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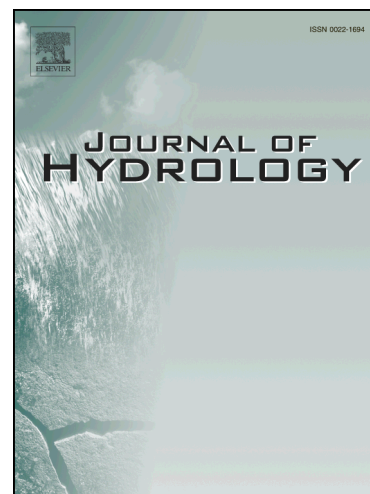
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Estimation of relative permeability curves using an improved Levenberg-Marquardt method with simultaneous perturbation Jacobian approximation

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Abstract: Relative permeability controls the flow of multiphase fluids in porous media. The estimation of relative permeability is generally solved by Levenberg-Marquardt method with finite difference Jacobian approximation (LM-FD). However, the method can hardly be used in large-scale reservoirs because of unbearably huge computational cost. To eliminate this problem, the paper introduces the idea of simultaneous perturbation to simplify the generation of the Jacobian matrix needed in the Levenberg-Marquardt procedure and denotes the improved method as LM-SP. It is verified by numerical experiments and then applied to laboratory experiments and a real commercial oilfield. Numerical experiment indicates that LM-SP uses only 16.1% computational cost to obtain similar estimation of relative permeability and prediction of production performance compared with LM-FD. Laboratory experiment also shows the LM-SP has a 60.4% decrease in simulation cost while a 68.5% increase in estimation accuracy compared with the earlier published results. This is mainly because LM-FD needs $2n$ (n is the number of controlling knots) simulations to approximate Jacobian in each iteration, while only 2 simulations are enough in basic LM-SP. The convergence rate and estimation accuracy of LM-SP can be improved by averaging several simultaneous perturbation Jacobian approximations but the computational cost of each iteration may be increased. Considering the estimation accuracy and computational cost, averaging two Jacobian approximations is recommended in this paper. As the number of unknown controlling knots increases from 7 to 15, the saved simulation runs by LM-SP than LM-FD increases from 114 to 1164. This indicates LM-SP is more suitable than LM-FD for multivariate problems. Field application further proves the applicability of LM-SP on large real field as well as small laboratory problems.

Keywords: multiphase flow; relative permeability; Levenberg-Marquardt method; simultaneous perturbation

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Jacobian approximation

1. Introduction

Relative permeability curves control the flow of fluids in porous media and they are the most important parameters in reservoir development. Reliable relative permeability curves are essential for numerical simulation technique to reproduce production history, predict reservoir behavior and design development plan. Generally, relative permeability curves can be obtained by laboratory experiments (Krause and Benson, 2015; Masihi et al., 2011), theoretical models (Ahmadi, 2015; Khaz'ali et al., 2011) and numerical estimation (Hussain et al., 2010; Taklimy and Rasaei, 2015). Laboratory experiments are time consuming and hard to operate. Theoretical models are limited to ideal assumptions and unsatisfactory accuracy. Numerical estimation considers the overall production history and geological properties. It has high estimation accuracy and can be used for large-scale commercial oilfields. Therefore, numerical estimation attracts more attention in estimating relative permeability curves.

Numerical estimation method modifies the control parameters of relative permeability model so that the simulation results matches production history. This process is one nonlinear least squares problem (Hou et al., 2015; Oliver and Chen, 2011). It can be solved by general optimization methods such as Newton's method, genetic algorithm and ensemble-based method (Chen and Oliver, 2013; Li et al., 2012; Zhang et al., 2016). However, there are more efficient solving methods owing to the special form of objective function of nonlinear least squares problems, including Gauss-Newton method and Levenberg-Marquardt method (LM) (Madsen et al., 2004; Nocedal and Wright, 2006). In fact, the LM method can be viewed as a damped version of Gauss-Newton method. The iteration direction of the LM method is closer to the Gauss-Newton direction near the optimal solution. However, it is closer to the steepest descent direction when the current solution is far away from the optimum. As a result, the robustness and convergence rate of the LM method is much higher and has been widely used in history matching problems (Awotunde and Horne, 2012; Iglesias and Dawson, 2013).

The Jacobian matrix is necessary for the LM method to solve nonlinear least squares problems. However, it happens that we cannot give formulae for Jacobian elements when estimating relative permeability curves, because the nonlinear commercial reservoir simulator works as a "black box" (Monfared et al., 2014; Shirangi, 2014). The secant version of the LM method is intended for this problem. The most widely used method is to

replace the elements of original Jacobian by finite difference approximations (Hou et al., 2012a, 2012b). For simplicity, the LM method with finite difference Jacobian approximation is denoted as the LM-FD method. It perturbs only one control variable at a time and thus each iteration needs $2n$ (n is the number of unknown control variables) simulations to calculate the finite difference Jacobian approximation. Since one single simulation in reservoir development may take several hours or even several days, the total computational cost is so huge that the use of the LM-FD method is limited to small-scale problems. In order to eliminate this drawback, this paper adopts the idea of simultaneous perturbation proposed by Spall (1992) to approximate the Jacobian. This method requires only 2 simulations to approximate all Jacobian elements in each iteration, and the computational cost is independent with the number of unknown control variables. Similar to the LM-FD method, we denote the improved LM method with simulations perturbation Jacobian approximation as the LM-SP method. Because the expectation of simultaneous perturbation Jacobian approximation is equal to the true matrix, we can improve the convergence rate and robustness of the LM-SP method by averaging several Jacobian approximations. This series of methods can be denoted as the LM-SP- β method, where β is the sampling number of simulations perturbation Jacobian approximations.

The paper chooses the cubic B-spline model (Li et al., 2010) to represent the relative permeability and uses the STARS module of CMG software to perform forward simulations. In this study, the reference permeability curves are given at first and the forward simulation is conducted to obtain the reference production data such as cumulative oil production, cumulative water production and injection pressure. Then, both the LM-SP and LM-FD methods are used to estimate the relative permeability by history matching the observed production performance. Their estimation accuracy and computational cost are compared and analyzed. We also study the effect of the sampling number of simultaneous perturbation Jacobian approximations and the segments of cubic B-spline model on the LM-SP method. Finally, the proposed LM-SP method is applied and verified on core flooding experiments as well as a large commercial oilfield.

2. Problem statement

2.1. Controlling parameter of relative permeability

The cubic B-spline model is used to represent relative permeability curves in this paper. Since there is no assumption regarding the shape of relative permeability curves, the cubic B-spline model is able to accurately represent almost any set of relative permeability curves, provided that sufficient number of controlling knots

are allowed. In order to introduce the cubic B-spline model, the dimensionless water saturation is firstly defined as the following equation:

$$S_{wD} = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}} \quad (1)$$

where S_{wD} denotes the dimensionless water saturation; S_w denotes the original water saturation; S_{wc} denotes the irreducible water saturation; S_{or} denotes the residual oil saturation.

