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The final size of a SARS epidemic model without quarantine

Sze-Bi Hsu [a], Lih-Ing W. Roeger [b,*]

[a] *Department of Mathematics, National Tsing Hua University, Hsinchu, Taiwan*
[b] *Department of Mathematics and Statistics, Box 41042, Texas Tech University, Lubbock, TX 79409, USA*

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Abstract

In this article, we present the continuing work on a SARS model without quarantine by Hsu and Hsieh [Sze-Bi Hsu, Ying-Hen Hsieh, Modeling intervention measures and severity-dependent public response during severe acute respiratory syndrome outbreak, SIAM J. Appl. Math. 66 (2006) 627–647]. An "acting basic reproductive number" ψ is used to predict the final size of the susceptible population. We find the relation among the final susceptible population size S_∞, the initial susceptible population S_0, and ψ. If $\psi > 1$, the disease will prevail and the final size of the susceptible, S_∞, becomes zero; therefore, everyone in the population will be infected eventually. If $\psi < 1$, the disease dies out, and then $S_\infty > 0$ which means part of the population will never be infected. Also, when $S_\infty > 0$, S_∞ is increasing with respect to the initial susceptible population S_0, and decreasing with respect to the acting basic reproductive number ψ.

Keywords: SARS; Final size; Epidemic models

1. Introduction

Severe acute respiratory syndrome (SARS) was first recognized as a global threat in mid-March 2003. The first known cases and the last case of SARS occurred in November 2002 and in July 2003. The international spread of SARS resulted in 8098 SARS cases in 26 countries, with 774 deaths [6]. It is believed that the transmission of SARS can be effectively controlled

* Corresponding author. Fax: +1 806 742 1112.
E-mail addresses: sbhsu@math.nthu.edu.tw (S.-B. Hsu), lih-ing.roeger@ttu.edu (L.-I.W. Roeger).

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by adhering to basic public health measures—rapid case detection, case isolation, contact tracing, good infection control (hand-washing and the use of personal protective equipment), and quarantine. Hsu and Hsieh [3] had modeled the SARS outbreak in Taiwan for year 2003 using the general models that include two different levels of quarantines, level A for those who were suspected of having close contact with a suspected SARS case and level B for those who traveled from affected areas after April 28, 2003. Their model equations are as follows:

$$\begin{aligned}
S' &= -\lambda(S, E, I, Q_A, Q_B, P, R, D)S, \\
E' &= \lambda(S, E, I, Q_A, Q_B, P, R, D)S - \mu E - q_1 E, \\
Q_A' &= q_1 E - \gamma_1 Q_A, \\
Q_B' &= Q(t) - \gamma_2 Q_B, \\
I' &= \mu E + \gamma_2 Q_B - (\sigma_1 + \rho_1 + \gamma_3) I, \\
P' &= \gamma_1 Q_A + \gamma_3 I - (\sigma_2 + \rho_2) P, \\
R' &= \sigma_1 I + \sigma_2 P, \\
D' &= \rho_1 I + \rho_2 P,
\end{aligned} \tag{1}$$

where λ function is the rate of incidence of infection and is given by

$$\begin{aligned}
&\lambda(S, E, I, Q_A, Q_B, P, R, D) \\
&\quad = \left(\frac{\beta c}{1 + a(P + R + D)}\right)\left(\frac{I + \alpha_A Q_A + \alpha_B Q_B + \alpha_P P}{S + E + I + \alpha_A Q_A + \alpha_B Q_B + \alpha_P P}\right).
\end{aligned}$$

The variables at time t are defined as the following:

S—the number of susceptible individuals.
E—the number of infected asymptomatic persons not under any quarantine.
Q_A—the number of infected asymptomatic persons under level A quarantine.
Q_B—the number of imported asymptomatic infected persons.
I—the number of infective persons with onset of symptoms but not isolated or quarantined.
P—the number of isolated probable SARS cases.
D—the cumulative number of SARS deaths.
R—the cumulative number of discharged SARS patients.

For this model, the time unit is in days. The initial conditions are $S(0) = S_0 > 0$, $I(0) = I_0 > 0$, $E(0) = Q_A(0) = Q_B(0) = P(0) = R(0) = D(0) = 0$. Since the duration of the outbreak was a short time, the total population is assumed to be constant: $S(t) + E(t) + Q_A(t) + Q_B(t) + I(t) + P(t) + R(t) + D(t) \equiv N = S_0 + I_0$.

