13}$$

By integrating the term $\dot{e}_i^{[k]}(\tau)$ in (13) by parts, we can obtain that for any $t \in [t_0, T]$,

$$x_i^{[k+1]}(t) - x_i^{[k]}(t) = B_i(t)\Gamma_i(t)e_i^{[k]}(t) - \int_{t_0}^{t} \frac{\partial}{\partial \tau}\big(\Phi_i(t, \tau)B_i(\tau)\Gamma_i(\tau)\big)e_i^{[k]}(\tau)\,d\tau + \sum_{j=1}^{N} \int_{t_0}^{t} \Phi_i(t, \tau)A_{ij}(\tau)\big(x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\big)\,d\tau. \tag{14}$$

For simplicity, we introduce the following definitions:

$$\phi_i(t, \tau) = \frac{\partial}{\partial \tau}\big(\Phi_i(t, \tau)B_i(\tau)\Gamma_i(\tau)\big), \qquad \psi_{ij}(t, \tau) = \Phi_i(t, \tau)A_{ij}(\tau). \tag{15}$$

Then, we can rewrite (14) as

$$x_i^{[k+1]}(t) - x_i^{[k]}(t) = B_i(t)\Gamma_i(t)e_i^{[k]}(t) - \int_{t_0}^{t} \phi_i(t, \tau)e_i^{[k]}(\tau)\,d\tau$$

$$+\sum_{j=1}^{N}\int_{t_0}^{t}\psi_{ij}(t,\tau)\big(x_j^{[k+1]}(\tau)-x_j^{[k]}(\tau)\big)\,d\tau. \tag{16}$$

Taking the norm of both sides of (16) and making use of the general properties of norms, we can obtain that for any $t\in[t_0,T]$,

$$\begin{aligned}\big\|x_i^{[k+1]}(t)-x_i^{[k]}(t)\big\| &\leqslant \big\|B_i(t)\Gamma_i(t)\big\|\big\|e_i^{[k]}(t)\big\|+\int_{t_0}^{t}\big\|\phi_i(t,\tau)\big\|\big\|e_i^{[k]}(\tau)\big\|\,d\tau\\ &\quad+\sum_{j=1}^{N}\int_{t_0}^{t}\big\|\psi_{ij}(t,\tau)\big\|\big\|x_j^{[k+1]}(\tau)-x_j^{[k]}(\tau)\big\|\,d\tau.\end{aligned} \tag{17}$$

Moreover, letting

$$\begin{aligned}\beta_i^c &:= \sup_{t\in[t_0,T]}\big\|B_i(t)\Gamma_i(t)\big\|,\quad i=1,2,\ldots,N,\\ \beta_i^d &:= \sup_{t,\tau\in[t_0,T]}\big\|\phi_i(t,\tau)\big\|,\quad i=1,2,\ldots,N,\\ \beta_{ij} &:= \sup_{t,\tau\in[t_0,T]}\big\|\psi_{ij}(t,\tau)\big\|,\quad i,j=1,2,\ldots,N,\end{aligned}$$

it follows from (17) that for any $t\in[t_0,T]$,

$$\begin{aligned}\big\|x_i^{[k+1]}(t)-x_i^{[k]}(t)\big\| &\leqslant \beta_i^c\big\|e_i^{[k]}(t)\big\|+\int_{t_0}^{t}\beta_i^d\big\|e_i^{[k]}(\tau)\big\|\,d\tau\\ &\quad+\sum_{j=1}^{N}\int_{t_0}^{t}\beta_{ij}\big\|x_j^{[k+1]}(\tau)-x_j^{[k]}(\tau)\big\|\,d\tau.\end{aligned} \tag{18}$$

