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A modified simple dynamic model: derived from the information embedded in observed streamflows

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Abstract

A zero-dimension hydrological model has been developed to simulate the discharge (Q) from watershed groundwater storage(S). The model is a modified version of the original model developed by Kirchner in 2009 which uses a unique sensitivity function, $\mathrm{g(Q)}$ to represent the relation between rate of flow recession and the instantaneous flow rate. The modified dynamic model instead uses a normalized sensitivity function $\mathrm{g}(\mathrm{Q}_{norm})$ which provides the model the flexibility to encompass the hysteretic effect of initial water storage on flow during recession periods. The sensitivity function is normalized based on a correlation function $F(Q)$ which implicitly quantifies the influence of initial storage conditions on recession flow dynamics. For periods of either positive or negative net recharge to groundwater the model applies a term similar in form to an analytical solution based on solution to the linearized Boussinesq equation. The combination of these two streamflow components, the recession component and the net recharge response, provides the model with the flexibility to realistically mimic the hysteresis in the Q vs. S relations for a watershed. The model is applied to the Sagehen Creek watershed, a hilly watershed located in the Sierra Mountains of California. The results show that the modified model has an improved performance to simulate the discharge dynamic encompassing a wide range of water storage (degree of wetness) representing an almost ten-fold variation in annual streamflow.

Keywords

Storage - discharge relationship; streamflow recession; zero-dimension dynamic model; subsurface flow

1. Introduction

. Recession flow analysis is a robust method to reveal possible Q vs. S relationships (Smakhtin, 2001; Tallaksen, 1995; Troch et al., 2013). One well-known recession flow analysis method for obtaining this relationship from streamflow data, first proposed by Brutsaert and Nieber (1977), is generally expressed as:

$$-\mathrm{d}Q/\mathrm{d}t = KQ^{\alpha} \tag{1}$$

where K and α are two parameters obtained from a recession flow analysis. Coupled with a water balance equation,

$$\mathrm{d}S/\mathrm{d}t = (R - ET - Q) \tag{2}$$

where R is recharge to and ET is evapotranspiration/water extraction from watershed subsurface storage, equation (1) has been widely used in the analysis of watershed storage and related hydrologic dynamic flux analysis.

As identified by Ewen (1997), it is known that a unique Q vs. S relationship is inadequate to represent the dynamics of watershed discharge and storage. The two parameters in equation (1) have been found to vary significantly among recession events, even within the same watershed. In attempts to overcome this issue, parallel or series simple models have been applied broadly in some previous recession flow analyses. Moore (1997) compared the performance of five model structures by application to the streamflow for a small forested watershed. Majone et al. (2010) presented a linear and nonlinear model to mimic the discharge from small Alpine watersheds. Bart and Hope (2014) used a parsimonious storage-discharge model consisting of two parallel stores to reveal the role of storage on streamflow recession. Rusjan and Mikoš (2015) divided streamflow into two components, a fast flow component and a slow flow component, and constructed the Q vs. S relationship separately for each. Stoelzle et al. (2015) examined the application of nine conceptual model structures to 25 watersheds to determine the best association with a particular watershed type. Shaw (2016) presented a conceptual model consisting of three parallel linear models and evaluated them for multiple wetting and drying periods. Based on the results of all these efforts it would seem that the storage-discharge relationship might be simulated better with a merging of multi-simple models. However, as suggested by Melsen et al. (2014), the merging of a number of simple models will tend to introduce more parameters and initial conditions to be specified *a priori*, and thus increase the uncertainty of model structure.

Hydrological models may be able to satisfactorily simulate observed flow processes just by providing “effective parameters”, even when the structure of the model does not correspond well with the conceptual structure of a watershed (Adamovic et al., 2015; Kirchner, 2006). However, such effective models will not generally perform well when tested outside the range of the data used for calibration of the effective parameters. Models that have structures compatible with the conceived structure of the watershed will instead be able to consistently outperform these effective parameter models.

So the question arises regarding how to identify or develop the correct hydrologic structure of a watershed. Sivapalan et al. (2003) suggested that observed time series such as streamflow have been shown to provide the information necessary for derivation of appropriate model structures, rather than only being used just for model parameter calibration for less than appropriate models. Consistent with this idea, Kirchner (2009) constructed the Q vs. S relationship by a sensitivity function, $g(Q)$, i.e.,

$$g(Q) = dQ/dS \tag{3}$$

The function $g(Q)$ has an advantage of not requiring a fixed form *a priori*, but rather it can be directly inferred from observed recession flow dynamics and thus has a flexible form in different watersheds, which may themselves have distinct hydrologic process structure. By combining the sensitivity function (3) and water balance equation (2), Kirchner constructed a simple model (hereinafter referred to as Kirchner model) for streamflow simulation. Following Kirchner, Teuling et al. (2010), Krier et al. (2012), Brauer et al. (2013), Melsen et al. (2014), and Adamovic et al. (2015) have applied the Kirchner model in a wide range of watersheds types and environments. As might be expected, it was also found that a unique $g(Q)$ had a limited ability to simulate hydrographs for a given watershed, especially for variable wetness conditions. Teuling et al. (2010) concluded the examined watershed behaved as a simple dynamic watershed only under wet conditions and did less so under dry conditions. Krier et al. (2012) tested the Kirchner model in 24 watersheds and found model performance much better systematically at higher soil moisture levels. Brauer et al. (2013) applied the Kirchner model in a less humid lowland watershed and also found poor performance during dry summer periods. Melsen et al. (2014) concluded the Kirchner model was not able to describe both the high and low flows with a single set of parameters after applying it to a small watershed. Adamovic et al. (2015) also found the Kirchner model worked especially well in wet conditions, and showed a poorer performance associated with dry initial conditions. These results confirmed that the wetness condition in a watershed has a significant influence on the watershed Q vs. S relationship.

Although efforts to define and quantify the Q vs. S relationship analysis has been ongoing for decades, many questions persist, including whether a superior model structure for baseflow simulation exists (Stoelzle et al., 2015), and whether the hydrologic characteristics of watersheds can really be drafted from streamflow recession flows (Stoelzle et al., 2013). The starting point of our current work is to develop a new model similar in structure to the Kirchner model in that the model consists of a single conceptual reservoir. The focus of this new model is that it needed to have the ability to account for differences in varied conditions of initial storage. Consistent with many other watersheds mentioned above, our recent research (Li and Nieber, 2016, unpublished report) into the streamflow for Sagehen Creek, located in the Sierra Mountains of California, has demonstrated the strong effect of initial storage distribution on baseflow recession. We also found a clear relationship between antecedent storage and the recession low flow associated with the recession event. In the present study, we introduce a new variable, "recession low flow" as a basis for modifying the Kirchner model. We do this in contrast to the approach of constructing a model composed of multiple series or parallel conceptual reservoirs (e.g., Bart and Hope, 2014; Rusjan and Mikoš, 2015; Shaw, 2016). Introducing a new variable to represent the whole system is an approach suggested by Ewen (1997) for improving the performance of hydrologic models.

Overall, there are two objectives for this paper. First to derive a model similar to the model of Kirchner (2009) but modified to improve the model performance for both wet and dry watershed conditions. Here we will refer to the model as a modified simple dynamic model (hereinafter referred to as MSD model). As with the Kirchner model, the MSD model simulates only the groundwater contribution to streamflow. The second objective is to test the MSD model performance by comparing model predictions with the groundwater component of flow derived from a well-parameterized model, GSFLOW, for the Sagehen Creek watershed (Markstrom et al., 2008).

2. Description of the study watershed and associated data

Sagehen Creek is located on the east side of the northern Sierra Nevada (Figure 1). The drainage area of the watershed is approximately 29.3 km^2. The land surface elevation in the watershed ranges from 1935 to 2653 m, and the average slope of surface is about 15.8%. Sagehen has been described as having a Mediterranean-type climate with cold, wet winters and warm dry summers (Manning et al., 2012). Mean annual temperature from 1980 to 2002 was 4℃ at an altitude of 2,545 m. Mean annual precipitation from 1960 to 1991 was 970 mm. Approximately 80% of precipitation falls as snow. Daily mean streamflow data are available for the gage located near the outlet of the watershed (gaging station number: 10343500). While most of the precipitation falls as snow, the streamflow is continuous throughout the year, supported by baseflow derived from rainfall and snowmelt infiltration, and surface runoff from snowmelt. Mean daily streamflow was approximately 0.33 m^3/s during the 16-year period of record (Markstrom et al., 2008).

