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Symbolic computation on the Darboux transformation for a generalized variable-coefficient higher-order nonlinear Schrödinger equation from fiber optics

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ABSTRACT

Considering the propagation of ultrashort pulse in the realistic fiber optics, a generalized variable-coefficient higher-order nonlinear Schrödinger equation is investigated in this paper. Under certain constraints, a new 3×3 Lax pair for this equation is obtained through the Ablowitz–Kaup–Newell–Segur procedure. Furthermore, with symbolic computation, the Darboux transformation and nth-iterated potential transformation formula for such a model are explicitly derived. The corresponding features of ultrashort pulse in inhomogeneous optical fibers are graphically discussed by the one- and two-soliton-like solutions.

1. Introduction

In the past decades, the nonlinear evolution equations (NLEEs) with variable coefficients have attracted extensive consideration in the physics and mathematics field [1–5]. Especially, due to the swift development of symbolic computation and wide potential applications of optical soliton in the long distance communication and all-optical ultrafast switching devices [6–10], various variable-coefficient nonlinear Schrödinger (NLS) models have been paid much attention to by a great many of researchers in various branches of physical and engineering sciences [11–21].

Recently, the variable-coefficient higher-order NLS (HNLS) models of various styles have become more and more interesting for optical fibers [13–18]. For the ultrashort pulse propagating in a realistic optical fiber [22–24], in addition to the group velocity dispersion (GVD) and self-phase modulation (SPM), it also suffers from other higher-order effects influenced by the spatial variations of the fiber parameters such as the third-order dispersion (TOD), self-steepening (also called dispersion Kerr) and self-frequency shift resulting from stimulated Raman scattering (SRS) [3]. Based on this consideration, the propagation of such ultrashort pulse with loss and/or gain effect can be governed by the following generalized variable-coefficient HNLS equation [24,25]

$$iu_z + A(z)u_{tt} + B|u|^2u + iM_1(z)u_{ttt} + iQ_1\big(|u|^2u\big)_t + (P + iP_1)u\big(|u|^2\big)_t + iH_1(z,t)u_t + \big[C(z,t) + iC_1\big]u = 0, \quad (1)$$

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where $u(z,t)$ is the complex envelope of the electrical field in the comoving frame, z and t respectively represent the normalized propagation distance along the fiber and retarded time, while all the variable coefficients are real analytic functions. $A(z)$, B and $M_1(z)$ denote the GVD, SPM and TOD, respectively. Q_1 is the self-steepening and P_1 is related to the delayed nonlinear response effects. $H_1(z,t)$ is the inverse of group velocity v_g, $C(z,t)$ accounts for the phase modulation and C_1 represents the amplification or absorption coefficient. For Eq. (1), some special examples can be listed as below:

- In a weakly inhomogeneous plasma, the governing equation modelling the propagation of envelope solitons is the following modified NLS equation [26],

$$iq_{t'} + q_{xx} + \big[2|q|^2 - F\big(x,t'\big)\big]q = 0, \tag{2}$$

  where $F(x,t')$ is the inhomogeneity effect, which is a special case of Eq. (1) with $u=q$, $z=t'$, $t=x$, $A(z)=1$, $B=2$, $C(z,t)=-F(x,t')$ and $M_1(z)=Q_1=P=P_1=H_1(z,t)=C_1=0$. When $F(x,t')=x$, Eq. (2) is also applicable for the description of the low-frequency plasma dynamics in the case of resonant absorption of electromagnetic waves in fully ionized inhomogeneous plasmas [27] and soliton excitation by an incident electromagnetic wave in an inhomogeneous overdense plasma [28].
- In arterial mechanics [29], treating the arteries as thinwalled, linearly tapered, prestressed elastic tubes and blood as an incompressible viscous fluid, the propagation of weakly nonlinear waves in such a fluid-filled elastic tube with variable radiuses obeys the dissipative NLS equation with variable coefficients [26,30],