Obviously, S_{wD} varies from 0 to 1. It is uniformly divided into n segments and thus $S_{wD,j} = j/n$ ($j=0, 1, \dots, n$). On this basis, the cubic B-spline form of relative permeability can be formulated as:

$$k_{rl}(S_{wD}) = \sum_{j=-3}^{n-1} C_{j+2}^l B_j(S_{wD}), l = o, w \quad (2)$$

where $k_{rl}(S_{wD})$ denotes the relative permeability of l -phase at dimensionless water saturation S_{wD} ; C_{j+2}^l denotes the controlling knots of cubic B-spline model of l -phase; $B_j(S_{wD})$ denotes the basic function of cubic B-spline model; n denotes the total number of controlling knots of l -phase.

Since traditional cubic B-spline model cannot pass through the controlling knots while the endpoints of relative permeability curves are very important, two mirror points denoted as C_{-1}^l and C_{n+1}^l are introduced into each B-spline curve. The mirror points are defined as:

$$C_{-1}^l = 2C_0^l - C_1^l \quad (3a)$$

$$C_{n+1}^l = 2C_n^l - C_{n-1}^l \quad (3b)$$

Suppose that S_{wc} and S_{or} have already been known. The oil permeability at irreducible water saturation is generally used as the reference value. In other words, $C_0^o = 1$ and $C_n^o = C_0^w = 0$. Therefore, the unknown controlling knots for water phase are C_j^w ($j=1, 2, \dots, n$), and the unknown controlling knots for oil phase are C_j^o ($j=1, 2, \dots, n-1$). Based on above analysis, there are a total of $2n-1$ unknown parameters to be estimated in this study.

In order to ensure the cubic B-spline model to be monotonic, many researchers use log transformation for the original controlling knots. Following Hou et al. (2012b), the pseudo-controlling knots are written as:

$$x_i = \ln \left(\frac{C_i^w - (2C_{i-1}^w - C_{i-2}^w)}{\frac{1}{2}(C_{i+1}^w + C_{i-1}^w) - C_i^w} \right), \quad 1 \leq i \leq n \quad (4a)$$

$$y_i = \ln \left(\frac{C_i^o - (2C_{i+1}^o - C_{i+2}^o)}{\frac{1}{2}(C_{i+1}^o + C_{i-1}^o) - C_i^o} \right), \quad 1 \leq i \leq n-1 \quad (4b)$$

where x_i denotes the pseudo-controlling knots of cubic B-spline model for water phase; y_i denotes the pseudo-controlling knots of cubic B-spline model for oil phase.

In this paper, the pseudo-controlling knots are estimated at first and then the original controlling knots of cubic B-spline model are obtained by solving equations (4a) and (4b).

2.2. Objective function

A good estimation of relative permeability should ensure that the predicted production performance matches well with the observed history. In order to evaluate the matching effect quantitatively, an objective function is established in the form of least squares error. It can be written as:

$$O(\mathbf{x}) = \frac{1}{2} \mathbf{R}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{R}(\mathbf{x}) \quad (5a)$$

$$\mathbf{R}(\mathbf{x}) = \mathbf{g}(\mathbf{x}) - \mathbf{d}_{obs}, \text{ where } \mathbf{g}(\mathbf{x}): \mathbf{R}^n \rightarrow \mathbf{R}^m \text{ with } m \geq n \quad (5b)$$

where \mathbf{x} denotes the vector of unknown controlling knots; $O(\mathbf{x})$ defined the least squares error between the predicted and observed production performance at the current estimation of relative permeability; $\mathbf{R}(\mathbf{x})$ denotes the vector of residuals between predicted and observed production performance; \mathbf{C}_D denotes the covariance matrix; $\mathbf{g}(\mathbf{x})$ denotes the vector of current production prediction; \mathbf{d}_{obs} denotes the vector of observed production performance; n denotes the total number of controlling knots; m denotes the total number of production data.

Mathematically, the estimation of relative permeability is to find the optimal controlling knots \mathbf{x}^* so that the objective function $O(\mathbf{x})$ approaches 0. According to the form of Eq. (5), this is a nonlinear least squares problem. It can be solved by general optimization methods, but because of the special form of the objective function there are more efficient solving methods such as the Gauss-Newton method, the LM method and the Dog Leg method. In this paper, the LM method is adopted and a speed-up procedure is proposed in order to

increase its convergence efficiency and reduce computational cost.

3. Solving methods

3.1. The basic Levenberg-Marquardt method

The LM method can be viewed as a damped version of the Gauss-Newton method. It is similar to the Gauss-Newton direction near the optimal solution, while it is closer to the steepest descent direction far away from the optimum. As a result, the robustness and convergence rate of the LM method is much higher and has been most widely used in nonlinear least squares problems. We will introduce the basic theories of the LM method at first.

Provided that the residuals $\mathbf{R}(\mathbf{x})$ has continuous second partial derivatives, its Taylor expansion can be written as:

$$\mathbf{R}(\mathbf{x} + \mathbf{h}) = \mathbf{R}(\mathbf{x}) + \mathbf{J}(\mathbf{x})\mathbf{h} + O(\|\mathbf{h}\|^2) \quad (6)$$

where, \mathbf{h} denotes the search step at the current iteration; $\|\mathbf{h}\|$ denotes the norm of \mathbf{h} ; $\mathbf{J}(\mathbf{x})$ denotes the Jacobian matrix of the residuals to the controlling knots, its $(i, j)^{\text{th}}$ element is defined as:

$$(\mathbf{J}(\mathbf{x}))_{ij} = \frac{\partial R_i}{\partial x_j}(\mathbf{x}), (i = 1, \dots, m; j = 1, \dots, n) \quad (7)$$

For small $\|\mathbf{h}\|$, Eq. (6) can be simplified as:

$$\mathbf{R}(\mathbf{x} + \mathbf{h}) \approx \mathbf{R}(\mathbf{x}) + \mathbf{J}(\mathbf{x})\mathbf{h} \quad (8)$$

Inserting Eq. (8) into Eq. (5) we can obtain the objective function as:

$$\begin{aligned} O(\mathbf{x} + \mathbf{h}) \approx L(\mathbf{h}) &= \frac{1}{2}[\mathbf{R}(\mathbf{x})^T + \mathbf{h}^T \mathbf{J}(\mathbf{x})^T] \mathbf{C}_D^{-1} [\mathbf{R}(\mathbf{x}) + \mathbf{J}(\mathbf{x})\mathbf{h}] \\ &= \frac{1}{2}[\mathbf{R}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{R}(\mathbf{x})] + \mathbf{h}^T \mathbf{J}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{R}(\mathbf{x}) + \frac{1}{2} \mathbf{h}^T \mathbf{J}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{J}(\mathbf{x}) \mathbf{h} \end{aligned} \quad (9)$$

Since the Gauss-Newton step satisfies the line search condition, it is formulated as:

$$\mathbf{h}_{gn} = \arg \min_{\mathbf{h}} \{L(\mathbf{h})\} \quad (10)$$

where, \mathbf{h}_{gn} denotes the search step used in the Gauss-Newton method.