The basic assumptions for the model are the following:

(1) A SARS-infective person is infective after onset of symptoms.
(2) A quarantined person is quarantined without symptoms (hence are not infective), becoming infective with reduced contact rates due to quarantine, and is isolated upon diagnosis.
(3) An infective person can infect others unless isolated as probable case with reduced contact rates depending on the effectiveness of isolation.
(4) A probable case is removed from isolation either by death or discharge.
(5) As people's behavior change caused by public response to the outbreak, the contact rate decreases with the increasing cumulated number of probable cases, deaths, and the removed.

(6) Homogeneous mixing with quarantine-adjusted incidence is assumed.
(7) Quarantine for level A is proportional to the number of infected asymptomatic persons.
(8) Imported cases are a function of time ($Q(t) = 0$, 1, or 2).

The parameters are defined as follows:

β—the average number of susceptible individuals infected by one infective individual per effective contact per day.
c—the per-capita effective contact number in the absence of an outbreak.
a—the effect of behavior change in reduction of contact due to cumulative numbers of probable cases, deaths, and the removed.
α_A, α_B, or α_P—the proportional reduction in infectivity of quarantined persons due to levels A, B quarantine (before isolation) and probable cases, respectively.
μ—the progression rate to onset of symptoms.
q_1—the proportion of recruitment of asymptomatic infected persons for level A quarantine.
γ_1, γ_2—the isolation rates of Q_A and Q_B, respectively.
γ_3—the isolation rate of infectives not under quarantine.
σ_1, σ_2—the respective discharged rate of infective cases and isolated probable cases.
ρ_1, ρ_2—the respective fatality rate of infective cases and isolated probable SARS patient.

For the general model of SARS (1), Hsu and Hsieh had derived the basic reproduction number and the full description of its dynamics was provided in [3].

For the worse case scenario when there are no quarantines and the probable cases are not isolated, i.e., $\alpha_A = \alpha_B = \alpha_P = 0$, then the model (1) becomes the following SARS model without quarantine [3]:

$$\begin{aligned}
S' &= \frac{-\beta I S}{E + I + S} \frac{c}{1 + a(P + R + D)}, \\
E' &= \frac{\beta I S}{E + I + S} \frac{c}{1 + a(P + R + D)} - \mu E, \\
I' &= \mu E - (\sigma_1 + \rho_1 + \gamma_3) I, \\
P' &= \gamma_3 I - (\sigma_2 + \rho_2) P, \\
R' &= \sigma_1 I + \sigma_2 P, \\
D' &= \rho_1 I + \rho_2 P.
\end{aligned} \tag{2}$$

Hsu and Hsieh [3] had done some analysis for this model. We will summarize their results. The equilibrium with the susceptible present for the system in (S, E, I, P, R, D) is $(S^*, 0, 0, 0, R^*, D^*)$ with $S^* + R^* + D^* = N$; the equilibrium with no susceptible present is $(0, 0, 0, 0, R^\#, D^\#)$ with $R^\# + D^\# = N$. The basic reproduction number $\mathcal{R}_0$ for this model is

$$\mathcal{R}_0 = \frac{\beta c}{(\sigma_1 + \rho_1 + \gamma_3)(1 + aN - aS^*)}.$$

If $\mathcal{R}_0 < 1$, the equilibrium with the susceptible present is locally asymptotically stable and if $\mathcal{R}_0 > 1$, unstable. When $S^* = 0$ in $\mathcal{R}_0$, we define

$$\psi = \frac{\beta c}{(\sigma_1 + \rho_1 + \gamma_3)(1 + aN)}. \tag{3}$$

Note that $0 < \psi \leqslant \mathcal{R}_0$. If $\psi > 1$, then $\mathcal{R}_0 > 1$ and the disease prevails. We will show that this is indeed the case. When $\psi > 1$, the final size S_∞ of the susceptible becomes zero. Eventually, everyone will be infected and either dies or recovers. Hsu and Hsieh [3] had obtained the following results relating to the parameter ψ.

Theorem 1. *For the SARS model without quarantine* (2)*, the solutions have the following asymptotic properties*: $S(t) \to S_\infty \geqslant 0$, $R(t) \to R_\infty > 0$, $D(t) \to D_\infty > 0$, $I(t) \to 0$, $E(t) \to 0$, *and* $P(t) \to 0$ *as* $t \to \infty$.