$$iU_\tau + \kappa_1 U_{\xi\xi} + \kappa_2|U|^2U + i\kappa_3\Theta\tau U_\xi + \big[\kappa_4\Theta^2\tau^2 + \kappa_5\Theta\xi - \kappa_6\Theta^2\tau\xi + i\kappa_7\big]U = 0, \tag{3}$$

  where U denotes the dynamical radial displacement upon such initial static deformation, τ and ξ are the stretched coordinates from the time and axial coordinates after static deformation. Θ accounts for the tapering angle, κ_1 and κ_2 are the arterial-system parameters, κ_3, κ_4, κ_5 and κ_6 stand for the contribution of variable radiuses, while κ_7 gives the contribution of dissipation resulting from the viscosity of the fluid. When $u=U$, $z=\tau$, $t=\xi$, $A(z)=\kappa_1$, $B=\kappa_2$, $H_1(z,t)=\kappa_3\Theta\tau$, $C(z,t)=\kappa_4\Theta^2\tau^2+\kappa_5\Theta\xi-\kappa_6\Theta^2\tau\xi$, $C_1=\kappa_7$ and $M_1(z)=Q_1=P=P_1=0$, Eq. (3) becomes a special case of Eq. (1).
- In view of various effects of higher-order, the ultrashort pulses propagating inside an optical fiber can be described by the following HNLS equation [22],

$$iu_z + \alpha_1 u_{tt} + \alpha_2|u|^2u + i\alpha_3 u + i\alpha_4 u_{ttt} + \alpha_5 u\big(|u|^2\big)_t + i\alpha_6\big(|u|^2u\big)_t = 0, \tag{4}$$

  where α_i $(i=1,2,\ldots,6)$ correspond to GVD, SPM, amplification or absorption coefficient, TOD, delayed nonlinear response effects and self-steepening, respectively. It can be found that Eq. (4) also is a special case of Eq. (1). As seen in Ref. [31], investigations on Eq. (4) for admitting soliton solutions under certain constraint conditions have become very fruitful.

Additionally, the Cauchy problem for Eq. (1) with $H_1(z,t)=C(z,t)=C_1=0$ has been considered [25]. The authors in Ref. [24] have investigated Eq. (1) by the Painlevé analysis method [32] and obtained the following integrable constraint conditions,

$$\begin{aligned} &P=0, \qquad Q_1=-2P_1=\sigma_1 B, \qquad M_1(z)=\sigma_1 A(z)/3,\\ &H_1(z,t)=\sigma_1 K(z)t + K_1(z), \qquad C(z,t)=K(z)t+K_2(z), \end{aligned} \tag{5}$$

where $K(z)=C_1/\sigma_1 + A'(z)/[2\sigma_1 A(z)]$ with σ_1 as an arbitrary constant, $K_1(z)$ and $K_2(z)$ are two arbitrary analytic functions. With this constraint conditions, a Lax pair (also called zero-curvature representation) for Eq. (1) has been presented in the 3×3 matrix form [24]. However, it is found that the entries of the principal diagonal in the 'time' part of the Lax pair are the same parameters, which implies that this linear eigenvalue problem is not suitable for constructing the Darboux transformation.

It is well known that the Lax pair plays a fundamental and important role in the soliton theory [33]. The Lax pair not only gives a scheme to solve the initial problem of a given NLEE through the inverse scattering method, but also is of vital importance in investigating the integrable properties of NLEEs such as the Hamiltonian structures, conservation laws, symmetry classes and Darboux transformations [2,23,34–39]. It is possible that there exist different Lax pairs for a given NLEE such as the modified Kadomtsev–Petviashvili equation [40,41], Gardner equation [42] and generalized dispersive long wave equation [40].

