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Large deviations for Markov bridges with jumps

Xiangfeng Yang

Department of Mathematics, Linköping University, SE-581 83 Linköping, Sweden

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ABSTRACT

In this paper, we consider a family of Markov bridges with jumps constructed from truncated stable processes. These Markov bridges depend on a small parameter $\hbar > 0$, and have fixed initial and terminal positions. We propose a new method to prove a large deviation principle for this family of bridges based on compact level sets, change of measures, duality and various global and local estimates of transition densities for truncated stable processes.

1. Markov bridges with jumps

In this section, let us recall the construction of one dimensional Markov bridges with jumps defined through Lévy processes in the sense of [6]. We first specify the family of Lévy processes considered throughout this paper.

1.1. Truncated stable processes

Suppose that $j(u) \in C_0^\infty(\mathbb{R})$ is a deterministic and symmetric function such that $0 \leqslant j(u) \leqslant 1$, $\operatorname{supp}(j) = \{u\colon\ |u| \leqslant c\}$ and $j(u) \equiv 1$ in $\{u\colon\ |u| \leqslant c/2\}$ for some $0 < c < +\infty$. On some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, we consider a family of one dimensional symmetric Lévy processes $\{\xi_t^\hbar\}_{t \geqslant 0}$ depending on a small parameter $\hbar > 0$ whose generator is

$$L^\hbar \varphi(x) = \frac{1}{\hbar} \int_{\mathbb{R}} \big[\varphi(x + \hbar u) - \varphi(x) - \hbar u \varphi'(x)\big] g(u)\, du$$

for $\varphi \in C_0^\infty(\mathbb{R})$ and each fixed $\hbar$, where $g(u) = j(u)|u|^{-1-\alpha}$ with $\alpha \in (1, 2)$. The processes $\{\xi_t^\hbar\}_{t \geqslant 0}$ are called *truncated stable processes* (see [8] and [10]). For each $\hbar > 0$, it follows from the Lévy–Khintchine formula that the characteristic function of $\xi_t^\hbar$ is

E-mail address: xiangfeng.yang@liu.se.

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$$\mathbb{E}\big[e^{-i\cdot x\cdot \xi_t^\hbar/\hbar}\big] = e^{-tV(x)/\hbar}, \quad x\in\mathbb{R} \text{ and } t\geqslant 0, \tag{1.1}$$

with $V(x) = -\int_{\mathbb{R}}[e^{-i\cdot x\cdot u} - 1 + i\cdot x\cdot u]g(u)\,du$.

We can realize $\{\xi_t^\hbar\}_{t\geqslant 0}$ as the coordinate process on the sample space $D[0,\infty)$ consisting of all functions from $[0,\infty)$ to $\mathbb{R}$ which are right continuous with left limits. In this setting, we write the coordinate process as $\xi_t(\omega) = \omega(t)$, and the law of $\{\xi_t^\hbar\}_{t\geqslant 0}$ on $D[0,\infty)$ as $\mathbb{P}^\hbar$. More precisely, for any measurable $A\subseteq D[0,\infty)$,

$$\mathbb{P}^\hbar\{\xi\in A\} = \mathbb{P}\big\{\xi^\hbar\in A\big\}. \tag{1.2}$$

Throughout this paper, we do not distinguish the coordinate process $\{\xi_t\}_{t\geqslant 0}$ under $\mathbb{P}^\hbar$ and the process $\{\xi_t^\hbar\}_{t\geqslant 0}$ under $\mathbb{P}$ since they have the same law on $D[0,\infty)$.

Under $\mathbb{P}^\hbar$, the law of the coordinate process $\{\xi_t\}_{t\geqslant 0}$ when started at $a\in\mathbb{R}$ is denoted as $\mathbb{P}_a^\hbar$, namely,

$$\mathbb{P}_a^\hbar\{\xi\in A\} = \mathbb{P}^\hbar\{\xi\in A \mid \xi_0 = a\} = \mathbb{P}\big\{\xi^\hbar\in A \bigm| \xi_0^\hbar = a\big\} =: \mathbb{P}_a\big\{\xi^\hbar\in A\big\}. \tag{1.3}$$

Let $\{\mathcal{F}_t\}_{t\geqslant 0}$ be the natural filtration of the coordinate process $\{\xi_t\}_{t\geqslant 0}$. The truncated stable processes $\{\xi_t^\hbar\}_{t\geqslant 0}$ under $\mathbb{P}$ have transition semigroups $\{P_t^\hbar\}_{t\geqslant 0}$ and transition densities $p_t(x,y,\hbar) > 0$ with respect to the Lebesgue measure dx (cf. [10]). In this case, the processes $\{\xi_t^\hbar\}_{t\geqslant 0}$ do not have jumps at fixed times: $\mathbb{P}_a\{\xi_t^\hbar = \xi_{t-}^\hbar\} = 1$ for any $t>0$; see [6] and the references therein.

1.2. Markov bridges

Based on the truncated stable processes $\{\xi_t^\hbar\}_{t\geqslant 0}$ under $\mathbb{P}$ (or equivalently $\{\xi_t\}_{t\geqslant 0}$ under $\mathbb{P}^\hbar$), a family of Markov bridge laws on $D[0,\infty)$ can be constructed in the sense of [6]. Fixed $a,b\in\mathbb{R}$, it is easy to check that $p_{1-t}(\xi_t^\hbar, b, \hbar)$ is a positive martingale under $\mathbb{P}_a$. Thus

$$\mathbb{Q}_{a,b}^\hbar(A) := \int_A p_{1-t}\big(\xi_t^\hbar, b, \hbar\big)\,d\mathbb{P}_a, \quad A\in\mathcal{F}_t,\ 0\leqslant t<1, \tag{1.4}$$

defines a family of finitely additive set functions on the algebra $\mathcal{G} = \bigcup_{0\leqslant t<1}\mathcal{F}_t$ such that each restriction $\mathbb{Q}_{a,b}^\hbar|_{\mathcal{F}_t}$ is σ-additive. It has been proved (cf. Proposition 1 in [6]) that $\mathbb{Q}_{a,b}^\hbar/p_1(a,b,\hbar)$ extends to a probability measure on $\mathcal{F}_{1-}$ which is the σ-algebra generated by $\mathcal{G}$. We use $\mathbb{P}_{a,b}^\hbar$ to denote such a law $\mathbb{Q}_{a,b}^\hbar/p_1(a,b,\hbar)$.