Theorem 2. *Consider system* (2)*. The parameter* ψ *is defined as in* (3)*.*

(i) *If* $\psi < 1$, *then* $S(t) \to S_\infty > 0$ *as* $t \to \infty$.
(ii) *If* $\psi > 1$, *then* $S(t) \to 0$ *as* $t \to \infty$.

The above two theorems state that the asymptotic dynamics are actually global so that we may write the equilibrium with the susceptible present or the equilibrium without the susceptible present to be $(S_\infty, 0, 0, 0, R_\infty, D_\infty)$ and $(0, 0, 0, 0, R_\infty, D_\infty)$ depending on the parameter ψ. We will show that although ψ is not a basic reproductive number, it acts like one. If $\psi > 1$, the disease will prevail. When $\psi > 1$, the final size S_∞ of the susceptible becomes zero. Eventually, everyone will be infected and either dies or recovers. In the following section, we will give details of the finding of the relation among S_0, S_∞, and ψ.

In the classical Kermack–Mckendric *SIR* model, the asymptotic state or the final size S_∞ satisfies a transcendental equation [4,5], so does S_∞ obtained in system (2). Diekmann et al. [1] also found the final size of epidemics in a closed population. Their model is described by a nonlinear Volterra integral equation of convolution type, just as the general Kermack–McKendrick model. Similar results that relate the final size and the basic reproductive number $\mathcal{R}_0$ can also be found in the book by Diekmann and Heesterbeek [2].

2. The final sizes of S_∞, R_∞, and D_∞

We can integrate the equations of P, R, and D in (2) from $t = 0$ to ∞. Since $P(\infty) = 0$, we have

$$\gamma_3 \int_0^\infty I(t)\,dt = (\sigma_2 + \rho_2) \int_0^\infty P(t)\,dt, \tag{4}$$

$$R_\infty = \sigma_1 \int_0^\infty I(t)\,dt + \sigma_2 \int_0^\infty P(t)\,dt, \tag{5}$$

and

$$D_\infty = \rho_1 \int_0^\infty I(t)\,dt + \rho_2 \int_0^\infty P(t)\,dt. \tag{6}$$

By substituting (4) into (5) and (6), we obtain

$$R_\infty = \left(\sigma_1 + \sigma_2 \frac{\gamma_3}{(\sigma_2 + \rho_2)}\right) \int_0^\infty I(t)\,dt,$$

and

$$D_\infty = \left(\rho_1 + \rho_2 \frac{\gamma_3}{(\sigma_2 + \rho_2)}\right) \int_0^\infty I(t)\,dt.$$

Let

$$r = \frac{\sigma_1 + \sigma_2 \frac{\gamma_3}{(\sigma_2+\rho_2)}}{\rho_1 + \rho_2 \frac{\gamma_3}{(\sigma_2+\rho_2)}}. \tag{7}$$

Then we have

$$\frac{R_\infty}{D_\infty} = r.$$

Let $V = S + E + I$. Then $P + R + D = N - V$, and system (2) can be simplified so that the two equations for S and V are

$$S' = \frac{-\beta I S}{V} \frac{c}{1 + a(N - V)},$$
$$V' = -(\rho_1 + \sigma_1 + \gamma_3) I.$$

Then we have

$$\frac{dS}{dV} = \frac{\beta c}{\rho_1 + \sigma_1 + \gamma_3} \frac{S}{V(1 + a(N - V))}.$$

Applying the method of separation of variables and integrating both sides of the equation from $t = 0$ leads to

$$\ln\left(\frac{S}{S_0}\right) = \psi \ln\left(\frac{V}{V_0(1 + a(N - V))}\right),$$

where ψ is defined as in (3). Since $V_0 = S_0 + E_0 + I_0 = N$, and $V_\infty = S_\infty$, S_∞ satisfies the following:

$$\frac{S_\infty}{S_0} = \left(\frac{S_\infty}{N(1 + aN - aS_\infty)}\right)^\psi. \tag{8}$$

We can show that the equation in x,

$$\frac{x}{S_0} = \left(\frac{x}{N(1 + aN - ax)}\right)^\psi \tag{9}$$

has only two roots at $x = 0$ and $x = S_\infty \in (0, S_0)$ when $\psi < 1$, as seen in Fig. 1.

Theorem 3. *Consider the equation in* (9).