The gradient of Eq. (9) is derived as:

$$L'(\mathbf{h}) = \mathbf{J}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{R}(\mathbf{x}) + \mathbf{J}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{J}(\mathbf{x}) \mathbf{h} \quad (11)$$

According to $L'(\mathbf{h}_{gn}) = 0$, the Gauss-Newton step can be calculated by the following equation:

$$\mathbf{J}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{J}(\mathbf{x}) \mathbf{h}_{gn} = -\mathbf{J}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{R}(\mathbf{x}) \quad (12)$$

Later, Levenberg and Marquardt provide a damped version of Gauss-Newton method, which is familiar to researchers as the LM method. The search step of the LM method can be calculated similarly by:

$$(\mathbf{J}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{J}(\mathbf{x}) + \lambda \mathbf{I}) \mathbf{h}_{lm} = -\mathbf{J}(\mathbf{x})^T \mathbf{C}_D^{-1} \mathbf{R}(\mathbf{x}) \quad (13)$$

where, λ denotes the damping parameter; \mathbf{I} denotes the identify matrix; \mathbf{h}_{lm} denotes the search step used in the LM method.

After obtaining \mathbf{h}_{lm} , the estimation of the controlling knots can be updated by:

$$\mathbf{x}_{new} = \mathbf{x} + \mathbf{h}_{lm} \quad (14)$$

where, \mathbf{x}_{new} denotes the newly updated estimation of the controlling knots

3.2. The LM method with finite difference Jacobian approximation

As mentioned in Section 3.1, Jacobian matrix is the key to using the LM method successfully. However, in many practical optimization problems including estimation of relative permeability, the objective function cannot be formulated analytically. So it is impossible to give the equation for the elements in Jacobian matrix. In order to deal with this problem, the secant version of the LM method is intended.

The finite difference is most widely used to approximate the elements of Jacobian matrix. The $(i, j)^{th}$ element can be approximated by:

$$(\mathbf{J}(\mathbf{x}))_{ij} \approx d_{ij}^{FD} = \frac{R_i(\mathbf{x} + \varepsilon \mathbf{e}_j) - R_i(\mathbf{x} - \varepsilon \mathbf{e}_j)}{2\varepsilon}, (i = 1, \dots, m; j = 1, \dots, n) \quad (15)$$

where, d_{ij}^{FD} denotes the finite difference gradient; \mathbf{e}_j denotes the unit vector in the j th coordinate direction; ε denotes the perturbation step.

For simplicity, the LM method with finite difference Jacobian approximation is denoted as the LM-FD method. As Eq. (15) indicates, 2 simulations are essential to calculate the central difference of the residuals to each controlling knot. So every iteration needs $2n$ (where n is the number of unknown controlling knots) simulations to approximate the Jacobian matrix by the finite difference method. The computational cost is huge

and proportional to the number of unknown controlling knots, which limits the use of the LM-FD method to only small-scale problems. As a result, more efficient Jacobian approximation method should be introduced into the procedure of the traditional LM method.

3.3. The LM method with simultaneous perturbation Jacobian approximation

Spall (1992) proposed simultaneous perturbation stochastic approximation method (SPSA). The SPSA method perturbs all unknown parameters at a time and the central difference approximations of the objective function to all variables can be calculated by only 2 simulations. So in every iteration, only 2 simulations are enough to approximate the Jacobian matrix, saving huge computational cost than the finite difference method. This paper integrates the idea of simultaneous perturbation into the basic LM method for calculating the Jacobian approximation.

On the basis of simultaneous perturbation, the $(i, j)^{\text{th}}$ element of the Jacobian can be approximated by:

$$(\mathbf{J}(\mathbf{x}))_{ij} \approx d_{ij}^{SP} = \frac{R_i(\mathbf{x} + \varepsilon \boldsymbol{\delta}) - R_i(\mathbf{x} - \varepsilon \boldsymbol{\delta})}{2\varepsilon \delta_j}, (i = 1, \dots, m; j = 1, \dots, n) \quad (16)$$

where, d_{ij}^{SP} denotes the simultaneous perturbation gradient; $\boldsymbol{\delta}$ denotes the n -dimensional vector of perturbation direction; δ_j denotes the j th element of $\boldsymbol{\delta}$, sampled from ± 1 Bernoulli distribution.

Similar to the LM-FD method (Hou et al., 2012b), the paper denotes the LM method with simultaneous perturbation Jacobian approximation as the LM-SP method. Spall (1992) proved that the expectation of SPSA is equal to the true gradient. So the convergence rate and robustness of the LM-SP method can be improved by averaging several Jacobian approximation samples. Here, the $(i, j)^{\text{th}}$ element of the Jacobian matrix is calculated by averaging several simultaneous perturbation gradient samples. It is formulated as:

$$(\mathbf{J}(\mathbf{x}))_{ij} \approx \frac{1}{\beta} \sum_{k=1}^{\beta} d_{ijk}^{SP}, (i = 1, \dots, m; j = 1, \dots, n) \quad (17)$$

where, β denotes the sampling number of the simultaneous perturbation gradients.

The series of the LM method with Jacobian approximation by Eq. (17) are denoted as the LM-SP- β method. From Eq. (16) and (17) we can see that only 2β (where $\beta \ll n$) simulations are required in each iteration and the computational cost is independent from the number of the controlling knots. So the computational efficiency of the LM-SP- β is much higher than the traditional LM-FD method.

The LM-SP- β method is verified and compared with the LM-FD method by estimation of relative permeability in this paper. During the iterations, the damping parameter λ is updated using the following rule:

if $O(\mathbf{x}_{new}) \geq O(\mathbf{x})$, $\lambda = \lambda \times 10$; however if $O(\mathbf{x}_{new}) < O(\mathbf{x})$, $\lambda = \lambda / 10$. The stopping criteria of the iteration is set as:

$$|O(\mathbf{x}_{new}) - O(\mathbf{x})| < \xi \text{ or } count > count_{max} \quad (18)$$

where, ξ denotes a small positive number; $count_{max}$ denotes the maximum allowed iteration steps.