In this paper, utilizing the Ablowitz–Kaup–Newell–Segur (AKNS) procedure [34], a new Lax pair for Eq. (1) with constraints (5) will be constructed. Based on this Lax pair, we will further construct the Darboux transformation and derive the nth-iterated potential transformation formula for Eq. (1) by iterating the Darboux transformation n times. Through the one- and two-soliton-like solutions of Eq. (1), the graphical discussions about the features of ultrashort pulse propagating in inhomogeneous optical fibers will be addressed at last.

2. Symbolic computation on the Lax pair and Darboux transformation

The Darboux transformation method, as an effective and computerizable procedure, has been widely applied to a class of variable-coefficient NLEEs [14–17] to derive a series of analytic solutions including the multi-soliton solutions from an initial solution. As demonstrated in Refs. [34–39], an obvious advantage of the Darboux transformation lies in that the iterative algorithm is purely algebraic and very computerizable by virtue of symbolic computation. In order to construct abundant analytic soliton-like solutions for Eq. (1), we will firstly derive a new Lax pair for Eq. (1) under constraints (5) and then construct its Darboux transformation, which can give rise to a general procedure to recursively generate abundant analytic solutions.

2.1. Lax pair

According to constraints (5), it can be found that the linear eigenvalue problem for Eq. (1) is of the form [2,3,34–36]

$$\Phi_t = \mathbf{U}\Phi = (\lambda U_0 + U_1)\Phi, \qquad \Phi_z = \mathbf{V}\Phi = \left(\lambda^3 V_0 + \lambda^2 V_1 + \lambda V_2 + V_3\right)\Phi, \tag{6}$$

where $\Phi = (\phi_1, \phi_2, \phi_3)^T$, T denotes the transpose of the matrix, and λ is a spectral parameter, while the matrices U_i and V_j $(i = 1, 2;\ j = 1, 2, 3)$ can be presented as

$$U_0 = e^{-C_1 z}\sqrt{\frac{B}{2A(z)}}\begin{pmatrix} -i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{pmatrix}, \qquad U_1 = \sqrt{\frac{B}{2A(z)}}\begin{pmatrix} 0 & \rho u & \rho^* u^* \\ -\rho^* u^* & 0 & 0 \\ -\rho u & 0 & 0 \end{pmatrix}, \tag{7}$$

$$V_2 = \frac{1}{3}Be^{-C_1 z}\begin{pmatrix} r_1 + i\sigma_1\sqrt{B/[2A(z)]}|u|^2 & \rho r_2 & -\rho^* r_2^* \\ -\rho^* r_2^* & r_1^* & -\rho^{*2} r_3^* \\ \rho r_2 & \rho^2 r_3 & r_1^* \end{pmatrix}, \tag{8}$$

$$V_3 = \begin{pmatrix} 0 & \rho r_4 & \rho^* r_4^* \\ -\rho^* r_4^* & r_5 & 0 \\ -\rho r_4 & 0 & r_5^* \end{pmatrix}, \qquad V_0 = \frac{2}{3}Be^{-2C_1 z}\sigma_1 U_0, \qquad V_1 = \frac{2}{3}Be^{-2C_1 z}\sigma_1 U_1, \tag{9}$$

with

$$\rho = e^{-i\{\int[-2A(z) - 3\sigma_1 K_1(z) + 3\sigma_1^2 K_2(z)]\,dz + 3\sigma_1 t\}/(3\sigma_1^2)},$$

$$r_1 = \frac{3i}{2\sqrt{2B}A(z)^{3/2}\sigma_1}\left\{2A(z)^2 + 2\sigma_1 A(z)\left[C_1 t + K_1(z)\right] + \sigma_1 A'(z)t\right\} + i\sigma_1\sqrt{B/\left[2A(z)\right]}|u|^2,$$

$$r_2 = u + i\sigma_1 u_t, \qquad r_3 = -i\sigma_1\sqrt{B/\left[2A(z)\right]}u^2,$$

