

Survey of 360° domain walls in magnetic heterostructures: Topology, chirality and current-driven dynamics

Mei Li^a, Jie Lu^{b,*}

^a Physics Department, Shijiazhuang University, Shijiazhuang, Hebei 050035, People's Republic of China

^b College of Physics and Hebei Advanced Thin Films Laboratory, Hebei Normal University, Shijiazhuang 050024, People's Republic of China

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ABSTRACT

Chirality and current-driven dynamics of topologically nontrivial 360° domain walls (360DWs) in magnetic heterostructures (MHs) are systematically investigated. For MHs with normal substrates, the static 360DWs are Néel-type with no chirality. While for those with heavy-metal substrates, the interfacial Dzyaloshinskii-Moriya interaction (iDMI) therein makes 360DWs prefer specific chirality. Under in-plane driving charge currents, as the direct result of “full-circle” topology a certain 360DW does not undergo the “Walker breakdown”-type process like a well-studied 180° domain wall as the current density increases. Alternatively, it keeps a fixed propagating mode (either steady-flow or precessional-flow, depending on the effective damping constant of the MH) until it collapses or changes to other types of soliton when the current density becomes too high. For both modes, modifications to the mobility of 360DWs by iDMI and spin-orbit torque are provided.

1. Introduction

The invention and great development of non-volatile magnetic nanodevices have led to a profound revolution in the information industry [1–3]. In these nanodevices, various magnetic solitons or magnetic domains they separate play the roles of 0 and 1 in binary world. In wide magnetic nanostrips, skyrmions/antiskyrmions [4–8], bimerons [9–13] and so on are two-dimensional (2D) isolated topologically nontrivial magnetic solitons surrounded by connected domains with uniform orientation. Under the standard definition of 2D topological charge

$$\mathcal{W}_{2D}(\mathbf{m}) = \frac{1}{4\pi} \int_{\mathbb{R}^2} \mathbf{m} \cdot \left(\frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y} \right) d(x, y), \quad (1)$$

in which $\mathbf{m}$ is a $\mathbb{R}^3$ unit magnetization field locating on the (x, y) plane, these solitons have an integer $\mathcal{W}_{2D}$ and they themselves are the information carriers. While in narrow enough nanostrips which are quasi-one dimensional (Q1D) systems, the most studied magnetic solitons are the 1D (Néel or Bloch) 180° domain walls (180DWs) bearing $\pm 1/2$ 1D topological charge, which is defined as

$$\mathcal{W}_{1D}(\mathbf{m}) = \frac{1}{2\pi} \int_{\mathbb{R}} \left(m_1 \frac{\partial m_2}{\partial \rho} - m_2 \frac{\partial m_1}{\partial \rho} \right) d\rho, \quad (2)$$

where $m_{1,2}$ are magnetization components in the wall plane and the Q1D systems are supposed to extend in ρ -direction. 180DWs separate two opposite oriented domains whose orientations can be defined as 0 and 1, meantime the wall motion leads to the transformation of information. Since the famous Walker analysis [14], tremendous progress has been made on statics and dynamics of 180DWs driven by various external stimuli [15–29]. The corresponding results have laid the foundation for many mature commercial and developing magnetic nanodevices.

Interestingly, even in Q1D systems we also have some kinds of isolated magnetic solitons which have integer $\mathcal{W}_{1D}$. Among them, the simplest ones are the so-called 360° domain walls (360DWs) in which the magnetization rotates over one full circle across the intermediate region thus bearing $\mathcal{W}_{1D} = \pm 1$. In the beginning of 1960s, 360DWs were first found to appear in the magnetization reversal process of thin films and their existence seem to be a nuisance since they may complicate the reversal process [30–32]. However, studies in the past three decades revealed that 360DWs themselves in 2D magnetic films have more interesting physics [33–38]. Now we know that 360DWs in lower dimensional systems can be qualified candidates to store and process information in magnetic nanodevices due to its “full-circle” topology. In magnetic nanorings with outer diameter around 1 micron, width of several hundred nanometers and thickness of several nanometers, two 180DWs can be forced toward one another by the applied circular

* Corresponding author.

E-mail address: jlu@hebtu.edu.cn (J. Lu).

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Oersted field to form a 360DW and then annihilated by increasing the field strength [39–44]. While in magnetic nanostrips tens to hundreds of nanometers wide and a few to tens of nanometers thick, 360DWs emerge from the combination of two neighboring 180DWs with the same $\mathcal{W}_{1D}$ due to the long-range magnetostatic interaction in relatively wider strips [45–47] or appropriate external magnetic fields in narrow ones [48–50]. From the viewpoint of application, the energy barrier of nucleating a single 360DW (local magnetization kink) from the single-domain ground states of nanostrips is much lower than a single 180DW since in the latter case one should reverse the magnetic moments in the entire half.

From the beginning of this century, a series of analytical works focus on the question whether 360DWs are genuine stable magnetization textures or just long-lived metastable states [35,51,52]. For 2D ferromagnetic (FM) films, the main results are as follows: (i) the magnetostatics is crucial for the existence of 360DWs; (ii) if the long-range component of magnetostatics is neglected, an appropriate external field must be applied to stabilize a 1D front of 360DW whose energy is independent of wall orientation [35]. As the films fade into narrow enough nanostrips, changes in boundary conditions further require that the external field should align with the easy axis to guarantee the existence of 360DWs. In recent device applications, narrow FM metallic nanostrips often serve as the central components of magnetic heterostructures (MHs) with heavy-metal (HM) substrates. Then the effects of interfacial Dzyaloshinskii-Moriya interaction (iDMI) [53,54] therein to the chirality preference of 360DWs need to be clarified.

Once nucleated, 360DWs in MHs can be driven by certain external stimuli. First, external magnetic fields along easy axis can not finish this job. This can be understood by our roadmap of field-driven domain wall motion since the Zeeman energy densities in the two domains on both sides of 360DWs are the same [16]. Then, current-induced motion of 360DWs becomes the next choice. Indeed, it is the most common way to implement and manipulate in real MHs. Numerical investigations on this issue have been widely preformed in the past decade [55–61]. Alternatively, there are few analytical studies due to the complexity from the coexistence of iDMI, spin-transfer torque (STT) and spin-orbit torque (SOT) therein. In this paper, by adopting the Lagrangian-based collective coordinate model (LB-CCM) and adequate wall ansatz, the current-driven dynamics of 360DWs is systematically explored which constitutes

the second part of this work.