On the space $(D[0,\infty), \mathcal{F}_{1-}, \mathbb{P}_{a,b}^\hbar)$, the coordinate process $\{\xi_t\}_{0\leqslant t<1}$ restricted on $0\leqslant t<1$ is a non-homogeneous strong Markov process with transition densities

$$p^{b,1}\big(z,s;z',t;\hbar\big) = \frac{p_{t-s}(z,z',\hbar)p_{1-t}(z',b,\hbar)}{p_{1-s}(z,b,\hbar)}, \quad 0<s<t<1,\ z,z'\in\mathbb{R}.$$

The coordinate process $\{\xi_t\}_{0\leqslant t<1}$ now starts from a and ends with b at $t=1^-$, namely

$$\mathbb{P}_{a,b}^\hbar\{\xi_0 = a, \xi_{1-} = b\} = 1. \tag{1.5}$$

Furthermore, $\{\mathbb{P}_{a,b}^\hbar\}_{b\in\mathbb{R}}$ is a regular version of the family of conditional probability distributions $\mathbb{P}_a^\hbar\{\cdot \mid \xi_{1-} = b\} = \mathbb{P}^\hbar\{\cdot \mid \xi_0 = a, \xi_{1-} = b\}$, $b\in\mathbb{R}$. Equivalently, in the spirit of (1.3), it is a regular version of $\mathbb{P}_a\{\cdot \mid \xi_{1-}^\hbar = b\} = \mathbb{P}\{\cdot \mid \xi_0^\hbar = a, \xi_{1-}^\hbar = b\}$, $b\in\mathbb{R}$.

The aim of this paper is to study large deviations of the family of laws $\{\mathbb{P}_{a,b}^\hbar\}_{\hbar>0}$ as $\hbar\to 0$ for two fixed numbers $a,b\in\mathbb{R}$. Previously, large deviations were obtained for various families of bridge processes having continuous trajectories, such as Brownian bridges and diffusion bridges. Section 2 contains a more detailed summary. In this paper, a new method is proposed to study large deviations of bridges, whose main ingredient is to consider compact level sets instead of closed sets. Because of the appearance of jumps, several technical difficulties arise in the study of large deviations of these bridges (see Section 2 for several specific difficulties). This paper is the first one to deal with large deviations of bridges with jumps. The main result of this paper is formulated in Section 2, whose proof is included in Section 3.

2. The main result

Since the laws $\{\mathbb{P}^{\hbar}_{a,b}\}_{\hbar>0}$ are on $(D[0,\infty),\mathcal{F}_{1-})$, it is natural to consider the coordinate process $\{\xi_t\}_{0\leqslant t<1}$ restricted on $0\leqslant t<1$, and the sample space restricted on $0\leqslant t<1$. More specifically, the following space is considered:

$$D^{a,b}[0,1)=\bigl\{\phi\colon[0,1)\to\mathbb{R},\ \text{right continuous with left limits},\ \phi(0)=a \text{ and } \phi\bigl(1^-\bigr)=b\bigr\}.$$

This space is equipped with the uniform topology $\|\cdot\|=\sup_{0\leqslant t<1}|\cdot|$. We actually focus on the laws $\{\mathbb{P}^{\hbar}_{a,b}\}_{\hbar>0}$ restricted on the space $(D^{a,b}[0,1),\mathcal{B})$ where $\mathcal{B}$ is the Borel σ-algebra generated by open sets in $D^{a,b}[0,1)$. For any set $A\in\mathcal{B}$, it is obvious that $A\in\mathcal{F}_{1-}$ and we understand $\mathbb{P}^{\hbar}_{a,b}\{\xi\in A\}$ as $\mathbb{P}^{\hbar}_{a,b}\{A\}$.

In order to formulate large deviations of the laws $\{\mathbb{P}^{\hbar}_{a,b}\}_{\hbar>0}$, we define a rate function

$$S^{a,b}(\phi)=\int_0^1 L\bigl(\phi'(t)\bigr)\,dt-d(a,b) \tag{2.1}$$

for absolutely continuous ϕ (otherwise $=\infty$), where $L(x)$ is the Legendre transformation of $H(y)=\int_{\mathbb{R}}(e^{yu}-1-yu)g(u)\,du$, namely,

$$L(x)=\sup_{y\in\mathbb{R}}\bigl[xy-H(y)\bigr], \tag{2.2}$$

and $d(a,b)$ represents the distance between a and b which is defined as

$$d(a,b)=\inf\left\{\int_0^1 L\bigl(\phi'(t)\bigr)\,dt\colon\ \phi(0)=a \text{ and } \phi\bigl(1^-\bigr)=b\right\}.$$

This $d(a,b)$ was introduced in [10] in the study of estimates of transition densities for jump processes. We now formulate a large deviation principle of $\{\mathbb{P}^{\hbar}_{a,b}\}_{\hbar>0}$ as the following theorem.

Theorem 2.1. *$S^{a,b}$ is a good rate function, namely, it is lower semi-continuous and the level set $\{\phi\in D^{a,b}[0,1)\colon\ S^{a,b}(\phi)\leqslant s\}$ is compact for each $s>0$. Furthermore,*

(1) *for any Borel measurable and open set $O\subseteq D^{a,b}[0,1)$,*

$$\liminf_{\hbar\to0^+}\hbar\log\mathbb{P}^{\hbar}_{a,b}\{O\}\geqslant-\inf_{\phi\in O}S^{a,b}(\phi); \tag{2.3}$$

(2) *for any Borel measurable and closed set $F\subseteq D^{a,b}[0,1)$,*

$$\limsup_{\hbar\to0^+}\hbar\log\mathbb{P}^{\hbar}_{a,b}\{F\}\leqslant-\inf_{\phi\in F}S^{a,b}(\phi). \tag{2.4}$$

Remark 2.1. It follows from Theorem 2.1 that the most probable trajectory of the coordinate process $\{\xi_t\}_{0\leqslant t<1}$ under $\mathbb{P}^{\hbar}_{a,b}$ as $\hbar\to 0$ is ϕ_0 such that $S^{a,b}(\phi_0)=0$. Since $H(y)$ is convex and the Legendre transformation preserves convexity, the function $L(x)$ is also convex. Thus there is at most one such trajectory. What is more, the existence of such ϕ_0 comes from the lower semi-continuity of $S^{a,b}(\phi)$. The problem of searching the most probable trajectory is related to the principle of least action in a mechanical system; see Section 1.2 in [12]. It also has connections with Monge's problem as well if the initial and terminal probability distributions are more regular instead of Dirac measures; see [13,17] and the references therein.