By multiplying both sides of (18) by $\exp\{-\gamma(t-t_0)\}$ where γ is any positive constant, and by making use of some manipulations, we can obtain that for any $t\in[t_0,T]$,

$$\begin{aligned}&\big\|x_i^{[k+1]}(t)-x_i^{[k]}(t)\big\|\exp\big\{-\gamma(t-t_0)\big\}\\ &\quad\leqslant \beta_i^c\big\|e_i^{[k]}(t)\big\|\exp\big\{-\gamma(t-t_0)\big\}+\beta_i^d\int_{t_0}^{t}\exp\big\{-\gamma(t-\tau)\big\}\big\|e_i^{[k]}(\tau)\big\|\exp\big\{-\gamma(\tau-t_0)\big\}\,d\tau\\ &\qquad+\sum_{j=1}^{N}\beta_{ij}\int_{t_0}^{t}\exp\big\{-\gamma(t-\tau)\big\}\big\|x_j^{[k+1]}(\tau)-x_j^{[k]}(\tau)\big\|\exp\big\{-\gamma(\tau-t_0)\big\}\,d\tau.\end{aligned} \tag{19}$$

Letting

$$\begin{aligned}\tilde{e}_i^{[k]}(t) &:= \sup_{\rho\in[t_0,t]}\Big(\big\|e_i^{[k]}(\rho)\big\|\exp\big\{-\gamma(\rho-t_0)\big\}\Big),\\ \tilde{z}_i^{[k]}(t) &:= \sup_{\rho\in[t_0,t]}\Big(\big\|x_j^{[k+1]}(\rho)-x_j^{[k]}(\rho)\big\|\exp\big\{-\gamma(\rho-t_0)\big\}\Big),\end{aligned}$$

it follows from (19) that for any $i\in\{1,2,\ldots,N\}$ and any $t\in[t_0,T]$,

$$\begin{aligned}
&\big\|x_i^{[k+1]}(t) - x_i^{[k]}(t)\big\| \exp\big\{-\gamma(t-t_0)\big\} \\
&\quad\leqslant \beta_i^c \tilde{e}_i^{[k]}(t) + \beta_i^d \tilde{e}_i^{[k]}(t) \int_{t_0}^{t} \exp\big\{-\gamma(t-\tau)\big\}\, d\tau + \sum_{j=1}^{N} \beta_{ij} \tilde{z}_j^{[k]}(t) \int_{t_0}^{t} \exp\big\{-\gamma(t-\tau)\big\}\, d\tau \\
&\quad\leqslant \big(\beta_i^c + \beta_i^d/\gamma\big)\tilde{e}_i^{[k]}(t) + (1/\gamma) \sum_{j=1}^{N} \beta_{ij} \tilde{z}_j^{[k]}(t). \qquad (20)
\end{aligned}$$

First of all, notice such a fact that for any real function $f(t)$ and for any nondecreasing real function $g(t)$,

$$f(t) \leqslant g(t), \quad t \in [t_0, T],$$

implies

$$\tilde{f}(t) = \Big(\sup_{\rho\in[t_0,t]} f(\rho) \Big) \leqslant g(t), \quad t \in [t_0, T].$$

Now, it is obvious from the definitions that for any $i \in \{1, 2, \ldots, N\}$, the functions $\tilde{e}_i^{[k]}(t)$ and $\tilde{z}_i^{[k]}(t)$ are some nondecreasing functions on t. It follows that for any $i \in \{1, 2, \ldots, N\}$, the right-hand side of inequality (20) is also nondecreasing. Therefore, in the light of the definitions of the functions $\tilde{e}_i^{[k]}(t)$ and $\tilde{z}_i^{[k]}(t)$ and the fact stated above, we find from (20) that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\tilde{z}_i^{[k]}(t) \leqslant \big(\beta_i^c + \beta_i^d/\gamma\big)\tilde{e}_i^{[k]}(t) + (1/\gamma) \sum_{j=1}^{N} \beta_{ij} \tilde{z}_j^{[k]}(t). \qquad (21)$$