The paper is organized as follows. First, the magnetic Lagrangian and dissipation functional of MHs are introduced in Section 2. Both perpendicular magnetic anisotropy (PMA) and in-plane magnetic anisotropy (IPMA) for the central FM metallic layers are considered. Then in Section 3.1 we define the proper ansatz for 1D topologically nontrivial 360DWs and introduce three typical candidates. After integrating over the long axis of MHs, a set of unified dynamical equations is obtained in Section 3.2 and serves as the startpoint of our work. In Section 3.3, chirality preference of 360DWs selected by iDMI is investigated. After that, the propagation modes of 360DWs under in-plane currents are systematically explored in Section 3.4. Also, for both modes the effects of iDMI and SOT to the dynamics of 360DWs are analytically calculated. Finally discussions and concluding remarks are provided in Section 4 and 5, respectively.

2. Formulism

A MH under consideration is shown in Fig. 1, which is composed of three layers: a HM substrate, a central FM metallic layer and a normal caplayer. We suppose that the MH is long and narrow enough so that it can be viewed as a Q1D system extended in the long axis. For central FM layers with PMA (which will be referred to as “PMA sytems”), the easy axis lies in z -axis (out-of-plane normal), the long axis of MH is along x -axis and $\mathbf{e}_y = \mathbf{e}_z \times \mathbf{e}_x$ being the hard axis. While for central FM layers with IPMA (named as “IPMA sytems”), the easy axis coincides with long axis and is defined as z -axis, the hard axis is along out-of-plane normal and denote as y -axis, at last $\mathbf{e}_x = \mathbf{e}_y \times \mathbf{e}_z$. In these two coordinate systems, the crystalline anisotropy energy density for PMA and IPMA systems shares the same form. However, the iDMI should be treated carefully since it is determined by the out-of-plane normal component of the magnetization vector. By setting “ $\mathbf{n}$ ” as the out-of-plane normal, the iDMI energy density can be written as [62]

$$\mathcal{E}_{\text{iDMI}} = D_i \{ [\mathbf{m}(\mathbf{r}) \cdot \mathbf{n}] \nabla \cdot \mathbf{m}(\mathbf{r}) - [\mathbf{m}(\mathbf{r}) \cdot \nabla] [\mathbf{m}(\mathbf{r}) \cdot \mathbf{n}] \}, \quad (3)$$

where D_i is the iDMI strength and $\mathbf{m}(\mathbf{r})$ is the unit magnetization vector of the central FM layer at position $\mathbf{r}$. Accordingly, the total magnetic energy density $\mathcal{E}_0$ includes the exchange, crystalline anisotropy, magnetostatic, Zeeman and iDMI contributions. When we focus on

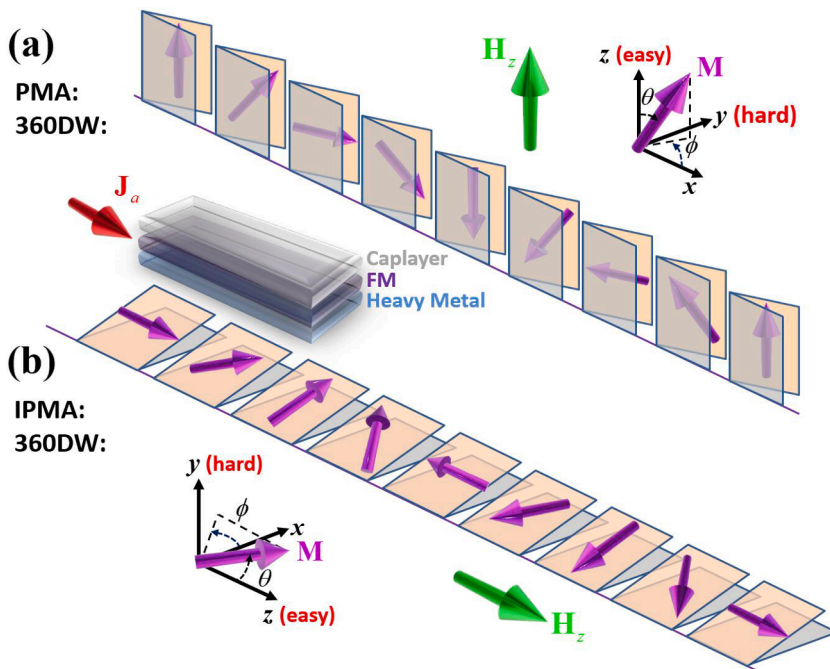


Fig. 1. Sketch of a MH in which a 360DW is formed in its central FM metallic layer with (a) PMA and (b) IPMA. A typical MH is composed of a three-layer structure: a HM substrate, a central FM metallic layer and a normal caplayer. The corresponding coordinate system is depicted at the up-right and bottom-left corners in the respective subfigure. In each case, the easy (hard) axis lies in the z (y)-direction. An external magnetic field $\mathbf{H}_z = H_z \mathbf{e}_z$ is applied to guarantee the existence of 360DWs. When in-plane charge current $\mathbf{J}_a = j_a \mathbf{e}_y$ is applied, magnetization vectors will be driven to tilt from their static locations meantime the 360DW will be driven to propagate along the long axis. Gray (orange) planes describe the planar φ -distribution of static (dynamical) magnetization texture.

360DWs around narrow MH centers (far from ends or corners), most of the magnetostatic energy can be described by local quadratic terms of $M_{x,y,z}$ by means of three average demagnetization factors $D_{x,y,z}$ [25], meantime the effects of long-range residue will be very weak thus can be neglected [16,48,63]. In addition, for Q1D systems $\nabla \equiv \frac{\partial}{\partial \rho} \mathbf{e}_\rho$ in which $\rho = x(z)$ for PMA (IPMA) systems. Thus one has

$$\mathcal{E}_0[\mathbf{m}] = A \left(\frac{\partial \mathbf{m}}{\partial \rho} \right)^2 + \mu_0 M_s^2 \left(-\frac{1}{2} k_E m_z^2 + \frac{1}{2} k_H m_y^2 \right) - \mu_0 M_s \mathbf{m} \cdot \mathbf{H}_z + \mathcal{E}_{\text{DMI}}, \quad (4)$$

in which A is the exchange stiffness, μ_0 is the permeability of vacuum, M_s is the saturation magnetization and $\mathbf{H}_z = H_z \mathbf{e}_z$ is the external magnetic field along the easy axis with the strength H_z . At last, k_E (k_H) denotes the total anisotropy coefficient along the easy (hard) axis of the central FM layer, namely $k_E = k_1 + (D_x - D_z)$ and $k_H = k_2 + (D_y - D_x)$ with $k_{1(2)}$ being the crystalline anisotropy coefficient in easy (hard) axis.

