



Analyticity of hybrid systems arising in visco and thermo elastic structures



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ABSTRACT

We consider two systems arising in a two-dimensional viscoelastic fluid-structure interaction, one of them coupled with a wave equation with interior damping of Kelvin–Voigt type. In the second model, the acoustic vibrations of the viscous fluid which fills the two-dimensional interior cavity are coupled with the mechanical vibrations of a one-dimensional thermoelastic beam. Therefore, the systems are related to the problem of the active control of noise in a cavity. Our main results are that the corresponding semigroups are analytic. In particular we show the exponential stability of the models.

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1. Introduction

In this paper we are concerned with the analyticity and the asymptotic behavior of models describing the interaction of acoustic vibrations with mechanical deformations, constituting the so called hybrid systems, since vibrations of different nature interact. Actually the acoustic vibrations of the viscous fluid which fills a two-dimensional interior cavity are coupled with the mechanical vibrations of an elastic structure located on the boundary of the cavity. Such models have as one of their main applications the problem of the active control of noise, as introduced in [3].

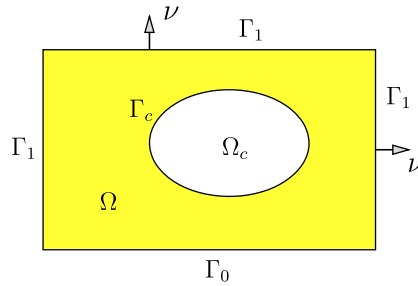
In order to state each of the two problems to be analyzed, let $\Omega_1 = (0, \ell) \times (0, \ell_1)$ be a two-dimensional rectangle with boundary $\partial\Omega_1 = \Gamma_0 \cup \Gamma_1$, where

$$\Gamma_1 = \{(0, y); y \in (0, \ell_1)\} \cup \{(x, \ell_1); x \in (0, \ell)\} \cup \{(\ell, y); y \in (0, \ell_1)\}$$

and

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Fig. 1. Two-dimensional structure Ω .

$$\Gamma_0 = \{(x, 0); x \in (0, \ell)\}.$$

Also we consider Ω_c be an open set of class C^2 such that $\Omega_c \subset \Omega_1$, with bounded boundary Γ_c , and $\Omega := \Omega_1 \setminus \overline{\Omega_c}$. Note that the boundary $\partial\Omega$ of Ω is given by

$$\partial\Omega = \Gamma_0 \cup \Gamma_1 \cup \Gamma_c.$$

A typical example of such a domain Ω is sketched in the Fig. 1.

The first model we analyze is

$$\rho\varphi_{tt} - \nabla \cdot (\alpha \nabla \varphi + \gamma_0 \nabla \varphi_t) = 0 \quad \text{in } \Omega \times (0, \infty) \quad (1.1)$$

$$\varphi = 0 \quad \text{on } \Gamma_c \times (0, \infty) \quad (1.2)$$

$$\frac{\partial \varphi}{\partial \nu} + \frac{\gamma_0}{\alpha} \frac{\partial \varphi_t}{\partial \nu} = 0 \quad \text{on } \Gamma_1 \times (0, \infty) \quad (1.3)$$

$$\frac{\partial \varphi}{\partial \nu} + \frac{\gamma_0}{\alpha} \frac{\partial \varphi_t}{\partial \nu} = w_t \quad \text{on } \Gamma_0 \times (0, \infty) \quad (1.4)$$

$$w_{tt} - (w + \gamma_1 w_t)_{xx} + \alpha \varphi_t = 0 \quad \text{on } \Gamma_0 \times (0, \infty) \quad (1.5)$$

$$w(0, t) = w(\ell, t) = 0 \quad \text{for } t > 0 \quad (1.6)$$

with initial conditions

$$\varphi(\mathbf{x}, 0) = \varphi_0(\mathbf{x}), \quad \varphi_t(\mathbf{x}, 0) = \varphi_1(\mathbf{x}) \quad \text{in } \Omega \quad (1.7)$$

$$w(x, 0) = w_0(x), \quad w_t(x, 0) = w_1(x) \quad \text{on } \Gamma_0 \quad (1.8)$$

where ν denotes the unit outward normal to Ω and ρ , α , γ_0 and γ_1 are positive constants.

As quoted in [3], [7], [9], [10] and references therein, the model (1.1)–(1.6) describes acoustic wave motion in a viscous fluid, with small amplitude. The boundary Γ_1 of the cavity Ω is taken to be the “hard” wall and Γ_0 the viscoelastic bottom of the cavity. The total energy associated to this system is given by

$$E(t) = \frac{1}{2} \int_{\Omega} \left(\rho |\varphi_t|^2 + \alpha |\nabla \varphi|^2 \right) d\mathbf{x} + \frac{1}{2} \int_{\Gamma_0} \left(|w_t|^2 + |w_x|^2 \right) dx$$

and straightforward calculation gives us

$$\frac{d}{dt} E(t) = -\gamma_0 \int_{\Omega} |\nabla \varphi_t|^2 d\mathbf{x} - \gamma_1 \int_{\Gamma_0} |w_{xt}|^2 dx \leq 0,$$

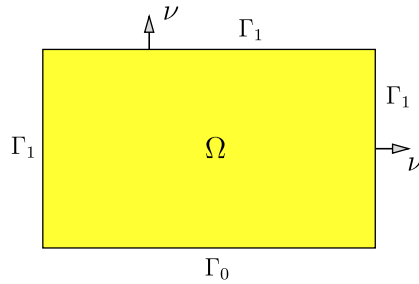


Fig. 2. The domain Ω for second problem.

which shows dissipative nature of the model. One of the aims of this paper is to prove that the semigroup associated to (1.1)–(1.8) is analytic.

The second problem we consider is over a rectangle $\Omega = (0, \ell) \times (0, \ell_1)$ with boundary $\partial\Omega = \Gamma_0 \cup \Gamma_1$, where

$$\Gamma_1 = \{(0, y); y \in (0, \ell_1)\} \cup \{(x, \ell_1); x \in (0, \ell)\} \cup \{(\ell, y); y \in (0, \ell_1)\}$$

and

$$\Gamma_0 = \{(x, 0); x \in (0, \ell)\},$$

as shown in the Fig. 2.

