J. Math. Anal. Appl. 362 (2010) 471–475

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Quantization dimension of random self-similar measures

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ARTICLE INFO

Article history:
Received 23 December 2008
Available online 20 August 2009
Submitted by B. Bongiorno

Keywords:
Quantization dimension
Random self-similar set
Random measure

ABSTRACT

In this paper, we study the quantization dimension of a random self-similar measure μ supported on the random self-similar set $K(\omega)$. We establish a relationship between the quantization dimension of μ and its distribution. At last we give a simple example to show that how to use the formula of the quantization dimension.

1. Introduction

The quantization problem consists in studying the L_r-error induced by the approximation of a given probability measure with discrete probability measures of finite supports. This problem originated in information theory and some engineering technology. Its history goes back to the 1940s [1]. Graf and Luschgy studied this problem systematically and gave a general mathematical treatment of it [3]. Two important objects in the quantization theory are the quantization coefficient and the quantization dimension.

Consider a random measure μ on $\mathbb{R}^d$, the nth quantization error of μ of order r is commonly defined as

$$V_{n,r}(\mu) = \inf\left\{\mathbb{E}\left(\int \min_{a\in\alpha} \|x-a\|^r \, d\mu(x)\right):\ \alpha \in \mathbb{R}^d,\ \operatorname{card}(\alpha) \leqslant n\right\}, \tag{1}$$

where $\|\cdot\|$ denotes Euclidean norm.

If the infimum in (1) is attained at some $\alpha \subset \mathbb{R}^d$ with $\operatorname{card}(\alpha) \leqslant n$, we call α an n-optimal set of μ of order r. The collection of all the n-optimal sets of order r is denoted by $C_{n,r}(\mu)$. The upper and lower quantization dimension of μ of order r are defined by

$$\overline{D}_r(\mu) := \limsup_{n\to\infty} \frac{r\log n}{-\log V_{n,r}(\mu)}; \qquad \underline{D}_r(\mu) := \liminf_{n\to\infty} \frac{r\log n}{-\log V_{n,r}(\mu)}.$$

If $\overline{D}_r(\mu)$, $\underline{D}_r(\mu)$ coincide, we call the common value the quantization dimension of μ of order r and denote it by $D_r(\mu)$.

Let $\{f_1,\ldots,f_N\}$ be an iterated function system of contractive similitude on $\mathbb{R}^d$ with contraction ratios $c_1,\ldots,c_N$. The corresponding self-similar set refers to the unique non-empty compact set E satisfying $E = \bigcup_{i=1}^N f_i(E)$. The self-similar measure associated with $\{f_1,\ldots,f_N\}$ and a given probability vector $(p_1,\ldots,p_N)$ is the unique Borel probability measure satisfying $\mu = \sum_{i=1}^N p_i \mu \circ f_i^{-1}$. We say that $\{f_1,\ldots,f_N\}$ satisfies the strong separation condition (SSC) if $f_i(E)$, $1 \leqslant i \leqslant N$, are pairwise disjoint. We say that $\{f_1,\ldots,f_N\}$ satisfies the open set condition (OSC) if there exists a non-empty open set U

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0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2009.08.026

such that $f_i(U) \subset U$ for all $i = 1, 2, \ldots, N$ and $f_i(U) \cap f_j(U) = \emptyset$ for any pair i, j with $1 \leqslant i \neq j \leqslant N$. Under the open set condition, Graf and Luschgy [4,5] proved that the quantization dimension of μ exists and equals D_r which is the solution of the following equation:

$$\sum_{i=1}^{N} \left(p_i c_i^r\right)^{\frac{D_r}{r+D_r}} = 1.$$

The above result was extended by Lindsay and Mauldin to the F-conformal measures associated with finitely many conformal maps [11]. Zhu extended this result to certain Cantor-like sets under a hereditary condition [12].