$$r_4 = \sqrt{B/\left[2A(z)\right]}\left[2iA(z)u_t - 2\sigma_1 B|u|^2 u - \sigma_1 A(z)u_{tt}\right]/3 - \frac{\sqrt{2B}}{12A(z)^{3/2}\sigma_1}\left\{4A(z)^2 + 6\sigma_1 A(z)\left[C_1 t + K_1(z)\right] + 3\sigma_1 A'(z)t\right\}u,$$

$$r_5 = \frac{B}{6}\left(\sigma_1 u^* u_t - \sigma_1 u u_t^* - 2i|u|^2\right),$$

where $K_1(z)$ and $K_2(z)$ have been explained before. It is easy to prove that the compatibility condition $\mathbf{U}_z - \mathbf{V}_t + [\mathbf{U}, \mathbf{V}] = 0$ can give rise to Eq. (1) with constraints (5).

2.2. Darboux transformation with symbolic computation

Using the knowledge of the Darboux transformation [36], we can construct the Darboux transformation for Eq. (1) as below,

$$\hat{\Phi} = \Gamma\Phi, \tag{10}$$

where Γ is a 3×3 nonsingular matrix to be determined. It requires that $\hat{\Phi}$ should also satisfy the linear eigenvalue problem (6), i.e.,

$$\hat{\Phi}_t = \hat{\mathbf{U}}\hat{\Phi} = (\lambda\hat{U}_0 + \hat{U}_1)\hat{\Phi}, \qquad \hat{\Phi}_z = \hat{\mathbf{V}}\hat{\Phi} = \left(\lambda^3\hat{V}_0 + \lambda^2\hat{V}_1 + \lambda\hat{V}_2 + \hat{V}_3\right)\hat{\Phi}, \tag{11}$$

where $\hat{U}_i$ and $\hat{V}_j$ $(i = 1, 2;\ j = 1, 2, 3)$ have the same forms as U_i and V_j except that the old potential $u(z, t)$ is replaced by the new potential $\hat{u}(z, t)$. In order to assure the invariance of the linear eigenvalue problem (6) under the Darboux transformation (10), we can get the following system

$$\Gamma_t + \Gamma\mathbf{U} - \hat{\mathbf{U}}\Gamma = 0, \tag{12}$$

$$\Gamma_z + \Gamma\mathbf{V} - \hat{\mathbf{V}}\Gamma = 0. \tag{13}$$

Based on the investigation in Refs. [36–39], the Darboux transformation can be presented as

$$\Gamma = \lambda I - S, \tag{14}$$

where I is the 3×3 identity matrix, S is a nonsingular matrix and its entries s_{ij} ($1 \leqslant i, j \leqslant 3$) are all analytic functions to be determined.

Combining expressions (12)–(14) and equating the coefficient matrices of the terms λ^i ($i = 0, 1, 2, 3, 4$) to be zero, give rise to

$$\hat{U}_0 = U_0, \qquad \hat{V}_0 = V_0, \tag{15}$$

$$\hat{U}_1 = U_1 + [U_0, S], \tag{16}$$

$$S_t + [S, U_0 S + U_1] = 0, \tag{17}$$

$$\hat{V}_j = V_j + \sum_{k=0}^{j-1} [V_{j-k}, S] S^k \quad (j = 1, 2, 3), \tag{18}$$

$$S_z + \left[S, V_0 S^3 + V_1 S^2 + V_2 S + V_3\right] = 0. \tag{19}$$

From expression (16), we can derive

$$\hat{u} = -2i s_{12} \rho^{-1} e^{-C_1 z} + u, \tag{20}$$

$$s_{31} = s_{12}, \qquad s_{21} = s_{13} = -s_{12}^*, \tag{21}$$

where expression (20) denotes the relation between the old potential and new one. In view of the set of strict constraints on the entries of S matrix, the nonsingular matrix S can be taken as the following form