Geology of the Sagehen Creek watershed consists of granodiorite bedrock overlain by volcanic deposits, which are overlain by till and alluvium. Although very little is known regarding the depths of different geologic formations, the volcanic deposits are estimated to range in thickness between 50 and 300m, while alluvium is estimated to range in thickness between 0 and 10m (Markstrom et al., 2008). The Sagehen Creek watershed is entirely forested except for scattered meadows along the stream. The tree species present in Sagehen Creek watershed are dominated by conifers including lodgepole pine, Jeffrey pine, sugar pine, western white pine, white fir, red fir, and mountain hemlock, but there is also a significant fraction of the deciduous species, quaking aspen (Faïn et al., 2011). A period of 16 water years, 01 October 1980 to 30 September 1996 was utilized in this research, because the 16 years encompass a wide range of wetness conditions. As is shown in Figure 2, the annual streamflow varied between 98 mm/year（1991 water year）and 974 mm/year (1982 water year). The study period was separated into two groups: 8 years calibration period (the gray bars illustrated in Figure 2, 1980, 1981, 1985, 1988, 1989, 1991, 1993, 1995) and 8 years validation period (the white bars illustrated in Figure 2, 1982, 1983, 1984, 1986, 1987, 1990, 1992, 1994).

The 16-year time period was previously evaluated as a documented example application of the GSFLOW model in the Sagehen Creek watershed (Markstrom et al., 2008). The GSFLOW model is composed of a surface water model, PRMS (Leavesley et al., 2005) coupled with MODFLOW (Harbaugh, 2005), a groundwater flow model. The PRMS model simulates the terrestrial components of the hydrologic cycle including interception, snowmelt, infiltration, deep percolation, interflow, evapotranspiration, surface runoff, and streamflow routing. The model imposes a distributed parameter approach by defining landscape hydrologic response units based on local soil, vegetation, and topography conditions. The deep percolation calculated by PRMS is then given as recharge input to the upper grid cells of the MODFLOW model. In the Markstrom et al. study the GSFLOW model was calibrated to simulate the observed daily streamflow given the observed meteorological data collected for the watershed, resulting with NSE (Nash and Sutcliffe, 1970) values of about 0.81. As such, the MSD model presented in this paper can be tested in detail by comparing with the groundwater component of flow synthesized by the GSFLOW model.

Recession analysis was carried out based on the observed streamflow in the calibration period. The variable, dQ/dt (=($Q_{t-\triangle t}$ - Q_t)/$\triangle$t) was calculated with consecutive daily flows, and plotted against the arithmetic mean (($Q_{t-\triangle t}$+Q_t)/2) of the corresponding flows (Brutsaert and Nieber, 1977). Recession streamflows were selected by the following criteria originally suggested by Kirchner (2009), Shaw and Riha (2012) and Stoelzle et al.(2013). Accept only data points associated with, 1) positive values of dQ /dt, and 2) values of the ratio $\frac{|AI-AET|}{Q}$, less than 0.1, where AI is the infiltration into the soil and AET is the actual daily evapotranspiration. These two criteria attempt to eliminate recession data affected by groundwater recharge and/or direct groundwater extraction by evapotranspiration. While we could have used a number of different methods to derive values of AI and AET, we decided to use simulated values from the GSFLOW model as it was calibrated for the Sagehen Creek watershed for the period of our study. In addition to the above two criteria, we also removed the data for the first day of each recession to avoid the possible influence of surface runoff on recessions. As a result, 15 individual recessions with at least continuous 4 days length were derived from the (8-year) calibration period daily data series. These individual recessions were then used to derive the model structure presented in section 4.

3. Simulation methods

3.1 Recession discharge normalization

Employing numerical solutions of groundwater flow in a vertical slice of sloping aquifers Rupp and Selker (2006) and Pauritsch et al. (2015) illustrated that at large times during baseflow recessions the plot of $-\mathrm{d}Q/dt$ vs. Q on a log-log scale, there exists a transition point at which the plotted line is nearly vertical and Q is a minimum value. We have also been able to demonstrate this within the GSFLOW model for the Sagehen Creek watershed (Li and Nieber, 2016, unpublished report). Illustration of this flow behavior during recession is presented in Figures 4a and 10 to be discussed later. The flow at this transition point we will call $Q_{\min}$. The minimum discharge is hypothesized to be a function of the initial storage in the aquifer at the start of the recession, and here this discharge will be treated as being a characteristic discharge, related to the watershed static properties and the initial storage. The effect of the spatial distribution of storage is not taken into account because the model is not spatially distributed.

Following the approach of Biswal and Marani (2014) we normalize the discharge in the sensitivity function, g(Q) so that the function will be sensitive to initial storage and thereby able to represent the range of Q vs. S relations that exist due to variations in antecedent storage conditions. The normalized discharge is then given by

$$Q_{norm}^{i} = Q^{i}/{Q^{i}}_{min} \tag{4}$$

where $Q^i = Q^i(t)$ is the variable discharge in ith recession event, ${Q^{i}}_{min}$ is the minimum discharge in ith recession event , and Q_{norm}^{i} is the normalized discharge based in the ith recession event.

3.2 Discharge simulation with R=0 and ET=0

Based on the watershed water balance, during a recession event when R and ET can be neglected in equation (2), the watershed storage variation is described by

$$\frac{\mathrm{dS}}{\mathrm{dt}} = -Q \tag{5}$$

As suggested by Kirchner (2009), the recession rate of discharge is calculated from equation (3) and (5)

$$\mathrm{d}Q/dt = \frac{dQ}{dS}\frac{dS}{dt} = \mathrm{g}(\mathrm{Q})(-Q) \tag{6}$$

Rather than use the discharge Q within the sensitivity function g(Q), we propose to use the normalized discharge from equation (4) to formulate a normalized sensitivity function, $\mathrm{g}(\mathrm{Q}_{norm})$

so that equation (6) becomes

$$\mathrm{d}Q/dt = \mathrm{g}(\mathrm{Q}_{norm})(-\mathrm{Q}) \tag{7}$$

The sensitivity function is a function of the normalized discharge, which itself is a function of initial storage, making the sensitivity function related to the initial storage.

Equation (7) can be solved by numerical integration, e.g., the Runge-Kutta method. To perform this solution, $Q^{i}{}_{min}$ related to the current recession event should be predicted *a priori*, and then Q_{norm} is calculated with equation (4). Since the discharge at the transition point is a function of the initial saturated thickness of an aquifer, and thereby related to the discharge at the start of the recession, it would make sense that Q_{min} should be predictable from the discharge, Q_0 , at the start of the recession. We therefore now assume there is some function, to represent this relationship

$$Q_{min} = F(Q) \tag{8}$$

where $F(Q)$ is some yet to be defined function of the discharge sequence leading up to the individual recession event.

3.3 Discharge simulation with R $\neq$0 and ET $\neq$0

Generally, $\mathrm{g}(\mathrm{Q}_{norm})$ would be used to simulate discharge with the influence of R and ET, by using

$$\mathrm{d}Q/dt = \mathrm{g}(\mathrm{Q}_{norm})(\mathrm{R} - \mathrm{ET} - \mathrm{Q}) \tag{9}$$

However, our initial application revealed that this MSD model is poor in simulating the hydrograph under conditions of active hydrologic fluxes (recharge R, evapotranspiration ET), even though we found the same model was able to simulate recession flows appropriately in the case of R=ET=0. This result might be expected because it makes sense that the recession sensitivity function should not apply to a rising hydrograph due to hysteresis in the Q vs. S relation. We thus expanded the model to improve the ability to simulate the streamflow under conditions of active hydrologic fluxes. We propose to perform discharge simulation during periods of nonzero R and ET using a simple transform for net recharge flux. The approach adopted was inspired by the analytical solution to the linearized Boussinesq equation presented by Huyck et al. (2005), which accounts for recession and recharge events.

The resulting equation is composed of a recession component and a component for net recharge-discharge response, and is expressed as,

$$\mathrm{Q} = Q_{rec} + \mathrm{c}' \ N_m \tag{10}$$

where, N_m is the net recharge directly to the groundwater storage, c' converts the net recharge into a groundwater discharge response, and Q_{rec} is the recession flow influenced by past recharge events and is computed by solution to equation (7) The parameter c' will be dependent on aquifer geometry and aquifer hydraulic properties.

The net recharge, N_m should be smaller than the net infiltration to the soil, because of the influence of the vadose zone storage capacity. The magnitude of N_m is related to the storage in the aquifer; the higher the storage, the smaller is the vadose zone storage, and the higher the value of N_m. Also, the higher the storage, the higher the value of Q_{min}. So N_m is further assumed to be related to $Q_{min,}$, i.e., $N_m \propto Q_{min}$. In the present research, the net infiltration flux (AI-AET) was calculated by the

GSFLOW model and is used as input to the dynamic model. As a result, we assume that N_m in equation (10) can be expressed as

$$N_m = c'' Q_{min}(\mathrm{AI} - \mathrm{AET}) \tag{11}$$

This then yields for equation (10),

$$\mathrm{Q} = Q_{rec} + cQ_{min}(\mathrm{AI} - \mathrm{AET}) \tag{12}$$

where in the present study the parameter c (equal to $c' c''$) is obtained based on calibration. As mentioned above, the parameter c' is related to aquifer characteristics, while the parameter c'' should be related to water storage (field capacity) and transmission (saturated hydraulic conductivity) properties of the vadose zone.

