Journal of Number Theory 136 (2014) 320–329

Contents lists available at ScienceDirect

Journal of Number Theory

www.elsevier.com/locate/jnt

A new quicker sequence convergent to Euler's constant

Dawei Lu

School of Mathematical Sciences, Dalian University of Technology, Dalian 116023, China

ARTICLE INFO

Article history:
Received 10 July 2013
Accepted 31 October 2013
Available online 11 December 2013
Communicated by David Goss

MSC:
11Y60
41A25
34E05

Keywords:
Euler's constant
Rate of convergence
Asymptotic expansion

ABSTRACT

In this paper, a new quicker sequence convergent to Euler's constant is provided. Finally, for demonstrating the superiority of our new convergent sequence over DeTemple's sequence, Vernescu's sequence and Mortici's sequences, some numerical computations are also given.

1. Introduction

It is well known that we often need to establish some new sequences to converge to some fundamental constants with increasingly higher speed. These convergent sequences and constants play a key role in many areas of mathematics and science in general, as theory of probability, applied statistics, physics, special functions, number theory, or analysis.

To the best of our knowledge, the most useful convergent sequence is

$$\gamma_n = \sum_{k=1}^{n} \frac{1}{k} - \ln n, \tag{1.1}$$

E-mail address: ludawei_dlut@163.com.

0022-314X/$ – see front matter
http://dx.doi.org/10.1016/j.jnt.2013.10.012

which converge towards the well-known Euler's constant

$$\gamma = 0.57721566490115328\ldots.$$

Up to now, many researchers made great efforts in the area of concerning the rate of convergence of the sequence $(\gamma_n)_{n\geqslant 1}$ and establishing faster sequences to converge to Euler's constant and had a lot of inspiring results. For example, in [3,4,13,14], the following estimates are established

$$\frac{1}{2n+1} < \gamma_n - \gamma < \frac{1}{2n}, \tag{1.2}$$

using interesting geometric interpretations. In [1,2], DeTemple introduced a faster convergent sequence $(R_n)_{n\geqslant 1}$ to γ as follows:

$$R_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln\left(n + \frac{1}{2}\right), \tag{1.3}$$

which decreases to γ with the rate of convergence n^{-2}, since

$$\frac{1}{24(n+1)^2} < R_n - \gamma < \frac{1}{24n^2}. \tag{1.4}$$

In [12], Vernescu provided the sequence

$$V_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} + \frac{1}{2n} - \ln n, \tag{1.5}$$

for which

$$\frac{1}{12(n+1)^2} < \gamma - V_n < \frac{1}{12n^2}. \tag{1.6}$$

Both (1.3) and (1.5) are slight modifications of Euler's sequences (1.1), but significantly improve the rate of convergence from n^{-1} to n^{-2}.

Recently, Mortici researched Euler's constant again, and provided some convergent sequences which are faster than (1.1), (1.3) and (1.5).

In [5], Mortici provided the following two sequences

$$u_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} + \frac{1}{(6-2\sqrt{6})n} - \ln\left(n + \frac{1}{\sqrt{6}}\right) \tag{1.7}$$

and

$$v_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} + \frac{1}{(6+2\sqrt{6})n} - \ln\left(n - \frac{1}{\sqrt{6}}\right). \tag{1.8}$$

Both sequences (1.7) and (1.8) were shown to converge to γ as n^{-3}.

Next, in [7], Mortici introduced the following class of sequences of the form

$$\mu_n(a,b)=\sum_{k=1}^{n}\frac{1}{k}+\ln\bigl(e^{a/(n+b)}-1\bigr)-\ln a, \tag{1.9}$$

where a, b are real parameters, $a>0$. Furthermore, they proved that among the sequences $(\mu_n(a,b))_{n\geqslant 1}$, the privileged one

$$\mu_n\left(\frac{\sqrt{2}}{2},\frac{2+\sqrt{2}}{4}\right)$$

offers the best approximations of γ, since

$$\lim_{n\to\infty}n^3\left(\mu_n\left(\frac{\sqrt{2}}{2},\frac{2+\sqrt{2}}{4}\right)-\gamma\right)=\frac{\sqrt{2}}{96}. \tag{1.10}$$