$$S = H \Lambda H^{-1}, \tag{22}$$

with

$$H = \begin{pmatrix} \phi_{11}(\lambda_1) & -\phi_{21}(\lambda_1) & 0 \\ \phi_{21}(\lambda_1) & \phi_{11}(\lambda_1) & -\phi_{11}(\lambda_1) \\ \phi_{31}(\lambda_1) & 0 & \phi_{11}(\lambda_1) \end{pmatrix}, \qquad \Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_1^* & 0 \\ 0 & 0 & \lambda_1^* \end{pmatrix}, \tag{23}$$

where $Re(\lambda_1) = 0$, $\phi_{31}(\lambda_1) = \phi_{21}(\lambda_1)$ and the first column of above matrix is the real vector solution of the Lax pair for the initial potential $u(z,t)$ with $\lambda = \lambda_1$. It can be found that the construction of the Darboux transformation for the 3×3 Lax pair (6) is consistent with that in Ref. [3].

With the aid of symbolic computation, it is easy to verify that expressions (21) is satisfied automatically and the identity of expressions (17)–(19) can also be proved. Therefore, the relation between the new potential $\hat{u}(z,t)$ and old potential $u(z,t)$, i.e., expression (20), turns out to be

$$\hat{u} = u + \frac{4e^{-C_1 z}}{\rho} \frac{Im(\lambda_1)\phi_{11}(\lambda_1)\phi_{21}(\lambda_1)}{\phi_{11}^2(\lambda_1) + 2\phi_{21}^2(\lambda_1)}, \tag{24}$$

from which, we know that a series of analytic solutions including the soliton-like solutions for Eq. (1) under constraints (5) can be derived by solving the linear eigenvalue problem (6) with an initial potential and performing tedious but not complicated algebraic operations [37–39].

In illustration, substituting the initial solution $u(z,t) = 0$ into Lax pair (6), we can obtain

$$\hat{u} = \sqrt{2}\kappa_1 e^{-C_1 z + i\{\int[-2A(z) - 3\sigma_1 K_1(z) + 3\sigma_1^2 K_2(z)]\,dz + 3\sigma_1 t\}/(3\sigma_1^2)} \operatorname{sech}\theta, \tag{25}$$

with

$$\theta = \sqrt{\frac{2B}{A(z)}}\kappa_1 e^{-C_1 z} t - \frac{1}{3\sigma_1}\int \sqrt{\frac{2B}{A(z)}}\kappa_1 e^{-C_1 z}\left[2B\kappa_1^2\sigma_1^2 e^{-2C_1 z} + 3\sigma_1 K_1(z) + 3A(z)\right] dz,$$

where $\lambda_1 = i\kappa_1$ and κ_1 is an arbitrary real constant. It can be seen that the optical pulse width and soliton amplitude are both related to the amplification or absorption coefficient C_1, and the velocity of the ultrashort optical soliton is associated with C_1, $A(z)$ and $K_1(z)$.

When $n = 2$, the two-soliton-like solution for Eq. (1) can be presented as

$$u(z,t) = \sqrt{2}e^{-C_1 z + i\{\int[-2A(z) - 3\sigma_1 K_1(z) + 3\sigma_1^2 K_2(z)]\,dz + 3\sigma_1 t\}/(3\sigma_1^2)} \times \frac{[\kappa_1 \cosh(2\varphi_2) - \kappa_2 \cosh(2\varphi_1)](\kappa_1^2 - \kappa_2^2)}{-2\kappa_1\kappa_2[\sinh(2\varphi_1)\sinh(2\varphi_2) + 1] + \cosh(2\varphi_1)\cosh(2\varphi_2)(\kappa_1^2 + \kappa_2^2)}, \tag{26}$$

with

$$\varphi_j = \sqrt{\frac{B}{2A(z)}}\kappa_j e^{-C_1 z} t - \frac{1}{6\sigma_1}\int \sqrt{\frac{2B}{A(z)}}\kappa_j e^{-C_1 z}\big[2B\kappa_j^2\sigma_1^2 e^{-2C_1 z} + 3\sigma_1 K_1(z) + 3A(z)\big]\,dz,$$

where κ_j are the imaginary part of spectral parameters λ_j ($j = 1, 2$). It is shown that the pulse width and group velocity of each soliton in the two-soliton-like solution are both related to the variable-coefficient parameters in the ultrashort soliton control system, which implies that abundant ultrashort soliton structures can be obtained by adjusting these distributed parameters.