The resulting set of equations to simulate streamflow with and without net infiltration is presented by coupling equations (4), (7), (8) and (12). Figure 3 illustrates a computational sequence for the simulation process as represented by the solution to these equations. The sequence starts with a long recession period (time period t_0 to t_j), followed by two periods (time period t_j and t_{j+2}) of positive net recharge, a short recession period (time period t_{j+2} and t_{j+3}), and then two periods of negative net recharge (time period t_{j+3} and t_{j+5}). For this sequence the first step involves the calculation of discharge recession based on equation (7) from t_0 to t_j a period with net infiltration equal to zero. Second, the increased discharge, caused by positive AI-AET, is calculated based on equation (12) for the periods t_j to t_{j+1}, and t_{j+1} to t_{j+2}. The recession during these two periods, both Q_{rec}^{j+1} and Q_{rec}^{j+2}, are calculated from equation (7) with appropriate initial conditions for each. Third, the discharge is calculated based on equation (7) from t_{j+2} to t_{j+3} during a period of net infiltration equal to zero. Fourth, the decreased discharge, caused by negative AI-AET, is calculated based on equation (12) over the periods t_{j+3} to t_{j+4} and t_{j+4} to t_{j+5}. Again, both Q_{rec}^{j+4} and Q_{rec}^{j+5} are calculated from equation (7). Last, the discharge is calculated from equation (7) after t_{j+5} where the net infiltration is zero. Note that during this simulation process, before using equation (7) to calculate recession discharge, the Q_{min} of each recession is calculated with equation (8), and then is used to calculate Q_{norm} with equation (4).

Up to this point the MSD model requires the specification of Q_{min}, the function $g(Q_{norm})$ and the constant c to fully characterize the system. It will be shown in section 4.1 that Q_{min} is determined from a regression analysis yielding three independent parameters, and that $g(Q_{norm})$ requires a regression analysis yielding an additional three independent parameters. The total number of parameters required then will be found to be seven.

Three details should be considered when implementing the simulation. One is the time lag in the catchment. As suggested by Kirchner (2009), the observed discharge at the gauge of watershed outlet should lag behind the discharge from the watershed storage to the channel network, since the streamflow requires time to travel along the channel. There is also another time lag, which is between the infiltration into the soil and recharge to the groundwater table. Thus, we assessed the travel time lag by the cross-correlation method proposed by Kirchner (2009). However, we didn't find an obvious time lag between daily net infiltration flux (AI-AET) and daily observed streamflow, suggesting the time lag does not exceed one day in most cases in Sagehen Creek. This is consistent with Kirchner (2009), Teuling et al. (2010), Krier et al. (2012), Adamovic et al. (2015) and Rusjan and Mikoš (2015). In their research, the time lag was found no more than several hours. Generally, the small time lag reflects the small size of the watershed with a shallow active aquifer. As a result, in our analysis, the time lag was neglected in the application of equation (12).

A second detail is that a minimum threshold value for discharge simulation is needed to avoid negative discharge values. It is noticed in equation (12) that if the AET is large enough compared to AI, the second term on the right can become negative, and this could lead to calculated negative discharges. To account for this a minimum discharge of 0.11 mm/day was set. This minimum value was selected because it is the minimum daily streamflow value for the 16-year streamflow record of Sagehen Creek.

The third detail relates to the fate of the net infiltration that does not become net recharge. The net infiltration is (AI-AET), and when (AI-AET)>0 on any particular day, only a portion of that net will become recharge, meaning that $c' \; ' \; Q_{min}$<1. The remainder of the net recharge will remain stored in the soil profile. The current version of the MSD model does not account for this residual net infiltration. In a model that accounts for all fluxes and not just the groundwater discharge this portion of net infiltration would be stored in the unsaturated zone and potentially could later contribute to groundwater recharge, evapotranspiration, interflow, return flow, or it could contribute to saturating the soil profile and thereby promoting surface runoff. Since the present model only accounts for the groundwater contribution to streamflow the contributions of this residual net infiltration to other hydrologic flows was not taken into account.

4. Results

4.1 Construction of $\mathbf{g}(\mathbf{Q}_{norm})$ and $\mathbf{F}(\mathbf{Q})$

When the magnitude of discharge is smaller than the precision of the stream gage, the calculated recession rate from observed data may lead to errors (Rupp and Selker, 2006). Following the method proposed by Rupp and Selker (2006), any unreasonable stream flow data in the selected 15 events were removed by using a variable Δt to make sure ΔQ exceeded a precision threshold. These individual recession curves are plotted on a log-log scale, shown in Figure 4a. The normalized data derived using the minimum stream flow for each recession are shown in Figure 4b（gray points）.

Following Kirchner (2009) and Krakauer and Temimi (2011), a binning procedure was conducted as described below. Beginning with the top 1% of the logarithmic Q_{norm}, calculate the corresponding mean and standard error for dQ_{norm}/dt in the range, and if the number of points in the range is less than 5 or the standard error is larger than the half of its mean, expand the range within the next 1% of the logarithmic range. Otherwise, keep the mean dQ_{norm}/dt and mean Q_{norm} for this bin and continue on to the next bin. This procedure resulted in 14 bins, each with mean dQ_{norm}/dt and mean Q_{norm} (the solid black dots in Figure 4b). A nonlinear least squares method was employed to fit these points, generating $g(Q_{norm})$ as

$$\ln\big(g(Q_{norm})\big) = 0.1944\ln(\ln(Q_{norm})) + 2.624\ln(Q_{norm}) - 2.975) \tag{13}$$

A power law relationship between antecedent streamflow and recession flow characteristic has been found in some prior research (Bart and Hope, 2014; Biswal and Nagesh Kumar, 2014). Our prior unpublished research (Li and Nieber, 2016, unpublished report) also shows a power law relationship between antecedent streamflow and Q_{min}. Thus, in our first try at conducting regression analysis, a power expression of $F(Q)$ was derived between the initial recession flow Q_0 and Q_{min}. The expression for $F(Q)$ is shown in Figure 5 and is of the form,

$$Q_{min} = r_1\overline{Q_n}^{r_2} + r_3 \tag{14}$$

with r_1, r_2, and r_3 are obtained by nonlinear least squares regression, and $\overline{Q_n}$ is the mean daily discharge for n days prior to the beginning of the recession. For the case of n=0, the values of r_1, r_2 and r_3 are 0.714, 0.852, and -0.001 respectively, and the R^2 for the regression was 0.95.

4.2 Parameter calibration and discharge simulation; MSD model vs. observed streamflow

The parameter c in equation (12) was calibrated by a trial-and-error procedure with the 8 years calibration period. The “best” value, 0.042 was selected with maximum NSE, a value of 0.558, a relatively mediocre performance. Then the discharge was simulated during the 8 years validation period, and the value of NSE was -0.201, indicating a very poor performance. We therefore tried different values of n for the function, equation (14), and we discovered the value of $n = 5$ (r_1=0.805, $r_2 = 0.552$, and $r_3 = -0.174$) provided the best fit for both calibration (NSE=0.692) and validation (NSE=0.769). The R^2 for the Q_{min} model is 0.78. The value of the parameter c for this case was 0.16.

The calibration and validation hydrographs for this case are shown in Figure 6a. The simulations generally reproduce the shape of the observed streamflow hydrographs both in the calibration and validation periods. As mentioned above the best model performance is obtained when n equals 5 instead of 0, even though Q_{min} has a better relationship with $\overline{Q_0}$ than $\overline{Q_5}$. Possible reasons for this unexpected result are outlined in the discussion section.

Although the model using $\overline{Q_5}$ for the Q_{min} model provides a better fit for the hydrograph, there still remains an obvious defect in that the model is unable to quantify the peak flows that occur. Missing the peak flows is a result of the MSD model not having a surface runoff component, since the equations account for aquifer discharge only.

In addition, as viewed in Figure 6b, a zoomed-in segment of Figure 6a, there is also some deficiency in the correspondence between the times for rises in hydrographs of the observed streamflow and the times of the positive net recharges associated with positive net infiltration flux. The net infiltration flux plotted in Figure 6b are derived from the GSFLOW model simulation and it is seen that the response of the observed streamflow hydrograph is not in sync with the net infiltration fluxes computed by GSFLOW. The MSD model used the net infiltration flux computed by GSFLOW and therefore one would not expect to see a correspondence between the MSD model simulated flows and the observed streamflows. Rather than use the observed streamflow for the comparison to the MSD model, we will in the next section use the groundwater flow component simulated by GSFLOW

4.3 Reevaluation of model performance; MSD model vs. GSFLOW simulated streamflow

To avoid the influence of surface runoff and net infiltration time series that are inconsistent with the observed streamflow, we used the subsurface flow simulated by GSFLOW as the 'observed streamflow' for comparison against the flow simulated by the MSD model. As before, the net infiltration calculated by GSFLOW was used as input to the MSD model.