The in-plane charge current flows along “ $\mathbf{e}_\rho$ ” with density j_a . As passing through the MH, the charge current splits into two parts. Suppose j_F (j_H) to be the component in FM (HM) layer. A simple circuit model delivers that $j_F = j_a(t_F + t_H)\sigma_F / (t_F\sigma_F + t_H\sigma_H)$ and $j_H = j_a(t_F + t_H)\sigma_H / (t_F\sigma_F + t_H\sigma_H)$, where t_F (t_H) and σ_F (σ_H) are the thickness and conductivity of the FM (HM) layer, respectively. For the most common FM metal (Co, Ni, Fe) and HM (Pt, Ta, Ir) materials, the conductivity varies from 10 to 20 ($\mu\Omega\text{m}$)⁻¹. For simplicity, we set $\sigma_F \approx \sigma_H$ thus $j_F = j_H = j_a$. The charge current component (j_H) in HM substrate will induce a spin current into the FM layer which is polarized in the direction of “ $\mathbf{m}_p \equiv \mathbf{n} \times \mathbf{e}_\rho$ ”, hence generate the SOT. On the other hand, in the global Cartesian coordinate system, the unit vector of magnetization in the FM layer can be fully described by its polar angle θ and azimuthal angle ϕ , as shown in Fig. 1. The resulting local spherical coordinate system is denoted as ($\mathbf{e}_m \equiv \mathbf{m}, \mathbf{e}_\theta, \mathbf{e}_\phi$). Then $\mathbf{m}_p$ can be decomposed as

$$\mathbf{m}_p = p_m \mathbf{e}_m + p_\theta \mathbf{e}_\theta + p_\phi \mathbf{e}_\phi. \quad (5)$$

Base on all these preparations, the Lagrangian density $\mathcal{L}$ and dissipation functional density $\mathcal{F}$ of this magnetization system are expressed as

$$\frac{\mathcal{L}}{\mu_0 M_s^2} = -\frac{\cos\theta}{\gamma M_s} \frac{\partial \phi}{\partial t} - \frac{B_J \phi}{\gamma M_s} \frac{\partial (\cos\theta)}{\partial \rho} + \frac{H_{\text{FL}}}{M_s} p_m - \frac{\mathcal{E}_0}{\mu_0 M_s^2}, \quad (6)$$

and

$$\frac{\mathcal{F}}{\mu_0 M_s^2} = \frac{\alpha}{2\gamma M_s} \left\{ \left[\frac{\partial}{\partial t} - \frac{\beta B_J}{\alpha} \frac{\partial}{\partial \rho} \right] \mathbf{m} \right\}^2 - \frac{H_{\text{ADL}}}{M_s} (\mathbf{m} \times \mathbf{m}_p) \cdot \frac{\partial \mathbf{m}}{\partial t}, \quad (7)$$

in which $\gamma = \mu_0 \gamma_e$ with γ_e being the electron gyromagnetic ratio, $B_J = \mu_B P j_a / (e M_s)$ with e, μ_B being respectively the absolute value of electron charge and Bohr magneton, P is the spin polarization of j_F , α is the Gilbert damping constant, β is the dimensionless coefficient describing the relative strength of the nonadiabatic STT over the adiabatic one, at last H_{FL} and H_{ADL} denotes the strength of field-like (FL) and anti-damping-like (ADL) SOT, respectively.

The dynamics of magnetization in the central FM layers of MHs is then described by the Lagrangian-Rayleigh equation

$$\frac{d}{dt} \left(\frac{\delta \mathcal{L}}{\delta \dot{X}} \right) - \frac{\delta \mathcal{L}}{\delta X} + \frac{\delta \mathcal{F}}{\delta \dot{X}} = 0, \quad (8)$$

in which X is any related local or collective coordinate. In particular, when $X = \theta$ and ϕ (the most common local coordinates), the resulting two equations can be combined to recover the familiar Landau-Lifshitz-Gilbert equation

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \mathbf{T}_{\text{STT}} + \mathbf{T}_{\text{SOT}}, \quad (9)$$

where $\mathbf{H}_{\text{eff}} = -(\mu_0 M_s)^{-1} \delta \mathcal{E}_0 / \delta \mathbf{m}$, and

$$\mathbf{T}_{\text{STT}} = B_J \frac{\partial \mathbf{m}}{\partial \rho} - \beta B_J \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial \rho}, \quad (10)$$

as well as

$$\mathbf{T}_{\text{SOT}} = -\gamma H_{\text{FL}} \mathbf{m} \times \mathbf{m}_p - \gamma H_{\text{ADL}} \mathbf{m} \times (\mathbf{m} \times \mathbf{m}_p). \quad (11)$$

However, $\theta(\rho, t)$ and $\phi(\rho, t)$ vary from point to point, hence generating a huge number of degrees of freedom. To obtain collective behaviors of magnetization system, LB-CCMs are adopted which need preset ansatz. In the beginning of next section, we will define adequate ansatz for topologically nontrivial 360DWs and introduce several typical trial profiles which contain reasonable collective coordinates. Based on them, a set of dynamical equations can be obtained, which lays the foundation of our work in this paper.

3. Results

3.1. Adequate ansatz for topologically nontrivial 360DWs

As we mentioned above, earlier studies confirm that in Q1D MHs if the long-range component of magnetostatics is neglected, then an external field along the easy axis is crucial for forming a 360DW. Accordingly, an analytical profile of static 360DWs has been provided based on the requirement that at equilibrium the $\mathbf{e}_\theta$ component of $\mathbf{H}_{\text{eff}}$ disappears [35]. In this solution the azimuthal angle takes a fixed value while the polar angle changes monotonously from 0 to π as ρ runs from one end of MH to the wall center and then decreases back to 0 as ρ goes further to the other end. This nonmonotonic behavior comes from the consideration that polar angles are defined in spherical coordinate system thus can not exceed π . However, we would like to point out that: a 360DW defined like this must be a topologically trivial one. In PMA (IPMA) systems, this corresponds to a “ $\uparrow \rightarrow \downarrow \rightarrow \uparrow$ ” (“ $\rightarrow \uparrow \leftarrow \uparrow$ ”) type wall which first rotates half a circle and then returns back, thus leading to $\mathcal{W}_{1D} = 0$.