4. Validation and application of the LM-SP method

4.1. Numerical experiment

4.1.1. Numerical simulation model

In order to verify the effectiveness and efficiency of the LM-SP method, an ideal 1-dimensional reservoir simulation model is built, as shown in Fig. 1. It is discretized into 40 blocks on a Cartesian grid with $40 \times 1 \times 1$ in x, y and z directions. The grid lengths are 0.5m, 5m, and 5m respectively in the three directions. There are one injection well and one production well in the model. Table 1 summaries the basic reservoir properties. As for the forward simulation, the reference permeability curves are represented by a cubic B-spline model with 6 segments. In other words, 13 controlling knots are used to characterize the permeability curves. The oil production rate and water injection rate are set as $0.3 \text{ cm}^3/\text{min}$. And the reservoir is developed for 1500 minutes in total. The injection pressure, cumulative oil production and cumulative water production are collected to be matched in the following inverse problem. On this basis, the LM-SP and LM-FD methods are used to estimate the 13 controlling knots of the cubic B-spline model by history matching the observed production performance. During the iterations, the stopping criteria are set as $\xi = 10^{-6}$ and $count_{max} = 100$. In this study, the forward simulation and the estimation of permeability curves are carried out on our personal computer (Intel Core i3-2130 CPU 3.40GHz; RAM 4.00GB).

4.1.2. Results and analyses

Fig. 2 compares the iteration process of the least squares error obtained by LM-FD, LM-SP-1 and LM-SP-2, respectively. Fig. 3 compares the estimation accuracy and computational cost observed by the three algorithms. As can be seen, the LM-FD method converges to the objective function of 0.010 after 41 iterations. The matching accuracy between the predicted and observed production performance is satisfactory, but the

computational cost is huge. It runs 947 numerical simulator and costs about 10.00 minutes. This is mainly because each iteration needs at least $2n+1$ (where n is the number of unknown controlling knots) simulations to approximate Jacobian matrix and judge stopping criteria. The LM-SP-1 method also needs 41 iterations to converge. However, because 2 simulations are enough to approximate the Jacobian and 1 simulations for judging stopping criteria, the total simulation runs are reduced to 124 and the computational time is only 1.47 minutes. Since the LM-SP-1 method uses only one simultaneous perturbation gradient, the Jacobian approximation is not accurate enough and the objective function converges to 0.017. Comparatively, the LM-SP-2 method converges to the least objective function of 0.0015 after only 25 iterations. This is owing to more accurate Jacobian approximation by averaging 2 simultaneous perturbation gradients for each Jacobian element. Although the LM-SP-2 method needs twice as many simulations as the LM-SP-1 method to approximate Jacobian in each iteration, the total simulations are just 137 and only 1.61 minutes are cost. The computational cost is only 16.1% of that using by the LM-FD method. So we can conclude that the LM-SP-2 method is able to achieve better estimation accuracy with much less computational cost compared with the LM-FD method.

Fig. 4 compares the estimation results of relative permeability obtained by LM-FD, LM-SP-1 and LM-SP-2, respectively. As can be seen, the estimation results are different although the three methods use identical initial guesses. When the water saturation is high, the water phase relative permeability estimated by the LM-FD method is of relatively lower accuracy. As for the LM-SP-1 method, the estimated oil phase relative permeability deviates from the reference curve at medium water saturation. To be exciting, both of the oil phase and water phase relative permeability estimated by the LM-SP-2 method match well with the reference curves. This is mainly because the LM-SP-2 method increases the approximation accuracy of Jacobian matrix by averaging 2 simulations perturbation gradients for each Jacobian element.

Fig. 5 compares the matching results of injection pressure, cumulative oil production and cumulative water production obtained by the three methods. From the figure, we can see that the cumulative oil production predicted by the LM-SP-1 method cannot match the observed data. However, the LM-FD and LM-SP-2 methods can match well with the observed cumulative oil production. As regards to the injection pressure and cumulative water production, all of the three methods have high matching accuracy.

As mentioned above, the LM-SP-2 method performs better than the LM-SP-1 method for estimating relative permeability because of averaging 2 simulations perturbation Jacobian approximations. So the problem here is finding the most suitable sampling number of simultaneous perturbation gradients to be averaged for

each Jacobian element. Fig. 6 illustrates the effect of sampling number on estimation error and computational cost of the LM-SP- β methods (where β denotes the sampling number). In this figure, the average estimation error of relative permeability is calculated by $\frac{1}{n} \sum_{i=1}^n |kr_i - kr_i^o|$, where kr and kr^o denotes the estimated and reference relative permeability respectively. The figure shows that the matching errors and convergence iterations of the LM-SP- β methods decreases firstly as the sampling number increases, and then tends to be stable after averaging 3 simultaneous perturbation samples. This is because the approximation accuracy of Jacobian matrix can be improved by averaging more simultaneous perturbation samples and the approximation accuracy is good enough after using 3 samples. Since the simulations used to approximate Jacobian matrix in each iteration is proportional to the sampling number. The total computational cost of the LM-SP- β methods increases rapidly with the sampling number in each iteration. Considering comprehensively the estimation accuracy and computational cost, averaging 2 simultaneous perturbation gradient samples is the best choice to approximate the Jacobian matrix for using in the LM method. As a result, the LM-SP-2 method is used in the following study of this paper.

Generally, the number of unknown parameters has a great effect on the computational cost used by different optimization methods to solve nonlinear least squares problems. In this paper, the number of unknown controlling knots is $2n-1$ where n is the segments of the cubic B-spline model. Fig. 7 compares the simulation runs used by the LM-FD and LM-SP-2 methods, in which the segments are set as 4, 5, 6, 7, 8 (i.e. the unknown controlling knots are 7, 9, 11, 13, 15). Since the simulations needed to approximate Jacobian is proportional to the number of controlling knots in the LM-FD method, the total computational cost increases rapidly as the segments increase. However, as for the LM-SP-2 method, the simulations used to approximate Jacobian is independent from the number of controlling knots. So the number of simulations needed by the LM-SP-2 method to converge is similar although the segments of the cubic B-spline model increases. We can also see that the LM-SP-2 method saves more and more computational cost than the LM-FD method as the number of segments increases. In other words, the LM-SP-2 method is much more suitable for large-scale nonlinear least squares problems including the estimation of relative permeability in actual oilfields where different permeability curves are used in different regions.

4.2. Laboratory experiment

In order to further prove the stability of the proposed LM-SP method, this paper studies the application of

the LM-SP-2 method in history matching the radial laboratory displacement experiments published by Hou et al. (2012b). The same objective function, relative permeability representation model and production performance are used in this paper. More details about the radial laboratory displacement experiments can be found in the publication. We also compare the estimation and prediction results obtained by the LM-SP-2 method with that in the original publication.

Fig. 8 compares the estimation results of relative permeability and Fig. 9 compares the matching results of production performance. As can be seen, the estimation of relative permeability and the prediction of cumulative oil production and cumulative water production obtained by the LM-SP-2 method are similar to the earlier published paper. However, the LM-SP-2 method has a much higher prediction accuracy of displacement pressure difference than that in Hou et al. (2012b).