The model calibration and validation were repeated by testing different $\overline{Q_n}$ in equation (14). The results of this test are summarized in Table 1. The best NSE values obtained were 0.925 for calibration and 0.933 for validation for a value of n=3 and c=0.151 (hereinafter referred to as simulation S_1). Figure 7a shows the simulated hydrograph both in the calibration and the validation period. It is clear that the simulation performance is considerably improved compared to the results shown in Figure 6a. The zoomed-in plot of the hydrograph presented in Figure 7b also shows a net infiltration time series that is consistent with the streamflow series simulated both by the MSD model and GSFLOW.

4.4 Comparison of the MSD model to the Kirchner model

We conducted another two simulations to test the model performance. One is a similar cross-validation proposed by Kirchner (2009). We exchanged calibration and validation period (hereinafter referred to as simulation S_2). This test prevents a circular simulation (Kirchner, 2009). Another is a whole period calibration, in which we used all 16 years of streamflow data to generate the $g(Q_{norm})$, $F(Q)$ and parameter c (hereinafter referred to as simulation S_3). This third test demonstrates the influence of data series length on model performance. All three simulations (S_1, S_2 and S_3) are compared with the Kirchner model.

The results of the comparison are summarized in Table 2. The MSD model has a stable satisfactory performance for both the cross-validation and the whole period calibration. The results indicate that the increase of data length has little influence on improvement of model performance. This suggests that the MSD model may be useful even in those watersheds not having long-duration observed time series. Consistent with prior research (Teuling et al., 2010), Krier et al. (2012), Brauer et al. (2013), Melsen et al. (2014), Adamovic et al. (2015) , the Kirchner model produces a good performance especially in wet water years, while a poorer performance in dry water years. The MSD model had higher simulation accuracy in dry water years when annual streamflow was less than 400 mm and also is shown to be superior for wet water years (as shown in Figure 8). We hypothesize that the reason for the improved performance with the MSD model is the non-unique (albeit implicit) storage-discharge relation embedded in the model, which provides for flexibility in the model to handle the wide range of initial conditions. This feature is missing from the Kirchner model (Xu et al., 2012).

In the original research of Kirchner (2009), thousands of hourly streamflow data were taken to generate the model parameters. Melsen et al. (2014) examined the length and timing of discharge data to obtain reliable parameters in Kirchner model and found a five-month hourly data series, approximately 3,600 values, was sufficient. The length of data source in these previous research efforts is much larger than the streamflow time series taken in the present research, approximately 150 values. Thus, it is possible to increase the performance of the Kirchner model with a much longer observed streamflow time series. However, as pointed out in Kirchner (2009), the Kirchner model is an average description of the behavior of the watershed, thus cannot be expected to achieve the ability to simulate both wet and dry watershed condition only by increasing the length of data series.

As shown in Table 2 and labeled on Figure 8, there are still two water years (1989 and 1991) with relatively small NSEs among the three simulations by the MSD model. This illustrates that although the performance of the dynamic model was improved over and above the original Kirchner model, some inaccuracy of the MSD model still exists. The simulated discharges for the 1989 water year are shown in Figure 9. It is apparent that the MSD model misses some high flows and low flows. It is expected that for a similar magnitude net infiltration (AI-AET) for conditions of different antecedent storage condition should lead to different discharge response. As is shown in Figure 9, comparing two infiltration segments with similar magnitude, the event labeled as I1 occurs under a higher antecedent storage than the event labeled as I2. For these GSFLOW appropriately simulated a higher peak discharge in response to the I1 event than for event I2. In contrast, the MSD model didn't provide similar responses, but instead produced a higher response to event I2. On the other hand, the

MSD model underestimated some recession flows, for example, from January to March in 1990 as shown in Figure 9. For this specific time series it is apparent that the Q_{min} as determined by equation (14) with $n = 3$ was underestimated. However, as large as the disagreement is for that time period, in general the MSD model is found to accurately simulate recession dynamics for most events. Even with these deficiencies, the NSE calculated with the MSD model was still higher than 0.5 for both these water years.

5. Discussion

5.1 Evaluation of the Q_{min} recession flow parameter

It is curious why the use of $\overline{Q_5}$ to calculate Q_{min} in the MSD model is better than the use of $\overline{Q_0}$ since the Q_{min} vs. $\overline{Q_0}$ regression equation has higher R^2 than the Q_{min} vs. $\overline{Q_5}$ regression equation. An explanation of this seeming inconsistency is presented in the following. Note that this explanation also applies to the comparison of the Q_{min} ($\overline{Q_o}$) equation to the Q_{min} ($\overline{Q_3}$) equation.

Among the simulation results it was observed that when using the Q_{min} ($\overline{Q_o}$) equation the recessions are simulated quite well but the peak flows are over-predicted, while in contrast when using the $\overline{Q_5}$ equation to predict Q_{min} ($\overline{Q_5}$) the recessions are under-predicted and the peak flows are better predicted. The reasons for this are apparent when one views the two graphs shown in Figure 5. The $Q_{min}(\overline{Q_o})$ equation will generally estimate a larger value of Q_{min} than the $Q_{min}(\overline{Q_5})$ equation. Since Q_{min} is best predicted by $\overline{Q_o}$ (R^2=0.95) one then expects the baseflow recessions will fit the observed values more closely, while since $Q_{min}(\overline{Q_5})$ predicts a lower Q_{min} the simulation using this Q_{min} will under-predict the recessions.

While the $\overline{Q_0}$ equation is more accurate at predicting the recession period, it appears not to be as accurate at predicting the net recharge and therefore the rising limb of the hydrograph. While it would be tempting to immediately use the $\overline{Q_0}$ equation in the recession limb, and the $\overline{Q_5}$ equation for prediction of net recharge, it is probably advisable to leave this issue for a follow-up study to determine a more appropriate way to predict net recharge.

5.2 Parameters required for the MSD model

As stated previously the MSD model presented here requires the specification of seven independent parameters. These parameters are the three parameters in the $g(Q_{norm})$ function, the parameter c, and the parameters r_1, r_2 and r_3 in the F(Q) function for Q_{min}. We should compare this parameter requirement to what is reportedly required for the original Kirchner model. For that model there were three parameters required for the $g(Q)$ function, and then one parameter for the estimation of evapotranspiration. Thus, our model requires three parameters in addition to the parameters required for the original simple dynamic model (Kirchner, 2009).

We might also compare the parameters required for the MSD model to that required for the approach using multiple linear reservoirs, an alternative formulation to the one proposed herein. The specification of a linear reservoir requires in general two independent parameters. As shown by others (Moore (1997), Shaw (2016)), the combination of linear reservoirs in series and/or in parallel can be used to represent more complex nonlinear response systems. While this alternative approach was not used here, it would require at least one parameter to estimate the groundwater recharge process, and then at least two parameters for each of the linear reservoirs. Thus after combining two or three reservoirs the number of parameters would already exceed the number of parameters required of the MSD model presented here.

It is certainly possible that the MSD model could be made even more attractive by eliminating the parameters required for the estimation of Q_{min}. This could be accomplished by relating Q_{min} to watershed and aquifer characteristics, and to the dynamics of initial condition for recessions. Such relations could be regionalized and then one could use a regional curve to arrive at the estimate of Q_{min} rather than deriving the F(Q) for each and every watershed of interest. Also, as mentioned already the parameter c should be related to the aquifer geometric and hydraulic properties, and also to the water withholding and transmission properties of the unsaturated zone. Thus it is recommended that future work involve dimensional analysis to formulate the parameter c based on watershed soil hydraulic properties and to aquifer geometric characteristics and hydraulic characteristics.

5.3 Hysteresis in the Q vs. S relation

For the simulation of flow recessions the MSD model solved equation (7) with the $g(Q_{norm})$function. Since Q_{norm} is determined by Q_{min} for any given initial recession condition, this leads to a family of multivalued (non-unique) recession curves as illustrated in Figure 10. Thus, the Q vs. S relation for the recession in the MSD model will not be unique. In contrast, the Kirchner model, the $g(Q)$ function is single-valued and as such the Q vs. S relation will be single-valued.

For conditions of positive or negative net recharge to the groundwater storage the Kirchner model solves the equation

$$dQ/dt = g(Q)(R - ET - Q) \tag{15}$$

which with a unique function $g(Q)$ also yields a unique relation between Q vs. S regardless of the magnitudes and trends of (R–ET). For the MSD model equation the case of positive or negative net recharge was simulated using the Huyck et al. (2005) model (equation (12)) coupled with the solution to equation (9). This approach leads to a non-unique Q vs. S relation.