One possible remedy is to add a fixed value π to the azimuthal angle when the polar angle crosses the South Pole. In principle this new set of polar and azimuthal angles is indeed the real spherical coordinates that realizes a topologically nontrivial “ $\uparrow \rightarrow \downarrow \leftarrow \uparrow$ ” (“ $\rightarrow \uparrow \leftarrow \downarrow$ ”) type wall for PMA (IPMA) systems, however it will artificially bring a discontinuity in exchange energy. For the convenience of comparisons below, we denote them as ϑ_{real} and φ_{real} . To remove the artificial discontinuity, we propose a monotonically increasing “0 to 2π ” polar angle profile meanwhile keep the azimuthal angle a constant value which are defined as $\vartheta_{\text{ansatz}}$ and φ_{ansatz} . We focus on the “ π to 2π ” part since this is the main region where differences occur. Obviously, we have

$$\vartheta_{\text{real}} = 2\pi - \vartheta_{\text{ansatz}}, \quad \varphi_{\text{real}} = \varphi_{\text{ansatz}} + \pi, \quad (12)$$

and they lead to the same magnetization components as follows

$$\begin{array}{ll} \text{real spherical} & \text{ansatz} \\ m_x : & \sin\vartheta_{\text{real}} \cos\varphi_{\text{real}} \equiv \sin\vartheta_{\text{ansatz}} \cos\varphi_{\text{ansatz}} \\ m_y : & \sin\vartheta_{\text{real}} \sin\varphi_{\text{real}} \equiv \sin\vartheta_{\text{ansatz}} \sin\varphi_{\text{ansatz}} \\ m_z : & \cos\vartheta_{\text{real}} \equiv \cos\vartheta_{\text{ansatz}} \end{array} \quad (13)$$

In addition, the polar and azimuthal profiles in our proposal are not bothered by discontinuities in continuous Heisenberg exchange interaction, meanwhile provide $\mathcal{W}_{1D} = +1$. Based on these facts, we conclude that a wall profile with a monotonically increasing “0 to 2π ” polar angle and a constant azimuthal angle should be an adequate ansatz for 360DWs.

In this work, we use three trial profiles of 360DWs to explore their chirality preference and current-driven dynamics. The first one is inspired by the work of Muratov in 2008 [35], but has been generalized to $[0, 2\pi)$ as we proposed above. By introducing the “traveling coordinate” $\xi \equiv \frac{\rho - q(t)}{\Delta(t)}$ where $q(t)$ and $\Delta(t)$ are respectively the center position and width of the 360DW, it can be written as

$$\vartheta = 2\cot^{-1} \left[\sqrt{\frac{h}{1+h}} \sinh \left(-\sqrt{1+h} \xi \right) \right], \quad \phi(\mathbf{r}, t) = \varphi(t), \quad (14)$$

in which $h \equiv \frac{H_z}{k_E M_s}$ and the “cot⁻¹” function takes the range of 0 to π . Note that Eq. (14) is accurate in the absence of driving current. When in-plane currents are applied, this solution becomes an approximation since it may not hold everywhere but it does grasp the main features of dynamical 360DWs. In particular, Eq. (14) clearly ascertains the conclusion that in the absence of external magnetic fields along the easy axis (i.e. $h = 0$), ϑ keeps a constant value thus 360DWs disappear. Also, we have two other options. The second trial profile is directly generalized from the Walker ansatz, which reads

$$\vartheta = 4\tan^{-1} e^\xi, \quad \phi(\mathbf{r}, t) = \varphi(t), \quad (15)$$

and the third one is

$$\vartheta = \frac{2\pi}{1 + e^{-\xi}}, \quad \phi(\mathbf{r}, t) = \varphi(t). \quad (16)$$

Obviously, the latter two do not depend on h , thus can not be rigorous even in the absence of driving currents. However, due to their mathematical simplicity, they can be used as references. In particular when $h = 1$ ($h = \pi^2/16$), $d\vartheta/d\xi$ at $\xi = 0$ in Eq. (14) coincides with that of Eq. (15) [Eq. (16)]. The corresponding ϑ profiles are plotted in Fig. 2. Also, curves with $h = 0.1$ and $h = 5$ have been appended to illustrate the dependence of polar angle profile in Eq. (14) on h : as h increases the effective width [not the parameter $\Delta(t)$] of 360DW is compressed. In addition, for a compact 360DW (under large h), the only inflection point locates at the wall center. While for small enough h (for example, “ $h = 0.1$ ” case) where the 360DW begins to decompose into two 180DWs, the number of inflection points increases to three, which are: the first 180DW center, the boundary point of two 180DWs and the second 180DW center. This transition tells us that the weaker the external field, the more obvious the tendency of a 360DW to split into two 180DWs.

3.2. Dynamical equations

In all three trial profiles, the wall center position $q(t)$, tilting angle $\varphi(t)$ and wall width $\Delta(t)$ are the three collective coordinates. In Eq. (8), by letting X take $q(t)$, $\varphi(t)$, $\Delta(t)$ successively, and integrating over the long axis of MHs (i.e. $\int_{-\infty}^{+\infty} d\rho$), a set of dynamical equations can be obtained and expressed in a unified form for both PMA and IPMA systems:

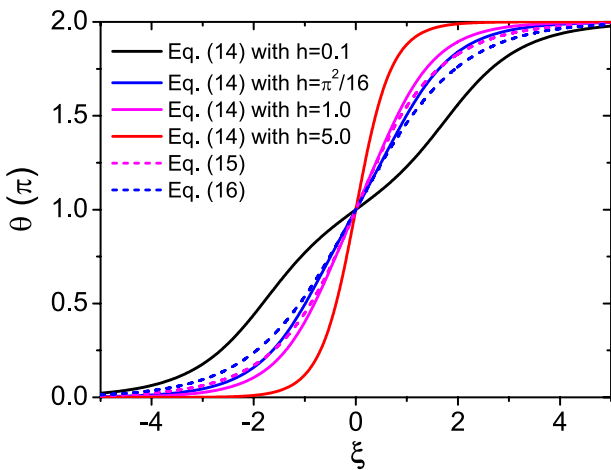


Fig. 2. Trial polar angle profiles in Eq. (14) - Eq. (16). Four solid curves are those from Eq. (14) with different h , while the magenta (blue) dashed curve shows Eq. (15) [Eq. (16)].

$$0 = (\alpha\dot{q} + \beta B_J) - \frac{2\pi}{I_1} \gamma H_{\text{ADL}} \Delta f(\varphi), \quad (17a)$$

$$0 = \frac{\alpha}{\gamma M_s} \dot{\varphi} + k_H \sin \varphi \cos \varphi + \frac{I_2}{I_3} \frac{1}{\gamma M_s} \frac{\dot{\Delta}}{\Delta} + \frac{2\pi}{I_3} \frac{D_1}{\mu_0 M_s^2 \Delta} \frac{df(\varphi)}{d\varphi}, \quad (17b)$$

$$\frac{\alpha I_4}{\gamma M_s} \frac{\dot{\Delta}}{\Delta} = \frac{I_2}{\gamma M_s} \dot{\varphi} - (k_E + k_H \sin^2 \varphi) I_5 - k_E I_2 h + \lambda \frac{l_0^2}{\Delta^2}, \quad (17c)$$

where an overdot means d/dt and $l_0 = \sqrt{2A/(\mu_0 M_s^2)}$. For PMA systems $f(\varphi) = \cos \varphi$ while for IPMA systems $f(\varphi) = -\sin \varphi$. The five integrals (I_1 to I_5) are defined in a general way without depending on the specific form of trial profiles (see the first column of Table 1). Their values and the parameter λ under each profile have been listed in the last three columns of Table 1. We also plot them in Fig. 3 as functions of h to show their evolution as h increases.