In this paper, we study the quantization dimensions of a random self-similar measure μ supported on random self-similar sets. We establish a relationship between the quantization dimension and its distribution and get the formula for the quantization dimension function of a random self-similar measure μ defined for the statistical contraction iterated function system $\{f_1, f_2, \ldots, f_N\}$ on $\mathbb{R}^d$ satisfying the *SSC*.

2. Definitions and notations

Let $(\Omega, \mathscr{F}, P)$ be a complete probability space, (E, ρ) be a Polish space, $\mathcal{K}(E)$ be the collection of all non-empty compact sets in E with Hausdorff metric η on it, that is, $\eta(K, L) = \sup\{\rho(x, L), \rho(K, y)\colon x \in K, \ y \in L\}$ for $K, L \in \mathcal{K}(E)$. It is well know that $(\mathcal{K}(E), \eta)$ is also a Polish space [6]. For any $f : E \to E$, $\mathrm{Lip}(f)$ denotes the Lipschitz coefficient of f [7, Definition 1.1]. $\mathrm{con}(E) := \{f\colon \mathrm{Lip}(f) < 1\}$, $\mathrm{sicon}(E) := \{f \in \mathrm{con}(E)\colon \exists r < 1$ such that $\rho(f(x), f(y)) = r\rho(x, y)$ for all $x, y \in E\}$ denotes the collection of all similar contraction operators from E to E. Let $M(\Omega, \mathcal{K}(E))$ denote the collection of all random elements from Ω to $\mathcal{K}(E)$, $\mathrm{con}(\Omega, E)$ (or $\mathrm{sicon}(\Omega, E)$) denote the collection of all statistical contraction (or statistically similar contraction) operators from Ω to $\mathrm{con}(E)$ (or $\mathrm{sicon}(E)$), that is, $f \in \mathrm{con}(\Omega, E)$ (or $\mathrm{sicon}(\Omega, E)$) if and only if f is a random element from Ω to $\mathrm{con}(E)$ (or $\mathrm{sicon}(E)$). $\mathrm{con}(E)$ (or $\mathrm{sicon}(E)$) carries the topology of pointwise convergence.

In what follows N is always a natural number and $N \geqslant 2$. We let $\varXi := \{1, 2, \ldots, N\}$ be an index set, $\varXi_k := \{(i_1, i_2, \ldots, i_k)\colon i_j \in \varXi, \ 1 \leqslant j \leqslant k\}$, $\varXi_\infty := \{(i_1, i_2, \ldots)\colon i_j \in \varXi, \ j \in \mathbb{N}\}$, $\varXi_0 := \{\emptyset\}$, $\varXi^* := \bigcup_{k \geqslant 0} \varXi_k$. For any $\sigma = (\sigma_1, \ldots, \sigma_k) \in \varXi_k$, we call the number k the length of σ and denote it by $|\sigma|$. For any $\sigma \in \varXi^* \cup \varXi_\infty$ with $|\sigma| \geqslant k$, we write $\sigma|_k = (\sigma_1, \ldots, \sigma_k)$. If $\sigma, \tau \in \varXi^*$ and $|\sigma| \leqslant |\tau|$, $\sigma = \tau|_{|\sigma|}$, we call σ a predecessor of τ and denote this by $\sigma \prec \tau$. The empty word is a predecessor of any finite or infinite word. We say σ, τ are incomparable if we have neither $\sigma \prec \tau$ nor $\tau \prec \sigma$. A finite set $\Gamma \subset \varXi^*$ is called a finite anti-chain if any two words σ, τ in Γ are incomparable. A finite anti-chain Γ is called maximal if any word $\sigma \in \varXi_\infty$ has a predecessor in Γ. For $k \geqslant 2$, $\sigma = (\sigma_1, \ldots, \sigma_k) \in \varXi_k$ and $i \in \varXi$, we define

$$\sigma^- := \sigma|_{k-1}, \qquad \sigma * i := (\sigma_1, \ldots, \sigma_k, i).$$