Fig. 10 compares the objective function and computational cost after the iteration converges. Compared with the LM-FD method, the LM-SP-2 method is able to converge to a smaller objective function value using much fewer simulations. As a result, much computational time can be saved by using the LM-SP-2 method. The above analyses further verify that the series of the LM-SP methods are good candidates for estimation of relative permeability curves and to solve other nonlinear least squares problems.

4.3. Field application

In this section, the method is applied to estimate the relative permeability curves for Gudao Dong oilfield in China. Gudao Dong is a large real oilfield which covers a region of 12.9 km^2 and has a net pay thickness of 17.7 m. The average porosity, permeability and initial oil saturation are 31.1%, $1138 \times 10^{-3} \mu\text{m}^2$ and 61.3%, respectively. The proven reserve of Gudao Dong is about $3969 \times 10^4 \text{ t}$. This oilfield has been developed since June 1975 and now there are 155 producers and 85 injectors (Hu, 2010). Fig. 11 shows the reservoir model and well locations of Gudao Dong oilfield.

Before estimation of relative permeability curves, the other parameters of the reservoir model such as porosity, permeability and water saturation of each grid have to be determined and they should be in accordance with the real oilfield. This is realized by the history matching procedure with the traditional trial and error method before the estimation study. For the convenience of accurately comparing the efficiency of the proposed LM-SP-2 and the pre-existing LM-FD, the forward simulation results of oil production rate, reservoir pressure and water cut from 1975 to 2008 by the manually history matched reservoir model of Gudao

Dong oilfield are actually used as the observed data sets to estimate the relative permeability curves. The injection and production rates of the total liquid (water and oil) are strictly constrained in the estimation study by the observed liquid rates from the reservoir model.

Fig. 12 compares the estimated relative permeability curves obtained by the two methods. As can be seen, the water phase relative permeability curves are almost the same. Although there is a little difference, the oil phase relative permeability curves are also very close between the proposed LM-SP-2 and the pre-existing LM-FD. The oil production rate, reservoir pressure and water cut predicted by the two relative permeability curves will be compared and analyzed.

Fig. 13-15 show the comparisons between the observed and predicted production performance by the two methods. Table 2 further quantifies the difference between the observed and the predicted data. In the table, the average errors are calculated by $\sum_{i=1}^m |p_i^o - p_i^p| / m$, where p_i^o and p_i^p denote the observed and predicted data respectively. The average predicted error of oil production rate in the LM-SP-2 method is 53.64 m³/d, which is a little larger than 49.05 m³/d obtained by the LM-FD method. However, the LM-SP-2 method obtains a smaller average predicted error of 1.52% in water cut, while that obtained by the LM-FD method is 1.79%. As for reservoir pressure, both of the two methods match well with the observed data. This is mainly because the injection and production rates of the total liquid (water and oil) are strictly constrained by the observed liquid rates from the reservoir model.

Table 3 compares the computational cost needed by the two methods. The LM-SP-2 method saves 284 simulations and 68.6% CPU time compared with the pre-existing LM-FD method, although the proposed method needs 5 more iterations to converge. As can be seen from Table 2 and Table 3, the proposed LM-SP-2 method can reduce computational cost without losing estimation quality compared with the pre-existing LM-FD method when dealing with large real-field problems. These results, integrated with those of Section 4.1 and 4.2, indicate the applicability of the proposed LM-SP method on large real-field problems as well as small scale-laboratory problems.

5. Conclusions

(1) The paper introduces a simultaneous perturbation method into the original Levenberg-Marquardt procedure in order to simplify the generation of the necessary Jacobian matrix. This reduces computational cost and increases estimation quality. So the contribution of the paper is an improved LM-SP method that gives

a speed-up of the pre-existing LM-FD algorithm. Research indicates the applicability and superiority of the proposed LM-SP method on large real-field problems as well as small scale-laboratory problems.

(2) The basic LM-SP method needs 2 simulations in each iteration to approximate the Jacobian matrix needed by the Levenberg-Marquardt procedure, while $2n$ (n is the number of unknown parameters) simulations are needed in the LM-FD method. As a result, the LM-SP method is able to converge to a similar value using less computational cost compared with the pre-existing LM-FD method. This advantage becomes more obvious with the increase of the number of unknown parameters.

(3) The Jacobian approximation in the proposed method involves stochastic sampling of perturbation steps. It is not always good enough and thus the convergence rate may be affected. This problem is solved by averaging several independent simultaneous perturbation approximations but the computational cost of each iteration is increased. Two independent approximations perform well in this paper to balance the estimate quality and the computational cost. But we still do not know how to obtain a best choice generally for different problems.

Acknowledgments

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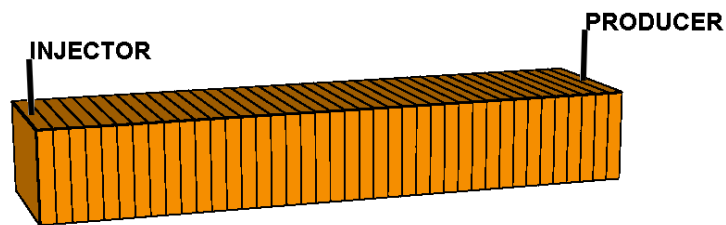


Fig. 1. The reservoir simulation model.

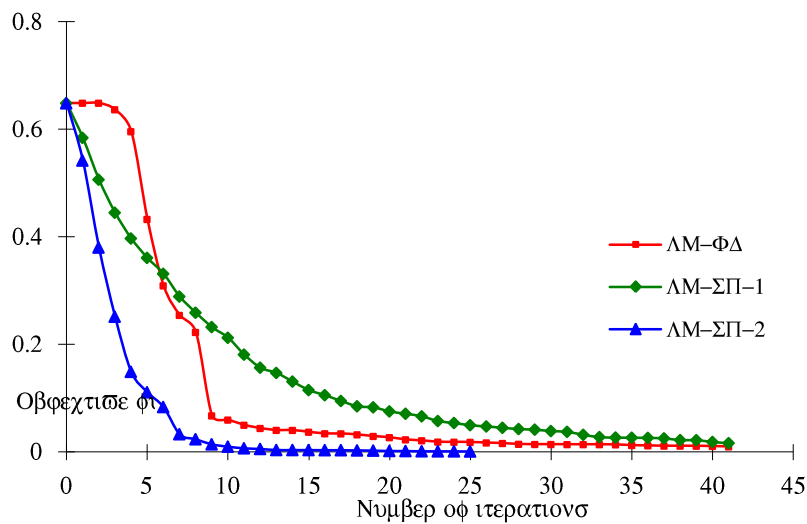


Fig. 2. Comparison of the objective function obtained by the three methods.