(i) *If $\psi < 1$, this equation has a root at zero and a unique positive root $x \in (0, S_0)$.*
(ii) *If $\psi \geqslant 1$, this equation has a root at zero and no roots in $(0, S_0]$.*

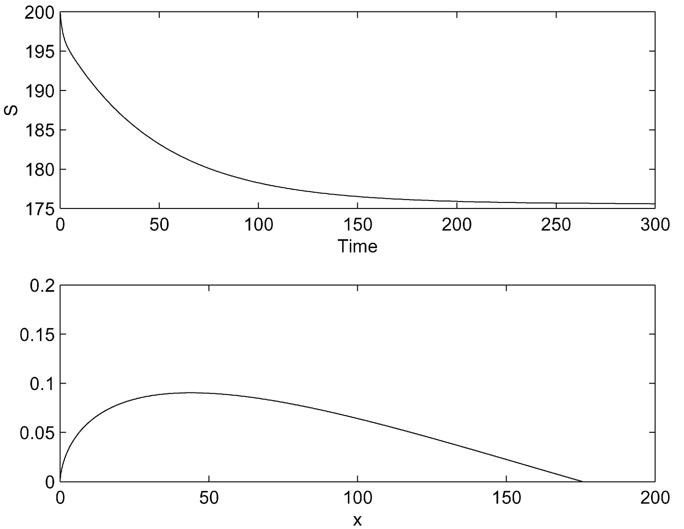

Fig. 1. The asymptotic example for system (2). The top figure shows the susceptible population approaching $S_\infty = 175.60$ when $\psi = 0.6768 < 1$. The lower figure shows the graph of the function $g(x) = (\frac{x}{N(1+aN-ax)})^{\psi} - \frac{x}{S_0}$ and its two roots, 0 and $S_\infty = 175.60$. We obtain the results by choosing the parameters $\beta = 0.3$ (person)(contact)$^{-1}$(day)$^{-1}$, $c = 2$ (contact)(person)$^{-1}$, $a = 0.0013$ (person)$^{-1}$, $\mu = 0.14$ (day)$^{-1}$, $\sigma_1 = \sigma_2 = 0.2$ (day)$^{-1}$, $\rho_1 = \rho_2 = 0.1$ (day)$^{-1}$, $\gamma_3 = 0.4$ (day)$^{-1}$, and the initial conditions $S_0 = 200$, $I_0 = 5$, and $E_0 = P_0 = R_0 = D_0 = 0$.

Proof. (i) $\psi < 1$. The roots of this equation are $x = 0$ and the roots of

$$N(1 + aN - ax) = S_0^{\frac{1}{\psi}} x^{1-\frac{1}{\psi}}. \tag{10}$$

Write $f(x) = S_0^{\frac{1}{\psi}} x^{1-\frac{1}{\psi}}$, then $\frac{d^2 f}{dx^2} = \frac{1}{\psi^2}(1-\psi)S_0^{\frac{1}{\psi}} x^{-1-\frac{1}{\psi}} > 0$. So f is convex and hence intersects each chord twice. When $x \geqslant 0$, Eq. (10) has a root in $(0, S_0)$ and a root in (S_0, ∞) so cannot have any more positive roots. The statement (i) follows in this case.

(ii) $\psi \geqslant 1$. The roots of this equation are $x = 0$ and the roots of (10). But for $x \in (0, S_0]$,

$$N(1 + aN - ax) \geqslant N\big(1 + a(N - S_0)\big) > S_0 \geqslant S_0^{\frac{1}{\psi}} x^{1-\frac{1}{\psi}}.$$

The statement (ii) follows. □

In case (i) of Theorem 2, since $S_\infty + R_\infty + D_\infty = N$, and R_∞ and D_∞ satisfy $R_\infty / D_\infty = r$, we can find all three values, S_∞, R_∞, and D_∞. In case (ii) of Theorem 2, all solutions approach the equilibrium $(0, 0, 0, 0, R_\infty, D_\infty)$, that is to say that eventually everyone gets infected and recovers or dies. Since $R_\infty + D_\infty = N$, it is also easy to find R_∞ and D_∞. Therefore, we have the following results.