Over Ω we consider the following hybrid system:

$$\rho\varphi_{tt} - \nabla \cdot (\alpha \nabla \varphi + \gamma_0 \nabla \varphi_t) = 0 \quad \text{in } \Omega \times (0, \infty) \quad (1.9)$$

$$\varphi = 0 \quad \text{on } \Gamma_1 \times (0, \infty) \quad (1.10)$$

$$\frac{\partial \varphi}{\partial \nu} + \frac{\gamma_0}{\alpha} \frac{\partial \varphi_t}{\partial \nu} = w_t \quad \text{on } \Gamma_0 \times (0, \infty) \quad (1.11)$$

$$w_{tt} + w_{xxxx} + \beta \theta_{xx} + \alpha \varphi_t = 0 \quad \text{on } \Gamma_0 \times (0, \infty) \quad (1.12)$$

$$\theta_t - \kappa \theta_{xx} - \beta w_{xxt} = 0 \quad \text{on } \Gamma_0 \times (0, \infty) \quad (1.13)$$

$$w(0, t) = w(\ell, t) = w_x(0, t) = w_x(\ell, t) = \theta(0, t) = \theta(\ell, t) = 0 \quad \text{for } t > 0 \quad (1.14)$$

where ν denotes the unit outward normal to Ω and ρ , α , γ_0 , β and κ are positive constants. The system satisfies the following initial conditions:

$$\varphi(\mathbf{x}, 0) = \varphi_0(\mathbf{x}), \quad \varphi_t(\mathbf{x}, 0) = \varphi_1(\mathbf{x}) \quad \text{in } \Omega \quad (1.15)$$

$$w(x, 0) = w_0(x), \quad w_t(x, 0) = w_1(x), \quad \theta(x, 0) = \theta_0(x) \quad \text{on } \Gamma_0. \quad (1.16)$$

Note that for this second problem another type of mechanism is interacting with the acoustic vibrations. Indeed, a thermoelastic beam is located in Γ_0 , the bottom of the cavity Ω , instead of a string with Kelvin–Voigt viscous damping. The total energy associated to this system is given by

$$\mathcal{E}(t) = \frac{1}{2} \int_{\Omega} \left(\rho |\varphi_t|^2 + \alpha |\nabla \varphi|^2 \right) d\mathbf{x} + \frac{1}{2} \int_{\Gamma_0} \left(|w_t|^2 + |w_{xx}|^2 + |\theta|^2 \right) dx.$$

It is easy to see that

$$\frac{d\mathcal{E}}{dt}(t) = -\gamma_0 \int_{\Omega} |\nabla \varphi_t|^2 d\mathbf{x} - \kappa \int_{\Gamma_0} |\theta_x|^2 dx \leq 0,$$

which ensures dissipative nature of the system. Our second main result is to show that the semigroup associated to (1.9)–(1.16) is also analytic.

The asymptotic behavior of solutions to hybrid acoustic systems was studied by Hansen and Zuazua [5] and Littman and Markus [7]. The authors proved for inviscid fluid the lack of exponential stability. Avalos and Lasiecka [2] consider a coupled system of hyperbolic and parabolic (Kelvin-Voigt type) PDE's arising in a given fluid / structure interaction. They proved the strong stability of the system. This result was improved by Avalos [1], where the author, assuming some geometrical conditions, shows the exponential stability of the system. Micu and Zuazua [10–12] also obtained results of boundary controllability and asymptotic behavior for the spectrum of hybrid systems arising in the control of noise.

The analyticity of the semigroup associated to fluid structure interactions models has been considered by R. Triggiani and J. Zhang [14], there the authors provide a first illustration of a heat-visco-elastic physically 2-dimensional plate interaction model, with *low*, physically hinged coupled boundary conditions on the plate at the interface, involving only the boundary bending moment operator. That is to say, the coupling occurs only at the interface between the two media, where each components evolves.

It seems to us that the analyticity of hybrid acoustic systems, similar to (1.1)–(1.6) and (1.9)–(1.14), was not considered before. So to fill this gap we study this topic here. The main result of this paper is to show the analyticity of the models, which in particular implies on three important properties. The first is the smoothing effect property over the initial data, that is no matter how irregular the initial data is, the solutions of both models are very smooth in a positive time. The second property is that the systems are exponentially stable. Finally, the systems enjoy the linear stability property, which means that the type of the semigroup is equals to the spectral bound of its infinitesimal operator.

The rest of the paper is organized as follows. In section 2, we show the well-posedness as well as the analyticity of model (1.1)–(1.8). In section 3, we show the same to model (1.9)–(1.16).

2. Analyticity of system (1.1)–(1.8)

To establish the well posedness of the model using the semigroup approach we denote

$$H_*^1(\Omega) = \{\varphi \in H^1(\Omega); \varphi = 0 \text{ on } \Gamma_c\}$$

and introduce the space of finite energy corresponding to (1.1)–(1.8) by

$$\mathcal{H} = H_*^1(\Omega) \times L^2(\Omega) \times H_0^1(\Gamma_0) \times L^2(\Gamma_0),$$

which is a Hilbert space with the norm

$$\|U\|_{\mathcal{H}}^2 = \int_{\Omega} \left(\alpha |\nabla \varphi|^2 + \rho |\psi|^2 \right) d\mathbf{x} + \int_{\Gamma_0} \left(|w_x|^2 + |v|^2 \right) dx, \quad (2.1)$$

for all $U = (\varphi, \psi, w, v) \in \mathcal{H}$. The coupling term that appears in (1.5) should be understood in the context of trace map, that is the map

$$\Upsilon : H^1(\Omega) \rightarrow H^{\frac{1}{2}}(\Gamma_0), \quad \Upsilon \psi = \begin{cases} \psi|_{\Gamma_0} & \text{on } \Gamma_0 \\ 0 & \text{on } \Gamma \setminus \Gamma_0 \end{cases}.$$

Let $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \rightarrow \mathcal{H}$ be the unbounded linear operator given by

$$\mathcal{A}(\varphi, \psi, w, v) = \left(\psi, \frac{1}{\rho} \nabla \cdot (\alpha \nabla \varphi + \gamma_0 \nabla \psi), v, (w + \gamma_1 v)_{xx} - \alpha \Upsilon \psi \right)$$

and

$$\mathcal{D}(\mathcal{A}) = \left\{ U = (\varphi, \psi, w, v) \in \mathcal{H}; \psi \in H_*^1(\Omega), v \in H_0^1(\Gamma_0), \frac{\partial \varphi}{\partial \nu} + \frac{\gamma_0}{\alpha} \frac{\partial \psi}{\partial \nu} = 0 \text{ on } \Gamma_1, \right. \\ \left. \alpha \varphi + \gamma_0 \psi \in H^2(\Omega), w + \gamma_1 v \in H^2(\Gamma_0), \frac{\partial \varphi}{\partial \nu} + \frac{\gamma_0}{\alpha} \frac{\partial \psi}{\partial \nu} = v \text{ on } \Gamma_0 \right\}.$$

As usual, we put $\Upsilon \psi = \psi|_{\Gamma_0}$ or $\Upsilon \psi = \psi$. With the previous definitions, system (1.1)–(1.8) is equivalent to the abstract Cauchy problem

$$U_t = \mathcal{A}U, \quad U(0) = U_0, \quad (2.2)$$

where $U_0 = (\varphi_0, \varphi_1, w_0, w_1) \in \mathcal{D}(\mathcal{A})$ and $U(t) = (\varphi(t), \varphi_t(t), w(t), w_t(t)) \in \mathcal{D}(\mathcal{A})$, $t > 0$.

