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Microscopic calculation of spin torques and forces

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Abstract

Spin torques, that is, effects of conduction electrons on magnetization dynamics, are calculated microscopically in the first order in spatial gradient and time derivative of magnetization. Special attention is paid to the so-called β-term and the Gilbert damping, α, in the presence of electrons' spin-relaxation processes, which are modeled by quenched magnetic impurities. Two types of forces that the electric/spin current exerts on magnetization are identified based on a general formula relating the force to the torque.

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1. Introduction

Manipulation of nanoscale magnetization by electric/spin current is of recent interest. One well-known effect of the current on magnetization dynamics is the spin-transfer torque [1,2]. It is expressed as $-(\boldsymbol{v}_s \cdot \boldsymbol{\nabla})\boldsymbol{n}$ for a continuous magnetic configuration $\boldsymbol{n}$ as described by the Landau–Lifshitz–Gilbert (LLG) equation [3]. Recently, another type of spin torque has been proposed on microscopic [4,5] and phenomenological [6–8] grounds. It has essentially the form $-\beta\boldsymbol{n} \times (\boldsymbol{v}_s \cdot \boldsymbol{\nabla})\boldsymbol{n}$, and modifies the magnetization dynamics significantly, especially in the case of a domain wall by acting as a force [6,7,9]. The purpose of the present study is to derive this new torque microscopically, by taking the electrons' spin-relaxation processes into account, and to give a general argument for the force [10,11] acting on magnetization texture. Some details of the former subject can be found in Ref. [12].

2. Spin torques

We consider a 'localized' ferromagnet consisting of localized d spins (of magnitude S) and conducting s electrons, which are coupled each other via the s–d exchange interaction, $H_{\rm sd} = -M\int d^3x\boldsymbol{n}\cdot\hat{\boldsymbol{\sigma}}$. We adopt a continuum description for the localized spin, $\boldsymbol{n}$, whose dynamics will then be described by the LLG equation:

$$\dot{\boldsymbol{n}} = \gamma_0 \boldsymbol{H}_{\rm eff} \times \boldsymbol{n} + \alpha_0 \dot{\boldsymbol{n}} \times \boldsymbol{n} + \boldsymbol{t}'_{\rm el}. \tag{1}$$

Here $\gamma_0\boldsymbol{H}_{\rm eff}$ and α_0 are an effective field and a Gilbert damping constant, respectively, coming from the spin part of the Hamiltonian. Effects of conduction electrons are contained in the spin torque $\boldsymbol{t}_{\rm el}(\boldsymbol{r}) = M\boldsymbol{n}(\boldsymbol{r}) \times @ \langle\hat{\boldsymbol{\sigma}}(\boldsymbol{r})\rangle_{\rm n.e.} \equiv (\hbar S/a^3)\boldsymbol{t}'_{\rm el}(\boldsymbol{r})$, which comes from $H_{\rm sd}$. Here $\langle\hat{\boldsymbol{\sigma}}(\boldsymbol{r})\rangle_{\rm n.e.}$ is the s-electron spin polarization, and a^3 is the volume per localized d spin. The spin torque is generally expressed as

$$\boldsymbol{t}'_{\rm el} = a'_0\dot{\boldsymbol{n}} + (\boldsymbol{a}' \cdot \boldsymbol{\nabla})\boldsymbol{n} + b'_0(\boldsymbol{n} \times \dot{\boldsymbol{n}}) + \boldsymbol{n} \times (\boldsymbol{b}' \cdot \boldsymbol{\nabla})\boldsymbol{n}, \tag{2}$$

in the first order in time derivative and spatial gradients. The coefficients, a'_μ and b'_μ, can be calculated from the linear response of $\langle\hat{\boldsymbol{\sigma}}(\boldsymbol{r})\rangle_{\rm n.e.}$ to small transverse fluctuations of $\boldsymbol{n}$ around the uniformly magnetized state [13]. For a 3D electron system under the influence of randomly distributed nonmagnetic and magnetic impurities with bare scattering amplitudes, u and $u_s\boldsymbol{S}_i \cdot \boldsymbol{\sigma}$, respectively, and concentrations,

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n_i and n_s, we obtain $a_0' = -\rho_s a^3/2S$, $\boldsymbol{a}' = (a^3/2eS)\boldsymbol{j}_s$,

$$b_0' = -\pi n_s u_s^2 \cdot \frac{a^3}{S}[2\overline{S_z^2}\nu_\uparrow\nu_\downarrow + \overline{S_\perp^2}(\nu_\uparrow^2 + \nu_\downarrow^2)], \tag{3}$$

and

$$\boldsymbol{b}' = \frac{\pi n_s u_s^2}{M} \cdot \frac{a^3}{2eS}[(\overline{S_\perp^2} + \overline{S_z^2})\nu_+ \boldsymbol{j}_s + (\overline{S_\perp^2} - \overline{S_z^2})\nu_- \boldsymbol{j}_c]. \tag{4}$$

