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Corrigendum

Corrigendum to "Global existence and blow-up for a mixed pseudo-parabolic p-Laplacian type equation with logarithmic nonlinearity" [J. Math. Anal. Appl. 478 (2019) 393–420]

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A B S T R A C T

We correct the proof of the infinite time blow-up of the solutions for a class of mixed pseudo-parabolic p-Laplacian type equation with logarithmic nonlinearity studied in [2].

1. Introduction and main results

As is well-known, in studying the infinite time blow-up of solutions, the first thing we need to do is to extend the existence time of solutions to infinity. In the proof of [2, Theorem 2], we proved this thing by showing the solutions cannot blow up in finite time. However, by J. M. Ball' paper (see [1]), the existence time of solutions may not be extended to infinity even if they cannot blow up in finite time. The main purpose of this paper is to fill this gap.

Theorem 1. *Assume [2, (1.3)] holds. Let $u = u(t)$ be a local weak solution to problem [2, (1.1)] with initial data $u_0 \in H_0^1(\Omega)$ satisfying $J(u_0) < d$ and $I(u_0) < 0$. Then u can be extended over time (the whole half line) and u blows up at $+\infty$, i.e., $\lim_{t\to+\infty} \|u(t)\|_{H_0^1} = +\infty$, where the definitions of J, I and d can be found in [2, (2.1)], [2, (2.2)], and [2, (2.4)] respectively.*

Proof. Firstly, we show u can be extended over time (the whole half line). By [1], we only need to prove that there exists a function $\varrho : [0, +\infty) \to [0, +\infty)$ such that $\varrho(t) < +\infty$ for all $t \in [0, +\infty)$ and $\|u(t)\|_{H_0^1} \le \varrho(t)$

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for all $t \in [0,T)$, where T is the maximal existence time of u. Since $J(u_0) < d$ and $I(u_0) < 0$, it follows from the proof of [2, Theorem 2] that $I(u(t)) < 0$ for $t \in [0,T)$. Let $G(t) = \int_0^t \|u(\tau)\|^2_{H_0^1(\Omega)} d\tau$. Similar to [2, (4.29)] and [2, (4.30)], we obtain

$$G'(t) = \|u(t)\|^2_{H_0^1} = \|u(t)\|_2^2 + \|\nabla u(t)\|_2^2 > 0, \quad G''(t) = -2I(u(t)) > 0, \quad t \in [0,T). \tag{1.1}$$

Since $1 < q \le 2$, we consider this proof for two situations: (i) $1 < q < 2$ and (ii) $q = 2$.

(i) When $1 < q < 2$, we can divide the proof into two cases (note that $G''(t) > 0$ for $t \in [0,T)$): (1) For all $t \in [0,T)$, there holds $G'(t) < e^{\frac{2}{2-q}}$; (2) There exists a $t_2 \in [0,T)$ such that $G'(t) \ge e^{\frac{2}{2-q}}$ for all $t \in [t_2,T)$. If (1) holds, let $\varrho(t) = e^{\frac{1}{2-q}}$ for $t \in [0,+\infty)$, then we know u can be extended over time (the whole half line). If (2) holds, since for any $b > 0$, it holds (note that $1 < q < 2$) $\log b \le \frac{1}{2-q} b^{2-q}$. Then we know $I(u) \ge \|\nabla u\|_p^p - \frac{1}{2-q}\|u\|_2^2$. Since $G'(t) \ge e^{\frac{2}{2-q}}$ for $t \in [t_2,T)$, we get $\log G'(t) \ge \frac{2}{2-q}$ for $t \in [t_2,T)$. Then by (1.1), we obtain

$$\begin{aligned} G'(t)\log G'(t) - G''(t) \ge & \frac{2}{2-q}\|u(t)\|^2_{H_0^1} + 2I(u(t)) \\ \ge & \frac{2}{2-q}\|u(t)\|_2^2 + 2\|\nabla u(t)\|_p^p - \frac{2}{2-q}\|u(t)\|_2^2 \ge 0, \quad t \in [t_2,T), \end{aligned}$$