So for periods of zero net recharge and for periods of nonzero net recharge the

MSD model will yield hysteretic relations between discharge and storage, while the Kirchner model yields a unique Q vs. S relation. Although the hysteresis in watersheds has been simulated with different approaches (Ewen and Birkinshaw, 2007; Camporese, et al., 2014; Tritz et al., 2011), as far as we know, only one paper accounting for hysteresis with the Kirchner model has appeared in the literature (Xu et al., 2012). In that paper Xu et al. (2012) represented the hysteresis in the storage-discharge relation by three piecewise simple linear sensitivity functions. Rather than using that approach, we instead expanded the Kirchner model to account for the hysteresis process by a normalized $g(Q_{norm})$ and an analytical solution presented by Huyck et al. (2005). The advantage of the hysteretic Q vs. S relation is that it provides the additional nonlinearity to the model to facilitate the flexibility needed to be able to handle the wide range of storage conditions. This is our qualification of why the MSD model is more adapt than the original Kirchner model in simulating the streamflows for the Sagehen Creek across a wide range of water storage conditions.

5.4 Limitations and prospects of the MSD model

In this study, the net infiltration flux (AI-AET) calculated by GSFLOW is used as input to the MSD model. However the hydrologic fluxes are generally not known, but would be determined from measurements or calculated using some acceptable method. Inaccuracy in the estimation of these hydrologic fluxes will introduce more uncertainty in the model application. Thus before the application of the MSD model in a given watershed, the hydrologic fluxes, such as actual evapotranspiration, infiltration of precipitation/snowmelt, and deep percolation to groundwater should be estimated based on some acceptable methods expressing a clear dependency on the state of the system.

A second limitation of the model is that it does not include an explicit soil water balance component. The current model calculates groundwater recharge implicitly, not accounting for possible carryover moisture between infiltration events. If the MSD model is eventually to be used for surface runoff computations in addition to the current ability to simulate groundwater discharge, it will be necessary to include a soil water balance component. An example of such a model is presented by Zhuo and Han (2016).

A third limitation is inherent in the simple form of the MSD model structure. Although the MSD model outlined shows some progress in simulating the hysteretic discharge behavior of the watershed, especially comparing with the original Kirchner model, as is stated in Section 4.4, there are still several water years with only fair model performance. In its present state of development the MSD model cannot be expected to have an ability of completely simulating the complex physical conditions in a real watershed. One example of the complex physical condition is the observation that the spatial distribution of storage in the Sagehen Creek watershed varies and this variation affects the storage-discharge relation. This observation was made using the simulations of the GSFLOW model (Li and Nieber, 2016, unpublished report). It seems that without a model that treats spatial variability it will not be possible to capture explicitly the spatial effect. Using different conceptual reservoirs to represent different spatial locations in the watershed could treat this spatial effect more explicitly. Whether such treatment can improve the simulation performance will need to be tested in a different study.

6. Conclusions

A modified Kirchner model, the Modified Simple Dynamic (MSD) model has been presented to simulate the streamflow discharge for the Sagehen Creek watershed, a watershed located in eastern California having a mean slope of 15.8%. As with the original Kirchner model the MSD model is composed of a single conceptual reservoir, except in the case of the MSD model the conceptual reservoir has a nonlinear Q vs. S relation. The three main features of the MSD model are: a sensitivity function ($g(Q_{norm})$) to represent the simple unique normalized discharge storage relationship, a simple correlation function (F(Q)) to represent the influence of different initial condition on recession flow dynamics, and a simple streamflow component proportional to net infiltration to represent the effect of active hydrologic fluxes. As a result, the MSD model has an ability to account for the hysteresis of the Q vs. S relation in a watershed.

After an application to Sagehen Creek and comparisons with a well parameterized model GSFLOW, also with Kirchner model, the following conclusions can be stated. First, the MSD model has a better performance than the original Kirchner model to simulate the discharge dynamic encompassing a wide range of wetness with almost a tenfold variability in annual streamflow. Second, two core governing equations ($g(Q_{norm})$ and F(Q)) in the MSD model can be derived from a few observed streamflow data directly by regression analysis, and then a parameter c associated with the transformation of groundwater recharge with streamflow response, needing calibration by fitting of streamflow data. Thus, we suggest the MSD model may be applicable easily to other watersheds. Third, the streamflow simulation is dependent on initial conditions, but also very sensitive to the magnitude of hydrological fluxes and therefore it is essential to have adequate estimates of these fluxes.

While the model does have an improved performance compared to the Kirchner model for the Sagehen Creek watershed, especially for drier years, it still appears to have some inaccuracies in the very driest years. This could be due to the non-unique spatial distribution of water storage having an effect on the watershed discharge. It is suggested that while employing multiple conceptual reservoirs to represent the watershed storage spatial distribution in this case might resolve the issue, the simplicity of a single nonlinear reservoir to represent the whole watershed response is arguably appealing.

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Figure 1. Map of the Sagehen Creek catchment

Figure 2. The annual streamflow, precipitation and actual evapotranspiration for Sagehen Creek, ranked by the streamflow magnitude from smallest to largest. The annual streamflow for the eight year calibration period is illustrated by gray bars and the eight year validation period is illustrated by white bars. Annual precipitation is shown as solid line. Annual actual evapotranspiration is shown as dash bars.

Figure 3. Simulation process of the modified dynamic model. The calculated discharge is illustrated with bold black line. Four bold dash lines illustrate the recession flow from previous discharge. The positive and negative AI-AET are shown as gray bars. In this illustration theyboth last for two consecutive time steps.

Figure 4. dQ/dt vs. Q plot for selected individual recessions. (a) The individual curves before normalization. The different colors illustrate designate distinct recessions. (b) The individual curves after normalization (gray dots), the binned means (black dots), and the best fit line calculated by nonlinear least squares method.

Figure 5. Plots of Q_{min} vs. $\overline{Q_n}$, with the best fit lines. The solid gray points are related to $\overline{Q_0}$, with a dashed fit line, whilethe solid black points are related to $\overline{Q_5}$, with a solid fit line.

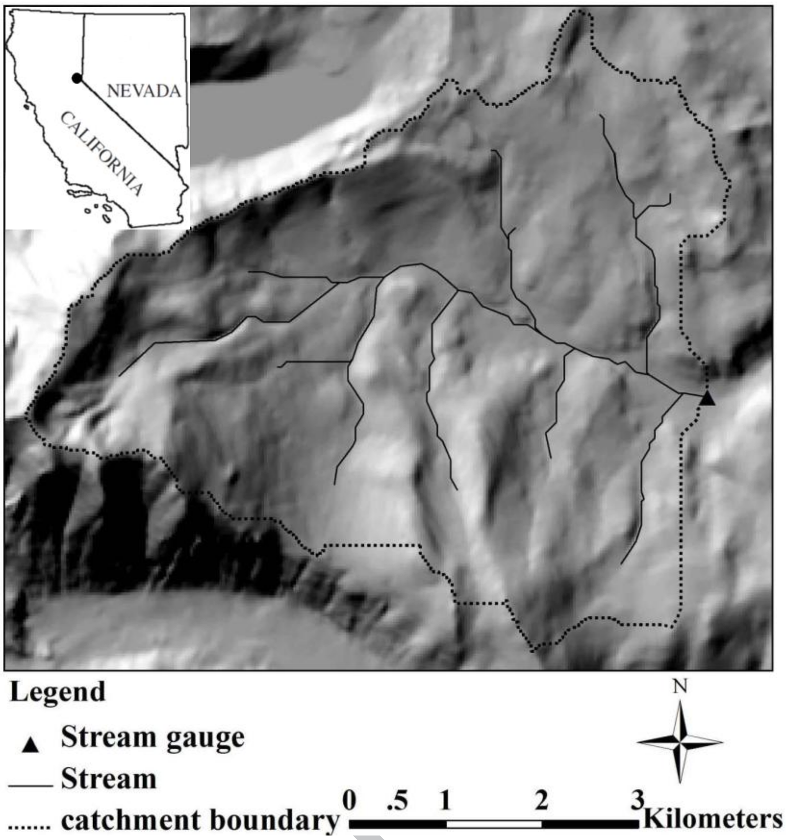
Figure 6. Simulated discharge time series by the modified model (dotted black curve), compared with observed streamflow (solid black curve). (a) a 16-year time period including both calibration and validation; (b) zoomed view of one segment, with daily GSFLOW-calculated net infiltration flux (AI-AET)(gray bars).

Figure 7. Simulated discharge time series by modified model (dotted black curve), compared with simulated subsurface flow discharge by GSFLOW (solid black curve). (a) a 16-year time period including both calibration and validation. (b) zoomed view of one segment, with daily net infiltration flux (AI-AET) (gray bars).

Figure 8. NSE vs. annual streamflow plots for the comparison of modified model and Kirchner model, a, b and c is the results of simulation S_1, S_2, S_3 respectively. Two black points labeled on the plots are related to 1989 and 1991 water years with much smaller NSEs.