Eqs. ((17a-c)) are the starting point for our investigations on chirality and current-driven dynamics of 360DWs in Q1D MHs. Before explicitly solving it, we would like to discuss its qualitative properties first. In the dynamical equations for 180DWs, the iDMI, FL-SOT and ADL-SOT are all present. However in Eq. (17a-c), the FL-SOT disappears. This can be understood based on the mathematical form of SOTs in Eq. (11). The main difference lies in the fact that the FL-term is linear to the magnetization $\mathbf{m}$ while the ADL-term is quadratic (thus is nonlinear). When integrating over the whole strip, the constant “ $-\gamma H_{\text{FL}} \mathbf{m}_p$ ” factor can be brought up, leaving “ $\mathbf{m}$ ” to be integrated over a full circle thus canceled out. However, this procedure fails for the ADL-term since the constant “ $-\gamma H_{\text{ADL}} \mathbf{m}_p$ ” factor can not be brought up there. This explains the presence (absence) of H_{ADL} (H_{FL}) in Eq. (17a-c). Similar analysis can be made to explain the presence of both H_{ADL} and H_{FL} in 180DW case. Furthermore, a general rule can be summarized as follows: *When dealing with current-driven dynamical equations of magnetic domain walls, only for “ 2π ” walls H_{FL} disappears; otherwise H_{ADL} and H_{FL} coexist.*

3.3. iDMI-induced chirality for static 360DWs

By choosing the easy-axis-oriented single-domain state as reference, then integrating over the entire Q1D MH, the “renormalized magnetic energy” E_0^{re} of the central FM layer reads

$$\frac{E_0^{\text{re}}}{\mu_0 M_s^2 S} = \frac{I_1}{2} \frac{l_0^2}{\Delta} + \left[\frac{I_3}{2} (k_E + k_H \sin^2 \varphi) + k_E I_2 h \right] \Delta + \frac{2\pi D_1 f(\varphi)}{\mu_0 M_s^2}, \quad (18)$$

where S is the cross section of the central FM layer. Combining with Eq. (17a-c) for $\dot{q} = 0$ (thus $H_{\text{ADL}} = 0$ and $B_J = 0$), the chirality preference of static 360DWs can be analyzed.

3.3.1. iDMI is absent

First we review the simplest case where the iDMI is absent ($D_1 = 0$). Physically this corresponds to MHs with normal substrates. Then the dynamical equations, as well as the renormalized magnetic energy for PMA and IPMA systems are the same. Since $H_{\text{ADL}} = 0$ and $B_J = 0$, Eq. (17a) provides $\dot{q} = 0$ meaning that the 360DW keeps static. A static wall also requires that $\dot{\varphi} = 0$ and $\dot{\Delta} = 0$. Putting them into Eq. (17b), one has $\sin 2\varphi = 0$ which means $\varphi = \frac{n\pi}{2}$. However, Eq. (18) clearly tells us that only $\varphi = n\pi$ (i.e. $\sin \varphi = 0$) minimizes E_0^{re} . Therefore, in the absence of iDMI, 360DWs should be Néel type, but have no chirality preference. At last, Eq. (17c) provides the static wall width Δ_0 as

$$\Delta_0 = \frac{l_0}{\sqrt{k_E}} \sqrt{\frac{\lambda}{I_2 h + I_5}}. \quad (19)$$

Note that Δ_0 should not be obtained from the direct minimization of the first two terms in Eq. (18) since the result may not satisfy the dynamical

Table 1

Summary of parameters in Eq. (17a-c): definitions and values based on the three trial profiles in Eqs. (14)–(16). First to fifth rows: Integrals I_1 to I_5 . Last row: Parameter λ .

Parameter:	Definition	Value on Eq. (14)	Value on Eq. (15)	Value on Eq. (16)
I_1 :	$\Delta \int_0^{2\pi} \frac{\partial \theta}{\partial \rho} d\theta$	$4\sqrt{1+h} + 2h \ln \frac{\sqrt{1+h}+1}{\sqrt{1+h}-1}$	8	$\frac{2}{3}\pi^2$
I_2 :	$\int_0^{2\pi} \sin \theta (-\xi) d\theta$	$2 \ln \frac{\sqrt{1+h}+1}{\sqrt{1+h}-1}$	4	$2 \int_0^{2\pi} \frac{1 - \cos t}{t} dt \approx 4.8753$
I_3 :	$\frac{1}{\Delta} \int_0^{2\pi} \frac{\sin^2 \theta}{\partial \theta / \partial \rho} d\theta$	$4\sqrt{1+h} - 2h \ln \frac{\sqrt{1+h}+1}{\sqrt{1+h}-1}$	$\frac{8}{3}$	$\int_0^{4\pi} \frac{1 - \cos t}{t} dt \approx 3.1144$
I_4 :	$\Delta \int_0^{2\pi} \frac{\partial \theta}{\partial \rho} \xi^2 d\theta$	$\frac{8h}{(1+h)^{3/2}} \int_0^{+\infty} \frac{\sqrt{1+x^2} (\sinh^{-1} x)^2}{\left(1 + \frac{h}{1+h} x^2\right)^2} dx$	$\frac{2}{3}\pi^2$	$(2\pi)^2 \int_0^{+\infty} \frac{x(\ln x)^2}{(1+x)^4} dx \approx 8.4870$
I_5 :	$\int_0^{2\pi} \sin \theta \cos \theta (-\xi) d\theta$		$\frac{I_3}{2}$	
λ :		$I_1 - hI_2 - I_5$	$I_1 - I_2$	$\frac{I_1}{2}$