Remark 2.2. For families of bridges without jumps, there are known results regarding the large deviation principles. In [9], a large deviation principle for a family of Brownian bridges on a Riemannian manifold was derived based on a Girsanov transformation involving minimal heat kernels on the manifold. Large deviations for Brownian bridges in Hölder norm were presented in [1] by using arguments of abstract Wiener spaces. For time-homogeneous diffusion bridges constructed in [15] and [16], large deviations were obtained as well relying on techniques of the analysis of arbitrarily small partitions of $[0,1]$. All of these approaches are to prove the lower bound (2.3) and the upper bound (2.4) directly which results in a number of technical difficulties (for instance, see the elaborate and technical proof of Lemma 2.4 in [9]). The method used in this paper is to prove these bounds indirectly (especially for the upper bound) based on suitable equivalent forms of large deviations (cf. Lemma 3.1). This seems to be more intuitive and promising. For this reason, in [19] large deviations of a family of time-inhomogeneous diffusion bridges and Bernstein bridges are studied in terms of this method.

Remark 2.3. Because of the appearance of jumps of the coordinate process $\{\xi_t\}_{0\leqslant t<1}$ under $\mathbb{P}^{\hbar}_{a,b}$, several technical difficulties arise in deriving large deviation principles. For instance, we no longer have an explicit form of $L(x)$ in the rate function $S^{a,b}(\phi)$. The global and local estimates of the transition densities $p_t(x,y,\hbar)$ are also more complicated than the ones associated with diffusion processes. We overcome these technical difficulties by large deviations for the family of the original truncated stable processes $\{\xi^{\hbar}_t\}_{t\geqslant 0}$ under $\mathbb{P}_a$, and several results regarding the estimates of $p_t(x,y,\hbar)$ in [10]. If the original Lévy processes $\{\xi^{\hbar}_t\}_{t\geqslant 0}$ have diffusion and drift components, then the corresponding bridges can be also constructed (in the sense of [6] or [14]) and similar large deviations are also expected. Although large deviations for these more general Lévy processes $\{\xi^{\hbar}_t\}_{t\geqslant 0}$ can be explicitly formulated, the large deviations for the corresponding bridges cannot be similarly obtained as in this paper due to the lack of appropriate estimates of the transition densities $p_t(x,y,\hbar)$.

Remark 2.4. Another different construction of bridges $\{\zeta^{\hbar}_t\}_{0\leqslant t\leqslant 1}$ based on the truncated stable processes $\{\xi^{\hbar}_t\}_{0\leqslant t\leqslant 1}$ is discussed in [14] in the framework of quantum mechanics. There, $\hbar>0$ is the Planck constant. This fact is the reason that we chose $\hbar$ as our parameter throughout the paper. The bridges $\{\zeta^{\hbar}_t\}_{0\leqslant t\leqslant 1}$ have fix initial (π_0) and terminal (π_1) distributions, and solve in the weak sense the stochastic integro-differential equations

$$d\zeta^{\hbar}_t=\int_{\mathbb{R}} u\left(\mu^{\hbar}(du,dt)-\frac{\eta^{\hbar}(t,\zeta^{\hbar}_{t-}+u)}{\eta^{\hbar}(t,\zeta^{\hbar}_{t-})}g(u)\,du\,dt\right)+\int_{\mathbb{R}} u\frac{\eta^{\hbar}(t,\zeta^{\hbar}_{t-}+u)-\eta^{\hbar}(t,\zeta^{\hbar}_{t-})}{\eta^{\hbar}(t,\zeta^{\hbar}_{t-})}g(u)\,du\,dt$$

where $\eta^{\hbar}(t,x)$ is the solution to $\frac{\partial\eta^{\hbar}}{\partial t}(t,x)=-L^{\hbar}\eta^{\hbar}(t,x)$ for $0\leqslant t<1$ subject to a terminal condition $\eta^{\hbar}(1,x)=\eta(x)$, and $\eta(x)$ is determined by the measures π_0 and π_1 in some sense. If $\eta^{*,\hbar}(t,x)$ is the solution to $\frac{\partial\eta^{*,\hbar}}{\partial t}(t,x)=L^{\hbar}\eta^{*,\hbar}(t,x)$ for $0<t\leqslant 1$ and $\eta^{*,\hbar}(0,x)=\eta^{*}(x)$ (which is also determined by π_0 and π_1), then the law of $\zeta^{\hbar}_t$ was proved to take the form $\eta^{\hbar}(t,x)\eta^{*,\hbar}(t,x)\,dx$. An advantage of this construction is that the bridges can be built on some Lévy processes which may not be absolutely continuous with respect to the Lebesgue measure, and the resulting bridges have the nice *Bernstein property* (see [2,11,5]). But the

sacrifice is that the marginal distributions π_0 and π_1 should be more regular than Dirac measures, which is the reason that we did not employ this construction in this paper.

3. Proof of Theorem 2.1

The first step of the proof is to shift large deviations from $\{\mathbb{P}^\hbar_{a,b}\}_{\hbar>0}$ to the original truncated stable processes $\{\xi^\hbar_t\}_{0\leqslant t<1}$ under $\mathbb{P}$ with the help of (1.4). To this end, we rewrite (1.4) in a more concrete way as, for any $A\in\mathcal{F}_t$ with $0\leqslant t<1$,

$$\mathbb{P}^\hbar_{a,b}\{A\} = \frac{1}{p_1(a,b,\hbar)}\int_A p_{1-t}\big(\xi^\hbar_t, b, \hbar\big)\,d\mathbb{P}_a = \frac{1}{p_1(a,b,\hbar)}\int_A p_{1-t}(\xi_t, b, \hbar)\,d\mathbb{P}^\hbar_a \tag{3.1}$$

where the second identity is from (1.3). Because of the singularity of $p_{1-t}(x,y,\hbar)$ at $t=1$, we thus break the time interval $[0,1)$ into two parts $[0,t]$ and $[t,1)$, and then send $t\to 1^-$.

Let $D^a[0,t]$ be the space of all right continuous functions on $[0,t]$ with left limits such that the value at 0 is a. This space is endowed with the uniform topology. The distance between a point $\phi\in D^a[0,t]$ and a set $E\subseteq D^a[0,t]$ is denoted as $\operatorname{dist}(\phi,E) := \inf_{\phi'\in E}\sup_{0\leqslant s\leqslant t}|\phi'(s)-\phi(s)|$. We first verify that the family of the original truncated stable processes $\{\xi^\hbar_s\}_{0\leqslant s\leqslant t}$ for any fixed $t>0$ satisfies a large deviation principle.