Lemma 2.1. *The operator \mathcal{A} generates a C_0 -semigroup of contractions on \mathcal{H} .*

Proof. Let us first note that \mathcal{A} is dissipative. Indeed, for any $U = (\varphi, \psi, w, v) \in \mathcal{D}(\mathcal{A})$,

$$(\mathcal{A}U, U)_{\mathcal{H}} = \alpha \int_{\Omega} (\nabla \psi \cdot \nabla \bar{\varphi} - \nabla \varphi \cdot \nabla \bar{\psi}) \, d\mathbf{x} - \gamma_0 \int_{\Omega} |\nabla \psi|^2 \, d\mathbf{x} \\ + \alpha \int_{\Gamma_0} (v \bar{\psi} - \psi \bar{v}) \, dx + \int_{\Gamma_0} (v_x \bar{w}_x - w_x \bar{v}_x) \, dx - \gamma_1 \int_{\Gamma_0} |v_x|^2 \, dx,$$

that is,

$$\operatorname{Re}(\mathcal{A}U, U)_{\mathcal{H}} = -\gamma_0 \int_{\Omega} |\nabla \psi|^2 \, d\mathbf{x} - \gamma_1 \int_{\Gamma_0} |v_x|^2 \, dx \leq 0. \quad (2.3)$$

Now we show that $0 \in \varrho(\mathcal{A})$ (the resolvent set of \mathcal{A}). That is, for any $\mathfrak{F} \in \mathcal{H}$, there exists only one solution $U \in \mathcal{D}(\mathcal{A})$ to $\mathcal{A}U = \mathfrak{F}$. In terms of the components,

$$\begin{cases} \psi = f_1 \\ \nabla \cdot (\alpha \nabla \varphi + \gamma_0 \nabla \psi) = \rho f_2 \\ v = f_3 \\ (w + \gamma_1 v)_{xx} - \alpha \psi = f_4, \end{cases} \quad (2.4)$$

where $\mathfrak{F} = (f_1, f_2, f_3, f_4) \in \mathcal{H}$ and φ, ψ, w and v satisfying the required boundary conditions. The variational formulation of (2.4) over $\mathcal{V} := H_*^1(\Omega) \times H_0^1(\Gamma_0)$ is given by

$$\alpha \int_{\Omega} \nabla \varphi \cdot \nabla \phi \, d\mathbf{x} + \int_{\Gamma_0} w_x u_x \, dx = -\rho \int_{\Omega} f_2 \phi \, d\mathbf{x} - \gamma_0 \int_{\Omega} \nabla f_1 \cdot \nabla \phi \, d\mathbf{x} + \alpha \int_{\Gamma_0} f_3 \phi \, dx \\ - \gamma_1 \int_{\Gamma_0} f_{3,x} u_x \, dx - \int_{\Gamma_0} (\alpha f_1 + f_4) u \, dx, \quad (2.5)$$

for all $(\phi, u) \in \mathcal{V}$.

The left hand side of (2.5) defines a continuous and coercive bilinear form $a : \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}$ while the right hand side defines a continuous linear form $F : \mathcal{V} \rightarrow \mathbb{R}$. As a consequence of the Lax–Milgram’s Lemma it

follows that (2.5) has a unique solution $(\varphi, w) \in \mathcal{V}$. Setting $\psi = f_1$, $v = f_3$ and manipulating identity (2.5) we can conclude that $U = (\varphi, \psi, w, v) \in \mathcal{D}(\mathcal{A})$ and satisfies (2.4). Moreover,

$$\|(\varphi, \psi, w, v)\|_{\mathcal{H}} \leq C\|(f_1, f_2, f_3, f_4)\|_{\mathcal{H}}. \quad \square$$

As a consequence of the above Lemma we have

Theorem 2.2. *Given $U_0 \in \mathcal{H}$ there exists a unique weak solution U to the problem (2.2) satisfying*

$$U \in C([0, +\infty), \mathcal{H}).$$

Furthermore, if $U_0 \in \mathcal{D}(\mathcal{A}^k)$, $k \in \mathbb{N}$, then the solution U of (2.2) satisfies

$$U \in \bigcap_{j=0}^k C^{k-j}([0, +\infty), \mathcal{D}(\mathcal{A}^j)).$$

The rest of this section is devoted to the proof of the analyticity of the semigroup associated to the hybrid system (1.1)–(1.8). The main tool we use is the following characterization of analytic semigroups due to Liu and Zheng [8], namely, a C_0 -semigroup of contractions $S(t) = e^{At}$ on a Hilbert space H is analytic if and only if

$$i\mathbb{R} \subseteq \varrho(A) \quad \text{and} \quad \limsup_{|\lambda| \rightarrow \infty} \|\lambda(i\lambda I - A)^{-1}\|_{\mathcal{L}(H)} < \infty, \quad (2.6)$$

where $\varrho(A)$ stands for the resolvent set of A , $i\mathbb{R} = \{i\lambda; \lambda \in \mathbb{R}\}$ and I is the identity operator in H . Note that conditions (2.6) imply the exponential stability of the semigroup. Indeed, a well-known result by Gearhart [4], Huang [6] and Prüss [13] asserts that a C_0 -semigroup of contractions $S(t) = e^{At}$ on a Hilbert space H is exponentially stable if and only if

$$i\mathbb{R} \subseteq \varrho(A) \quad \text{and} \quad \limsup_{|\lambda| \rightarrow \infty} \|(i\lambda I - A)^{-1}\|_{\mathcal{L}(H)} < \infty.$$

The main result of this section is the following.