which implies $(\log G'(t))' \le \log G'(t)$ for $t \in [t_2,T)$. Since $\log G'(t) > 0$, by Gronwall's inequality, we get $\log G'(t) \le e^{t-t_2}\log G'(t_2)$ for $t \in [t_2,T)$, i.e., $\|u(t)\|^2_{H_0^1} \le \|u(t_2)\|^{2e^{t-t_2}}_{H_0^1}$ for $t \in [t_2,T)$. Let $\varrho(t) = \|u(t)\|_{H_0^1}$ if $0 \le t < t_2$; and $\varrho(t) = \|u(t_2)\|^{e^{t-t_2}}_{H_0^1}$ if $t_2 \le t < +\infty$. Then we know u can be extended over time (the whole half line).

(ii) When $q = 2$, we can also divide the proof into two cases: (3) For all $t \in [0,T)$, there holds $G'(t) < e$; (4) There exists a $t_2 \in [0,T)$ such that $G'(t) \ge e$ for all $t \in [t_2,T)$. If (3) holds, let $\varrho(t) = \sqrt{e}$ for $t \in [0,+\infty)$, then we know u can be extended over time (the whole half line). If (4) holds, by (1.1), the definition of $I(u)$, logarithmic Sobolev inequality (see [2, (3.5)]), and $G'(t) = \|u(t)\|^2_{H_0^1} \ge e$ for all $t \in [t_2,T)$, we get

$$\begin{aligned} G'(t)\log G'(t) - G''(t) = & \|u(t)\|^2_{H_0^1}\log\|u(t)\|^2_{H_0^1} + 2\|\nabla u(t)\|_p^p - 2\int_\Omega u^2(t)\log|u(t)|dx \\ \ge & \|u(t)\|_2^2 \log\|u(t)\|^2_{H_0^1} + \|\nabla u(t)\|_2^2 \log\|u(t)\|^2_{H_0^1} \\ & - \|u(t)\|_2^2\log\|u(t)\|_2^2 + n\left(1+\log a\right)\|u(t)\|_2^2 - \frac{a^2}{\pi}\|\nabla u(t)\|_2^2 \\ \ge & \left(1 - \frac{a^2}{\pi}\right)\|\nabla u(t)\|_2^2 + n\left(1+\log a\right)\|u(t)\|_2^2, \quad t \in [t_2,T). \end{aligned}$$

Taking $a = \sqrt{\pi}$ in the above inequality, then there holds $G'(t)\log G'(t) - G''(t) \ge 0$ for $t \in [t_2,T)$, which implies $(\log G'(t))' \le \log G'(t)$ for $t \in [t_2,T)$. Since $\log G'(t) > 0$ (note $G'(t) \ge e$ for all $t \in [t_2,T)$), by Gronwall's inequality, we get $\log G'(t) \le e^{t-t_2}\log G'(t_2)$ for $t \in [t_2,T)$, i.e., $\|u(t)\|^2_{H_0^1} \le \|u(t_2)\|^{2e^{t-t_2}}_{H_0^1}$ for $t \in [t_2,T)$. Let $\varrho(t) = \|u(t)\|_{H_0^1}$ if $0 \le t < t_2$; and $\varrho(t) = \|u(t_2)\|^{e^{t-t_2}}_{H_0^1}$ if $t_2 \le t < +\infty$. Then we know u can be extended over time (the whole half line).

Secondly, we show u blows up at infinity. Since u can be extended over time (the whole half line), by [2, (4.31)], we know $\|u(t)\|^2_{H_0^1} \ge \|u_0\|^2_{H_0^1} + C_0 t$ for $t \in [0,+\infty)$, which implies that u blows up at $+\infty$ directly. □

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