Lemma 3.1. *On $D^a[0,t]$, the family of truncated stable processes $\{\xi^\hbar_s\}_{0\leqslant s\leqslant t}$ defined in Section 1.1 under $\mathbb{P}_a$ satisfies a large deviation principle with a rate function*

$$S^t(\phi) = \int_0^t L\big(\phi'(s)\big)\,ds$$

for absolutely continuous ϕ (otherwise $=\infty$), where L is defined in (2.2). Namely,

(i) *For any Borel measurable and open set $O\subseteq D^a[0,t]$,*

$$\liminf_{\hbar\to 0^+}\hbar\log\mathbb{P}_a\big\{\xi^\hbar\in O\big\}\geqslant -\inf_{\phi\in O}S^t(\phi);$$

(ii) *For any Borel measurable and closed set $F\subseteq D^a[0,t]$,*

$$\limsup_{\hbar\to 0^+}\hbar\log\mathbb{P}_a\big\{\xi^\hbar\in F\big\}\leqslant -\inf_{\phi\in F}S^t(\phi).$$

An equivalent formulation to (i) *and* (ii) *is as follows:*

(I) *For any $\delta>0$, $\gamma>0$ and $s_0>0$, there exists $\hbar_0>0$ such that*

$$\mathbb{P}_a\Big\{\sup_{0\leqslant s\leqslant t}\big|\xi^\hbar_s-\phi(s)\big|<\delta\Big\}\geqslant\exp\big\{-\hbar^{-1}\big[S^t(\phi)+\gamma\big]\big\}$$

for any $\hbar<\hbar_0$ and any $\phi\in\Phi^t(s_0)$ which is defined as

$$\Phi^t(s) = \big\{\phi\in D^a[0,t]:\ S^t(\phi)\leqslant s\big\},\quad \text{for } s>0.$$

(II) *For any $\delta>0$, $\gamma>0$ and $s_0>0$, there exists $\hbar_0>0$ such that*

$$\mathbb{P}_a\big\{\operatorname{dist}\big(\xi^\hbar,\Phi^t(s)\big)\geqslant\delta\big\}\leqslant\exp\big\{-\hbar^{-1}(s-\gamma)\big\}$$

for any $\hbar<\hbar_0$ and any $s\leqslant s_0$.

Proof. The equivalence between (i) + (ii) and (I) + (II) is from Section 3.3 in [7]. We consider the measure $\mu(du) := u^2 g(u)\,du$ where $g(u)$ is defined in Section 1.1. This measure is bounded $\mu(\mathbb{R}) < \infty$, and has a strict support $[-c, c]$. Thus Theorem 4.1.1 in [18] tells that the family $\{\xi_s^\hbar\}_{0\leqslant s\leqslant t}$ satisfies this large deviation principle. It also follows from the same result that $\Phi^t(s)$ is a compact set for each $s > 0$. □

For the proof of Theorem 2.1, we need one more property of the rate function $S^{a,b}$.

Lemma 3.2. *The rate function $S^{a,b}$ defined by (2.1) is lower semi-continuous and the level set $\Phi^{a,b}(s) = \{\phi \in D^{a,b}[0,1)\colon\ S^{a,b}(\phi) \leqslant s\}$ is compact for each $s > 0$.*

Proof. The idea of the proof is to change our consideration from $[0, 1)$ to $[0, 1]$ and then apply several known results on $[0, 1]$ from [18].

Step 1. We first prove that the level set $\Phi^{a,b}(s)$ is uniformly equicontinuous, that is, for any $\epsilon > 0$, there is $\delta(\epsilon) > 0$ such that

$$\bigl|\phi(x) - \phi(y)\bigr| \leqslant \delta \quad \text{whenever } |x - y| < \epsilon,\ \text{for all } \phi \in \Phi^{a,b}(s).$$

To this end, let us verify a fact that $L(x)$ grows to infinity faster than a linear speed. Note that an upper estimate of $H(y)$ for positive large enough y is $H(y) \leqslant \frac{c'}{y} e^{cy}$ with a constant c' depending on c which is in the definition of truncated stable processes in Section 1. Thus in the definition of $L(x)$, we simply take $y = \frac{1}{2c}\log x$ for large enough x to have $L(x) \geqslant x \cdot \frac{1}{2c}\log x - \frac{c'\cdot 2c}{\log x} \cdot x^{1/2}$.

Now from the convexity of L and Jensen's inequality, it follows that, for any $\phi \in \Phi^{a,b}(s)$, $x, y \in [0, 1)$ (without loss of generality, assuming $y > x$),

$$d(a,b) + s \geqslant \int_0^1 L\bigl(\phi'(t)\bigr)\,dt \geqslant \int_x^y L\bigl(\phi'(t)\bigr)\,dt \geqslant L\left(\frac{1}{y-x}\int_x^y \phi'(t)\,dt\right) = L\left(\frac{\phi(y) - \phi(x)}{y - x}\right).$$

Since $L(x)$ grows to infinity faster than a linear speed, the ratio $\frac{|\phi(y)-\phi(x)|}{|y-x|}$ has to be bounded uniformly for ϕ, x and y. Namely, with some $M > 0$,

$$\frac{|\phi(y) - \phi(x)|}{|y - x|} \leqslant M, \quad \text{for all } \phi \in \Phi^{a,b}(s).$$

Step 2. This step is to prove that $S^{a,b}$ is lower semi-continuous. The level set $\Phi^{a,b}(s)$ is uniformly equicontinuous, therefore it can be continuously extended to $[0, 1]$, and the extended level set $\widetilde{\Phi}^{a,b}(s)$ is still uniformly equicontinuous. The rate function $S^{a,b}(\widetilde{\phi})$ remains the same as $S^{a,b}(\phi)$. It has been proved in [18] (cf. Theorems 3.1.1 and 4.1.1 therein) that on $[0, 1]$ the function $S^{a,b}$ is lower semi-continuous, so is on $[0, 1)$. This also implies that $\widetilde{\Phi}^{a,b}(s)$ is compact.

Step 3. It follows from *Step 2* that $\Phi^{a,b}(s)$ is a closed set. Furthermore, for any sequence $\{\phi_n\} \subseteq \Phi^{a,b}(s)$, the extended sequence $\{\widetilde{\phi}_n\} \subseteq \widetilde{\Phi}^{a,b}(s)$ has a convergent subsequence because of the compactness of $\widetilde{\Phi}^{a,b}(s)$, thus $\{\phi_n\}$ also has a convergent subsequence. This implies that $\Phi^{a,b}(s)$ is compact. □

With the help of Lemmas 3.1 and 3.2, we are now ready to prove Theorem 2.1.