Theorem 2.3. *Under the conditions of Theorem 2.2, the corresponding semigroup $S(t) = e^{At}$ of the system (1.1)–(1.8) is analytic and, consequently, exponentially stable.*

Proof. To prove the analyticity of $S(t)$ it suffices to show that $i\mathbb{R} \subseteq \varrho(\mathcal{A})$ and boundedness in (2.6) holds. In fact, let us suppose that $i\mathbb{R} \subseteq \varrho(\mathcal{A})$ it is not true. Then there exists $\omega \in \mathbb{R}$ with $\|\mathcal{A}^{-1}\|_{\mathcal{L}(\mathcal{H})}^{-1} \leq |\omega|$ such that

$$\{i\lambda; |\lambda| < |\omega|\} \subset \varrho(\mathcal{A}) \quad \text{and} \quad \sup_{\lambda} \left\{ \|(i\lambda - \mathcal{A})^{-1}\|_{\mathcal{L}(\mathcal{H})}; |\lambda| < |\omega| \right\} = \infty. \quad (2.7)$$

But this means that there exists a sequence λ_n in \mathbb{R} which satisfies

$$|\lambda_n| < |\omega|, \quad \lim_{n \rightarrow \infty} \lambda_n = \omega < \infty \quad (2.8)$$

and a sequence $U_n = (\varphi_n, \psi_n, w_n, v_n)$ in $\mathcal{D}(\mathcal{A})$ with $\|U_n\|_{\mathcal{H}} = 1$ such that

$$\|(i\lambda_n I - \mathcal{A})U_n\|_{\mathcal{H}} \rightarrow 0, \quad \text{as } n \rightarrow \infty. \quad (2.9)$$

In terms of its components, the limit (2.9) gives the following convergence

$$i\lambda_n\varphi_n - \psi_n \rightarrow 0 \quad \text{in } H_*^1(\Omega), \quad (2.10)$$

$$i\rho\lambda_n\psi_n - \nabla \cdot (\alpha\nabla\varphi_n + \gamma_0\nabla\psi_n) \rightarrow 0 \quad \text{in } L^2(\Omega), \quad (2.11)$$

$$i\lambda_n w_n - v_n \rightarrow 0 \quad \text{in } H_0^1(\Gamma_0), \quad (2.12)$$

$$i\lambda_n v_n - (w_n + \gamma_1 v_n)_{xx} + \alpha\psi_n \rightarrow 0 \quad \text{in } L^2(\Gamma_0). \quad (2.13)$$

First we obtain, from (2.3) and (2.9),

$$\gamma_0 \int_{\Omega} |\nabla\psi_n|^2 d\mathbf{x} + \gamma_1 \int_{\Gamma_0} |v_{n,x}|^2 dx = \operatorname{Re}((i\lambda_n I - \mathcal{A})U_n, U_n)_{\mathcal{H}} \rightarrow 0. \quad (2.14)$$

It turns out that, thanks to Poincaré inequality,

$$\psi_n \rightarrow 0 \quad \text{in } L^2(\Omega) \quad \text{and} \quad v_n \rightarrow 0 \quad \text{in } L^2(\Gamma_0). \quad (2.15)$$

On the other hand, from (2.10) and (2.14) we obtain

$$\varphi_n \rightarrow 0 \quad \text{in } H_*^1(\Omega) \quad (2.16)$$

and, from (2.12) and (2.14),

$$w_n \rightarrow 0 \quad \text{in } H_0^1(\Gamma_0). \quad (2.17)$$

Thus, (2.15)–(2.17) contradict $\|U_n\|_{\mathcal{H}} = 1$, therefore $i\mathbb{R} \subseteq \varrho(\mathcal{A})$.

Next we show the estimate (2.6). To this end we consider the resolvent equation

$$i\lambda U - \mathcal{A}U = \mathfrak{F} \quad \text{in } \mathcal{H}, \quad (2.18)$$

where $U = (\varphi, \psi, w, v) \in \mathcal{D}(\mathcal{A})$ and $\mathfrak{F} = (f_1, f_2, f_3, f_4) \in \mathcal{H}$. So we have

$$i\lambda\varphi - \psi = f_1, \quad (2.19)$$

$$i\lambda\rho\psi - \nabla \cdot (\alpha\nabla\varphi + \gamma_0\nabla\psi) = \rho f_2, \quad (2.20)$$

$$i\lambda w - v = f_3, \quad (2.21)$$

$$i\lambda v - (w + \gamma_1 v)_{xx} + \alpha\psi = f_4. \quad (2.22)$$

By taking the inner product in \mathcal{H} of (2.18) with U one has

$$i\lambda\|U\|_{\mathcal{H}}^2 - (\mathcal{A}U, U)_{\mathcal{H}} = (\mathfrak{F}, U)_{\mathcal{H}}. \quad (2.23)$$

Thus, combining the real part of the above identity and (2.3) yields

$$\gamma_0 \int_{\Omega} |\nabla\psi|^2 d\mathbf{x} + \gamma_1 \int_{\Gamma_0} |v_x|^2 dx \leq \|\mathfrak{F}\|_{\mathcal{H}}\|U\|_{\mathcal{H}} \quad (2.24)$$

which, thanks to trace theorem, also provides us

$$\int_{\Gamma_0} |\psi|^2 dx \leq c \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \quad (2.25)$$

From (2.19) and (2.24), we get

$$|\lambda|^2 \int_{\Omega} |\nabla \varphi|^2 d\mathbf{x} \leq c \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}} + c \|\mathfrak{F}\|_{\mathcal{H}}^2. \quad (2.26)$$

Similarly, from (2.21) and (2.24),

$$|\lambda|^2 \int_{\Gamma_0} |w_x|^2 dx \leq c \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}} + c \|\mathfrak{F}\|_{\mathcal{H}}^2. \quad (2.27)$$

Multiplying by $-i\bar{\psi}$ the equation (2.20) and integrating over Ω yields

$$\lambda \rho \int_{\Omega} |\psi|^2 d\mathbf{x} - i\alpha \int_{\Omega} \nabla \varphi \cdot \nabla \bar{\psi} d\mathbf{x} - i\gamma_0 \int_{\Omega} |\nabla \psi|^2 d\mathbf{x} + i\alpha \int_{\Gamma_0} v \bar{\psi} dx = -i\rho \int_{\Omega} f_2 \bar{\psi} d\mathbf{x}$$

Taking the real part and using (2.24) and (2.25), we obtain

$$\rho |\lambda|^2 \int_{\Omega} |\psi|^2 d\mathbf{x} \leq c |\lambda| \|\mathfrak{F}\|_{\mathcal{H}}^{\frac{1}{2}} \|U\|_{\mathcal{H}}^{\frac{3}{2}} + c |\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \quad (2.28)$$

Similarly, multiplying by $-i\bar{v}$ the equation (2.22) and integrating on Γ_0 yields

$$\lambda \int_{\Gamma_0} |v|^2 dx - i \int_{\Gamma_0} w_x \bar{v}_x dx - i\gamma_1 \int_{\Gamma_0} |v_x|^2 dx - i\alpha \int_{\Gamma_0} \psi \bar{v} dx = -i \int_{\Gamma_0} f_4 \bar{v} dx.$$

Hence, proceeding as above we have

$$|\lambda|^2 \int_{\Gamma_0} |v|^2 dx \leq c |\lambda| \|\mathfrak{F}\|_{\mathcal{H}}^{\frac{1}{2}} \|U\|_{\mathcal{H}}^{\frac{3}{2}} + |\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \quad (2.29)$$