Definition 2.1. (See [8,9].) Suppose $\{f_1, f_2, \ldots, f_N\} \subset \mathrm{con}(\Omega, E)$, $K(\omega, \omega_1, \ldots, \omega_N) \in M(\Omega^{N+1}, \mathcal{K}(\varepsilon))$, we call $K(\omega, \omega_1, \ldots, \omega_N)$ a random self-similar set (R.S.S.S.): if there exists a set Ω_0 with $P(\Omega_0) = 1$ such that $K(\omega, \omega_1, \ldots, \omega_N) = \bigcup_{i=1}^N f_i^{(\omega)}(K(\omega, \omega_1, \ldots, \omega_N))$ for all $\widetilde{\omega} := (\omega, \omega_1, \ldots, \omega_N) \in \Omega_0^{N+1}$.

Sometimes we write $K(\omega, \omega_1, \ldots, \omega_N) = K(\omega)$ for simplification.

Remark 2.1. If the R.S.S.S. $K(\omega, \omega_1, \ldots, \omega_N)$ do not depend on $\widetilde{\omega}$, then K is the self-similar set defined in [1].

We let $E := [0, 1]^d$ and denote $f_\sigma^{(\omega)} := f_{\sigma_1}^{(\omega)} \circ f_{\sigma_2}^{(\omega_{\sigma_1})} \circ \cdots \circ f_{\sigma_k}^{(\omega_{\sigma_{k-1}})}$, $l_\sigma^{(\omega)} := \mathrm{Lip}(f_\sigma^{(\omega)}) = \prod_{i=1}^k \mathrm{Lip}(f_{\sigma_i}^{\omega_{\sigma_{i-1}}})$ (where $\widetilde{\omega} \in \Omega_0^{N+1}$, $\sigma = (\sigma_1, \ldots, \sigma_k) \in \varXi_k$, $\omega_{\sigma_0} = \omega$), $p_\sigma := p_{\sigma_1} \cdots p_{\sigma_k}$, $E_\sigma := f_\sigma(E)$.

The random self-similar measure μ with respect to the probability vector $(p_1, p_2, \ldots, p_N)$ is denoted as follow: If $(f_1, f_2, \ldots, f_N) \subset \mathrm{con}(\Omega, E)$ are random vectors of similitude from $\mathbb{R}^d$ into itself with contraction ratios $\mathrm{Lip}(f_i)$ $(i \in \varXi)$ distributed according to $(p_1, p_2, \ldots, p_N)$, then

$$\mu(\cdot) = \sum_{i=1}^N p_i \mu\left(\left(f_i^{(\omega)}\right)^{-1}(\cdot)\right), \quad \widetilde{\omega} \in \Omega_0^{N+1}.$$

Then $K(\omega) = \operatorname{supp} \mu$ is the attractor of $(f_1, f_2, \ldots, f_N)$ [10].

In this paper, we need to give some more definitions and notations.

$\mathbb{E}^{N+1}$ denotes the expectation operator for P^{N+1}. For $\sigma \in \varXi_k$, we define $\tilde{h}_k(\sigma) := p_\sigma (l_\sigma^{(\omega)})^r$ and $h_k(\sigma) := \mathbb{E}^{N+1}(\tilde{h}_k(\sigma))$, $\widetilde{\omega} \in \Omega_0^{N+1}$. Note that the sequence $\{\tilde{h}_k(\sigma)\}$ is monotone. So the sequence of random variables $\{\tilde{h}_k(\sigma)\}$ converge with probability 1 as $k \to \infty$ to a random variable $\tilde{h}(\sigma)$ such that

$$\mathbb{E}^{N+1}\left(\tilde{h}(\sigma)\right) = \mathbb{E}^{N+1}\Big(\lim_{k\to\infty} \tilde{h}_k(\sigma)\Big) = \lim_{k\to\infty} \mathbb{E}^{N+1}\left(\tilde{h}_k(\sigma)\right) = \lim_{k\to\infty} h_k(\sigma).$$

Set $h(\sigma) := \mathbb{E}^{N+1}(p_\sigma (l_\sigma^{(\omega)})^r)$ for $\sigma \in \varXi^*$, and $l := \min\{\mathbb{E}^{N+1}(p_i (\mathrm{Lip}\, f_i^{(\omega)})^r)\colon i \in \varXi, \ \widetilde{\omega} \in \Omega_0^{N+1}\}$.