3.1. Proof of the lower bound (2.3)

For any Borel measurable and open set $O \subseteq D^{a,b}[0, 1)$ and a point $\phi_*(\cdot) \in O$ with $S^{a,b}(\phi_*) < \infty$ and $Ball_\delta(\phi_*) \subseteq O$ for some $\delta > 0$, we define $(0 \leqslant t < 1)$

$$O_t = \Big\{\phi \in D^{a,b}[0,1)\colon \sup_{0\leqslant s\leqslant t}\big|\phi(s)-\phi_*(s)\big| < \delta\Big\};$$
$$O^t = \Big\{\phi \in D^{a,b}[0,1)\colon \sup_{t\leqslant s< 1}\big|\phi(s)-\phi_*(s)\big| \geqslant \delta\Big\}.$$

It is then clear that $O_t \subseteq O \cup O^t$ and

$$\mathbb{P}^{\hbar}_{a,b}\{O\} + \mathbb{P}^{\hbar}_{a,b}\{O^t\} \geqslant \mathbb{P}^{\hbar}_{a,b}\{O_t\}. \tag{3.2}$$

On the set O_t, it is from (1.5) that

$$\begin{aligned}\liminf_{\hbar\to 0^+}\hbar\log\mathbb{P}^{\hbar}_{a,b}\{O_t\} &= \liminf_{\hbar\to 0^+}\hbar\log\mathbb{P}^{\hbar}_{a,b}\Big\{\xi_0=a, \xi_{1^-}=b, \sup_{0\leqslant s\leqslant t}\big|\xi_s-\phi_*(s)\big|<\delta\Big\}\\ &= \liminf_{\hbar\to 0^+}\hbar\log\mathbb{P}^{\hbar}_{a,b}\Big\{\sup_{0\leqslant s\leqslant t}\big|\xi_s-\phi_*(s)\big|<\delta\Big\}.\end{aligned}$$

Now we set $A = \{\omega\colon \sup_{0\leqslant s\leqslant t}|\xi_s(\omega)-\phi_*(s)|<\delta\}$. Then it is clear that $A\in\mathcal{F}_t$. Therefore it follows from the transformation (3.1) that for large enough n,

$$\begin{aligned}\liminf_{\hbar\to 0^+}\hbar\log\mathbb{P}^{\hbar}_{a,b}\{O_t\} &= \liminf_{\hbar\to 0^+}\hbar\log\mathbb{P}^{\hbar}_{a,b}\{A\}\\ &= \liminf_{\hbar\to 0^+}\hbar\log\int\limits_{\{\sup_{0\leqslant s\leqslant t}|\xi_s-\phi_*(s)|<\delta\}}\frac{p_{1-t}(\xi_t,b,\hbar)}{p_1(a,b,\hbar)}\,d\mathbb{P}^{\hbar}_a\\ &\geqslant \liminf_{\hbar\to 0^+}\hbar\log\int\limits_{\{\sup_{0\leqslant s\leqslant t}|\xi_s-\phi_*(s)|<\frac{1}{n}\}}\frac{p_{1-t}(\xi_t,b,\hbar)}{p_1(a,b,\hbar)}\,d\mathbb{P}^{\hbar}_a\\ &= \liminf_{\hbar\to 0^+}\hbar\log\int\limits_{\{\sup_{0\leqslant s\leqslant t}|\xi^{\hbar}_s-\phi_*(s)|<\frac{1}{n}\}}\frac{p_{1-t}(\xi^{\hbar}_t,b,\hbar)}{p_1(a,b,\hbar)}\,d\mathbb{P}_a\\ &= \liminf_{\hbar\to 0^+}\hbar\log\int\limits_{\{\sup_{0\leqslant s\leqslant t}|\xi^{\hbar}_s-\phi_*(s)|<\frac{1}{n}\}}p_{1-t}\big(\xi^{\hbar}_t,b,\hbar\big)\,d\mathbb{P}_a\\ &\quad-\liminf_{\hbar\to 0^+}\hbar\log p_1(a,b,\hbar).\end{aligned} \tag{3.3}$$

The limit $\liminf_{\hbar\to 0^+}\hbar\log p_1(a,b,\hbar) = -d(a,b)$ which is from the main theorem in [10]. Now we will prove that in (3.3) the limit

$$\liminf_{\hbar\to 0^+}\hbar\log\int\limits_{\{\sup_{0\leqslant s\leqslant t}|\xi^{\hbar}_s-\phi_*(s)|<\frac{1}{n}\}}p_{1-t}\big(\xi^{\hbar}_t,b,\hbar\big)\,d\mathbb{P}_a \geqslant -\int\limits_0^t L\big(\phi_*'(s)\big)\,ds. \tag{3.4}$$

To prove (3.4), we need a lower bound for $p_{1-t}(x,b,\hbar)$ as follows. For x in any fixed compact set, $\theta>0$ and $0\leqslant t<1$ being any fixed values, the following holds

$$p_{1-t}(x,b,\hbar) \geqslant \exp\Big\{-\frac{1}{\hbar}\big(d(x,b)+2\theta\big)\Big\}\cdot\hbar^{-1/2}\cdot c(\hbar,\theta,t) \tag{3.5}$$

when $\hbar$ (depending on θ and t) is small enough, where $c(\hbar,\theta,t)$ is a constant depending on $\hbar$, θ, t, and satisfying $\lim_{\hbar\to 0^+}c(\hbar,\theta,t) = c(\theta,t) > 0$ for each fixed $\theta>0$ and fixed $0\leqslant t<1$. This lower bound is from [10]. Then applying (3.5) to the first limit in the last identity in (3.3) yields

