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Water exclusion from tunnel cavities spanning a water table

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Abstract

The vertical downward flow of water in unsaturated soils is perturbed by the presence of cavities. Analysis of the flow problem has shown that there is for a given situation a critical shape of a cavity for it to exclude water entry when the soil–water pressure at the cavity wall is everywhere the air-entry value for the soil and the cavity wall is a streamline. In this note cavities spanning a water table are considered when the water table is caused by artesian pressure in a water-bearing substratum. Building on previous analysis, critical shapes for such cavities are obtained for particular circumstances, thus showing that situations can exist for cavities not to fill with water although protruding below the water table. It is noted that the analysis used to obtain a solution to this problem is the same as that for the groundwater seepage problem of water movement to a water-bearing substratum under pressure from infinite ponded-water regions separated by a long island strip.

Keywords: Tunnel cavity; Water exclusion; Drainage; Capillary fringe; Water table; Water-bearing permeable substratum

1. Introduction

Philip and his colleagues (Knight et al., 1989; Philip, 1989a,b; Philip 1990; Philip et al., 1989a,b) reported analytical studies on the perturbation of vertical downward flow of water around cavities in unsaturated soils. Their work was concerned with the flow in 'Gardner' soils in which the hydraulic conductivity of unsaturated soils is described by an exponential function of the soil–water pressure. They showed that the shape of the cavity was important as regards whether the cavity would exclude water (the subcritical condition) when the soil–water pressure on all parts of the cavity wall was less than atmospheric. The cavity would leak water (the supercritical condition) when the soil–water pressure on any part of the cavity wall was greater than atmospheric. A critical shape exists when the soil–water pressure on all parts of the cavity wall is just less than atmospheric. For two-dimensional tunnel cavities this shape is parabolic (Philip et al., 1989b).

Youngs (2002) and Youngs et al. (2004) considered the critical shape of a tunnel cavity situated in

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the saturated capillary fringe of a Green and Ampt soil on top of a freely draining gravel substratum. The critical shape approximated to the parabolic shape predicted for Gardner soils, but was slightly smaller. They used conformal mapping to solve the two-dimensional problem of potential theory that describes the flow in the capillary fringe which, although at a negative soil–water pressure, is saturated with a uniform hydraulic conductivity.

Youngs' (2002) analysis assumes a horizontal upper boundary of the capillary fringe that implies a non-uniform flux input over this surface, so that this is an approximation to the exact analysis for uniform accretion given by Youngs et al. (2004). If, instead of the freely draining gravel substratum maintaining a water table at the base of the capillary fringe, a water table occurs in the soil above as a result of the water-bearing permeable substratum being under pressure, it might be surmised that a cavity spanning the water table would fill up to the level of the water table. However, consideration of the effect those such cavities have on water flow show that there are situations in which water is excluded. Here we use Youngs' (2002) analysis and show that in particular circumstances it leads simply to a critical shape of tunnel cavities spanning a water table that will not leak water.

2. The critical shape of tunnel cavities

We consider the situation of vertical downward flow of water through a capillary fringe in a uniform soil that overlies a very permeable, water-bearing stratum under pressure. In the absence of a cavity with a vertical flow at a rate q the water table is at a height w above the substratum, given by

$$w = \frac{P_a}{1 - q'} \tag{1}$$

where P_a is the pressure head of the water in the substratum and $q' = q/K$ with K the hydraulic conductivity of the saturated soil that extends to the top of the capillary fringe. The height of the capillary fringe H above the water table is

$$H = \frac{|P|}{1 - q'} \tag{2}$$

where P (negative in value) is the air-entry value of the soil–water pressure head at the top of the capillary fringe. Thus if $P_a = |P|$, $w = H$ and the water table divides the saturated flow region in half with the tension-saturated region above and the groundwater under positive pressure below.

We now consider a tunnel cavity that spans the water table in the above situation. We consider separately the capillary fringe above, assuming this to be bounded at the top by a horizontal surface at height $y = H$, where the soil–water pressure head is P, and the groundwater region below bounded at the bottom by the interface with a very permeable, horizontal substratum at depth $y = -w(= -H)$, where the soil–water pressure head is P_a, with $P_a = |P|$. We consider further that the tunnel cavity is of critical shape when water is just excluded from the cavity with the soil–water pressure on the cavity wall, which is a streamline, atmospheric. The boundary conditions are thus those shown in Fig. 1, where we see that the boundary conditions in the region above the water table are the same as those considered in Youngs' (2002) solution to the water exclusion problem. The potential distribution, as well as the shape of the cavity, is symmetrical about the water table. Thus, the potential and streamline patterns above and below the water table are thus given as a solution of a free boundary problem for