For each $n \geqslant 1$, we define

$$\Gamma_n := \left\{\sigma \in \Xi^*:\ h(\sigma^-) \geqslant \frac{l}{n} > h(\sigma)\right\}. \tag{2}$$

The set Γ_n is crucial in the calculation of the quantization dimension. We remark that the definition of Γ_n is motivated by Graf and Luschgy's work on the quantization for self-similar distributions [4]. For each $n \in \mathbb{N}$, according to the definition l, the set Γ_n is non-empty and finite. Moreover, for each n, Γ_n is a finite maximal anti-chain.

3. Main result

We need to give some lemmas. Let $[x]$ denote the largest integer less than or equal to x. We begin with the following simple lemma which is an immediate consequence of the definitions.

Lemma 3.1. *(See [12, Lemma 2].) Let $l, \lambda, \xi > 0$. For $\phi(n) := [\lambda(n/l)^{\xi}]$, we have*

$$\overline{D}_r(\mu) = \limsup_{n\to\infty} \frac{r\log\phi(n)}{-\log V_{\phi(n),r}(\mu)}, \qquad \underline{D}_r(\mu) = \liminf_{n\to\infty} \frac{r\log\phi(n)}{-\log V_{\phi(n),r}(\mu)}.$$

Let $(A)_\epsilon$ denote the ϵ-neighborhood of a set A. By the *SSC*, there exists a constant $\beta > 0$ such that for any $\sigma \in \Xi_k$, we have

$$\min_{i\neq j}\left\{\operatorname{dist}(E_{\sigma*i}, E_{\sigma*j}):\ i, j \in \Xi\right\} \geqslant \beta \max\left\{|E_{\sigma*i}|,\ i \in \Xi\right\}. \tag{3}$$

For $\alpha \in C_{m,r}(\mu)$ and $\sigma \in \Gamma_n$, we define

$$\alpha_\sigma := \alpha \cap (E_\sigma)_{\beta|E_\sigma|/8}.$$

The following Lemma 3.2 and Lemma 3.3 will be crucial in the proof of Theorem 3.4. We can prove directly these lemmas using the same method in [12].

Lemma 3.2. *There exists a constant $L \geqslant 1$ such that for any $m \leqslant \operatorname{card}(\Gamma_n)$, $\alpha \in C_{m,r}(\mu)$ and all $\sigma \in \Gamma_n$ we have $\operatorname{card}(\alpha_\sigma) \leqslant L$.*

Lemma 3.3. *Let $\widetilde{L} \geqslant 1$ be an integer and α an arbitrary subset of $\mathbb{R}^d$ with cardinality $\widetilde{L}$. Then there exists a constant $D > 0$ such that for any $\sigma \in \Xi^*$, we have*

$$\int_{E_\sigma} \min_{a\in\alpha} \|x-a\|^r\, d\mu(x) \geqslant Dh(\sigma).$$

We now state our main result.