$$\begin{aligned}
&\liminf_{\hbar\to 0^+} \hbar \log \int\limits_{\{\sup_{0\leqslant s\leqslant t} |\xi_s^\hbar - \phi_*(s)| < \frac{1}{n}\}} p_{1-t}\big(\xi_t^\hbar, b, \hbar\big)\, d\mathbb{P}_a \\
&\quad \geqslant \liminf_{\hbar\to 0^+} \hbar \log \int\limits_{\{\sup_{0\leqslant s\leqslant t} |\xi_s^\hbar - \phi_*(s)| < \frac{1}{n}\}} \exp\left\{-\frac{1}{\hbar}\big(d\big(\xi_t^\hbar, b\big) + 2\theta\big)\right\} \cdot \hbar^{-1/2} \cdot c(\hbar, \theta, t)\, d\mathbb{P}_a \\
&\quad \geqslant \liminf_{\hbar\to 0^+} \hbar \log \int\limits_{\{\sup_{0\leqslant s\leqslant t} |\xi_s^\hbar - \phi_*(s)| < \frac{1}{n}\}} \exp\left\{-\frac{1}{\hbar}\big(\alpha(n,t) + 2\theta\big)\right\} \cdot \hbar^{-1/2} \cdot c(\hbar, \theta, t)\, d\mathbb{P}_a \\
&\quad = \liminf_{\hbar\to 0^+} \hbar \log \int\limits_{\{\sup_{0\leqslant s\leqslant t} |\xi_s^\hbar - \phi_*(s)| < \frac{1}{n}\}} \exp\left\{-\frac{1}{\hbar}\big(\alpha(n,t) + 2\theta\big)\right\} d\mathbb{P}_a \\
&\quad \geqslant \liminf_{\hbar\to 0^+} \hbar \log \left[\exp\left\{-\frac{1}{\hbar}\big(\alpha(n,t) + 2\theta\big)\right\} \mathbb{P}_a \left\{\sup_{0\leqslant s\leqslant t} \big|\xi_s^\hbar - \phi_*(s)\big| < \frac{1}{n}\right\}\right]
\end{aligned} \tag{3.6}$$

for $\alpha(n,t) \to 0$ as $n \to \infty$ and $t \to 1^-$ which is from the fact that $(x,y) \mapsto d(x,y)$ is continuous (cf. [4] and [10]). Now in (3.6), the last limit is equal to

$$-\big(\alpha(n,t) + 2\theta\big) + \liminf_{\hbar\to 0^+} \hbar \log \mathbb{P}_a \left\{\sup_{0\leqslant s\leqslant t} \big|\xi_s^\hbar - \phi_*(s)\big| < \frac{1}{n}\right\} \geqslant -\big(\alpha(n,t) + 2\theta\big) - \int_0^t L\big(\phi_*'(s)\big)\, ds$$

where the last inequality is from the large deviations of the truncated stable processes $\{\xi_s^\hbar\}_{0\leqslant s\leqslant t}$; namely (I) in Lemma 3.1.

Taking into account preceding inequalities, we now take $\lim_{t\to 1^-}$ in (3.3) to get

$$\begin{aligned}
\lim_{t\to 1^-} \liminf_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\{O_t\} &\geqslant -\lim_{t\to 1^-} \lim_{n\to\infty} \big(\alpha(n,t) + 2\theta\big) - \lim_{t\to 1^-} \int_0^t L\big(\phi_*'(s)\big)\, ds + d(a,b) \\
&\geqslant -\lim_{t\to 1^-} \int_0^t L\big(\phi_*'(s)\big)\, ds + d(a,b) = -S^{a,b}(\phi_*).
\end{aligned} \tag{3.7}$$

The last inequality in (3.7) is obtained by sending $\theta \to 0$ and using the fact that $\alpha(n,t) \to 0$ as $n \to \infty$ and $t \to 1^-$. From (3.2) and (3.7), it is clear that the lower bound (2.3) is proved if

$$\lim_{t\to 1^-} \lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\big\{O^t\big\} = -\infty. \tag{3.8}$$

For convenience, we prove the limit (3.8) in the following lemma which will be also used in the proof of the upper bound (2.4).

Lemma 3.3.

$$\lim_{t\to 1^-} \lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\big\{O^t\big\} = -\infty.$$

Proof. In order to prove this lemma, we need a notation from [6]. For each fixed $\hbar > 0$, if there is another right continuous Markov process $\{\hat{\xi}_t^\hbar\}_{t\geqslant 0}$ with left limits associated with a transition semigroup $\{\hat{P}_t^\hbar\}_{t\geqslant 0}$ such that

$$\int_{\mathbb{R}} f(x)P_t^\hbar g(x)\,dx = \int_{\mathbb{R}} \hat{P}_t^\hbar f(x)g(x)\,dx$$

for all $t > 0$ and all positive Borel functions f and g, then we say $\hat{\xi}^\hbar$ is in duality with $\xi^\hbar$ relative to the Lebesgue measure. Corollary 1 in [6] states that the $\mathbb{P}_{a,b}^\hbar$-law of the time-reversed process $\{\xi_{(1-t)^-}\}_{0\leqslant t<1}$ is the bridge law $\hat{\mathbb{P}}_{b,a}^\hbar$ from b to a constructed in the same way as in Section 1.2 based on the dual process $\{\hat{\xi}_t^\hbar\}_{t\geqslant 0}$.

According to Section II.2 in [3], the dual of a Lévy process $\xi^\hbar$ is always $\hat{\xi}^\hbar = -\xi^\hbar$. Our Lévy process $\xi^\hbar$ considered in this paper has better properties since it is symmetric, thus the characteristic exponent is real-valued (cf. (1.1)), and in this case the dual $\hat{\xi}^\hbar$ has the same law as $\xi^\hbar$.

Based on these observations, we have

$$\begin{aligned}
\lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\{O^t\} &= \lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\Big\{\xi_0 = a, \xi_{1^-} = b, \sup_{t\leqslant s<1}\big|\xi_s - \phi_*(s)\big| \geqslant \delta\Big\} \\
&= \lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\Big\{\sup_{t\leqslant s<1}\big|\xi_s - \phi_*(s)\big| \geqslant \delta\Big\} \\
&= \lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\Big\{\sup_{0\leqslant s<1-t}\big|\xi_{(1-s)^-} - \phi_*\big((1-s)^-\big)\big| \geqslant \delta\Big\} \\
&= \lim_{\hbar\to 0^+} \hbar \log \hat{\mathbb{P}}_{b,a}^\hbar\Big\{\sup_{0\leqslant s<1-t}\big|\hat{\xi}_s - \phi_*(1-s)\big| \geqslant \delta\Big\} \\
&= \lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{b,a}^\hbar\Big\{\sup_{0\leqslant s<1-t}\big|\xi_s - \phi_*(1-s)\big| \geqslant \delta\Big\},
\end{aligned}$$

where the fourth equality is from Corollary 1 in [6] and the fact that ϕ_* is continuous (since $S^{a,b}(\phi_*) < \infty$ at the beginning of Section 3.1), and the last equality follows from the fact that the dual has the same law as the original process. Applying the transformation (3.1), we obtain

$$\lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\{O^t\} = \lim_{\hbar\to 0^+} \hbar \log\bigg(\frac{1}{p_1(b,a,\hbar)} \int\limits_{\{\sup_{0\leqslant s<1-t} |\xi_s^\hbar - \phi_*(1-s)| \geqslant \delta\}} p_t\big(\xi_{1-t}^\hbar, a, \hbar\big)\, d\mathbb{P}_b\bigg).$$