It is their works that motivate our study. In this paper, starting from the well-known sequence γ_n and DeTemple's sequence $(R_n)_{n\geqslant 1}$, based on the early works of Mortici and DeTemple, we provide some new classes of convergent sequence for Euler's constant as follows:

Theorem 1.1. *For Euler's constant, we have the following convergent sequence,*

$$r_n=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n-\cfrac{a_1}{n+\cfrac{a_2n}{n+\cfrac{a_3n}{n+\cfrac{a_4n}{n+\ddots}}}}, \tag{1.11}$$

where

$$a_1=\frac{1}{2},\qquad a_2=\frac{1}{6},\qquad a_3=-\frac{1}{6},\qquad a_4=\frac{3}{5},\qquad \ldots.$$

Let

$$r_n^{(1)}=r_n=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n-\frac{a_1}{n}; \tag{1.12}$$

$$r_n^{(2)}=r_n=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n-\frac{a_1}{n+a_2}; \tag{1.13}$$

$$r_n^{(3)}=r_n=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n-\frac{a_1}{n+\frac{a_2n}{n+a_3}}. \tag{1.14}$$

We also have

$$\lim_{n\to\infty}n^2\bigl(r_n^{(1)}-\gamma\bigr)=-\frac{1}{12}; \tag{1.15}$$

$$\lim_{n\to\infty} n^3\bigl(r_n^{(2)} - \gamma\bigr) = -\frac{1}{72}; \tag{1.16}$$

$$\lim_{n\to\infty} n^4\bigl(r_n^{(3)} - \gamma\bigr) = \frac{1}{120}. \tag{1.17}$$

It is easy to see that $r_n^{(1)} = V_n$ and (1.12) is equivalent to (1.5). Comparing with DeTemple's sequence $(R_n)_{n\geqslant 1}$, Vernescu's sequence $(V_n)_{n\geqslant 2}$, Mortici's sequences $(u_n)_{n\geqslant 1}$, $(v_n)_{n\geqslant 1}$ and $\mu_n(\frac{\sqrt{2}}{2}, \frac{2+\sqrt{2}}{4})$, $(r_n^{(3)})_{n\geqslant 1}$ improves the rate of convergence from n^{-2} and n^{-3} to n^{-4}. In fact, if we need, using Theorem 1.1, we can obtain other convergent sequences which are faster than $r_n^{(3)}$.

Furthermore, for $r_n^{(2)}$ and $r_n^{(3)}$, we also have the following conclusion:

Theorem 1.2. *For all natural numbers n,*

$$\frac{1}{72(n+1)^3} < \gamma - r_n^{(2)} < \frac{1}{72n^3}; \tag{1.18}$$

$$\frac{1}{120(n+1)^4} < r_n^{(3)} - \gamma < \frac{1}{120(n-1)^4}. \tag{1.19}$$

For obtaining Theorem 1.1, we need the following lemma which was used in [6–11] and very useful for construction of convergent sequence.

Lemma 1.1. *If $(x_n)_{n\geqslant 1}$ is convergent to zero and there exists the limit*

$$\lim_{n\to\infty} n^s(x_n - x_{n+1}) = l \in [-\infty, +\infty], \tag{1.20}$$

with $s > 1$, then

$$\lim_{n\to\infty} n^{s-1} x_n = \frac{l}{s-1}. \tag{1.21}$$

Lemma 1.1 was firstly proved by Mortici in [9]. From Lemma 1.1, we can see that the speed of convergence of the sequence $(x_n)_{n\geqslant 1}$ increases together with the value s satisfying (1.20).

The rest of this paper is arranged as follow. In Section 2, we provide the proof of Theorem 1.1. In Section 3, the proof of Theorem 1.2 is given. In Section 4, we give some numerical computations which demonstrate the superiority of our new convergent sequences over DeTemple's sequence, Vernescu's sequence and Mortici's sequences.