Moreover, if we define that for any $t \in [t_0, T]$,

$$\tilde{z}^{[k]}(t) = \max_i \big\{\tilde{z}_i^{[k]}(t);\ i = 1, 2, \ldots, N\big\},$$

then, from (21) we can further have that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\tilde{z}_i^{[k]}(t) \leqslant \big(\beta_i^c + \beta_i^d/\gamma\big)\tilde{e}_i^{[k]}(t) + \sum_{j=1}^{N} (\beta_{ij}/\gamma) \tilde{z}^{[k]}(t). \qquad (22)$$

Similarly, it is obvious from the definition of $\tilde{z}^{[k]}(t)$ that the right-hand side of inequality (22) is nondecreasing on time t. Therefore, from the fact stated above, we can obtain that for any $t \in [t_0, T]$,

$$\tilde{z}^{[k]}(t) \leqslant \big(\beta_i^c + \beta_i^d/\gamma\big)\tilde{e}_i^{[k]}(t) + \sum_{j=1}^{N} (\beta_{ij}/\gamma) \tilde{z}^{[k]}(t), \qquad (23)$$

where γ is any positive constant. Letting γ be chosen such that for any $i \in \{1, 2, \ldots, N\}$,

$$\gamma - \beta_i > 0,$$

where $\beta_i := \sum_{j=1}^{N} \beta_{ij}$. Then, we can find from (23) that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\tilde{z}^{[k]}(t) \leqslant \frac{\gamma\beta_i^c + \beta_i^d}{\gamma - \beta_i} \tilde{e}_i^{[k]}(t). \qquad (24)$$

On the other hand, in the light of the definition of the output error $e_i(t)$ and (16), we can have that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$e_i^{[k+1]}(t) = \big[I_i - C_i(t)B_i(t)\Gamma_i(t)\big]e_i^{[k]}(t) + \int_{t_0}^{t} C_i(t)\phi_i(t,\tau)e_i^{[k]}(\tau)\,d\tau - \sum_{j=1}^{N}\int_{t_0}^{t} C_i(t)\psi_{ij}(t,\tau)\big(x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\big)\,d\tau. \tag{25}$$

Taking the norm of both sides of (25) and making use of the general properties of norms, we can obtain that for any $t \in [t_0, T]$,

$$\big\|e_i^{[k+1]}(t)\big\| \leqslant \big\|I_i - C_i(t)B_i(t)\Gamma_i(t)\big\|\big\|e_i^{[k]}(t)\big\| + \delta_i^c\beta_i^d\int_{t_0}^{t}\big\|e_i^{[k]}(\tau)\big\|\,d\tau + \sum_{j=1}^{N}\delta_i^c\beta_{ij}\int_{t_0}^{t}\big\|x_j^{[k+1]}(\tau) - x_j^{[k]}(\tau)\big\|\,d\tau, \tag{26}$$

where

$$\delta_i^c = \sup_{t\in[t_0,T]}\big\|C_i(t)\big\|, \quad i = 1, 2, \ldots, N.$$

Similar to the method employed above, by multiplying both sides of inequality (26) by $\exp\{-\gamma(t - t_0)\}$ and by making use of some trivial manipulations, we can obtain from (26) that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\begin{aligned}
&\big\|e_i^{[k+1]}(t)\big\|\exp\big\{-\gamma(t-t_0)\big\}\\
&\quad\leqslant \big\|I_i - C_i(t)B_i(t)\Gamma_i(t)\big\|\tilde{e}_i^{[k]}(t) + \big(\delta_i^c\beta_i^d/\gamma\big)\tilde{e}_i^{[k]}(t) + \sum_{j=1}^{N}\big(\delta_i^c\beta_{ij}/\gamma\big)\tilde{z}_j^{[k]}(t)\\
&\quad\leqslant \big(\big\|I_i - C_i(t)B_i(t)\Gamma_i(t)\big\| + \delta_i^c\beta_i^d/\gamma\big)\tilde{e}_i^{[k]}(t) + \sum_{j=1}^{N}\big(\delta_i^c\beta_{ij}/\gamma\big)\tilde{z}^{[k]}(t).
\end{aligned} \tag{27}$$