Theorem 4. *Consider system* (2). *The parameters ψ and r are defined as in* (3) *and* (7)*, respectively.*

(i) *If $\psi < 1$, then*

$$S_\infty > 0, \quad R_\infty = \frac{r(N - S_\infty)}{1 + r}, \quad \text{and} \quad D_\infty = \frac{N - S_\infty}{1 + r}.$$

(ii) *If* $\psi > 1$, *then*

$$S_\infty = 0, \quad R_\infty = \frac{rN}{1+r}, \quad \textit{and} \quad D_\infty = \frac{N}{1+r}.$$

3. S_∞ decreases with ψ, increases with S_0 and a

If $\psi < 1$, then $S_\infty > 0$, we can show that S_∞ decreases as the parameter ψ increases, $dS_\infty/d\psi < 0$. We can also show that S_∞ increases as the parameter a increases or as S_0 increases. That is, we can show that $dS_\infty/da > 0$ and $dS_\infty/dS_0 > 0$. Let

$$g = \frac{S_\infty}{N(1 + aN - aS_\infty)}. \tag{11}$$

Note that if $\psi < 1$, we have $0 < S_\infty < N$, so that $0 < g < 1$. We will need the following lemma.

Lemma 1. *Consider Eq.* (8). *If* $\psi < 1$, *then*

$$1 - \frac{\psi(1 + aN)}{1 + aN - aS_\infty} > 0.$$

Proof. From Eq. (10), we know that the function $f = S_0^{\frac{1}{\psi}} x^{1-\frac{1}{\psi}}$ is convex and there is only one positive root of Eq. (10), S_∞, in $(0, S_0)$. Therefore,

$$f'(S_\infty) < \frac{d}{dx}\big(N(1 + aN - ax)\big)\big|_{x=S_\infty} = -aN,$$

i.e.,

$$\left(\frac{1}{\psi} - 1\right)\left(\frac{S_0}{S_\infty}\right)^{\frac{1}{\psi}} > aN.$$

Using Eq. (8), we obtain

$$\begin{aligned}
&\left(\frac{1}{\psi} - 1\right)\frac{N(1 + aN - aS_\infty)}{S_\infty} > aN \\
&\iff \quad (1-\psi)(1 + aN - aS_\infty) > a\psi S_\infty \\
&\iff \quad 1 + aN - aS_\infty - \psi(1 + aN) > 0 \\
&\iff \quad 1 - \frac{\psi(1 + aN)}{1 + aN - aS_\infty} > 0. \qquad \Box
\end{aligned}$$

When $\psi < 1$, $S_\infty > 0$. If we consider S_∞ as a function of ψ, then S_∞ is a decreasing function.

Theorem 5. *Consider Eq.* (8). *Let* $\psi < 1$ *and* S_∞ *be a function of the parameter* ψ. *Then*

$$\frac{dS_\infty}{d\psi} < 0.$$

Proof. Let $S_\infty = S_\infty(\psi)$ and $g = g(\psi)$ be defined as in (11). Then we have

$$\frac{g'(\psi)}{g(\psi)} = \frac{S'_\infty(\psi)(1 + aN)}{S_\infty(\psi)(1 + aN - aS_\infty(\psi))}. \tag{12}$$

Equation (8) is now

$$\frac{S_\infty(\psi)}{S_0} = g(\psi)^\psi.$$

Differentiating the equation with respect to the parameter ψ yields

$$\begin{aligned}\frac{S'_\infty(\psi)}{S_0} &= g(\psi)^\psi\left(\ln g(\psi) + \psi\frac{g'(\psi)}{g(\psi)}\right)\\ &= \frac{S_\infty(\psi)}{S_0}\left(\ln g(\psi) + \psi\frac{g'(\psi)}{g(\psi)}\right).\end{aligned}$$

Applying g'/g in (12) and simplifying, we have

$$S'_\infty = S_\infty \ln g(\psi) + S'_\infty\frac{\psi(1+aN)}{1+aN-aS_\infty},$$

i.e.,

$$\left(1 - \frac{\psi(1+aN)}{1+aN-aS_\infty}\right)S'_\infty = S_\infty \ln g(\psi).$$

The right side of the equation is negative because $0 < g(\psi) < 1$. By Lemma 1, we have $S'_\infty(\psi) < 0$. $\square$

The result in this theorem does not surprise us, since we have mentioned in the introduction that the parameter ψ is acting like a basic reproductive number. Therefore, increasing ψ should also increase the epidemic and hence it decreases the final size of the susceptible. Similarly, the following theorem says that S_∞ increases with the initial number of the susceptible.