Analogously, we can also derived the nth-iterated potential transformation formula through continuously iterating the Darboux transformation

$$u_n = u + \frac{4e^{-C_1 z}}{\rho}\sum_{j=1}^{n}\frac{Im(\lambda_j)\phi_{1,j}(\lambda_j)\phi_{2,j}(\lambda_j)}{A_j}, \tag{27}$$

with

$$\phi_{m,j+1} = \big(\lambda_{j+1} - \lambda_j^*\big)\phi_{m,j}(\lambda_{j+1}) - \frac{B_j}{A_j}\big(\lambda_j - \lambda_j^*\big)\phi_{m,j}(\lambda_j),$$
$$A_j = \phi_{1,j}^2(\lambda_j) + 2\phi_{2,j}^2(\lambda_j),$$
$$B_j = \phi_{1,j}(\lambda_j)\phi_{1,j}(\lambda_{j+1}) + 2\phi_{2,j}(\lambda_j)\phi_{2,j}(\lambda_{j+1}),$$

where $m = 1, 2, 3$, $\phi_{3,j}(\lambda_j) = \phi_{2,j}(\lambda_j)$ and $[\phi_{1,j}(\lambda_j), \phi_{2,j}(\lambda_j), \phi_{3,j}(\lambda_j)]^T$ corresponds to the real vector solution of the Lax pair with $\lambda_j = i\kappa_j$ ($\kappa_i \neq \kappa_j$; $i \neq j$) for potential $u_{j-1}(z,t)$ ($j = 1, 2, 3, \ldots, n$). In conclusion, we have constructed the Darboux transformation for Eq. (1) with constraints (5) and further obtained the relationship between the new and old potentials. Consequently, the multi-soliton-like solutions of Eq. (1) can be explicitly derived through the general procedure presented by expression (27).

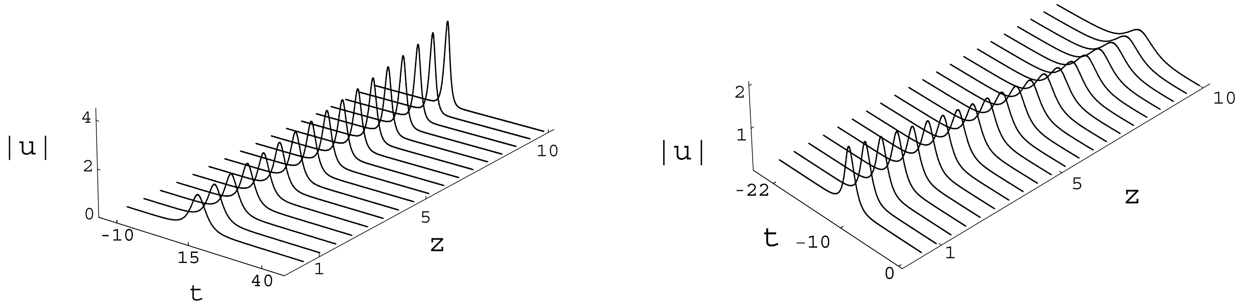
3. Discussions

For the ultrashort optical soliton propagating in a realistic optical fiber, in order to simulate the particular applications and analyze physical properties of these multi-soliton-like solutions of Eq. (1) often requires an advisable choice of the free parameters and functions involved in the ultrashort soliton control systems such as the GVD, TOD, SPM, self-steepening and amplification (absorption) parameters. In the following picture drawing and qualitative analysis, we can choose some special values for those parameters in line with the nonuniform backgrounds of the variable-coefficient HNLS model under investigation.