From (2.26)–(2.29) we arrive at

$$|\lambda|^2 \|U\|_{\mathcal{H}}^2 \leq \varepsilon |\lambda|^2 \|U\|_{\mathcal{H}}^2 + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}} + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}^2,$$

this means that, for large values of $|\lambda|$,

$$|\lambda| \|U\|_{\mathcal{H}} \leq C \|\mathfrak{F}\|_{\mathcal{H}}, \quad (2.30)$$

where $C > 0$ is a constant that does not depend on λ , U and \mathfrak{F} . Hence (2.6) is satisfied and the semigroup $S(t)$ is analytic. \square

3. Analyticity of system (1.9)–(1.16)

As in the previous section, we set

$$H_*^1(\Omega) = \{\varphi \in H^1(\Omega); \varphi = 0 \text{ on } \Gamma_1\}$$

and introduce the space of finite energy corresponding to (1.9)–(1.16) by

$$\mathcal{H} = H_*^1(\Omega) \times L^2(\Omega) \times H_0^2(\Gamma_0) \times L^2(\Gamma_0) \times L^2(\Gamma_0),$$

which is a Hilbert space with the norm

$$\|U\|_{\mathcal{H}}^2 = \int_{\Omega} \left(\alpha |\nabla \varphi|^2 + \rho |\psi|^2 \right) d\mathbf{x} + \int_{\Gamma_0} \left(|w_{xx}|^2 + |v|^2 + |\theta|^2 \right) dx, \quad (3.1)$$

for all $U = (\varphi, \psi, w, v, \theta) \in \mathcal{H}$. We denote $\Upsilon : H^1(\Omega) \rightarrow H^{\frac{1}{2}}(\Gamma_0)$ the trace map. In \mathcal{H} , we consider the unbounded linear operator $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \rightarrow \mathcal{H}$ given by

$$\mathcal{A}(\varphi, \psi, w, v, \theta) = \left(\psi, \frac{1}{\rho} \nabla \cdot (\alpha \nabla \varphi + \gamma_0 \nabla \psi), v, -w_{xxxx} - \beta \theta_{xx} - \alpha \Upsilon \psi, \kappa \theta_{xx} + \beta v_{xx} \right)$$

whose domain is

$$\mathcal{D}(\mathcal{A}) = \left\{ U = (\varphi, \psi, w, v, \theta) \in (H_*^1(\Omega))^2 \times (H^4(\Gamma_0) \cap H_0^2(\Gamma_0)) \times H_0^2(\Gamma_0) \times \right. \\ \left. (H^2(\Gamma_0) \cap H_0^1(\Gamma_0)); \alpha \varphi + \gamma_0 \psi \in H^2(\Omega) \text{ and } \frac{\partial \varphi}{\partial \nu} + \frac{\gamma_0}{\alpha} \frac{\partial \psi}{\partial \nu} = v \text{ on } \Gamma_0 \right\}.$$

With the previous definitions, we see that system (1.9)–(1.16) is equivalent to

$$U_t = \mathcal{A}U, \quad U(0) = U_0, \quad (3.2)$$

where $U_0 = (\varphi_0, \varphi_1, w_0, w_1, \theta_0) \in \mathcal{D}(\mathcal{A})$ and $U(t) = (\varphi, \varphi_t, w, w_t, \theta) \in \mathcal{D}(\mathcal{A})$, $t > 0$.

Therefore, well-posedness of the system (1.9)–(1.16) is guaranteed by the following Lemma.

Lemma 3.1. *The operator \mathcal{A} generates a C_0 -semigroup of contractions in \mathcal{H} .*

Proof. For any $U \in \mathcal{D}(\mathcal{A})$ we get

$$\begin{aligned} (\mathcal{A}U, U)_{\mathcal{H}} &= \alpha \int_{\Omega} (\nabla \psi \cdot \nabla \bar{\varphi} - \nabla \varphi \cdot \nabla \bar{\psi}) d\mathbf{x} - \gamma_0 \int_{\Omega} |\nabla \psi|^2 d\mathbf{x} \\ &\quad + \alpha \int_{\Gamma_0} (v \bar{\psi} - \psi \bar{v}) dx + \int_{\Gamma_0} (v_{xx} \bar{w}_{xx} - w_{xx} \bar{v}_{xx}) dx \\ &\quad + \beta \int_{\Gamma_0} (\theta_x \bar{v}_x - v_x \bar{\theta}_x) dx - \kappa \int_{\Gamma_0} |\theta_x|^2 dx. \end{aligned}$$

Hence,

$$\operatorname{Re}(\mathcal{A}U, U)_{\mathcal{H}} = -\gamma_0 \int_{\Omega} |\nabla \psi|^2 d\mathbf{x} - \kappa \int_{\Gamma_0} |\theta_x|^2 dx, \quad (3.3)$$

which proves that \mathcal{A} is dissipative. To conclude the proof we show that $0 \in \varrho(\mathcal{A})$ (the resolvent of \mathcal{A}). That is, for any $\mathfrak{F} \in \mathcal{H}$, there exists only one $U \in \mathcal{D}(\mathcal{A})$ solution of $\mathcal{A}U = \mathfrak{F}$. In fact, for any $\mathfrak{F} = (f_1, f_2, f_3, f_4, f_5) \in \mathcal{H}$ we solve the system

$$\begin{cases} \psi = f_1 \\ \nabla \cdot (\alpha \nabla \varphi + \gamma_0 \nabla \psi) = \rho f_2 \\ v = f_3 \\ -w_{xxxx} - \beta \theta_{xx} - \alpha \psi = f_4 \\ \kappa \theta_{xx} + \beta v_{xx} = f_5, \end{cases} \quad (3.4)$$

which is equivalent to find (φ, w, θ) solution of

$$\begin{cases} \nabla \cdot (\alpha \nabla \varphi + \gamma_0 \nabla f_1) = \rho f_2 & \text{in } \Omega \\ w_{xxxx} + \beta \theta_{xx} = -\alpha f_1 - f_4 & \text{on } \Gamma_0 \\ \kappa \theta_{xx} = -\beta f_{3,xx} + f_5 & \text{on } \Gamma_0. \end{cases} \quad (3.5)$$

A direct application of Lax–Milgram’s Lemma on the space $(H_*^1(\Omega) \times H_0^2(\Gamma_0) \times H_0^1(\Gamma_0))^2$ solves the above system and then (3.4). Moreover, there exists constant $C > 0$ such that

$$\|(\varphi, \psi, w, v, \theta)\|_{\mathcal{H}} \leq C \|(f_1, f_2, f_3, f_4, f_5)\|_{\mathcal{H}}. \quad \square$$

Finally, we show the main result of this section.