Furthermore, substituting (24) into (27) yields

$$\big\|e_i^{[k+1]}(t)\big\|\exp\big\{-\gamma(t-t_0)\big\} \leqslant \left[\rho_i(t) + \frac{1}{\gamma}\delta_i^c\beta_i^d\left(1 + \frac{\beta_i}{\gamma-\beta_i}\right) + \frac{\delta_i^c\beta_i^c\beta_i}{\gamma-\beta_i}\right]\tilde{e}_i^{[k]}(t). \tag{28}$$

It is obvious from the definitions of $\tilde{e}_i^{[k]}(t)$ and $\rho_i(t)$ that the right-hand side of inequality (28) is nondecreasing on t. Therefore, we find from (28) that for any $i \in \{1, 2, \ldots, N\}$ and any $t \in [t_0, T]$,

$$\tilde{e}_i^{[k+1]}(t) \leqslant \left[\rho_i(t) + \frac{1}{\gamma}\delta_i^c\beta_i^d\left(1 + \frac{\beta_i}{\gamma-\beta_i}\right) + \frac{\delta_i^c\beta_i^c\beta_i}{\gamma-\beta_i}\right]\tilde{e}_i^{[k]}(t).$$

That is,

$$\tilde{e}_i^{[k+1]}(t) \leqslant \eta_i(t)\tilde{e}_i^{[k]}(t), \quad i \in \{1, 2, \ldots, N\}, \tag{29}$$

where

$$\eta_i(t) = \rho_i(t) + \rho_i(\gamma), \quad i \in \{1, 2, \ldots, N\}, \tag{30}$$

and where $\rho_i(t)$ has been defined in (10) and $\rho_i(\gamma)$ is given by the following equation:

$$\rho_i(\gamma) = \frac{1}{\gamma}\delta_i^c \beta_i^d \left(1 + \frac{\beta_i}{\gamma - \beta_i}\right) + \frac{\delta_i^c \beta_i^c \beta_i}{\gamma - \beta_i}. \tag{31}$$

If the condition described by (9) and (10) is satisfied, it is obvious from (31) that there exists a positive constant γ^* such that for any $\gamma \geqslant \gamma^*$ and any $t \in [t_0, T]$, $\eta_i(t) < 1$. Therefore, we can obtain from (29) that for any $t \in [t_0, T]$,

$$\lim_{k\to\infty} e_i^{[k]}(t) = 0, \quad i \in \{1, 2, \ldots, N\}.$$

Thus, we can complete the proof of this theorem. □

Remark 3.1. It is obvious from (10) and Assumption 2.2 that one can always choose a decentralized learning gain matrix for each subsystem such that the condition given in (9) is satisfied. Therefore, under Assumption 2.2, the existence of decentralized local iterative learning control laws is well guaranteed.

Remark 3.2. In this paper, we have employed a class of D-type iterative learning control laws described by (6) with (7), which may regarded as the first-order updating laws. It is not difficult, from the method used in the proof of Theorem 3.1, that the result obtained here is extended to the problem of decentralized iterative learning control with high-order updating laws for large scale interconnected systems. For instance, the iterative learning control law described by (6) may be extended to more high-order case as follows.

$$u_i^{[k+1]}(t) = \sum_{j=1}^{\mu} \left\{ u_i^{[k-j+1]}(t) + \Gamma_i^{[k-j+1]}(t) \dot{e}_i^{[k-j+1]}(t) \right\}, \quad i \in \{1, 2, \ldots, N\},$$

where μ is the order of the updating law.

Remark 3.3. Here, the problem of decentralized iterative learning control is considered for a class of linear time-varying large scale dynamical systems. However, the method proposed in this paper can be applied to a class of nonlinear large scale dynamical systems under some conditions (e.g., Lipschitz condition) to construct some types of decentralized local iterative learning control laws.