Now a relaxed upper bound of $p_t(x,a,\hbar)$ can be derived based on Section 3 in [10] as follows. For any fixed $\theta > 0$ and $0 < t \leqslant 1$, the following holds

$$p_t(x,a,\hbar) \leqslant c_1(t)\cdot \hbar^{-c_2(t)} \cdot \exp\big\{\theta/(2\hbar)\big\} \tag{3.9}$$

when $\hbar$ (depending on θ and t) is small enough, where c_1 and c_2 are two positive constants depending on t. Therefore, for t close enough to 1 from the left,

$$\begin{aligned}
\lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_{a,b}^\hbar\{O^t\} &\leqslant d(a,b) + \lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_b\Big\{\sup_{0\leqslant s\leqslant 1-t}\big|\xi_s^\hbar - \phi_*(1-s)\big| \geqslant \delta\Big\} \\
&\leqslant d(a,b) + \lim_{\hbar\to 0^+} \hbar \log \mathbb{P}_b\Big\{\sup_{0\leqslant s\leqslant 1-t}\big|\xi_s^\hbar - b\big| \geqslant \delta/4\Big\}
\end{aligned} \tag{3.10}$$

From the upper bound of large deviations (i.e. (ii) in Lemma 3.1) for the family of $\{\xi_s^\hbar\}_{0\leqslant s\leqslant 1-t}$, it follows that

$$\limsup_{\hbar\to 0^+} \hbar \log \mathbb{P}_b\Big\{\sup_{0\leqslant s\leqslant 1-t}\big|\xi_s^\hbar - b\big| \geqslant \delta/4\Big\} \leqslant -\inf_{\phi\in A_{1-t}} \int_0^{1-t} L\big(\phi'(s)\big)\, ds$$

where the closed set A_{1-t} is defined as

$$A_{1-t} = \{\phi \in D^b[0,1-t]\colon\ |\phi(1-t_\delta) - b| \geqslant \delta/4 \text{ for some } 1-t_\delta \in [0,1-t]\}.$$

Because of the convexity of $L(x)$, we have

$$\begin{aligned}
\inf_{\phi\in A_{1-t}} \int_0^{1-t} L(\phi'(s))\,ds &\geqslant \inf_{\phi\in A_{1-t}} \int_0^{1-t_\delta} L(\phi'(s))\,ds \\
&\geqslant \inf_{\phi\in A_{1-t}} L\left(\int_0^{1-t_\delta} \phi'(s)/(1-t_\delta)\,ds\right)\cdot(1-t_\delta) \\
&= \inf_{\phi\in A_{1-t}} L\big(|\phi(1-t_\delta)-b|/(1-t_\delta)\big)\cdot(1-t_\delta) \\
&\to \infty \quad \text{as } 1-t_\delta \to 0^+
\end{aligned} \tag{3.11}$$

where the convergence to infinity in the last step comes from the facts that $|\phi(1-t_\delta) - b| \geqslant \delta/4$ and $L(x)$ grows to infinity faster than a linear speed (see the proof of Lemma 3.2). The proof is complete by combining (3.11) and (3.10). □

3.2. Proof of the upper bound (2.4)

Based on the rate function $S^{a,b}$ in (2.1), we define the following set for each $s > 0$,

$$\Phi(s) = \{\phi\colon [0,1) \to \mathbb{R},\ \text{right continuous with left limits},\ \phi(0) = a \text{ and } S^{a,b}(\phi) \leqslant s\}.$$

Applying the same arguments as in Lemma 3.2, we see that $\Phi(s)$ is compact. The proof of the upper bound is entirely based on these compact sets. For a fixed $0 \leqslant t < 1$ and a small $\delta > 0$ (which will be specified later), we rewrite $\{\mathrm{dist}(\xi, \Phi(s))\}$ as

$$\{\mathrm{dist}(\xi,\Phi(s)) \geqslant \delta\} = \{\mathrm{dist}(\xi_{[0,t]},\Phi_{[0,t]}(s)) \geqslant \delta\} \cup \{\mathrm{dist}(\xi_{[t,1)},\Phi_{[t,1)}(s)) \geqslant \delta\}$$

where $\Phi_{[0,t]}(s)$ (resp. $\Phi_{[t,1)}(s)$) denotes the collection of all elements in $\Phi(s)$ restricted on $[0,t]$ (resp. $[t,1)$), and $\xi_{[0,t]}$ (resp. $\xi_{[t,1)}$) denotes the path of ξ restricted on $[0,t]$ (resp. $[t,1)$). Therefore

$$\mathbb{P}^\hbar_{a,b}\{\mathrm{dist}(\xi,\Phi(s)) \geqslant \delta\} \leqslant \mathbb{P}^\hbar_{a,b}\{\mathrm{dist}(\xi_{[0,t]},\Phi_{[0,t]}(s)) \geqslant \delta\} + \mathbb{P}^\hbar_{a,b}\{\mathrm{dist}(\xi_{[t,1)},\Phi_{[t,1)}(s)) \geqslant \delta\}. \tag{3.12}$$

We first deal with the probability $\mathbb{P}^\hbar_{a,b}\{\mathrm{dist}(\xi_{[0,t]},\Phi_{[0,t]}(s)) \geqslant \delta\}$. First, it is clear that the set $\{\mathrm{dist}(\xi_{[0,t]},\Phi_{[0,t]}(s)) \geqslant \delta\} \in \mathcal{F}_t$. Therefore from the transformation (3.1), we obtain

$$\mathbb{P}^\hbar_{a,b}\{\mathrm{dist}(\xi_{[0,t]},\Phi_{[0,t]}) \geqslant \delta\} = \frac{1}{p_1(a,b,\hbar)} \int_{\{\mathrm{dist}(\xi^\hbar_{[0,t]},\Phi_{[0,t]}(s))\geqslant\delta\}} p_{1-t}(\xi^\hbar_t, b, \hbar)\,d\mathbb{P}_a.$$