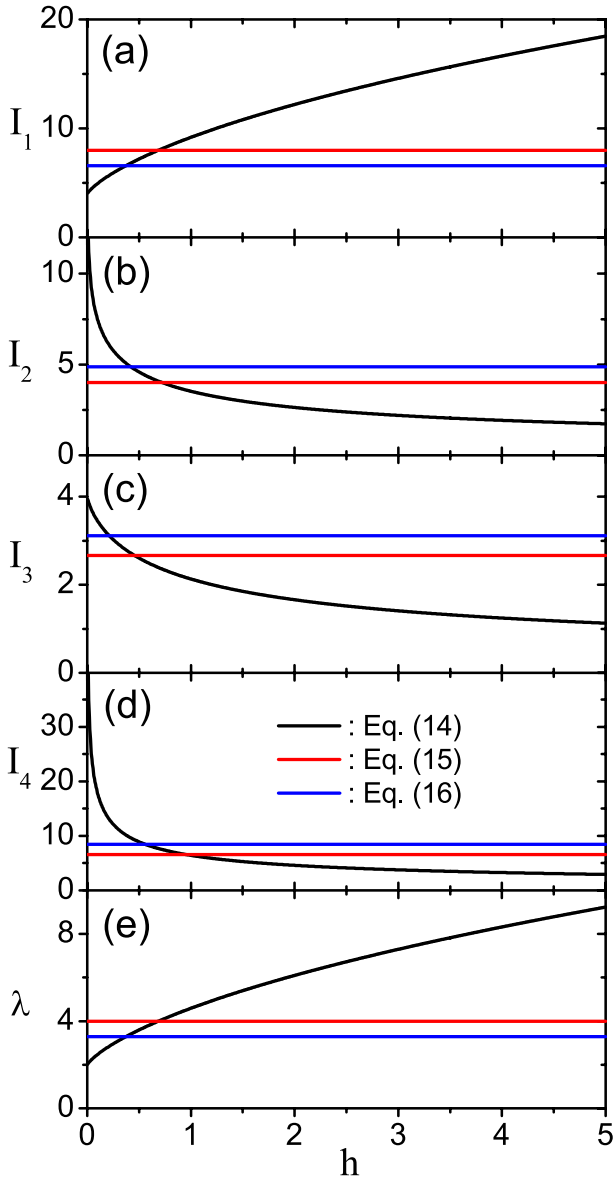


Fig. 3. Evolution of $I_{1,2,3,4}$ and λ as h increases for Eq. (14) - Eq. (16). Note that I_5 is omitted since it is always half of I_3 .

equations. This argument also holds when iDMI appears.

3.3.2. PMA systems with iDMI

For PMA systems, $f(\varphi) = \cos \varphi$. The combination of Eq. (17b) and the static requirement ($\dot{\varphi} = 0$ and $\dot{\Delta} = 0$) leads to

$$\begin{cases} \text{case(a)} : \sin \varphi = 0 \text{ or} \\ \text{case(b)} : \cos \varphi = \frac{2\pi D_i}{k_H I_3 \mu_0 M_s^2 \Delta} \end{cases} \quad (20)$$

To determine which solution provides the real tilting angle, we must compare the corresponding “renormalized magnetic energy” in Eq. (18). For case (a), $\sin \varphi = 0 \Leftrightarrow \varphi = n\pi$. However, the existence of iDMI [the last term in Eq. (18)] breaks the twofold degeneracy of E_0^c upon azimuthal angle. To minimize E_0^c , one must have

$$\cos \varphi = -\text{sgn}(D_i), \quad (21)$$

where “sgn” denotes the sign function. Correspondingly in this case the renormalized magnetic energy becomes

$$\frac{(E_0^c)_a}{\mu_0 M_s^2 S} = \frac{I_1}{2} \frac{I_0^2}{\Delta} + k_E \left(\frac{I_3}{2} + I_2 h \right) \Delta - \frac{2\pi |D_i|}{\mu_0 M_s^2}. \quad (22)$$

For case (b), direct calculation yields

$$\frac{(E_0^c)_b}{\mu_0 M_s^2 S} = \frac{I_1}{2} \frac{I_0^2}{\Delta} + k_E \left(\frac{I_3}{2} + I_2 h \right) \Delta + \left[\frac{(2\pi D_i)^2}{2k_H I_3 (\mu_0 M_s^2)^2 \Delta} + \frac{I_3 k_H \Delta}{2} \right]. \quad (23)$$

Obviously for any positive Δ , we always have $(E_0^c)_a < (E_0^c)_b$. Therefore for PMA systems, Eq. (21) provides the real azimuthal angle of 360DWs, which presents definite chirality uniquely determined by iDMI. This can be understood more intuitively from the perspective of effective fields. For PMA systems, the iDMI energy density in Eq. (3) leads to the following effective field

$$\mathbf{H}_{\text{iDMI}} = -\frac{1}{\mu_0} \frac{\delta \mathcal{E}_{\text{iDMI}}}{\delta \mathbf{M}} = -\frac{2D_i}{\mu_0 M_s} \left[\left(\frac{\partial m_x}{\partial x} \right) \mathbf{e}_z - \left(\frac{\partial m_z}{\partial x} \right) \mathbf{e}_x \right]. \quad (24)$$

Clearly the x -component leads to the chirality of 360DWs.

At last, by putting Eq. (21) into Eq. (17c), the static wall width is found to be the same as that in Eq. (19). For the first trial profile [see Eq. (14)], I_2, I_5 and λ are all functions of h . One can easily check that $\sqrt{\lambda/(I_2 h + I_5)} = 1$, which means that Δ_0 is independent on h . This is reasonable since in this profile Δ and h are independent variables. While for the other two profiles, I_2, I_5 and λ are constants. Then the wall width will be compressed when an external field in the easy axis appears, which is also reasonable since h is absent in these two.

3.3.3. IPMA systems with iDMI

For IPMA systems, parallel discussions can be performed. For brevity, we only list the main results here. Since $f(\varphi) = -\sin\varphi$, the static condition then provides

$$\begin{cases} \text{case (a')} : & \cos\varphi = 0 \quad \text{or} \\ \text{case (b')} : & \sin\varphi = \frac{2\pi D_i}{k_H I_3 \mu_0 M_s^2 \Delta} \end{cases} \quad (25)$$

For case (a'), the iDMI-induced chirality selects $\sin\varphi = \text{sgn}(D_i)$. However after simple calculation, it is easy to find that the renormalized magnetic energy in case (b') is lower than that in case (a'). Therefore the correct static azimuthal angle for IPMA systems should be the one in case (b'). Also, the wall acquires definite chirality determined by the iDMI.