Theorem 3.4. *Let $K(\omega)$ be the random self-similar set and $(f_1, f_2, \ldots, f_N) \subset \operatorname{con}(\Omega, E)$ satisfies the SSC. Let μ be a random self-similar measure supported on $K(\omega)$ with respect to probability vector $(p_1, p_2, \ldots, p_N)$. Then with probability 1, the random self-similar measure μ has $\overline{D}_r(\mu) = \underline{D}_r(\mu) = D_r$, where D_r is the solution of the expectation equation*

$$\sum_{i=1}^{N}\left[\mathbb{E}^{N+1}\left(p_i\left(\operatorname{Lip} f_i^{(\omega)}\right)^r\right)\right]^{\frac{D_r}{r+D_r}} = 1. \tag{4}$$

Proof. From Lemma 3.1 in [2], we easy see that there exists a unique $D_r \in (0, +\infty)$ satisfying (4). By virtue of the identical distribution we also have

$$\sum_{i=1}^{N}\left[\mathbb{E}^{N+1}\left(p_i\left(l_{\sigma*i}^{(\omega)}\right)^r\right)\right]^{\frac{D_r}{r+D_r}} = 1 \quad \text{for } \sigma \in \Xi_k,\ \widetilde{\omega} \in \Omega_0^{N+1}.$$

By induction, for any $k \geqslant 1$, we can see

$$\sum_{\sigma\in\Xi_k}\left[\mathbb{E}^{N+1}\left(p_\sigma\left(l_\sigma^{(\omega)}\right)^r\right)\right]^{\frac{D_r}{r+D_r}} = 1, \quad \widetilde{\omega} \in \Omega_0^{N+1}.$$

We select a $\tau \in \Gamma_n$ then we have

$$\sum_{i=1}^{N}\left[\mathbb{E}^{N+1}\left(p_{\tau^-*i}\left(l^{(\omega)}_{\tau^-*i}\right)^r\right)\right]^{\frac{D_r}{r+D_r}}=\left[\mathbb{E}^{N+1}\left(p_{\tau^-}\left(l^{(\omega)}_{\tau^-}\right)^r\right)\right]^{\frac{D_r}{r+D_r}}\cdot\sum_{i=1}^{N}\left[\mathbb{E}^{N+1}\left(p_i\left(l^{(\omega)}_{\tau^-*i}\right)^r\right)\right]^{\frac{D_r}{r+D_r}}$$
$$=\left[\mathbb{E}^{N+1}\left(p_{\tau^-}\left(l^{(\omega)}_{\tau^-}\right)^r\right)\right]^{\frac{D_r}{r+D_r}},\quad \widetilde{\omega}\in\Omega_0^{N+1}.$$

From Γ_n is a finite maximal anti-chain we could have

$$\sum_{\sigma\in\Gamma_n}\left(h(\sigma)\right)^{\frac{D_r}{r+D_r}}=\sum_{\sigma\in\Xi_{k_n}}\left[\mathbb{E}^{N+1}\left(p_\sigma\left(l^{(\omega)}_\sigma\right)^r\right)\right]^{\frac{D_r}{r+D_r}}$$
$$=1,$$

where $\widetilde{\omega}\in\Omega_0^{N+1}$, and $k_n=\max|\sigma|$ for any $\sigma\in\Gamma_n$.

The central part of the proof is estimates for $\operatorname{card}(\Gamma_n)$.

Note that

$$1=\sum_{\sigma\in\Gamma_n}\left(h(\sigma)\right)^{\frac{D_r}{r+D_r}}$$
$$\geqslant l^{\frac{D_r}{r+D_r}}\sum_{\sigma\in\Gamma_n}\left(h\left(\sigma^-\right)\right)^{\frac{D_r}{r+D_r}}$$
$$\geqslant l(l/n)^{\frac{D_r}{r+D_r}}\operatorname{card}(\Gamma_n).$$

Hence we have $\operatorname{card}(\Gamma_n)\leqslant l^{\frac{-2D_r-r}{r+D_r}}n^{\frac{D_r}{r+D_r}}$. Let $\phi(n):=[l^{\frac{-2D_r-r}{r+D_r}}n^{\frac{D_r}{r+D_r}}]$. For each $\sigma\in\Gamma_n$, we choose an arbitrary point of E_σ and denote by α the set of these points. Note that $\phi(n)\geqslant\operatorname{card}(\Gamma_n)$. We deduce