Theorem 3.2. *The semigroup $S(t) = e^{At}$ of the system (1.9)–(1.16) is analytic.*

Proof. To prove the analyticity of $S(t)$ we show that (2.6) holds. To show $i\mathbb{R} \subseteq \varrho(\mathcal{A})$ we use a contradiction argument as in the proof of Theorem 2.3. So, there exist a sequence λ_n and $U_n = (\varphi_n, \psi_n, w_n, v_n, \theta_n)$ in $\mathcal{D}(\mathcal{A})$ with $\|U_n\|_{\mathcal{H}} = 1$ such that

$$\lim_{n \rightarrow \infty} \lambda_n = \lambda < \infty, \quad \|(i\lambda_n I - \mathcal{A})U_n\|_{\mathcal{H}} \rightarrow 0, \quad \text{as } n \rightarrow \infty. \quad (3.6)$$

In terms of its components, the limit (3.6) gives the following convergence

$$i\lambda_n \varphi_n - \psi_n \rightarrow 0 \quad \text{in } H_*^1(\Omega), \quad (3.7)$$

$$i\rho \lambda_n \psi_n - \nabla \cdot (\alpha \nabla \varphi_n + \gamma_0 \nabla \psi_n) \rightarrow 0 \quad \text{in } L^2(\Omega), \quad (3.8)$$

$$i\lambda_n w_n - v_n \rightarrow 0 \quad \text{in } H_0^2(\Gamma_0), \quad (3.9)$$

$$i\lambda_n v_n + w_{n,xxxx} + \beta \theta_{n,xx} + \alpha \psi_n \rightarrow 0 \quad \text{in } L^2(\Gamma_0), \quad (3.10)$$

$$i\lambda_n \theta_n - \kappa \theta_{n,xx} - \beta v_{n,xx} \rightarrow 0 \quad \text{in } L^2(\Gamma_0). \quad (3.11)$$

From (3.3) and (3.6) we get

$$\gamma_0 \int_{\Omega} |\nabla \psi_n|^2 \, d\mathbf{x} + \kappa \int_{\Gamma_0} |\theta_{n,x}|^2 \, dx = \operatorname{Re}((i\lambda_n I - \mathcal{A})U_n, U_n)_{\mathcal{H}} \rightarrow 0. \quad (3.12)$$

Therefore, using Poincaré Inequality we have

$$\psi_n \rightarrow 0 \quad \text{in } L^2(\Omega) \quad \text{and} \quad \theta_n \rightarrow 0 \quad \text{in } L^2(\Gamma_0) \quad (3.13)$$

and, due to trace theorem,

$$\psi_n \rightarrow 0 \quad \text{in } L^2(\Gamma_0). \quad (3.14)$$

From (3.7) and (3.12) we obtain

$$\varphi_n \rightarrow 0 \quad \text{in } H_*^1(\Omega) \quad (3.15)$$

and, from (3.11) and (3.13),

$$\kappa\theta_{n,xx} + \beta v_{n,xx} \rightarrow 0 \quad \text{in } L^2(\Gamma_0). \quad (3.16)$$

Multiplying the above sequence by $\overline{\kappa\theta_n + \beta v_n}$ we conclude that $\kappa\theta_n + \beta v_n$ converges strongly in $H^1(\Gamma_0)$. Thus using (3.12) we see that

$$v_n \rightarrow 0 \quad \text{in } H^1(\Gamma_0). \quad (3.17)$$

Finally, multiplying sequence (3.10) by $\overline{w_n}$ and using the above convergence we obtain

$$w_n \rightarrow 0 \quad \text{in } H_0^2(\Gamma_0). \quad (3.18)$$

Thus (3.13)–(3.15) and (3.17)–(3.18) contradict $\|U\|_{\mathcal{H}} = 1$. Hence $i\mathbb{R} \subseteq \varrho(\mathcal{A})$. Finally, we show the second part of (2.6). To do that we consider the resolvent equation $i\lambda U - \mathcal{A}U = \mathfrak{F}$, for $\mathfrak{F} = (f_1, f_2, f_3, f_4, f_5) \in \mathcal{H}$ and $U = (\varphi, \psi, w, v, \theta) \in D(\mathcal{A})$, which in terms of its components is equivalent to

$$i\lambda\varphi - \psi = f_1, \quad (3.19)$$

$$i\lambda\rho\psi - \nabla \cdot (\alpha\nabla\varphi + \gamma_0\nabla\psi) = \rho f_2, \quad (3.20)$$

$$i\lambda w - v = f_3, \quad (3.21)$$

$$i\lambda v + w_{xxxx} + \beta\theta_{xx} + \alpha\psi = f_4 \quad (3.22)$$

$$i\lambda\theta - \kappa\theta_{xx} - \beta v_{xx} = f_5. \quad (3.23)$$

By taking the inner product in \mathcal{H} to the resolvent equation with U one has

$$i\lambda\|U\|_{\mathcal{H}}^2 - (\mathcal{A}U, U)_{\mathcal{H}} = (\mathfrak{F}, U)_{\mathcal{H}}. \quad (3.24)$$

Thus, combining the real part of the above identity and (3.3) yields

$$\gamma_0 \int_{\Omega} |\nabla\psi|^2 \, d\mathbf{x} + \kappa \int_{\Gamma_0} |\theta_x|^2 \, dx \leq \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \quad (3.25)$$

The trace theorem also provides us

$$\int_{\Gamma_0} |\psi|^2 \, dx \leq c\|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \quad (3.26)$$

From (3.19) and (3.25) we get

$$|\lambda|^2 \int_{\Omega} |\nabla\varphi|^2 \, d\mathbf{x} \leq c\|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}} + c\|\mathfrak{F}\|_{\mathcal{H}}^2. \quad (3.27)$$

On the other hand, from (3.21) we have

$$|\lambda|^2 \int_{\Gamma_0} |w_{xx}|^2 dx \leq \left| -i\lambda \int_{\Gamma_0} v_{xx} \overline{w}_{xx} dx \right| + |\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \quad (3.28)$$

Multiplying by $-i\lambda\overline{\psi}$ the equation (3.20) and integrating over Ω yields

$$\begin{aligned} \rho|\lambda|^2 \int_{\Omega} |\psi|^2 d\mathbf{x} &= i\lambda\alpha \int_{\Omega} \nabla\varphi \cdot \nabla\overline{\psi} d\mathbf{x} + i\lambda\gamma_0 \int_{\Omega} |\nabla\psi|^2 d\mathbf{x} \\ &\quad - i\lambda\alpha \int_{\Gamma_0} v\overline{\psi} dx - i\lambda\rho \int_{\Omega} f_2\overline{\psi} d\mathbf{x} \end{aligned}$$

from which, taking the real part and using (3.25)–(3.26), we obtain

$$\rho|\lambda|^2 \int_{\Omega} |\psi|^2 d\mathbf{x} \leq c|\lambda| \|\mathfrak{F}\|_{\mathcal{H}}^{\frac{1}{2}} \|U\|_{\mathcal{H}}^{\frac{3}{2}} + c|\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \quad (3.29)$$