The upper bound estimate for $p_t(x,a,\hbar)$ in (3.9) implies that

$$\begin{aligned}
\limsup_{\hbar\to0^+} \hbar\log\mathbb{P}^\hbar_{a,b}\{\mathrm{dist}(\xi_{[0,t]},\Phi_{[0,t]}) \geqslant \delta\} &\leqslant d(a,b) + \limsup_{\hbar\to0^+}\hbar\log\mathbb{P}_a\{\mathrm{dist}(\xi^\hbar_{[0,t]},\Phi_{[0,t]}(s)) \geqslant \delta\} \\
&\leqslant d(a,b) + \limsup_{\hbar\to0^+}\hbar\log\mathbb{P}_a\{\mathrm{dist}(\xi^\hbar_{[0,1]},\Phi^1(s)) \geqslant \delta\} \leqslant -s
\end{aligned} \tag{3.13}$$

where the last inequality is from large deviations for the family of truncated stable processes $\{\xi_s^\hbar\}_{0\leqslant s\leqslant 1}$ (namely, (II) in Lemma 3.1), and $\Phi^1(s)$ is defined in Lemma 3.1. The reasoning is that $\xi^\hbar$ is well defined on $[0,1]$ instead of $[0,t]$, and every function $\phi \in \Phi(s)$ can be extended to $[0,1]$ by its left limit at 1 without changing the value of $S^{a,b}(\phi)$. Furthermore, that fact that $S^{a,b}(\phi) = S^1(\phi) - d(a,b)$ is used. This form of large deviations involving the compact sets $\Phi^1(s)$ instead of closed sets are discussed in [18] and [7], together with several other different equivalent forms of large deviations.

Now for the second probability $\mathbb{P}^\hbar_{a,b}\{\operatorname{dist}(\xi_{[t,1)}, \Phi_{[t,1)}(s)) \geqslant \delta\}$ in (3.12), we choose a function ϕ_0 such that $S^{a,b}(\phi_0) = 0$. The existence of such ϕ_0 has been explained in Section 2. Then

$$\begin{aligned}\mathbb{P}^\hbar_{a,b}\bigl\{\operatorname{dist}\bigl(\xi_{[t,1)}, \Phi_{[t,1)}(s)\bigr) \geqslant \delta\bigr\} &= \mathbb{P}^\hbar_{a,b}\Bigl\{\operatorname{dist}\bigl(\xi_{[t,1)}, \Phi_{[t,1)}(s)\bigr) \geqslant \delta,\ \sup_{t\leqslant s<1}\bigl|\xi_s - \phi_0(s)\bigr| < \delta\Bigr\}\\ &\quad + \mathbb{P}\Bigl\{\operatorname{dist}\bigl(\xi_{[t,1)}, \Phi_{[t,1)}(s)\bigr) \geqslant \delta,\ \sup_{t\leqslant s<1}\bigl|\xi_s - \phi_0(s)\bigr| \geqslant \delta\Bigr\}.\end{aligned}$$

It is easy to see that $\{\operatorname{dist}(\xi_{[t,1)}, \Phi_{[t,1)}(s)) \geqslant \delta, \sup_{t\leqslant s<1}|\xi_s - \phi_0(s)| < \delta\} \subseteq \{\phi_0 \notin \Phi(s)\}$, thus

$$\begin{aligned}\mathbb{P}^\hbar_{a,b}\Bigl\{\operatorname{dist}\bigl(\xi_{[t,1)}, \Phi_{[t,1)}(s)\bigr) \geqslant \delta,\ \sup_{t\leqslant s<1}\bigl|\xi_s - \phi_0(s)\bigr| < \delta\Bigr\} &\leqslant \mathbb{P}^\hbar_{a,b}\bigl\{\phi_0 \notin \Phi(s)\bigr\}\\ &\leqslant \mathbb{P}^\hbar_{a,b}\bigl\{S^{a,b}(\phi_0) > s\bigr\} = 0.\end{aligned}$$

Thus the second probability

$$\begin{aligned}&\lim_{t\to 1^-}\limsup_{\hbar\to 0^+}\hbar\log\mathbb{P}^\hbar_{a,b}\bigl\{\operatorname{dist}\bigl(\xi_{[t,1)}, \Phi_{[t,1)}(s)\bigr) \geqslant \delta\bigr\}\\ &\quad\leqslant \lim_{t\to 1^-}\limsup_{\hbar\to 0^+}\hbar\log\mathbb{P}^\hbar_{a,b}\Bigl\{\operatorname{dist}\bigl(\xi_{[t,1)}, \Phi_{[t,1)}(s)\bigr) \geqslant \delta,\ \sup_{t\leqslant s<1}\bigl|\xi_s - \phi_0(s)\bigr| \geqslant \delta\Bigr\}\\ &\quad\leqslant \lim_{t\to 1^-}\limsup_{\hbar\to 0^+}\hbar\log\mathbb{P}^\hbar_{a,b}\Bigl\{\sup_{t\leqslant s<1}\bigl|\xi_s - \phi_0(s)\bigr| \geqslant \delta\Bigr\} = -\infty \qquad (3.14)\end{aligned}$$

where the last identity is from Lemma 3.3.

Now for a closed set F, we define $s = \inf_{\phi\in F} S^{a,b}(\phi) - \gamma$ for an arbitrarily small positive γ. Then there exists a small $\delta > 0$ such that

$$\begin{aligned}\limsup_{\hbar\to 0^+}\hbar\log\mathbb{P}^\hbar_{a,b}\{\xi \in F\} &\leqslant \lim_{t\to 1^-}\limsup_{\hbar\to 0^+}\hbar\log\mathbb{P}^\hbar_{a,b}\bigl\{\operatorname{dist}\bigl(\xi, \Phi(s)\bigr) \geqslant \delta\bigr\}\\ &\leqslant \lim_{t\to 1^-}\limsup_{\hbar\to 0^+}\hbar\log\mathbb{P}^\hbar_{a,b}\bigl\{\operatorname{dist}\bigl(\xi_{[0,t]}, \Phi_{[0,t]}(s)\bigr) \geqslant \delta\bigr\}\\ &\leqslant -s = -\inf_{\phi\in F} S^{a,b}(\phi) + \gamma = -\inf_{\phi\in F} S^{a,b}(\phi), \quad \text{by sending } \gamma \to 0, \qquad (3.15)\end{aligned}$$

where in the second inequality (3.12) and (3.14) are used, and in the third inequality (3.13) is used. Thus (3.15) completes the proof.

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