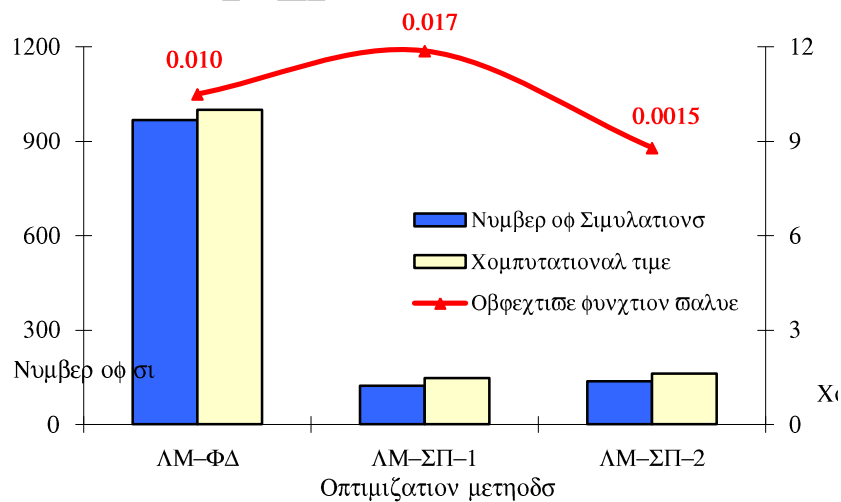


Fig. 3. Comparison of the computational cost and objective function obtained by the three methods.

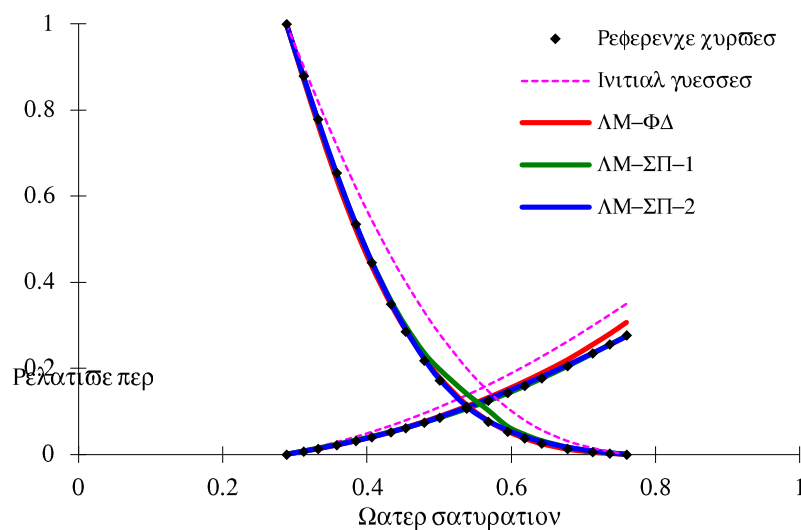


Fig. 4. Estimation results of relative permeability obtained by the three methods.

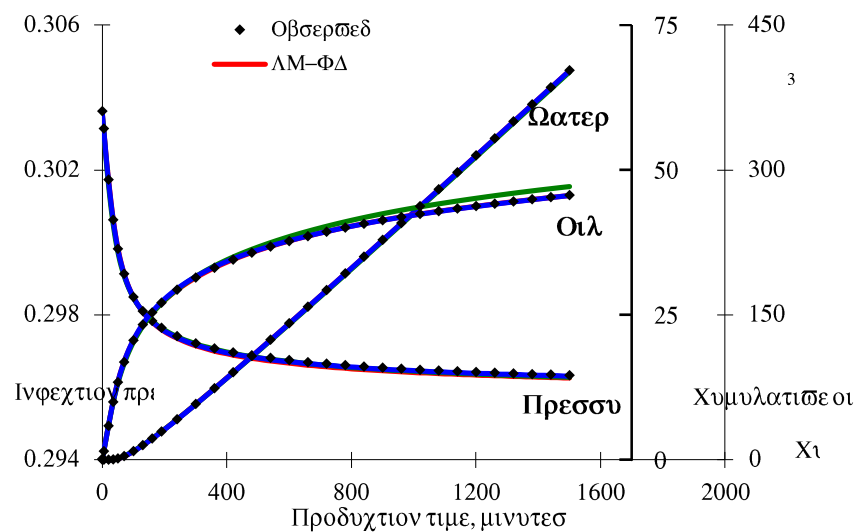


Fig. 5. Matching results of production performance obtained by the three methods.

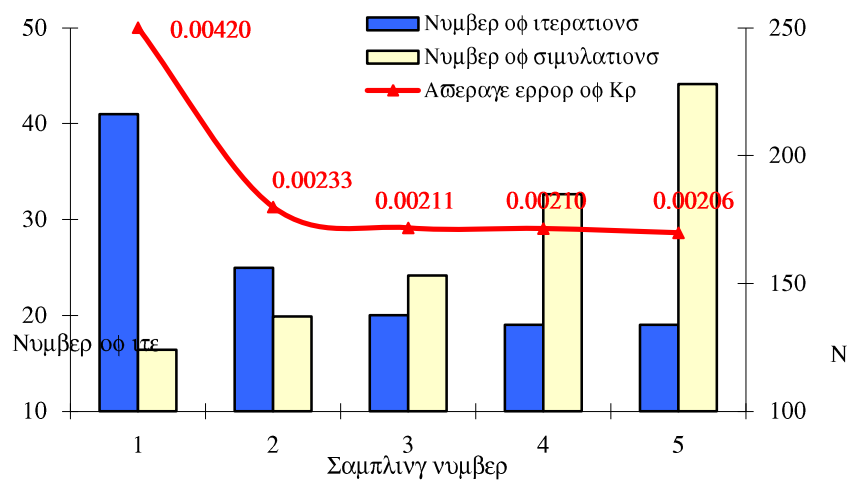


Fig. 6. The effect of sampling number on estimation error and computational cost of LM-SP- β methods.

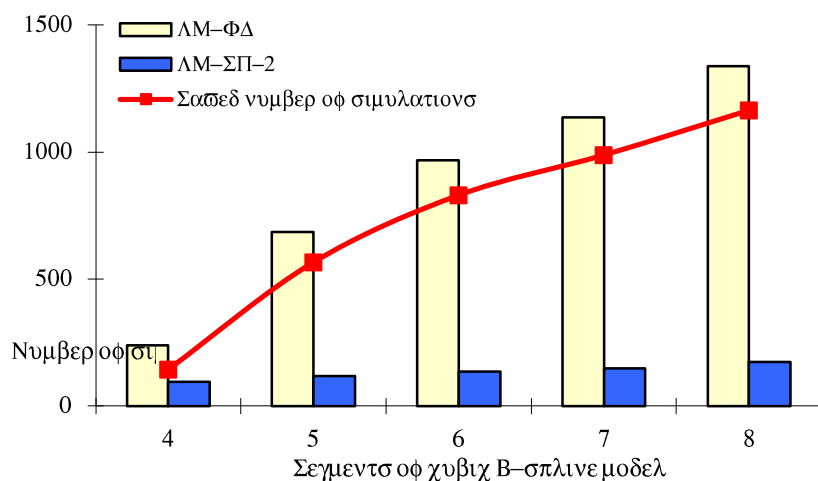


Fig. 7. The effect of the number of unknown parameters on computational cost of the LM-SP-2 method.