Here $\rho_s = n_\uparrow - n_\downarrow$ is the equilibrium s-electron spin polarization, $\boldsymbol{j}_s = \boldsymbol{j}_\uparrow - \boldsymbol{j}_\downarrow [\equiv -(2eS/a^3)\boldsymbol{v}_s]$ is the spin current, $\boldsymbol{j}_c = \boldsymbol{j}_\uparrow + \boldsymbol{j}_\downarrow$ is the charge current, and $\nu_\pm = \nu_\uparrow \pm \nu_\downarrow$ with ν_σ being the Fermi-level density of states (DOS) for spin-σ electrons. We have taken a quenched average for the impurity spin direction as $\overline{S_{i,\alpha}S_{j,\beta}} = \delta_{ij}\delta_{\alpha\beta}\overline{S_\alpha^2}$ with $\overline{S_x^2} = \overline{S_y^2} \equiv \overline{S_\perp^2}$. As seen, only the spin scattering (u_s), causing spin relaxation, contributes to b_0' and $\boldsymbol{b}'$, and the potential scattering (u) does not [14]. We put $\alpha = -b_0'$ (Gilbert damping) and $\boldsymbol{b}' = -\beta\boldsymbol{v}_s$ (defining β). In terms of longitudinal ($\tau_L^{-1} = 4\pi n_s u_s^2 \overline{S_\perp^2}\nu_+/\hbar$) and transverse ($\tau_T^{-1} = 2\pi n_s u_s^2(\overline{S_\perp^2} + \overline{S_z^2})\nu_+/\hbar$) spin-relaxation rates, we have

$$\alpha = \frac{a^3\hbar\nu_+}{4S}\left[(1 - P_\nu^2)\frac{1}{\tau_T} + P_\nu^2\frac{1}{\tau_L}\right], \tag{5}$$

$$\beta = \frac{\hbar}{2M}\left[\left(1 - \frac{P_\nu}{P_j}\right)\frac{1}{\tau_T} + \frac{P_\nu}{P_j}\frac{1}{\tau_L}\right], \tag{6}$$

where $P_j = j_s/j_c$ is the polarization of the current, and $P_\nu = \nu_-/\nu_+$ is the DOS asymmetry. For "isotropic" impurities with $\tau_L = \tau_T \equiv \tau_s$, we have $\alpha = a^3\hbar\nu_+/(4S\tau_s)$, and $\beta = \hbar/(2M\tau_s)$. Our model was intended to be one of the microscopic realizations of the phenomenology of Ref. [6], but now proves to give a different result for α, namely, with the factor ν_+ replacing n_0/M of Ref. [6]. Furthermore, we could not confirm the relation, $\alpha = \beta$, demonstrated in Refs. [8,13].

For a magnetization varying rapidly in space, we have in addition a spatially oscillating torque, $\boldsymbol{t}_{osc}$, due to electron reflection [4,11]. This torque has the same algebraic form as the β-term but is spatially nonlocal.

A note on itinerant ferromagnets; in this case, we can also derive an LLG equation of the same form, whose coefficients are obtained from the above results (for 'localized' ferromagnets) by the replacement $2S \to \rho_s a^3$ [12].

3. Forces

The β-term acts as a force (like magnetic field) for a specific case of a rigid domain wall [6,7]. We consider here a generalization of it, and derive a general expression for the force acting on a fixed but arbitrary magnetization texture.

Following a standard procedure in field theory, the energy-momentum tensor, especially the momentum density $\rho_{P,i}$, of the magnetization is obtained as

$$\rho_{P,i} = -\frac{\hbar S}{a^3}(\partial_i\phi)(\cos\theta - 1). \tag{7}$$

We define the force acting on the magnetization by Newton's equation of motion as $F_i = (\mathrm{d}/\mathrm{d}t)\int \mathrm{d}^3x\rho_{P,i}$. After doing a partial integration (neglecting surface terms), and then using the LLG equation, $(\hbar S/a^3)\dot{\boldsymbol{n}} = \boldsymbol{t}_{tot}$, we obtain

$$F_i = -\int \mathrm{d}^3x[\boldsymbol{n} \times (\partial_i\boldsymbol{n})] \cdot \boldsymbol{t}_{tot}. \tag{8}$$

This is a general formula relating the force to the torque. If we write the torque as $\boldsymbol{t}_{tot} = \boldsymbol{h} \times \boldsymbol{n}$ in terms of some effective field $\boldsymbol{h}$, Eq. (8) gives a well-known expression, $F_i = \int \mathrm{d}^3x(\partial_i\boldsymbol{n}) \cdot \boldsymbol{h}$ [15].

Each term of the spin torque (2) exerts a different type of force. The spin-transfer torque exerts a 'transverse' force

$$F_{ST,i} = -\frac{\hbar}{2e}j_{s,\ell}\int \mathrm{d}^3x[(\partial_i\boldsymbol{n}) \times (\partial_\ell\boldsymbol{n})] \cdot \boldsymbol{n} \tag{9}$$

for $\boldsymbol{n}$ subtending a finite solid angle (spin chirality). One such example is the magnetic vortex in a ferromagnetic dot, whose dynamics driven by electric/spin current is of recent interest [16–18]. The β-term leads to the force

$$F_{\beta,i} = -\beta\frac{\hbar}{2e}j_{s,\ell}\int \mathrm{d}^3x(\partial_i\boldsymbol{n}) \cdot (\partial_\ell\boldsymbol{n}). \tag{10}$$

Similar but nonlocal ('bilocal') expression results from $\boldsymbol{t}_{osc}$.

The reactions to the forces, $\boldsymbol{F}_{ST}$ and $\boldsymbol{F}_\beta$, will affect the s-electrons' motion, hence their transport, and lead to the Hall effect (due to spin chirality) [19–21] and the dissipative resistivity, respectively. Such a relation between forces and transport coefficients has been noted in the study of a domain wall [11], and will further be reported elsewhere.

In summary, we have presented a microscopic calculation of the spin torques, especially the Gilbert damping and the so-called β-term, on the basis of a microscopic model and controlled approximations. Two types of current-induced forces have been identified based on a general relation between the force and the torque.

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