Figure 9. Simulated discharge time series for 1989 water year by modified model (dotted black curve), compared with simulated subsurface flow discharge by GSFLOW (solid black curve), with daily net infiltration flux (AI-AET) (gray bars).

Figure 10. Plots of the g(Q) function for the modified dynamics model and the Kirchner model (broken line). Data are derived from the simulation S1 flows, considering only those flows corresponding to periods without significant AI and AET. The g(Q) function for the modified model is based on a normalized function that then provides for a family of functions each of which corresponds to a different antecedent storage condition. The Kirchner model is a single function that represents average conditions for the period of calibration.


NEVADA
CALIFORNIA
Legend
Stream gauge
Stream
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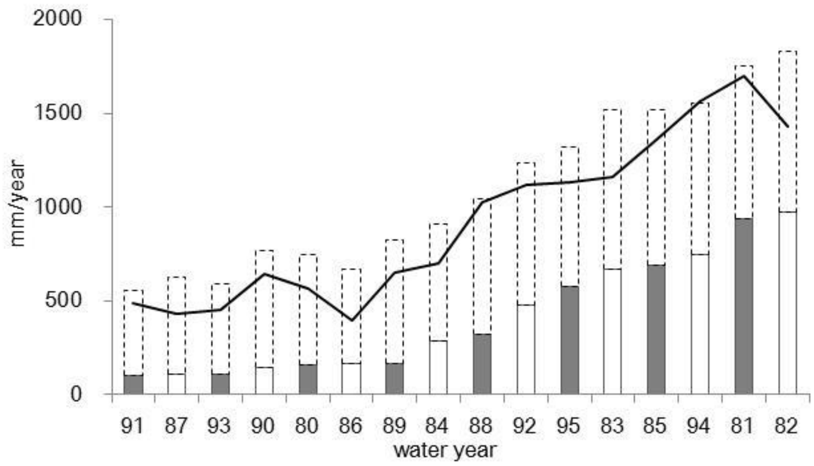
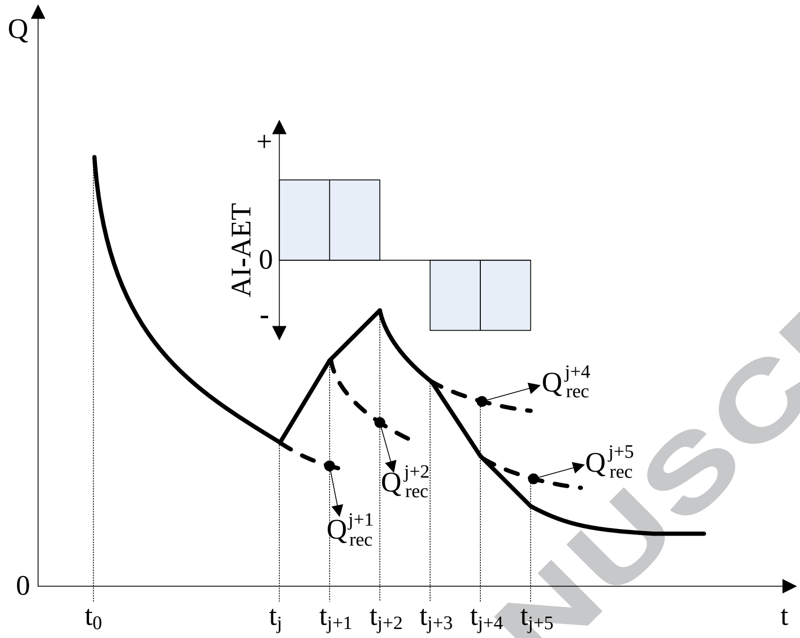
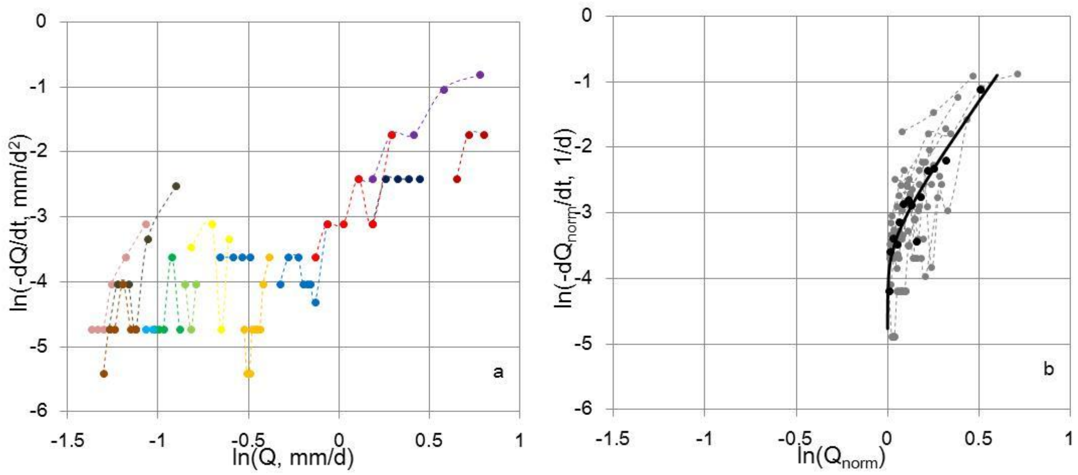
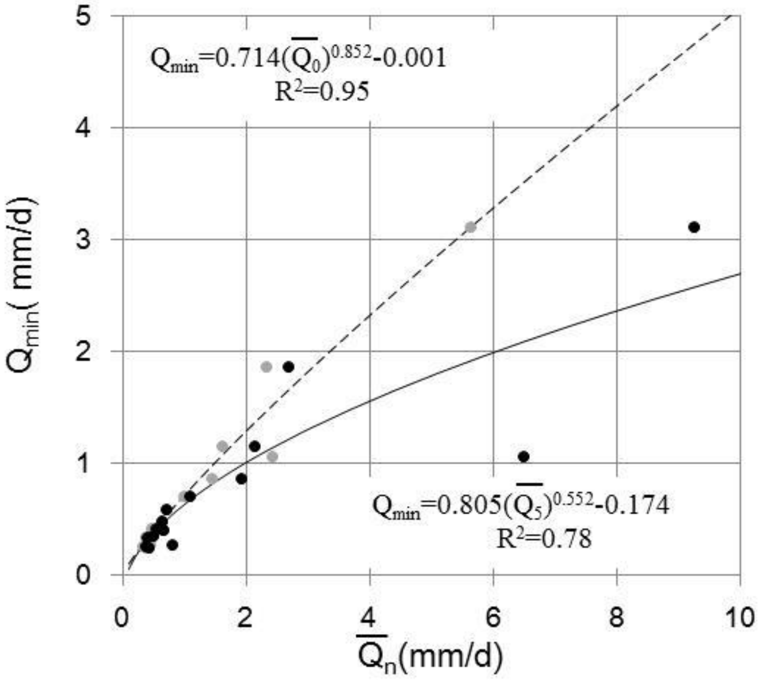

2000
1500
1000
500
0
mm/year
91
87
93
90
80
86
89
84
88
92
95
83
85
94
81
82
water year