$$V_{\phi(n),r}(\mu)=\mathbb{E}^{N+1}\left(V_{\phi(n),r}(\mu)\right)$$
$$\leqslant\mathbb{E}^{N+1}\left(\sum_{\sigma\in\Gamma_n}\int_{E_\sigma}\min_{a\in\alpha}\|x-a\|^r\,d\mu(x)\right)$$
$$\leqslant\mathbb{E}^{N+1}\left(\sum_{\sigma\in\Gamma_n}\mu(E_\sigma)|E_\sigma|^r\right)$$
$$\leqslant\mathbb{E}^{N+1}\left\{l^{-1}(n/l)^{\frac{D_r}{r+D_r}}(l/n)\right\}$$
$$=l^{-1}(n/l)^{-\frac{r}{r+D_r}}.$$

Thus by Lemma 3.1, with probability 1 we have

$$\overline{D}_r(\mu)=\limsup_{n\to\infty}\frac{r\log\phi(n)}{-\log V_{\phi(n),r}(\mu)}\leqslant D_r.$$

Next we show the reverse inequalities. Note that

$$1=\sum_{\sigma\in\Gamma_n}\left(h(\sigma)\right)^{\frac{D_r}{r+D_r}}$$
$$\leqslant(l/n)^{\frac{D_r}{r+D_r}}\operatorname{card}(\Gamma_n).$$

Hence $\operatorname{card}(\Gamma_n)\geqslant(n/l)^{\frac{D_r}{r+D_r}}$. Let $\alpha\in C_{[(n/l)^{\frac{D_r}{r+D_r}}],r}(\mu)$. For each $\sigma\in\Gamma_n$, let $\upsilon_1,\ldots,\upsilon_{L_1}$ be the centers of the L_1 closed balls with radii $\beta|E_\sigma|/(8M)$ which cover E_σ, and define

$$\widetilde{\alpha}_\sigma:=\alpha_\sigma\cup\{\upsilon_1,\ldots,\upsilon_{L_1}\}.$$

Thus for $\sigma\in\Gamma_n$ and all $x\in E_\sigma$, we have

$$\min_{a\in\alpha}\|x-a\|\geqslant\min_{a\in\widetilde{\alpha}_\sigma}\|x-a\|.$$

By Lemma 3.2, $\operatorname{card}(\widetilde{\alpha}_\sigma)\leqslant L+L_1$. By Lemma 3.3 we deduce that

$$
\begin{aligned}
V_{[(n/l)^{\frac{D_r}{r+D_r}}],r}(\mu) &= \mathbb{E}^{N+1}\left(\sum_{\sigma\in\Gamma_n}\int_{E_\sigma}\min_{a\in\alpha}\|x-a\|^r\,d\mu(x)\right)\\
&\geqslant \mathbb{E}^{N+1}\left(\sum_{\sigma\in\Gamma_n}\int_{E_\sigma}\min_{a\in\widetilde{\alpha}_\sigma}\|x-a\|^r\,d\mu(x)\right)\\
&\geqslant D\sum_{\sigma\in\Gamma_n}h(\sigma)\\
&\geqslant Dl(n/l)^{\frac{D_r}{r+D_r}}(l/n) = Dl(n/l)^{-\frac{r}{r+D_r}}.
\end{aligned}
$$

Thus by Lemma 3.1, for $\phi(n) := [(n/l)^{\frac{D_r}{r+D_r}}]$, with probability 1 we have

$$\underline{D}_r(\mu) = \liminf_{n\to\infty}\frac{r\log\phi(n)}{-\log V_{\phi(n),r}(\mu)} \geqslant D_r.$$