Again, putting the result in case (b') back into Eq. (17c), the static wall width for IPMA systems is

$$\Delta'_0 = \frac{l_0}{\sqrt{k_E}} \sqrt{\frac{\lambda}{I_2 h + I_5}} \sqrt{1 - \frac{k_H I_5}{\lambda} \left(\frac{2\pi D_i}{k_H I_3 \mu_0 M_s^2 l_0} \right)^2}. \quad (26)$$

Compared with that in PMA case [see Eq. (19)], a quadratic correction term of D_i appears. For the second and third trial profiles, it does not change the dependence trend of wall width on h . However, for the first profile a problem emerges since now Δ depends on h . This means that in IPMA systems, Eq. (14) is not as good as it is in PMA systems. The reason lies in the fact that in IPMA systems the hard axis is along y-axis (rather than x-axis) since it is the thinnest direction of the strip thus has the largest demagnetization factor. Despite this, Eq. (14) does grasp the main features of 360DWs in IPMA systems and should be a good ansatz to explore their statics and dynamics.

3.4. Current-driven 360DW dynamics

When in-plane currents are applied, 360DWs will be driven to propagate along the long axis of MHs. Generally, in Eq. (17a) the presence of “ H_{ADL} ” term will change the wall's mobility from the pure STT-driven result by means of $f(\varphi)$. To acquire the time-evolution of φ , Eqs. (17ab) and (17c) provide

$$\frac{d(2\varphi + \kappa)}{\Gamma - \frac{4\pi D_i}{k_H I_3 \mu_0 M_s^2 \Delta} \frac{df(\varphi)}{d\varphi} - \sin(2\varphi + \kappa)} = \chi dt, \quad (27)$$

with

$$\begin{aligned} \kappa &= \arctan \frac{I_2 I_5}{\alpha I_3 I_4}, \\ \Gamma &= \frac{2I_2}{\alpha k_H I_3 I_4} \left[\frac{I_5 k_H}{2} + k_E (I_2 h + I_5) - \lambda \frac{l_0^2}{\Delta^2} \right], \\ \chi &= k_H \gamma M_s \left[\alpha + \frac{(I_2)^2}{\alpha I_3 I_4} \right]^{-1} > 0. \end{aligned} \quad (28)$$

This is the fundamental equation when dealing with current-driven 360DW dynamics.

3.4.1. iDMI is absent

First we consider the simplest case where the iDMI is absent ($D_i = 0$), which corresponds to a 360DW residing in a MH with a normal substrate. Note that at this moment the SOT is also absent. Now Eq. (27) can be directly integrated out and the result depends on the value of Γ .

When $|\Gamma| < 1$,

$$\tan\left(\varphi + \frac{\kappa}{2}\right) = \frac{1}{\Gamma} - \frac{\sqrt{1-\Gamma^2}}{\Gamma} \frac{C_1 e^{\sqrt{1-\Gamma^2}\chi t} + 1}{C_1 e^{\sqrt{1-\Gamma^2}\chi t} - 1}, \quad (29)$$

with

$$C_1 = \frac{\Gamma \tan\left(\varphi_0 + \frac{\kappa}{2}\right) - 1 - \sqrt{1-\Gamma^2}}{\Gamma \tan\left(\varphi_0 + \frac{\kappa}{2}\right) - 1 + \sqrt{1-\Gamma^2}},$$

and $\varphi = \varphi_0$ at $t = 0$. Obviously when $t \rightarrow +\infty$ the azimuthal angle approaches the following value

$$\varphi_\infty = \arctan\left(\frac{1}{\Gamma} - \frac{\sqrt{1-\Gamma^2}}{\Gamma}\right) - \frac{\kappa}{2}. \quad (30)$$

This means that in this case the 360DW will eventually fall into the “steady-flow” mode. By letting $\dot{\varphi} = 0$ and $\dot{\Delta} = 0$, we know that the wall propagates with a constant velocity $-\beta B_J/\alpha$ and a fixed width

$$\Delta(\varphi_\infty) = \frac{l_0}{\sqrt{k_E}} \sqrt{\frac{\lambda}{I_2 h + I_5 + (k_H/k_E) I_5 \sin^2 \varphi_\infty}}. \quad (31)$$

When $|\Gamma| > 1$,

$$\varphi = \arctan\left[\frac{\sqrt{\Gamma^2-1}}{\Gamma} \tan\left(\frac{\sqrt{\Gamma^2-1}}{2} \chi t + C_2\right) + \frac{1}{\Gamma}\right] - \frac{\kappa}{2}, \quad (32)$$

with

$$C_2 = \arctan \frac{\Gamma \tan\left(\varphi_0 + \frac{\kappa}{2}\right) - 1}{\sqrt{\Gamma^2-1}}.$$

Now the azimuthal angle rotates periodically with the period

$$T_0 = \frac{4\pi}{\chi \sqrt{\Gamma^2-1}}, \quad (33)$$

which means that the 360DW takes a “precessional-flow” mode with the same constant velocity $-\beta B_J/\alpha$ and a periodically changing width.

It is worth noting that for a certain 360DW as the current density increases the wall always takes a specific mode (either steady-flow or precessional-flow) rather than going through a process of mode change, which is quite different from the commonly studies 180DWs. This is the direct consequence of the “full-circle” topology that 360DWs hold. To understand this phenomena, the most intuitive way is to check the integrated torques from external driving currents in $\mathbf{e}_\varphi$ -direction, which is perpendicular to the φ -plane. For a 180DW, due to the half-circle topology of magnetization rotation across the wall, the residual STT and SOT are both nonzero. The balance between these current-induced torques with that from internal anisotropy (“ $k_H \sin 2\varphi$ ” term) results in the “Walker limit”, which separates the steady and precessional flows of 180DWs. However, for 360DWs the “full-circle” topology leads to the complete elimination of integrated STT and SOT (half-circle positive and half-circle negative), leaving only those from internal anisotropy and iDMI in $\mathbf{e}_\varphi$ -direction. Therefore the “Walker breakdown” phenomena is absent for 360DWs, and only the internal magnetic parameters determine which mode the 360DW will fall into. This is similar to the elimination of skyrmion Hall effect [64,65] from current-induced Magnus force in antiferromagnetic (AFM) skyrmions [66,67], which are composed of two FM skyrmions and coupled by strong AFM exchange. However, for our 360DWs in narrow MHs the inter-180DW coupling is much weaker, thus leads to the instability of 360DWs under not-too-high current densities. At last, interestingly both modes share the same wall mobility which is equal to that in steady-flow mode of 180DWs. This explains nearly all existing numerical observations before 360DWs change to other magnetic solitons (for example vortices) under relatively high currents [55,60,61].