2. Proof of Theorem 1.1

Based on the argument of Theorem 2.1 in [10] or Theorem 5 in [11], we need to find the value $a_1 \in R$ which produces the most accurate approximation of the form

$$r_n^{(1)} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n - \frac{a_1}{n}. \tag{2.1}$$

To measure the accuracy of this approximation, a method is to say that an approximation (2.1) is better as $r_n^{(1)} - \gamma$ faster converges to zero. Using (2.1), we have

$$r_n^{(1)} - r_{n+1}^{(1)} = \ln\left(1 + \frac{1}{n}\right) - \frac{1}{n+1} + \frac{a_1}{n+1} - \frac{a_1}{n}. \tag{2.2}$$

Developing in power series in $1/n$, we have

$$r_n^{(1)} - r_{n+1}^{(1)} = \left(\frac{1}{2} - a_1\right)\frac{1}{n^2} + \left(a_1 - \frac{2}{3}\right)\frac{1}{n^3} + \left(\frac{3}{4} - a_1\right)\frac{1}{n^4} + O\left(\frac{1}{n^5}\right). \tag{2.3}$$

From Lemma 1.1, we know that the speed of convergence of the sequence $(r_n^{(1)} - \gamma)_{n\geqslant 1}$ is even higher as the value s satisfying (1.20). Thus, using Lemma 1.1, we have:

(i) If $a_1 \neq 1/2$, then the rate of convergence of the sequence $(r_n^{(1)} - \gamma)_{n\geqslant 1}$ is n^{-1}, since

$$\lim_{n\to\infty} n\left(r_n^{(1)} - \gamma\right) = \frac{1}{2} - a_1 \neq 0.$$

(ii) If $a_1 = 1/2$, then from (2.3), we have

$$r_n^{(1)} - r_{n+1}^{(1)} = -\frac{1}{6}\frac{1}{n^3} + O\left(\frac{1}{n^4}\right)$$

and the rate of convergence of the sequence $(r_n^{(1)} - \gamma)_{n\geqslant 1}$ is n^{-2}, since

$$\lim_{n\to\infty} n^2\left(r_n^{(1)} - \gamma\right) = -\frac{1}{12}.$$

We know that the fastest possible sequence $(r_n^{(1)})_{n\geqslant 1}$ is obtained only for $a_1 = 1/2$.

Next, we define the sequence $(r_n^{(2)})_{n\geqslant 1}$ by the relation

$$r_n^{(2)} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n - \frac{\frac{1}{2}}{n + a_2}. \tag{2.4}$$

Using the similar method from (2.1) to (2.3), we have

$$\begin{aligned} r_n^{(2)} - r_{n+1}^{(2)} = {} & \left(a_2 - \frac{1}{6}\right)\frac{1}{n^3} + \left(\frac{1}{4} - \frac{3}{2}a_2 - \frac{3}{2}a_2^2\right)\frac{1}{n^4} \\ & + \left(2a_2^3 + 3a_2^2 + 2a_2 - \frac{3}{10}\right)\frac{1}{n^5} + O\left(\frac{1}{n^6}\right). \end{aligned} \tag{2.5}$$

The fastest possible sequence $(r_n^{(2)})_{n\geqslant 1}$ is obtained only for $a_2 = 1/6$. Then, from (2.5), we have

$$r_{n,}^{(2)} - r_{n+1}^{(2)} = -\frac{1}{24}\frac{1}{n^4} + O\left(\frac{1}{n^5}\right)$$

and the rate of convergence of the sequence $(r_n^{(2)} - \gamma)_{n \geqslant 1}$ is n^{-3}, since

$$\lim_{n\to\infty} n^3\left(r_n^{(2)} - \gamma\right) = -\frac{1}{72}.$$

Thirdly, we define the sequence $(r_n^{(3)})_{n \geqslant 1}$ by the relation

$$r_n^{(3)} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n - \frac{\frac{1}{2}}{n + \frac{\frac{1}{6}n}{n+a_3}}. \tag{2.6}$$

Using the similar method from (2.1) to (2.3), we have

$$r_n^{(3)} - r_{n+1}^{(3)} = -\left(\frac{1}{4}a_3 + \frac{1}{24}\right)\frac{1}{n^4} + \left(\frac{11}{18}a_3 + \frac{1}{3}a_3^2 + \frac{17}{135}\right)\frac{1}{n^5} + O\left(\frac{1}{n^6}\right). \tag{2.7}$$