From (3.27)–(3.29) we get, for $|\lambda| \geq 1$,

$$\begin{aligned} |\lambda|^2 \|U\|_{\mathcal{H}}^2 &\leq c|\lambda| \|\mathfrak{F}\|_{\mathcal{H}}^{\frac{1}{2}} \|U\|_{\mathcal{H}}^{\frac{3}{2}} + c|\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}} + c\|\mathfrak{F}\|_{\mathcal{H}}^2 \\ &\quad + \underbrace{|\lambda|^2 \int_{\Gamma_0} |v|^2 dx}_{:=I_1} + \underbrace{\left| -i\lambda \int_{\Gamma_0} v_{xx} \overline{w}_{xx} dx \right|}_{:=I_2} + \underbrace{|\lambda|^2 \int_{\Gamma_0} |\theta|^2 dx}_{:=I_3}. \end{aligned} \quad (3.30)$$

To estimate I_3 we consider (3.21) to obtain

$$\|v\|_{H^2(\Gamma_0)} \leq |\lambda| \|U\|_{\mathcal{H}} + \|\mathfrak{F}\|_{\mathcal{H}}. \quad (3.31)$$

Using Gagliardo–Nirenberg inequalities we have

$$\|v\|_{H^1(\Gamma_0)} \leq c \|v\|_{H^2(\Gamma_0)}^{\frac{1}{2}} \|v\|_{L^2(\Gamma_0)}^{\frac{1}{2}}. \quad (3.32)$$

Then, inserting (3.31) into (3.32), we get

$$\|v\|_{H^1(\Gamma_0)}^2 \leq \tilde{c} (|\lambda| \|U\|_{\mathcal{H}}^2 + \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}). \quad (3.33)$$

Hence, using (3.25) and (3.33) we have for any $\varepsilon > 0$

$$\begin{aligned} \beta|\lambda| \int_{\Gamma_0} |v_x| |\theta_x| dx &\leq \frac{\varepsilon}{4\tilde{c}} |\lambda| \|v\|_{H^1(\Gamma_0)}^2 + C(\varepsilon) |\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}} \\ &\leq \frac{\varepsilon}{4} |\lambda|^2 \|U\|_{\mathcal{H}}^2 + C(\varepsilon) |\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \end{aligned} \quad (3.34)$$

Thus, multiplying equation (3.23) by $-i\lambda\overline{\theta}$, taking real part and using (3.34) we get

$$I_3 \leq \beta|\lambda| \int_{\Gamma_0} |v_x| |\theta_x| dx + |\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}} \leq \varepsilon |\lambda|^2 \|U\|_{\mathcal{H}}^2 + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}^2, \quad \forall \varepsilon > 0. \quad (3.35)$$

From (3.23) and (3.25) we obtain

$$\beta \int_{\Gamma_0} |v_x|^2 dx \leq \frac{|\lambda|}{4\varepsilon} \int_{\Gamma_0} |\theta|^2 dx + \varepsilon |\lambda| \|U\|_{\mathcal{H}}^2 + \frac{\beta}{2} \int_{\Gamma_0} |v_x|^2 dx + C \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}.$$

Therefore, we have for any $\varepsilon > 0$

$$\frac{\beta}{2} \int_{\Gamma_0} |v_x|^2 dx \leq \frac{|\lambda|}{4\varepsilon} \int_{\Gamma_0} |\theta|^2 dx + 2\varepsilon |\lambda| \|U\|_{\mathcal{H}}^2 + \frac{C(\varepsilon)}{|\lambda|} \|\mathfrak{F}\|_{\mathcal{H}}^2. \quad (3.36)$$

Note that (3.35) implies that there exists $C(\varepsilon) > 0$ such that

$$|\lambda| \int_{\Gamma_0} |\theta|^2 dx \leq \varepsilon^2 |\lambda| \|U\|_{\mathcal{H}}^2 + \frac{C(\varepsilon)}{|\lambda|} \|\mathfrak{F}\|_{\mathcal{H}}^2. \quad (3.37)$$

Inserting (3.37) into (3.36) we arrive at

$$\frac{\beta}{2} \int_{\Gamma_0} |v_x|^2 dx \leq \varepsilon c |\lambda| \|U\|_{\mathcal{H}}^2 + \frac{C(\varepsilon)}{|\lambda|} \|\mathfrak{F}\|_{\mathcal{H}}^2.$$

We can conclude, in short, that for any $\varepsilon > 0$

$$\|v\|_{H^1(\Gamma_0)} \leq \varepsilon |\lambda|^{\frac{1}{2}} \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|^{\frac{1}{2}}} \|\mathfrak{F}\|_{\mathcal{H}}. \quad (3.38)$$

To estimate I_1 , we set $v = v_1 + v_2$, where v_1 satisfies

$$\begin{cases} i\lambda v_1 - v_{1,xx} = f_4 - \alpha\psi \\ v_1(0) = v_1(l) = 0 \end{cases} \quad (3.39)$$

and v_2 is such that

$$i\lambda v_2 = -w_{xxxx} - \beta\theta_{xx} - v_{1,xx}. \quad (3.40)$$

From (3.39) and (3.26) we deduce that

$$|\lambda| \|v_1\|_{L^2(\Gamma_0)} + |\lambda|^{\frac{1}{2}} \|v_1\|_{H^1(\Gamma_0)} + \|v_1\|_{H^2(\Gamma_0)} \leq c \left(\|\mathfrak{F}\|_{\mathcal{H}} + \|\mathfrak{F}\|_{\mathcal{H}}^{\frac{1}{2}} \|U\|_{\mathcal{H}}^{\frac{1}{2}} \right), \quad (3.41)$$

this means that for any $\varepsilon > 0$ and any $|\lambda| \geq 1$, the following inequality holds:

$$|\lambda| \|v_1\|_{L^2(\Gamma_0)} + |\lambda|^{\frac{1}{2}} \|v_1\|_{H^1(\Gamma_0)} + \|v_1\|_{H^2(\Gamma_0)} \leq \varepsilon |\lambda| \|U\|_{\mathcal{H}} + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}. \quad (3.42)$$