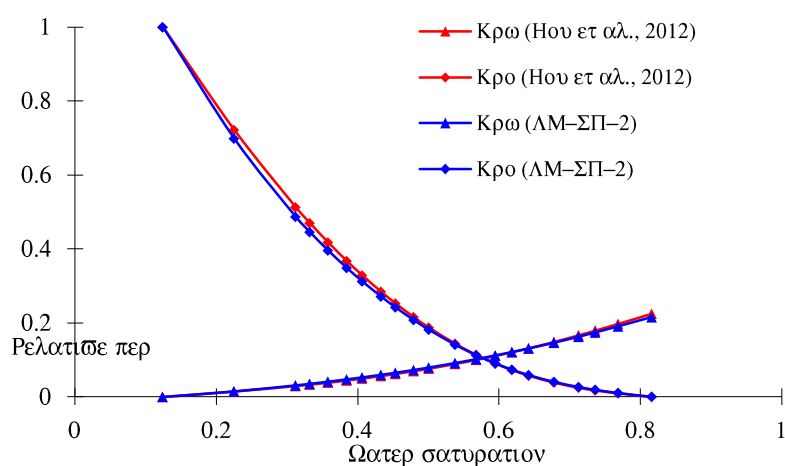


Fig. 8. Comparison of estimation results of relative permeability.

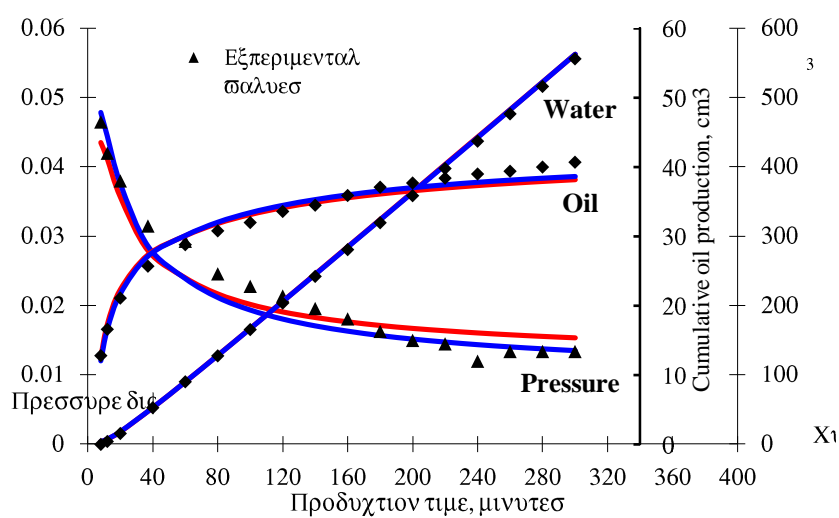


Fig. 9. Comparison of prediction results of production performance.

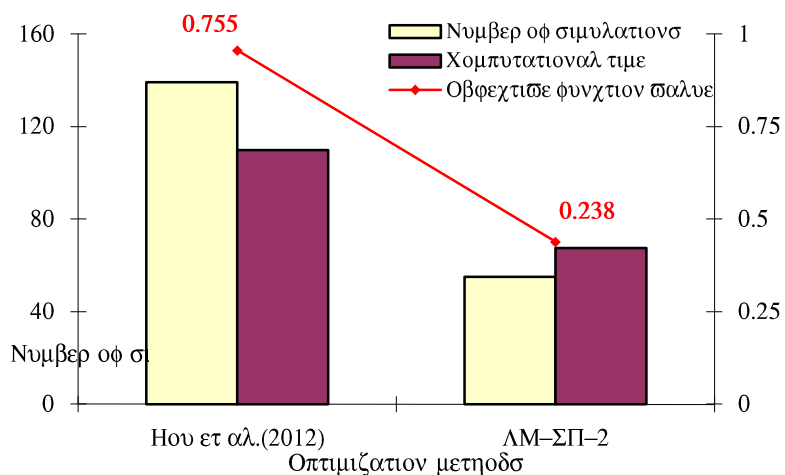


Fig. 10. Comparison of the computational cost and objective function values.

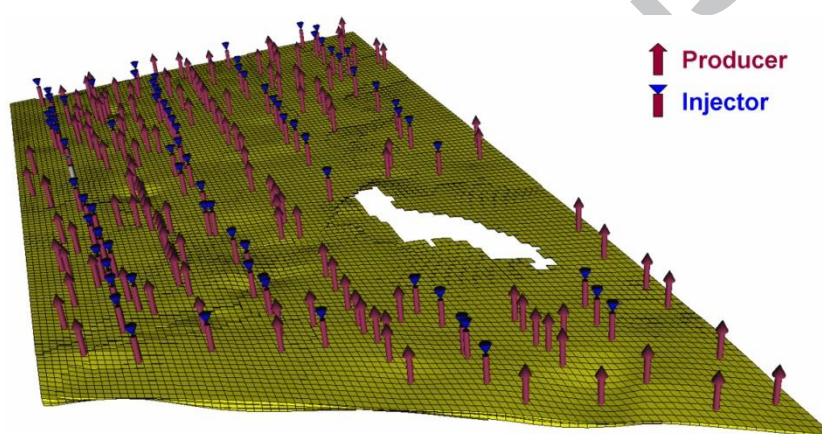


Fig. 11. The reservoir model and well locations of Gudao Dong Oilfield

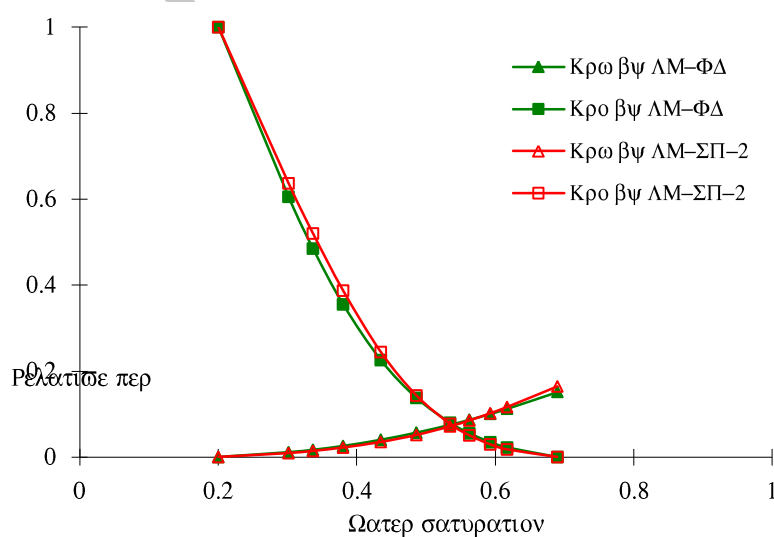


Fig. 12. Comparison of the estimated relative permeability curves.