Remark 3.4. It is well known that the λ-norm has been employed to develop some types of iterative learning control laws for composite systems (see, e.g., [11–13], and references therein). In general, for any real function $h(t)$ given on $[t_0, T]$, its λ-norm is defined by

$$\left\| h(t) \right\|_\lambda = \sup_{t\in[t_0,T]} \left(\mathrm{e}^{-\lambda t} \left\| h(t) \right\| \right).$$

Note that the functions $\tilde{e}_i^{[k]}(t)$ and $\tilde{z}_i^{[k]}(t)$, introduced in the proof of Theorem 3.1, are different from the λ-norm. In fact, by employing the same function $h(t)$, the new function introduced in this paper can be described by

$$\tilde{h}(t) = \sup_{\rho\in[t_0,t]} \left(\mathrm{e}^{-\gamma(\rho - t_0)} \left\| h(\rho) \right\| \right),$$

where $t \in [t_0, T]$ and γ is any positive function. It is obvious from the definitions of $\|h(t)\|_\lambda$ and $\tilde{h}(t)$ that the λ-norm, i.e., $\|h(t)\|_\lambda$, is a positive constant, but the function $\tilde{h}(t)$ is a nondecreasing function on the time t. It is by using such a property of the nondecreasing function that we can deal with successfully the problem of decentralized iterative learning control for large scale interconnected systems.

4. An illustrative example

In this section, in order to illustrate the application of the iterative learning control laws presented in the preceding section, we consider the two identical pendulums which are coupled by a spring and subject to two distinct inputs [21,22] as shown in Fig. 1. We choose the state vectors as

$$x_1(t) = \begin{bmatrix} \theta_1(t) & \dot{\theta}_1(t) \end{bmatrix}^\top, \qquad x_2(t) = \begin{bmatrix} \theta_2(t) & \dot{\theta}_2(t) \end{bmatrix}^\top.$$

Then, the systems can be described by

$$\frac{dx_1(t)}{dt} = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u_1(t) + \begin{bmatrix} 0 & 0 \\ -\frac{ka^2}{ml^2} & 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{ml^2} & 0 \end{bmatrix} x_2(t),$$

$$y_1(t) = [\,1 \quad 1\,] x_1(t), \tag{32a}$$

$$\frac{dx_2(t)}{dt} = \begin{bmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u_2(t) + \begin{bmatrix} 0 & 0 \\ \frac{ka^2}{ml^2} & 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 & 0 \\ -\frac{ka^2}{ml^2} & 0 \end{bmatrix} x_2(t),$$

$$y_2(t) = [\,1 \quad 1\,] x_2(t), \tag{32b}$$

where k and g are spring and gravity constants. As pointed out in [21,22], since the position a of the spring can change along the full length l of the pendulums, the uncertainties in the interconnections are represented by making $a(t)$ an unknown function of time. Here, it is obvious that $a(t)/l \in [0, 1]$.

For simulation, we give the following parameters:

$$\frac{g}{l} = 1.0, \qquad \frac{1}{ml^2} = 1.0, \qquad \frac{k}{m} = 2.0, \qquad \frac{a}{l} = 0.5.$$

For the decentralized local iterative learning control laws given in (6) and (7), in the light of the condition described by (9) with (10), we select the learning gains as follows:

$$\Gamma_1 = 0.8, \qquad \Gamma_2 = 0.9.$$

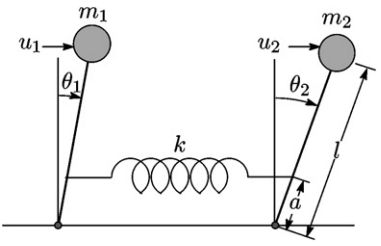


Fig. 1. The coupled inverted pendulums.

