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Note

# Letter to the editor

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ABSTRACT

This letter shows how the main result contained in a paper recently appeared in the Journal of Multivariate Analysis was in fact a particular case of a more general theorem published three years before.



The Journal of Multivariate Analysis has recently published a paper, [3], containing a characterization of the multivariate normality of a random vector based on the measure of the set of the normal one-dimensional marginals that a non-normal vector can have.

The goal of this letter is to bring to your attention a more general result, that of [1], that was previously known and that the authors and referees were apparently unaware of.

The two results have been obtained using quite different techniques. While the result in [3] follows from a study of the properties of the moments of a random vector with density, the one in [1] uses the random projection techniques introduced in [2].

Next we state the main results of [3] (Theorem 1) and of [1] (Theorem 3.6) for comparison.

**Theorem** (*Shao and Zhou*)**.** *A $p$-dimensional random vector $X$ with a Lebesgue density is not normally distributed if and only if the set of vectors $u$ in the unit $(p-1)$-dimensional sphere such that $u^TX$ is normally distributed, has measure 0 with respect to the uniform measure on the unit sphere.*

**Theorem** (*Cuesta-Albertos et al.*)**.** *Let $\mathbb{H}$ be a separable Hilbert space with scalar product $\langle \cdot, \cdot \rangle$ and let $X$ be an $\mathbb{H}$-valued random element. Let $\mu$ be a dissipative measure on $\mathbb{H}$. If the set*

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$$\{u \in \mathbb{H} : \langle u, X\rangle \text{ is normal}\}$$

*has positive $\mu$-measure, then $X$ is normal.*

From here, the following corollary immediately follows.

**Corollary.** *Under the same hypotheses as in the preceding theorem, it happens that $X$ is normal if and only if the set*

$$A = \{u \in \mathbb{H} : \langle u, X\rangle \text{ is normal}\}$$

*has $\mu$-measure zero.*

The definition of dissipative measure appears in [1]. It is a trivial fact that, in the finite dimensional case, every absolutely continuous distribution is dissipative. As a consequence, this result includes the case in which we are in a $p$-dimensional space and we choose the vector $u$ with a uniform distribution on the unit ball. Since the projection of $X$ on $u$ is the same as the projection of $X$ on $u/\|u\|$ we have that the corollary includes Theorem 1 in [3] with the advantage that not only is this corollary valid in separable Hilbert spaces, but also it requires no assumption on the distribution of the random vector $X$. Thus, this result broadly contains Theorem 1 of [3].

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