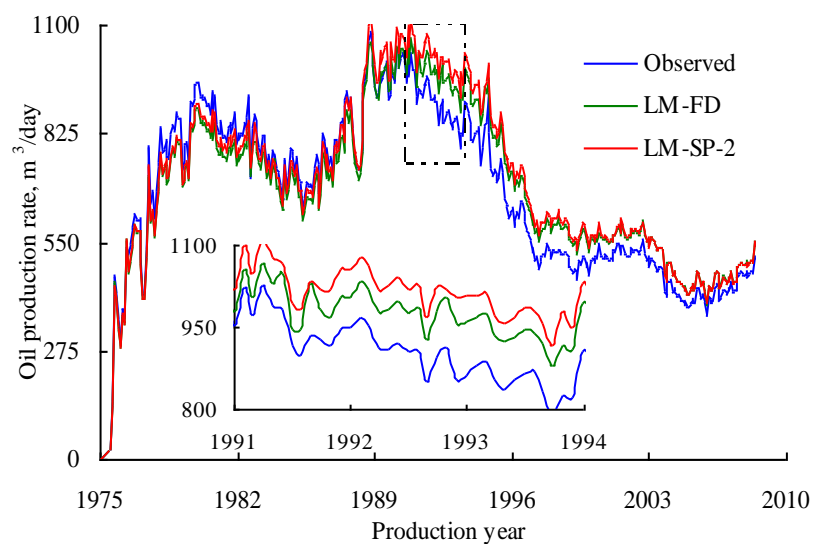


Fig. 13. Comparison of the predicted oil production rate.

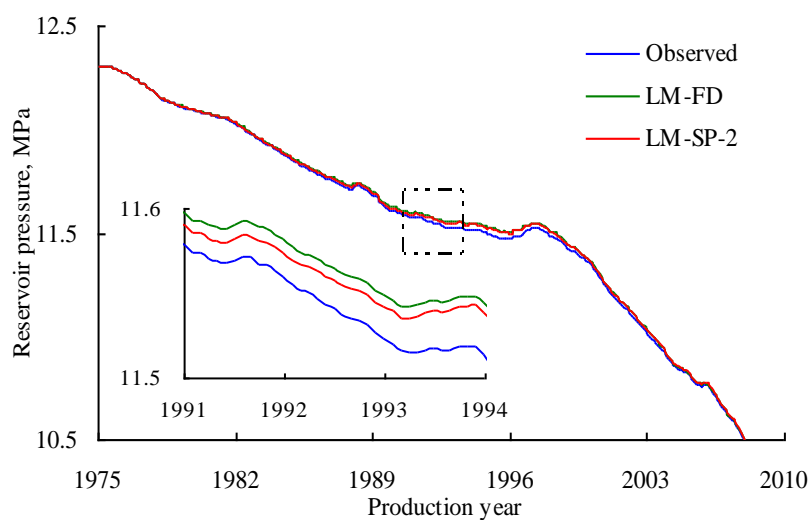


Fig. 14. Comparison of the predicted reservoir pressure.

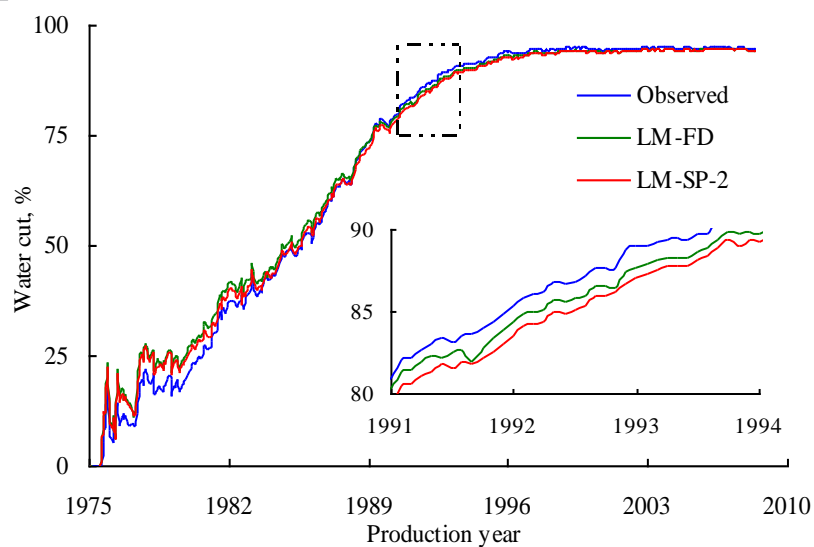


Fig. 15. Comparison of the predicted water cut.

Table 1 The geological properties of the reservoir simulation model.

Parameters	Values	Parameters	Values
Average porosity, %	30	Initial oil saturation, %	71.15
Average permeability, $10^{-3}\mu\text{m}^2$	1000	Crude oil density, g/cm^3	0.904
Reservoir temperature, $^{\circ}\text{C}$	45	Crude oil viscosity, $\text{mPa}\cdot\text{s}$	20
Initial pressure, MPa	10	Water viscosity, $\text{mPa}\cdot\text{s}$	0.45

Table 2 Fitting errors between the observed and predicted production performance

Methods	Fitting errors					
	Oil production rate, m^3/d		Reservoir pressure, MPa		Water cut, %	
	Max	Mean	Max	Mean	Max	Mean
LM-FD	108.17	49.05	0.033	0.014	7.72	1.79
LM-SP-2	154.27	53.64	0.028	0.011	6.51	1.52

Table 3 Comparison of the computational cost used by LM-FD and LM-SP-2

Computational cost	LM-FD	LM-SP-2
Number of iterations	17	22
Number of simulations	395	111
CPU time, hours	801.5	251.6

Highlights

1. An improved LM method with simultaneous perturbation Jacobian approximation is built.
2. The LM-SP method is verified by both numerical and radial displacement experiments.
3. The LM-SP method can obtain better estimation results using less computational cost.
4. The convergence of LM-SP can be speeded by averaging several Jacobian approximations.
5. The LM-SP method is very suitable for large-scale nonlinear least squares problems.