In additions, for iterative schemes (6) and (7), we give in this simulation the following initial conditions:

$$u_1^{[0]}(t) = 1.0, \qquad x_{11}^{[0]}(0) = 0.0, \qquad x_{12}^{[0]}(0) = 1.0,$$
$$u_2^{[0]}(t) = 1.0, \qquad x_{21}^{[0]}(0) = 0.0, \qquad x_{22}^{[0]}(0) = 2.0.$$

For system (32), the desired local output trajectories $y_i^m(t)$, $i = 1, 2$, are given as

$$y_1^m(t) = \sin(0.1t), \qquad y_2^m(t) = \sin(0.2t),$$

where $t \in [0, T]$ and $T = 1.0$.

Here, let the final local tracking errors be defined as

$$\hat{e}_i(k) = \sup_{t\in[0,T]} \left|e_i^{[k]}(t)\right|, \quad \hat{x}_i(k) = \left\|x_i^{[k]}(0)\right\|, \quad i = 1, 2.$$

Then, the results of a simulation are shown in Figs. 2 and 3 for this coupled identical pendulum system with the chosen parameter settings.

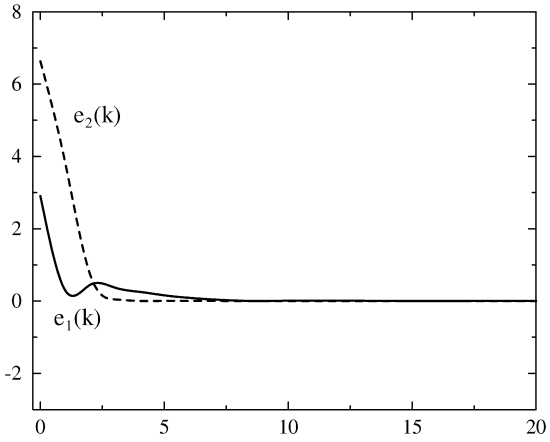


Fig. 2. The tracking error bounds $\hat{e}_i(k)$, $i = 1, 2$.

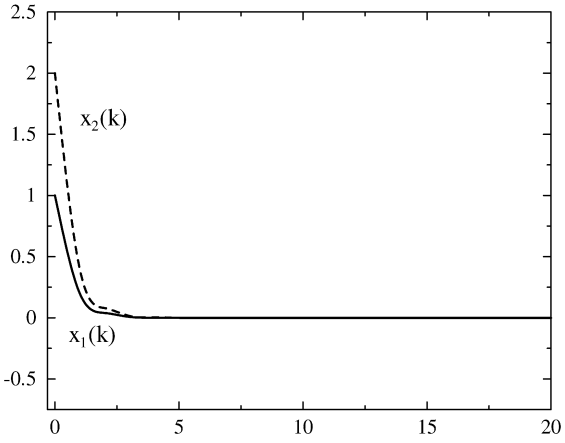


Fig. 3. The learning processes of initial state $\hat{x}_i(k)$, $i = 1, 2$.

It can be observed from Fig. 2 that by using the proposed decentralized local iterative learning control laws, we can guarantee the asymptotic convergence of the local output error between the given desired local output and the actual local output for all $t \in [t_0, T]$. In particular, from Fig. 3 we can also know that the decentralized local initial state learning schemes are effective. That is, the initial state for each subsystem tracks finally the desired one.

5. Concluding remarks

The problem of decentralized iterative learning control for a class of large scale interconnected systems has been discussed. Here, the considered large scale systems have been assumed to be linear time-varying, and the interconnections between each subsystem to be unknown. A method has been presented whereby a class of decentralized local iterative learning control schemes is constructed. It has also been shown that under some conditions, the constructed decentralized local iterative learning controllers can guarantee the asymptotic convergence of the local output error between the given desired local output and the actual local output of each subsystem through the iterative learning process. The method proposed in this paper can be extended to nonlinear large scale interconnected systems to construct some decentralized local iterative learning control schemes.

Finally, a numerical example is given to demonstrate the synthesis procedure for the proposed decentralized local iterative learning control schemes. It is shown from the example and the results of its simulation that the results obtained in the paper are effective and feasible. Therefore, our results may be expected to have some applications to practical decentralized iterative learning control problems of large scale interconnected systems.

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