In virtue of these various variable coefficients and functions in expressions (25) and (26), abundant ultrashort optical soliton structures can be presented through appropriately adjusting these distributed parameters. From expression (25), we know that the soliton amplitude is invariant with $C_1 = 0$. However, when $C_1 \neq 0$, the amplification or absorption coefficient has a great effect on the wave profile [43]. As studied in the realistic optical systems, based on the realization of the decreasing GVD in a fiber [43], we can choose the parameters like GVD, nonlinearity and TOD according to the results in Refs. [14,17].

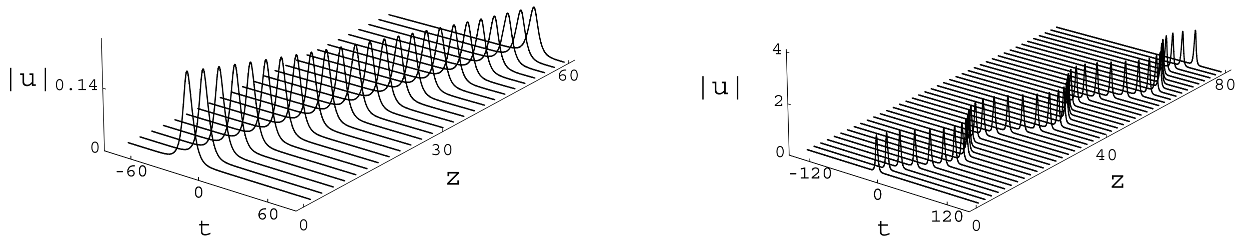
For expression (25), if parameter $C_1 < 0$, the fiber media is dispersion increasing, and $C_1 > 0$ corresponds to dispersion decreasing case, which can be clearly caught from Fig. 1. Due to the influence of the amplification (absorption) coefficient C_1, it is shown that the soliton amplitude exponentially grows (attenuates) with the velocity $v = e^{0.1z}$ ($v = e^{-0.1z}$) (see Fig. 1). Otherwise, the optical soliton width exponentially compresses (expands) in Fig. 1(a) (Fig. 1(b)). Fig. 2(a), with proper parameters and functions chosen as listed in its caption, exhibits a special case of the compress optical pulse with changeless width. Fig. 2(b) displays the effect of coefficient function $K_1(z)$ on the one-soliton pulse solution surface without amplification or absorption coefficient, from which we can see that the velocity is periodically variable with the pulse soliton propagating in the fiber media.

Considering the effect of C_1 and $K_1(z)$ synchronously, Fig. 3 provides us with the visual evolution plot of expression (25) with different signs of the parameter C_1. For expression (26), abundant ultrashort two-soliton pulses structures can also be presented. For instance, when choosing the coefficient $K_1(z)$ with different values, Fig. 4 shows the elastic interactions between two ultrashort optical pulse solitons.



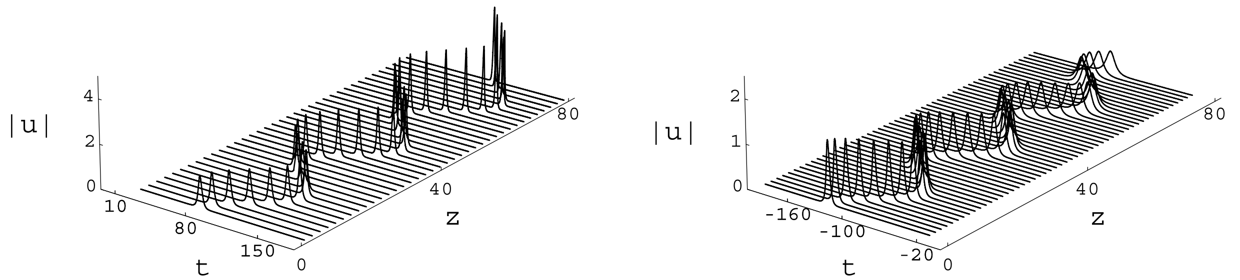
(a) (b)

Fig. 1. The evolution plot of one pulse soliton given by expression (25) with $A(z) = \kappa_1 = \sigma_1 = 1$ and $K_1(z) = K_2(z) = 0$. (a) $B = 0.1$ and $C_1 = -0.1$; (b) $B = 1$ and $C_1 = 0.1$.