The fastest possible sequence $(r_n^{(3)})_{n \geqslant 1}$ is obtained only for $a_3 = -1/6$. Then, from (2.7), we have

$$r_n^{(3)} - r_{n+1}^{(3)} = \frac{1}{30}\frac{1}{n^5} + O\left(\frac{1}{n^6}\right)$$

and the rate of convergence of the sequence $(r_n^{(3)} - \gamma)_{n \geqslant 1}$ is n^{-4}, since

$$\lim_{n\to\infty} n^4\left(r_n^{(3)} - \gamma\right) = \frac{1}{120}.$$

By induction, we have $a_4 = 3/5, \ldots,$ the new sequence (1.11) can be obtained.

3. Proof of Theorem 1.2

Based on the argument of Theorem in [1] or the method in [2], first, we prove (1.18). It is easy to have

$$\gamma - r_n^{(2)} = \sum_{k=n}^{\infty}\left(r_{k+1}^{(2)} - r_k^{(2)}\right) = \sum_{k=n}^{\infty} f(k), \tag{3.1}$$

where

$$f(k) = \frac{1}{k+1} - \frac{3}{6k+7} + \frac{3}{6k+1} - \ln\left(1 + \frac{1}{k}\right).$$

Next, we have

$$f'(x) = -\frac{x^2 + \frac{8}{9}x - \frac{49}{216}}{6(x+\frac{7}{6})^2(x+1)^2(x+\frac{1}{6})^2 x}. \tag{3.2}$$

For the upper bound in (1.18), we have

$$-f'(x) \leqslant \frac{x+\frac{8}{9}}{6(x+\frac{7}{6})^2(x+1)^2(x+\frac{1}{6})^2} \leqslant \frac{1}{6(x+\frac{7}{6})^2(x+1)(x+\frac{1}{6})^2}. \tag{3.3}$$

Combining (3.3) and

$$\left(x+\frac{7}{6}\right)^2(x+1)\left(x+\frac{1}{6}\right)^2 - \left(x+\frac{1}{2}\right)^5 = \frac{7}{6}x^4 + \frac{7}{3}x^3 + \frac{155}{108}x^2 + \frac{79}{324}x + \frac{17}{2592} > 0,$$

we have

$$-f'(x) \leqslant \frac{1}{6(x+\frac{1}{2})^5}. \tag{3.4}$$

Since $f(\infty) = 0$, we have

$$\begin{aligned} f(k) &= -\int_k^\infty f'(x)\,dx \leqslant \frac{1}{6}\int_k^\infty \left(x+\frac{1}{2}\right)^{-5} dx \\ &= \frac{1}{24}\left(k+\frac{1}{2}\right)^{-4} \leqslant \frac{1}{24}\int_k^{k+1} x^{-4}\,dx, \end{aligned} \tag{3.5}$$

where we use the following fact

$$\int_k^{k+1} x^{-4}\,dx - \left(k+\frac{1}{2}\right)^{-4} = \frac{40k^4 + 80k^3 + 51k^2 + 11k + 1}{3k^3(k+1)^3(2k+1)^4} > 0,$$

in the last inequality in (3.5). Combining (3.1) and (3.5), we have

$$\gamma - r_n^{(2)} \leqslant \sum_{k=n}^\infty \frac{1}{24}\int_k^{k+1} x^{-4}\,dx = \frac{1}{24}\int_n^\infty x^{-4}\,dx = \frac{1}{72n^3}. \tag{3.6}$$

For the lower bound, combining (3.2), we have

$$-f'(x) \geqslant \frac{1}{6(x+1)^5}, \tag{3.7}$$

where we use the following fact, for $x \geqslant 1$,

$$\left(x^2+\frac{8}{9}x-\frac{49}{216}\right)(x+1)^3-\left(x+\frac{7}{6}\right)^2\left(x+\frac{1}{6}\right)^2x$$
$$=\frac{11}{9}x^4+\frac{707}{216}x^3+\frac{533}{216}x^2+\frac{221}{1296}x-\frac{49}{216}\geqslant 0.$$