As $v_2 = v - v_1$, we also obtain from (3.38) and (3.42)

$$\|v_2\|_{H^1(\Gamma_0)} \leq \varepsilon |\lambda|^{\frac{1}{2}} \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|^{\frac{1}{2}}} \|\mathfrak{F}\|_{\mathcal{H}}, \quad (3.43)$$

for all $\varepsilon > 0$ and $|\lambda| \geq 1$. We return now to the identity (3.40) from which we get

$$|\lambda| \|v_2\|_{H^{-2}(\Gamma_0)} \leq c \left(\|U\|_{\mathcal{H}} + \|v_1\|_{L^2(\Gamma_0)} \right), \quad (3.44)$$

thus, thanks to (3.42),

$$\|v_2\|_{H^{-2}(\Gamma_0)} \leq c \left(\frac{1}{|\lambda|} \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|^2} \|\mathfrak{F}\|_{\mathcal{H}} \right). \quad (3.45)$$

Then, applying interpolation, from estimates (3.43) and (3.45) we deduce

$$\begin{aligned} \|v_2\|_{H^{-1}(\Gamma_0)} &\leq c \|v_2\|_{H^{-2}(\Gamma_0)}^{\frac{2}{3}} \|v_2\|_{H^1(\Gamma_0)}^{\frac{1}{3}} \\ &\leq c \left(\frac{1}{|\lambda|} \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|^2} \|\mathfrak{F}\|_{\mathcal{H}} \right)^{\frac{2}{3}} \left(\varepsilon |\lambda|^{\frac{1}{2}} \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|^{\frac{1}{2}}} \|\mathfrak{F}\|_{\mathcal{H}} \right)^{\frac{1}{3}}, \end{aligned}$$

which provides us, thanks to Young's inequality,

$$\|v_2\|_{H^{-1}(\Gamma_0)} \leq \varepsilon c \frac{1}{|\lambda|^{\frac{1}{2}}} \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|^{\frac{3}{2}}} \|\mathfrak{F}\|_{\mathcal{H}}. \quad (3.46)$$

Using interpolation again, along with (3.43) and (3.46), we find

$$\begin{aligned} \|v_2\|_{L^2(\Gamma_0)} &\leq c \|v_2\|_{H^{-1}(\Gamma_0)}^{\frac{1}{2}} \|v_2\|_{H^1(\Gamma_0)}^{\frac{1}{2}} \\ &\leq c \left(\varepsilon \frac{1}{|\lambda|^{\frac{1}{2}}} \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|^{\frac{3}{2}}} \|\mathfrak{F}\|_{\mathcal{H}} \right)^{\frac{1}{2}} \left(\varepsilon |\lambda|^{\frac{1}{2}} \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|^{\frac{1}{2}}} \|\mathfrak{F}\|_{\mathcal{H}} \right)^{\frac{1}{2}} \\ &\leq \varepsilon c \|U\|_{\mathcal{H}} + \frac{C(\varepsilon)}{|\lambda|} \|\mathfrak{F}\|_{\mathcal{H}}, \end{aligned}$$

or equivalently

$$|\lambda| \|v_2\|_{L^2(\Gamma_0)} \leq \varepsilon c |\lambda| \|U\|_{\mathcal{H}} + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}. \quad (3.47)$$

Therefore, inequalities (3.42) and (3.47) lead to

$$|\lambda| \|v\|_{L^2(\Gamma_0)} \leq |\lambda| (\|v_1\|_{L^2(\Gamma_0)} + \|v_2\|_{L^2(\Gamma_0)}) \leq \varepsilon c |\lambda| \|U\|_{\mathcal{H}} + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}},$$

in short, it is shown that for any $\varepsilon > 0$ and $|\lambda| \geq 1$,

$$I_1 = |\lambda|^2 \int_{\Gamma_0} |v|^2 dx \leq \varepsilon |\lambda|^2 \|U\|_{\mathcal{H}}^2 + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}^2. \quad (3.48)$$

Finally, to estimate I_2 we use equation (3.22) to get

$$\begin{aligned} -i\lambda \int_{\Gamma_0} v_{xx} \overline{w}_{xx} dx &= -i\lambda \int_{\Gamma_0} v (i\lambda \overline{v} - \beta \overline{\theta}_{xx} - \alpha \overline{\psi} + \overline{f_4}) dx \\ &= |\lambda|^2 \int_{\Gamma_0} |v|^2 dx - i\lambda \beta \int_{\Gamma_0} v_x \overline{\theta}_x dx + i\lambda \alpha \int_{\Gamma_0} v \overline{\psi} dx - i\lambda \int_{\Gamma_0} v \overline{f_4} dx. \end{aligned} \quad (3.49)$$

Thus, taking into account (3.26), (3.34) and (3.48) we obtain, from (3.49),

$$I_2 \leq \varepsilon c |\lambda|^2 \|U\|_{\mathcal{H}}^2 + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}^2 + c |\lambda| \|\mathfrak{F}\|_{\mathcal{H}}^{\frac{1}{2}} \|U\|_{\mathcal{H}}^{\frac{3}{2}} + |\lambda| \|\mathfrak{F}\|_{\mathcal{H}} \|U\|_{\mathcal{H}}. \quad (3.50)$$

Inserting into (3.30) estimates (3.35), (3.48) and (3.50) we have

$$|\lambda|^2 \|U\|_{\mathcal{H}}^2 \leq c|\lambda| \|\mathfrak{F}\|_{\mathcal{H}}^{\frac{1}{2}} \|U\|_{\mathcal{H}}^{\frac{3}{2}} + \varepsilon c |\lambda|^2 \|U\|_{\mathcal{H}}^2 + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}^2. \quad (3.51)$$

Using

$$|\lambda| \|\mathfrak{F}\|_{\mathcal{H}}^{\frac{1}{2}} \|U\|_{\mathcal{H}}^{\frac{3}{2}} \leq \varepsilon |\lambda|^2 \|U\|_{\mathcal{H}}^2 + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}^2, \quad \forall \varepsilon > 0, \quad |\lambda| \geq 1$$

into (3.51), we conclude that there exists $C(\varepsilon) > 0$ such that

$$|\lambda|^2 \|U\|_{\mathcal{H}}^2 \leq \varepsilon c |\lambda|^2 \|U\|_{\mathcal{H}}^2 + C(\varepsilon) \|\mathfrak{F}\|_{\mathcal{H}}^2. \quad (3.52)$$

This means that, for large values of $|\lambda|$, and choosing small $\varepsilon > 0$,

$$|\lambda| \|U\|_{\mathcal{H}} \leq C \|\mathfrak{F}\|_{\mathcal{H}}, \quad (3.53)$$

where $C > 0$ is a constant independent of λ , U and \mathfrak{F} . Hence (2.6) is satisfied and the semigroup $S(t)$ is analytic. \square

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