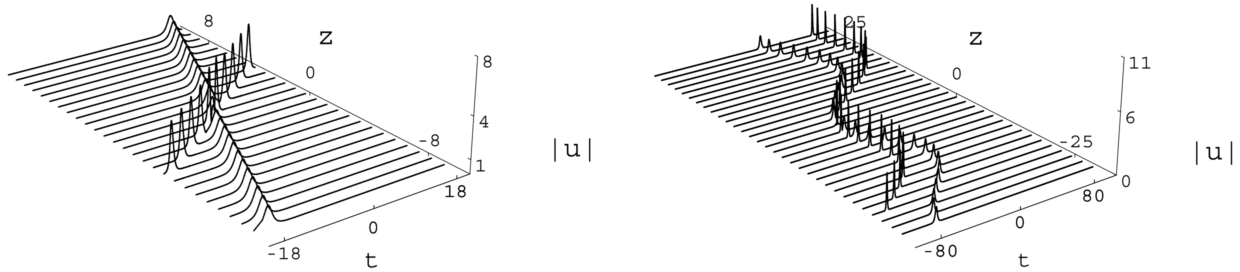
(a) (b)

Fig. 2. The evolution plots show the effect of $A(z)$ and $K_1(z)$ on the ultrashort pulse soliton profile. (a) $\sigma_1 = B = K_1(z) = 1$, $\kappa_1 = \sqrt{2}/10$, $C_1 = 0.01$, $A(z) = e^{-2C_1 z}/2$, and $K_2(z) = e^{-2C_1 z}/(3\sigma_1^2)$; (b) $A(z) = \kappa_1 = \sigma_1 = 1$, $B = 0.2$, $C_1 = K_2(z) = 0$ and $K_1(z) = 5\sin(0.2z)$.



(a) (b)

Fig. 3. The evolution plot of one ultrashort pulse soliton given by expression (25) with periodic influence for $B = 0.2$, $K_1(z) = 5\sin(0.2z)$, $A(z) = \kappa_1 = \sigma_1 = 1$ and $K_2(z) = 0$. (a) $C_1 = -0.01$; (b) $C_1 = 0.01$.



(a) (b)

Fig. 4. The evolution plot of two ultrashort pulse solitons via expression (26) with $k_1 = 1$, $k_2 = 2.5$, $C_1 = K_2(z) = 0$ and $\sigma_1 = A(z) = 1$. (a) $B = 1$ and $K_1(z) = 0$; (b) $B = 0.6$ and $K_1(z) = 5\sin(0.2z)$.

4. Conclusions

In this paper, we have symbolically studied the integrable properties of a generalized variable-coefficient HNLS model describing the propagation of ultrashort pulse in the real inhomogeneous fiber. Through employing the Ablowitz–Kaup–Newell–Segur procedure, a 3×3 Lax pair for this equation has been obtained under the corresponding Painlevé constraint condition. It is shown that the above Lax pair is different with that in the existing literature [24], which is not suitable for constructing the Darboux transformation. Based on the 3×3 Lax pair presented in this paper, the multi-soliton-like solutions for Eq. (1) have been obtained by the Darboux transformation method. The corresponding potential applications in the inhomogeneous ultrashort soliton control system have been graphically discussed by the one- and two-soliton-like solutions through controlling the GVD, TOD, SPM, self-steepening and amplification (absorption) parameters.

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