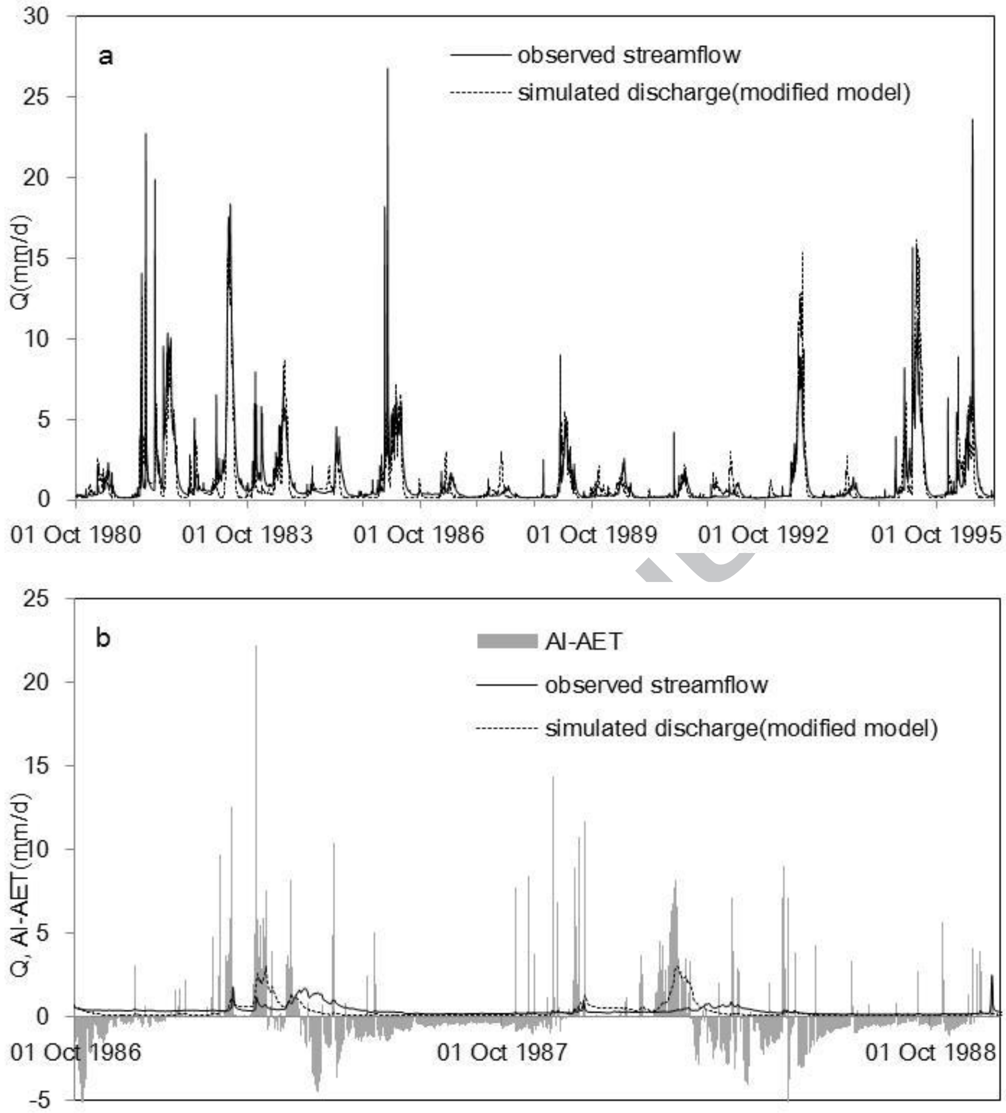


Q
0
t
AI-AET
+
0
-
t_0
t_j
t_{j+1}
t_{j+2}
t_{j+3}
t_{j+4}
t_{j+5}
Q_{rec}^{j+1}
Q_{rec}^{j+2}
Q_{rec}^{j+4}
Q_{rec}^{j+5}

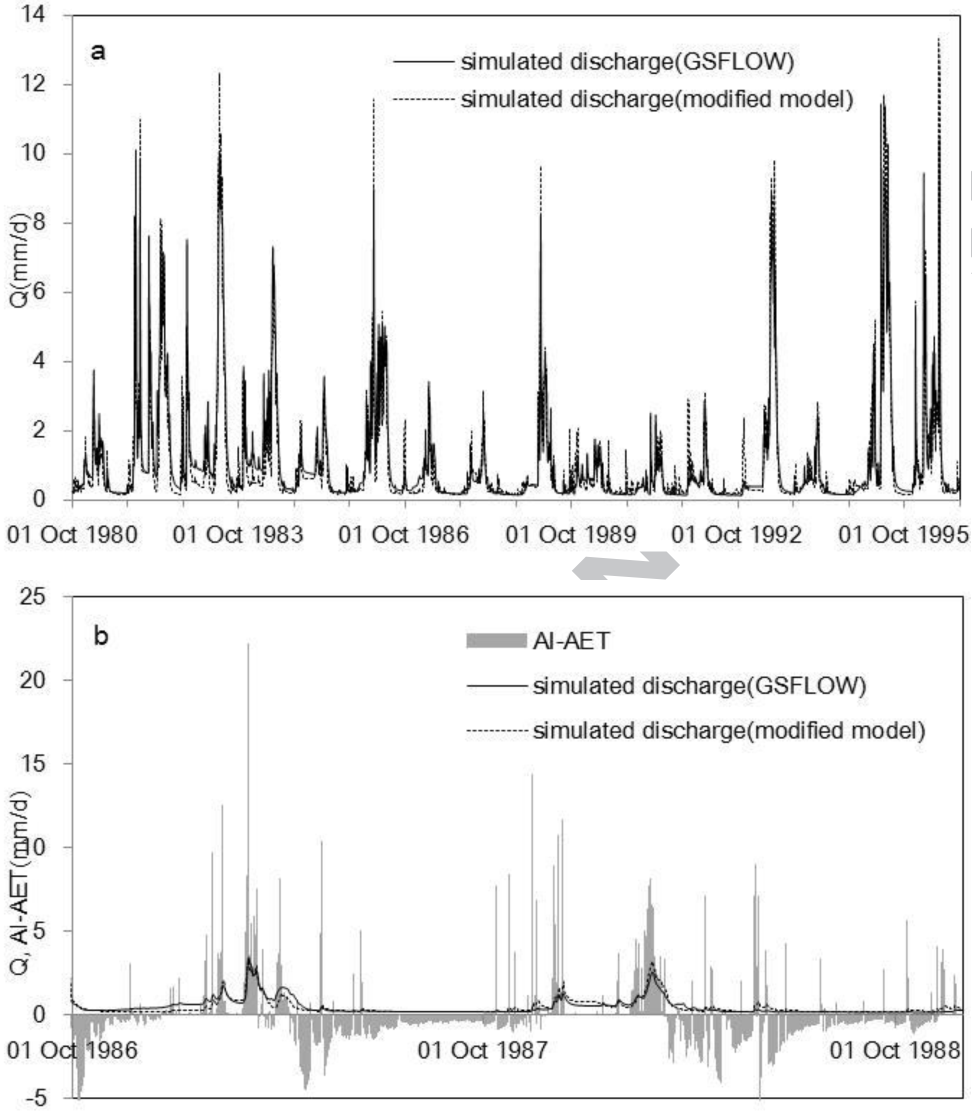

ln(-dQ/dt, mm/d^2)
ln(Q, mm/d)
a
ln(-dQ$_{norm}$/dt, 1/d)
ln(Q$_{norm}$)
b

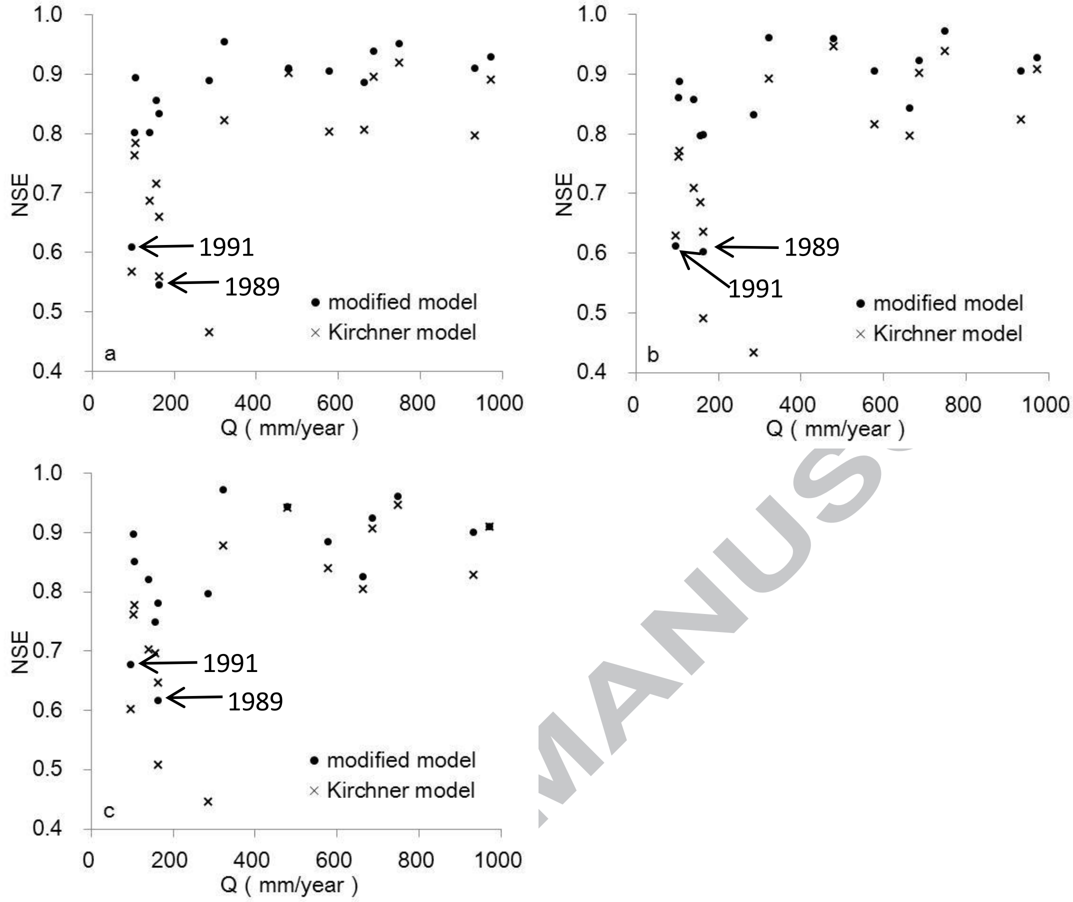


$Q_{min}=0.714(\overline{Q_0})^{0.852}-0.001$
$R^2=0.95$
$Q_{min}=0.805(\overline{Q_5})^{0.552}-0.174$
$R^2=0.78$
Q_{min}(mm/d)
$\overline{Q_n}$(mm/d)
0
1
2
3
4
5
0
2
4
6
8
10