Therefore with probability 1 we have

$$\overline{D}_r(\mu) = \underline{D}_r(\mu) = D_r. \qquad \square$$

Example 3.5. Let $E=[0,1]$, $T_i(t) = ct + \frac{i}{3}$ $(i=0,1,2)$, $\Omega = ((T_0,T_1),(T_0,T_2),(T_1,T_2))$, $P((T_i,T_j)) = \frac{1}{3}$, $(f_1^{(T_i,T_j)}, f_2^{(T_i,T_j)}) = (T_i,T_j)$, π_i be the ith coordinate operator from $\operatorname{con}(E)^2$ to $\operatorname{con}(E)$, $F(T_i)=i$, $\Gamma_n(\omega) = \{F(\pi_1(\omega)), F(\pi_1(\omega))\}^n$ $(n\geqslant 1,\ \omega\in\Omega)$, and

$$K_c(\omega) := \lim_{n\to\infty}\bigcup_{(\sigma|n)\in\Gamma_n(\omega)} T_{\sigma_1}\circ T_{\sigma_2}\circ\cdots\circ T_{\sigma_n}(E)\quad \big(\omega\in\Omega,\ \sigma=(\sigma_1,\sigma_2,\ldots),\ \sigma_i\in\{0,1,2\}\big).$$

We call $K_c(\omega)$ the random Cantor set [9]. Let c be a random variable with uniform distribution on the interval $(0,\frac{1}{3})$, μ be a random self-similar measure supported on $K_c(\omega)$ with respect to the probability vector $(1/2,1/2)$, then from Theorem 3.4 we can get

$$
\begin{aligned}
1 &= 2\left(\mathbb{E}\left(\frac{1}{2}c^r\right)\right)^{\frac{D_r}{r+D_r}}\\
&= 2\left(\int_0^{\frac{1}{3}} 3\cdot\frac{1}{2}c^r\,dc\right)^{\frac{D_r}{r+D_r}}\\
&= 2\left(\frac{1}{2(r+1)3^r}\right)^{\frac{D_r}{r+D_r}}.
\end{aligned}
$$

We deduce that $D_r = \frac{r\log 2}{\log(r+1)+r\log 3}$.

Acknowledgments

The authors would like to thank an anonymous referee for valuable comments and suggestions. Research is supported by the National Science Foundation of China (10671180), the Education Foundation of Jiangsu Province (08KJB110003) and Jiangsu University (JJ08B024).

References

[1] J.A. Bucklew, G.L. Wise, Multidimensional asymptotic quantization with rth power distortion measures, IEEE Trans. Inform. Theory 28 (1982) 239–247.
[2] A.J. Deng, D.H. Hu, The Hausdorff dimension of generalized statistically self-similar sets, Acta Math. Sci. Ser. A 23 (5) (2003) 554–564.
[3] S. Graf, H. Luschgy, Foundations of Quantization for Probability Distributions, Lecture Notes in Math., vol. 1730, Springer, 2000.
[4] S. Graf, H. Luschgy, The quantization dimension of self-similar probabilities, Math. Nachr. 241 (2002) 103–109.
[5] S. Graf, H. Luschgy, The quantization of the Cantor distribution, Math. Nachr. 183 (1997) 113–133.
[6] D.H. Hu, The necessary and sufficient conditions for various self-similar sets and their dimension, Stochastic Process. Appl. 90 (2000) 243–262.
[7] D.H. Hu, Probability properties and fractal properties of statistically recursive sets, Sci. China Ser. A 44 (6) (2001) 742–761.
[8] D.H. Hu, X.M. Zhang, The random shift set and random sub-self-similar set, Acta Math. Sci. Ser. B 27 (2) (2007) 267–273.
[9] D.H. Hu, X.M. Zhang, The dimension for random sub-self-similar set, Acta Math. Sci. Ser. B 27 (3) (2007) 561–573.
[10] J.R. Liang, Random Markov-self-similar measures, J. Math. Anal. Appl. 98 (2002) 113–130.
[11] L.J. Lindsay, R.D. Mauldin, Quantization dimension for conformal function system, Nonlinearity 15 (1) (2002) 189–199.
[12] S.G. Zhu, Quantization dimension of probability measures supported on Cantor-like sets, J. Math. Anal. Appl. 338 (2008) 742–750.