Theorem 6. *Consider Eq.* (8). *Let $\psi < 1$ and S_∞ be a function of S_0. Then*

$$\frac{dS_\infty}{dS_0} > 0.$$

Proof. Assume that S_∞ is a function of S_0, then $g = g(S_0)$. Differentiating Eq. (8) with respect to S_0 yields

$$\begin{aligned}\frac{1}{S_0^2}\big(S_0 S'_\infty(S_0) - S_\infty\big) &= \psi\left(\frac{S_\infty}{N(1+aN-aS_\infty)}\right)^{\psi-1}\left(\frac{1+aN}{N(1+aN-aS_\infty)^2}\right)S'_\infty(S_0)\\ &= \frac{1}{S_0}\left(\frac{\psi(1+aN)}{1+aN-aS_\infty}\right)S'_\infty(S_0).\end{aligned}$$

The last equation comes from substituting Eq. (8) into the right-hand side. Therefore, we have

$$\left(1 - \frac{\psi(1+aN)}{1+aN-aS_\infty}\right)S'_\infty(S_0) = \frac{S_\infty}{S_0} > 0.$$

Using Lemma 1, we have $S'_\infty(S_0) > 0$. $\square$

In the SARS model (2), the parameter a describes the effect of behavior change in reduction of contact due to the cumulative number of probable cases. If a increases, number of contacts will be reduced, the epidemic will be not as severe, and so the final size of the susceptible S_∞ increases. The result is the following theorem.

Theorem 7. *Consider Eq.* (8). *Let $\psi < 1$ and S_∞ be a function of the parameter a. Then*

$$\frac{dS_\infty}{da} > 0.$$

Proof. Let $S_\infty = S_\infty(a)$, $\psi = \psi(a)$, and $g = g(a)$. Then

$$\frac{g'(a)}{g(a)} = \frac{S'_\infty(1+aN) - S_\infty(N - S_\infty)}{S_\infty(1+aN-aS_\infty)} \quad \text{and} \quad \frac{\psi'(a)}{\psi(a)} = -\frac{N}{1+aN}.$$

Then Eq. (8) is now

$$\frac{S_\infty(a)}{S_0} = g(a)^{\psi(a)}.$$

Differentiating the equation with respect to the parameter a yields

$$\begin{aligned}\frac{S'_\infty(a)}{S_0} &= g(a)^{\psi(a)}\left(\psi'(a)\ln g(a) + \psi(a)\frac{g'(a)}{g(a)}\right)\\ &= \frac{S_\infty(a)}{S_0}\left(\psi'(a)\ln g(a) + \psi(a)\frac{g'(a)}{g(a)}\right).\end{aligned}$$

Substitute $g'(a)/g(a)$ into the above equation and rearrange. We have

$$\left(1 - \frac{\psi(1+aN)}{1+aN-aS_\infty}\right)S'_\infty(a) = S_\infty\psi\left(\frac{\psi'}{\psi}\ln g - \frac{N-S_\infty}{1+aN-aS_\infty}\right).$$

If we can show that the right-hand side of the above equation is positive, and since by Lemma 1, the factor in front of $S'_\infty(a)$ is positive, then we should obtain the result that $S'_\infty(a) > 0$. Therefore, we need to show that

$$\frac{\psi'}{\psi}\ln g - \frac{N-S_\infty}{1+aN-aS_\infty} > 0,$$

i.e.,

$$\frac{N}{1+aN}\ln\frac{S_\infty}{N(1+aN-aS_\infty)} + \frac{N-S_\infty}{1+aN-aS_\infty} < 0. \tag{13}$$

Let

$$h(x) = \frac{N}{1+aN}\ln\frac{x}{N(1+aN-ax)} + \frac{N-x}{1+aN-ax}.$$

Then $h(0+) = -\infty$ and $h(N) = 0$. We can show that

$$h'(x) = \frac{(1+aN)(N-x)}{x(1+aN-ax)^2} > 0, \quad \text{for all } x \in (0, N).$$

The function $h(x)$ is increasing on $(0, N)$ from $-\infty$ to 0, and therefore $h(S_\infty) < 0$ which is the inequality (13). □

4. Conclusion

We consider the final size of a SARS epidemic model without quarantine. Our model is not an integral equations model, unlike other papers [1,4,5] of similar results which describe the epidemics using integral equations.

We use an "acting basic reproductive number" ψ to predict the final size of the epidemics, and there is a relation among the final size of the susceptible S_∞, the initial susceptible S_0, and the parameter ψ. We show that if $\psi > 1$, the disease stays in the population until all population is infected and recovers or dies. If $\psi < 1$, then the final size of the susceptible is greater than zero, $S_\infty > 0$. S_∞ decreases with the parameter ψ, increases with the initial size of the susceptible S_0, and increases with the parameter a which is the measure of the effect of behavior change in reduction of contact due to the cumulation number or probable cases.

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