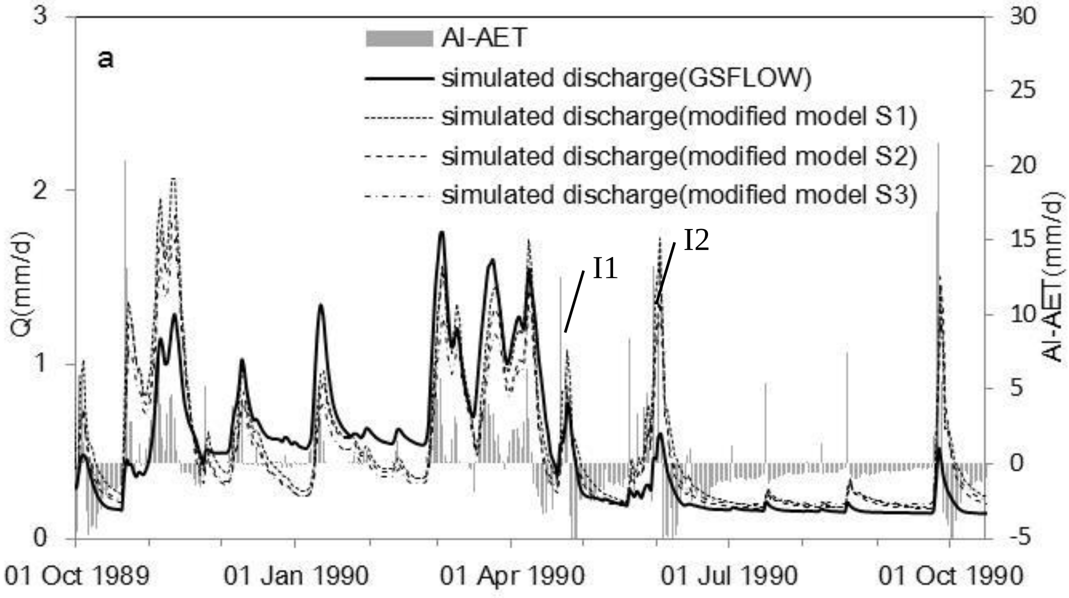


a
observed streamflow
simulated discharge(modified model)
Q(mm/d)
30
25
20
15
10
5
0
01 Oct 1980
01 Oct 1983
01 Oct 1986
01 Oct 1989
01 Oct 1992
01 Oct 1995
b
AI-AET
observed streamflow
simulated discharge(modified model)
Q, AI-AET(mm/d)
25
20
15
10
5
0
-5
01 Oct 1986
01 Oct 1987
01 Oct 1988


a
simulated discharge(GSFLOW)
simulated discharge(modified model)
Q(mm/d)
14
12
10
8
6
4
2
0
01 Oct 1980
01 Oct 1983
01 Oct 1986
01 Oct 1989
01 Oct 1992
01 Oct 1995
b
AI-AET
simulated discharge(GSFLOW)
simulated discharge(modified model)
Q, AI-AET (mm/d)
25
20
15
10
5
0
-5
01 Oct 1986
01 Oct 1987
01 Oct 1988



NSE
Q (mm/year)
1991
1989
modified model
Kirchner model
a
b
c



a
AI-AET
simulated discharge(GSFLOW)
simulated discharge(modified model S1)
simulated discharge(modified model S2)
simulated discharge(modified model S3)
I1
I2
Q(mm/d)
AI-AET(mm/d)
0
1
2
3
-5
0
5
10
15
20
25
30
01 Oct 1989
01 Jan 1990
01 Apr 1990
01 Jul 1990
01 Oct 1990

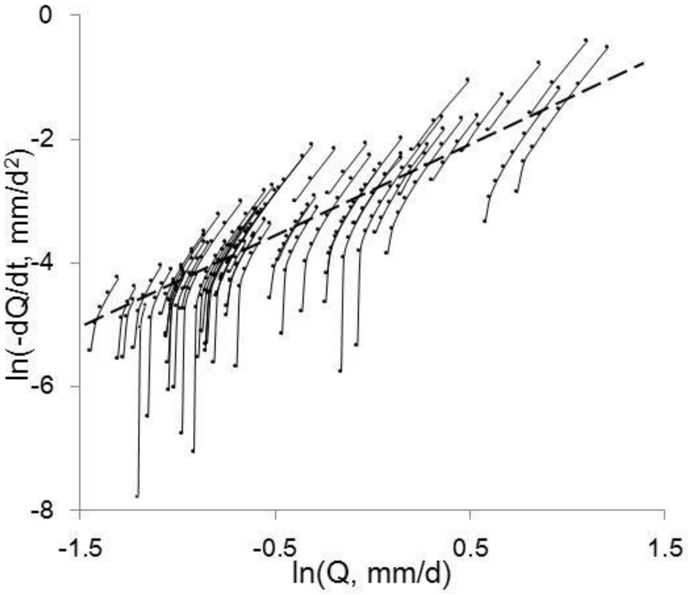

0
-2
-4
-6
-8
-1.5
-0.5
0.5
1.5
ln(-dQ/dt, mm/d²)
ln(Q, mm/d)

Table 1. Parameters values and NSE with different $\overline{Q_n}$ in function F(Q)

$\overline{Q_n}$	r_1	r_2	r_3	c	NSE	
					calibration	validation
$\overline{Q_0}$	0.714	0.852	-0.001	0.039	0.664	-0.560
$\overline{Q_1}$	0.771	0.717	-0.092	0.066	0.763	0.549
$\overline{Q_2}$	0.691	0.590	-0.069	0.128	0.924	0.914
$\overline{Q_3}$	0.757	0.515	-0.136	0.151	0.925	0.933
$\overline{Q_4}$	0.725	0.565	-0.104	0.143	0.921	0.885
$\overline{Q_5}$	0.805	0.552	-0.174	0.140	0.885	0.809
$\overline{Q_6}$	0.635	0.685	-0.023	0.130	0.870	0.570
$\overline{Q_7}$	0.480	0.854	0.101	0.123	0.795	-0.749
$\overline{Q_8}$	0.330	1.092	0.223	0.111	0.642	-50.277
$\overline{Q_9}$	0.196	1.430	0.319	0.080	0.297	-1.246
$\overline{Q_{10}}$	0.089	1.970	0.388	0.091	0.257	0.252

Table 2. NSE of three sets of simulations for individual water years*

water year	annual streamflow (mm)	Simulation S_1		Simulation S_2		Simulation S_3	
		MSD model	Kirchner model	MSD model	Kirchner model	MSD model	Kirchner model
1980	156.8	0.854	0.714	0.797	0.685	0.749	0.696
1981	934.3	0.909	0.796	0.904	0.824	0.900	0.828
1982	973.7	0.928	0.890	0.927	0.907	0.909	0.909
1983	664.0	0.886	0.806	0.843	0.796	0.826	0.805
1984	285.7	0.889	0.464	0.831	0.433	0.796	0.446
1985	686.2	0.939	0.895	0.921	0.901	0.924	0.906
1986	163.4	0.833	0.659	0.797	0.634	0.781	0.646
1987	105.0	0.801	0.763	0.860	0.761	0.897	0.762
1988	321.8	0.955	0.821	0.960	0.892	0.972	0.878
1989	164.7	0.545	0.558	0.602	0.491	0.616	0.507
1990	141.5	0.801	0.686	0.857	0.709	0.820	0.702
1991	97.6	0.609	0.566	0.612	0.628	0.676	0.602
1992	479.2	0.909	0.902	0.958	0.946	0.944	0.942
1993	106.1	0.893	0.783	0.887	0.771	0.850	0.778
1994	748.4	0.951	0.919	0.971	0.938	0.960	0.946
1995	577.8	0.904	0.803	0.905	0.815	0.883	0.839
calibration	-	0.925	0.847	0.941	0.907	0.923	0.894
validation	-	0.933	0.892	0.920	0.864	-	-

* S_1 refers to the simulation results in section 4.3, S_2 refers to the cross-validation simulation results, and S_3 refers to the whole period calibration result.

(1) Normalized relation for rate of change of recession flow versus recession flow.

(2) Regression model relating minimum recession flow and antecedent flow.

3) Propose simple streamflow generation component proportional to net infiltration.