Combining (3.7), we have

$$f(k)=-\int_k^\infty f'(x)\,dx\geqslant\frac{1}{6}\int_k^\infty(x+1)^{-5}\,dx=\frac{1}{24}(k+1)^{-4}\geqslant\frac{1}{24}\int_{k+1}^{k+2}x^{-4}\,dx. \tag{3.8}$$

Combining (3.1) and (3.8), we have

$$\gamma-r_n^{(2)}\geqslant\sum_{k=n}^\infty\frac{1}{24}\int_{k+1}^{k+2}x^{-4}\,dx=\frac{1}{24}\int_{n+1}^\infty x^{-4}\,dx=\frac{1}{72(n+1)^3}. \tag{3.9}$$

Combining (3.6) and (3.9), we complete the proof of (1.18).

Next, we prove (1.19). It is easy to have

$$r_n^{(3)}-\gamma=\sum_{k=n}^\infty\left(r_k^{(3)}-r_{k+1}^{(3)}\right)=\sum_{k=n}^\infty g(k), \tag{3.10}$$

where

$$g(k)=\ln\left(1+\frac{1}{k}\right)-\frac{1}{k+1}+\frac{6k+5}{12(k+1)^2}-\frac{6k-1}{12k^2}.$$

Next, we have

$$g'(x)=-\frac{1}{6x^3(x+1)^3}. \tag{3.11}$$

For the upper bound in (1.19), we have

$$-g'(x)\leqslant\frac{1}{6x^6}. \tag{3.12}$$

Since $g(\infty)=0$, combining (3.12), we have

$$g(k)=-\int_k^\infty g'(x)\,dx\leqslant\frac{1}{6}\int_k^\infty x^{-6}\,dx=\frac{1}{30}k^{-5}\leqslant\frac{1}{30}\int_{k-1}^k x^{-5}\,dx. \tag{3.13}$$

Table 1
Simulations for R_n, V_n, u_n and v_n.

n	$R_n-\gamma$	$V_n-\gamma$	$u_n-\gamma$	$v_n-\gamma$
10	3.7733×10^{-4}	-8.3250×10^{-4}	-2.1179×10^{-5}	2.4228×10^{-5}
25	6.4061×10^{-5}	-1.3331×10^{-4}	-1.4127×10^{-6}	1.4909×10^{-6}
50	1.6337×10^{-5}	-3.3332×10^{-5}	-1.7901×10^{-7}	1.8390×10^{-7}
100	4.1252×10^{-6}	-8.3333×10^{-6}	-2.2528×10^{-8}	2.2833×10^{-8}
250	6.6401×10^{-7}	-1.3333×10^{-6}	-1.4476×10^{-9}	1.4555×10^{-9}
1000	4.1625×10^{-8}	-8.3333×10^{-8}	-2.2665×10^{-11}	2.2696×10^{-11}

Combining (3.10) and (3.13), we have

$$r_n^{(3)}-\gamma\leqslant\sum_{k=n}^{\infty}\frac{1}{30}\int_{k-1}^{k}x^{-5}\,dx=\frac{1}{30}\int_{n-1}^{\infty}x^{-5}\,dx=\frac{1}{120(n-1)^4}. \tag{3.14}$$

For the lower bound, we have

$$-g'(x)\geqslant\frac{1}{6(x+1)^6}. \tag{3.15}$$

Combining (3.15), we have

$$g(k)=-\int_{k}^{\infty}g'(x)\,dx\geqslant\frac{1}{6}\int_{k}^{\infty}(x+1)^{-6}\,dx=\frac{1}{30}(k+1)^{-5}\geqslant\frac{1}{30}\int_{k+1}^{k+2}x^{-5}\,dx. \tag{3.16}$$

Combining (3.10) and (3.16), we have

$$r_n^{(3)}-\gamma\geqslant\sum_{k=n}^{\infty}\frac{1}{30}\int_{k+1}^{k+2}x^{-5}\,dx=\frac{1}{30}\int_{n+1}^{\infty}x^{-5}\,dx=\frac{1}{120(n+1)^4}. \tag{3.17}$$

Combining (3.14) and (3.17), we complete the proof of (1.19).