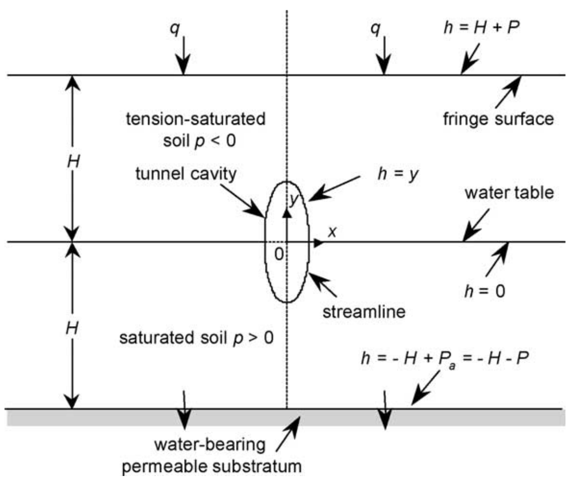

Fig. 1. A tunnel cavity of critical shape spanning the water table above a water-bearing permeable substratum under pressure.

a holomorphic function by [Youngs, 2002, Eq. (8)]

$$y - ix = \frac{(-P)}{\pi}\cos^{-1}\left[\frac{2\cos\left(\frac{\pi(\varphi+i\psi)}{K(H+P)}\right) + (1-t_L)}{1+t_L}\right] - \frac{\varphi + i\psi}{K} \quad (3)$$

with the origin of the coordinates (x,y) at the centre of the tunnel. In Eq. (3) φ is the seepage velocity potential (equal to $-Kh$, where h is the hydraulic head) and ψ is the stream function, and t_L is given by

$$t_L = \cos\left(\frac{\pi(-L)}{H+P}\right) \quad (4)$$

where $(0,L)$ are the coordinates of the apex of the tunnel. The negative value of the inverse trigonometric function in Eq. (3) is taken for calculations in the region below the water table. Thus, for the situation with $w=H$ the critical oval-shaped cavity tunnel is obtained from Eq. (3) when $\psi=0$ [Youngs, 2002, Eq. (9)]

$$x = \frac{(-P)}{\pi}\cosh^{-1}\left[\frac{2\cos\left(\frac{-\pi y}{H+P}\right) + (1-t_L)}{1+t_L}\right] \quad (5)$$

with the positive value of the inverse hyperbolic function taken when $0<y<H$, and the negative value when $0>y>-w(=-H)$. In Fig. 2 the streamline pattern around the critical oval-shaped tunnel cavity is illustrated. As H becomes large, $t_L \to 1$ so that x becomes zero for all values of y, showing that the cavity becomes a slit of negligible width.

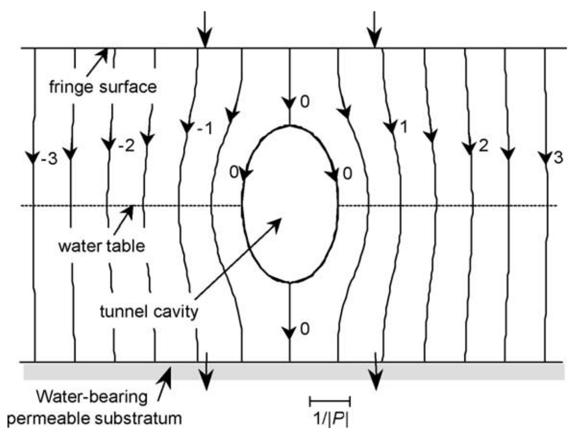

Fig. 2. The streamline pattern around a critical oval-shaped tunnel cavity spanning a water table with $L/|P|=2.0$ and $H/|P|=4.0$ giving $q_0=0.6$ K and $q_\infty=0.75$ K. Numbers by the streamlines are values of $\psi/(K|P|)$.

With the assumed horizontal upper capillary fringe surface, the flux density across the fringe surface at $y=H$ is non-uniform with a minimum value q_0 at $x=0$ given by [Youngs, 2002, Eq. (13)]

$$q_0 = \left|\frac{\partial\psi}{\partial x}\right|_{x=0,\ y=H} = \frac{K}{1 + \frac{(-P)}{(H+P)}\sqrt{\frac{2}{(1+t_L)}}} \quad (6)$$

and a maximum q_∞ at $x\to\infty$ given by [Youngs, 2002, Eq. (14)]

$$q_\infty = \frac{K(H+P)}{H} \quad (7)$$

$q_0 \to q_\infty$ as the fringe thickness H increases.

3. Discussion

A general assumption in groundwater hydrology is that water will enter any cavity dug below the water-table level. The analysis given here shows that this is not necessarily the case when there is vertical seepage to a water-bearing substratum under pressure. A critical shape for water exclusion from a tunnel cavity that spans a water table when the pressure of water in the substratum has the same value as the air-entry value of the soil, can be calculated using Youngs' (2002) analysis for critical cavity shapes situated above a freely draining permeable substratum. This critical shape is seen to be oval-shaped. Increasing the cavities beyond the critical shape would initiate water entry. This work indicates that caution must be taken in assuming water-table levels in soils are shown by levels of water in excavated holes.

This note addresses the problem of water exclusion from cavities in soils. It is interesting to note that the boundary conditions for the flow in the region below the water table in this problem are the same as those for the problem of seepage of water to a water-bearing substratum under pressure from infinite ponded-water regions separated by a long island strip. Thus, the analysis given here applies also to this groundwater problem and so is of more general interest.

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