4. Numerical computation

In this section, we give two tables to demonstrate the superiority of our new convergent sequences $(\gamma_n^{(2)})_{n\geqslant1}$ and $(\gamma_n^{(3)})_{n\geqslant1}$ over DeTemple's sequence $(R_n)_{n\geqslant1}$, Vernescu's sequence $(V_n)_{n\geqslant1}$ and Mortici's sequences $(\mu_n(\frac{\sqrt{2}}{2},\frac{2+\sqrt{2}}{4}))_{n\geqslant1}$, $(u_n)_{n\geqslant1}$ and $(v_n)_{n\geqslant1}$, respectively.

Combining Theorem 1.1 and Theorem 1.2, we have Tables 1 and 2.

Table 2
Simulations for μ_n, $r_n^{(2)}$, $r_n^{(3)}$.

n	$\mu_n(\frac{\sqrt{2}}{2}, \frac{2+\sqrt{2}}{4}) - \gamma$	$r_n^{(2)} - \gamma$	$r_n^{(3)} - \gamma$
10	1.1807×10^{-5}	-1.2832×10^{-5}	8.2941×10^{-7}
25	8.6183×10^{-7}	-8.6169×10^{-7}	2.1317×10^{-8}
50	1.1265×10^{-7}	-1.0941×10^{-7}	1.3331×10^{-9}
100	1.4402×10^{-8}	-1.3782×10^{-8}	8.3329×10^{-11}
250	9.3431×10^{-10}	-8.8616×10^{-10}	2.1333×10^{-12}
1000	1.4698×10^{-11}	-1.3878×10^{-11}	8.3333×10^{-15}

Acknowledgments

The author is supported by the National Natural Sciences Foundation of China (grant number 11101061 and 11371077), Research Foundation for Doctor of Liaoning Province (grant number 20121016) and the Fundamental Research Funds for the Central Universities (grant number DUT12LK16 and DUT13JS06).

References

[1] D.W. DeTemple, A quicker convergences to Euler's constant, Amer. Math. Monthly 100 (5) (1993) 468–470.
[2] D.W. DeTemple, A geometric look at sequences that converge to Euler's constant, College Math. J. 37 (2006) 128–131.
[3] C. Mortici, A. Vernescu, An improvement of the convergence speed of the sequence $(\gamma_n)_{n\geqslant 1}$ converging to Euler's constant, An. Stiint. Univ. "Ovidius" Constanta 13 (1) (2005) 97–100.
[4] C. Mortici, A. Vernescu, Some new facts in discrete asymptotic analysis, Math. Balkanica (N.S.) 21 (3–4) (2007) 301–308.
[5] C. Mortici, Optimizing the rate of convergence in some new classes of sequences convergent to Euler's constant, Anal. Appl. (Singap.) 8 (1) (2010) 99–107.
[6] C. Mortici, Very accurate estimates of the polygamma functions, Asymptot. Anal. 68 (3) (2010) 125–134.
[7] C. Mortici, A quicker convergence toward the gamma constant with the logarithm term involving the constant e, Carpathian J. Math. 26 (1) (2010) 86–91.
[8] C. Mortici, On new sequences converging towards the Euler–Mascheroni constant, Comput. Math. Appl. 59 (8) (2010) 2610–2614.
[9] C. Mortici, Product approximations via asymptotic integration, Amer. Math. Monthly 117 (5) (2010) 434–441.
[10] C. Mortici, A new Stirling series as continued fraction, Numer. Algorithms 56 (1) (2011) 17–26.
[11] C. Mortici, A continued fraction approximation of the gamma function, J. Math. Anal. Appl. 402 (2) (2013) 405–410.
[12] A. Vernescu, A new accelerate convergence to the constant of Euler, Gazeta Mat. Ser. A, Bucharest 104 (4) (1999) 273–278 (in Romanian).
[13] A. Vernescu, The order of convergence of the sequence that defines the constant of Euler, Gazeta Mat. Ser. A, Bucharest 88 (10–11) (1983) 380–381 (in Romanian).
[14] R.M. Young, Euler's constant, Math. Gaz. 75 (472) (1991) 187–190.