Furthermore, we provide the sufficient but non-necessary condition for the steady flow of 360DWs. Under the presupposition $|\Gamma| < 1$, by putting the wall width [see Eq. (31)] back into the definition of Γ [see Eq. (28)], we obtain

$$\Gamma = \frac{I_2 I_5}{\alpha I_3 I_4} \cos 2\varphi_\infty. \quad (34)$$

Thus the sufficient but non-necessary condition for $|\Gamma| < 1$ should be $I_2 I_5 / (\alpha I_3 I_4) < 1$, which corresponds to $\alpha > \alpha_c \equiv I_2 I_5 / (I_3 I_4)$. For the second and third trial profiles, one has $\alpha_c = 3/\pi^2 \approx 0.304$ and $\alpha_c = 0.287$, respectively. While for the first profile, α_c is the function of h . We have plotted their dependence on h in Fig. 4. One can clear see that for all three cases α_c has an upper limit $3/\pi^2$ even when h increases to 5 which is a quite high value in real experiments. In many MHs, existing measurements show that the effective damping in FM strips is enhanced from 0.001–0.01 to 0.3–0.9 [68,69]. This guarantees that experimentally steady-flows of 360DWs should be quite common. As for precessional flow, since the explicit form of wall width is hard to obtain, thus it is difficult to obtain the definite range of its existence. However, from the above discussion we can reasonably infer that for sufficient small α , 360DWs precess. This prediction needs to be verified by future experiments and numerical simulations.

At last we would like to qualitatively explain why large enough damping leads to steady-flow mode of 360DWs. The DW motion process in narrow MHs can be simplified as follows: external stimuli (by means of various torques) first rotate the wall's φ -plane a little bit, then damping process reorientates the magnetization distribution within the φ -plane (thus dissipates extra magnetic energy, leading to DW motion along strip axis) and meantime tends to bring the φ -plane back, and so on so forth. Suppose at some moment the wall's φ -plane is at an instantaneous and local energy minima. Although it is pushed away a little bit by external torques, large enough damping can quickly pull the φ -plane back to the original potential valley thus hold the DW to “swing” a little around the equilibrium tilting plane, which is indeed the steady flow mode. If the damping is too weak, the φ -plane is not yet able to be pulled back to its original potential valley by damping and continues to rotate further apart, therefore results in the precessional flow. In fact, this rough but vivid argument also applies to 180DWs where “Walker breakdown” can happen: for a given damping strength, if the external driving fields or currents are low enough, their rotation tendency to the φ -plane can be eliminated easily by damping. However, when they are stronger than a certain critical value (Walker limit), damping cannot match thus precession begins.

3.4.2. iDMI and ADL-SOT are present

Next we study the effects of iDMI and ADL-SOT on the current-driven dynamics of 360DWs in Q1D MHs with HM substrates. In the presence of iDMI, in principle the azimuthal angle φ can not be integrated out explicitly from Eq. (27). Recently a phase diagram has been drawn to show how the types of solutions are determined by the DMI and the anisotropic parameters [70]. However in real MHs, generally the iDMI is

weaker than other magnetic interactions, thus can be reasonably viewed as a small quantity. Depending on the value of Γ , different approximate treatments will be used.

When $|\Gamma| < 1$ or under the stronger condition $\alpha > \alpha_c$, at the lowest level of approximation the 360DW should eventually propagate like a rigid body with the final azimuthal angle φ_∞ , width $\Delta(\varphi_\infty)$ and velocity

$$\dot{q} = -\frac{\beta}{\alpha} B_J + \frac{2\pi}{\alpha I_1} \gamma H_{\text{ADL}} \Delta(\varphi_\infty) f(\varphi_\infty). \quad (35)$$

Obviously, the wall mobility is modified by the second term. However the effect of iDMI is totally submerged since it has been dropped when obtaining φ_∞ . Note that the form of $\Delta(\varphi_\infty)$ in Eq. (31) is not affected by this dropping.

When $|\Gamma| > 1$, the 360DW precesses. In this case for a physical quantity O , its time average

$$\langle O \rangle \equiv \frac{1}{T} \int_0^T X(t) dt = \frac{1}{T} \int_0^{2\pi} \frac{X}{\dot{\varphi}} d\varphi \quad (36)$$

corresponds to experimental observables, where T is the precession period. Under the assumption of small iDMI, we calculate the time-averaged wall velocity $\langle \dot{q} \rangle$. First the period T is replaced by T_0 in Eq. (33). Then the approximation $(1-x)^{-1} \approx 1+x+x^2$ for $|x| < 1$ is used to simplify $1/\dot{\varphi}$ in Eq. (27) hence the integral in Eq. (36) can be calculated. After standard algebra, we have

$$\langle \dot{q} \rangle = -\frac{\beta}{\alpha} B_J - \eta \frac{\sqrt{\Gamma^2 - 1}}{\Gamma^3} \gamma H_{\text{ADL}} \frac{D_i}{\mu_0 M_s^2} \frac{4\pi^2 \cos \kappa}{\alpha k_H I_1 I_3}, \quad (37)$$

where $\eta = +1$ (-1) for PMA (IPMA) systems. Clearly, Eq. (37) provides the effects of both ADL-SOT and iDMI on the wall velocity in precessional flows.

4. Discussions

First, one should note that the premise of all our analytical results is the existence of 360DWs. The constant mobility (whether adjusted by iDMI and SOT or not) upon current increase is the direct manifestation of the wall's “full-circle” topology. On the other hand, 360DWs have lower stability than 180DWs since they are bonded from two 180DWs by relatively weak magnetostatics or Zeeman interaction. Under not-too-high currents, the FL-SOT, which lies out of the φ -plane of a 360DW and points oppositely on the two constituent 180DWs, should twist the wall's φ -plane and leads to instability of the entire 360DW. In addition, the even stronger external stimuli would destroy thoroughly the 360DW profiles, thereby greatly change the mobility of domain walls (not 360DWs any more). This explains the huge reduction of 360DW mobility under relatively high currents in existing numerics [55,60,61]. Another interesting departure from ideal 360DW profile happens in heterostructures composed of numerous repeats of “heavy-metal (H)/magnetic (M)/symmetry breaking (S)” structure [71]. 360DWs therein would show different behaviors with what we have proposed here in a single H/M/S repeat (for example, the reappearance of Walker breakdown, etc.). The main reason is the “magnetostatically induced chirality twists along the structure normal”, as well as the effective field component of iDMI along the same direction.

Second, our analytics presented here is based on “0 to 2π ” monotonic profiles of polar angle. If ϑ is no longer monotonic but its overall change across the wall region keeps 2π ($\mathcal{W}_{1D} = +1$ still holds), then the results will be unchanged. In addition, for a 360DW with $\mathcal{W}_{1D} = -1$ mathematically its profile can be transfer to that with $\mathcal{W}_{1D} = +1$, except for an increase by π in the azimuthal angle. The following procedure is similar to what we have presented in the main text and will not provide new physics, so we won't repeat it. On the other hand, we should admit that our LB-CCM has lost some details about magnetization dynamics, especially some internal “soft” modes that the three collective

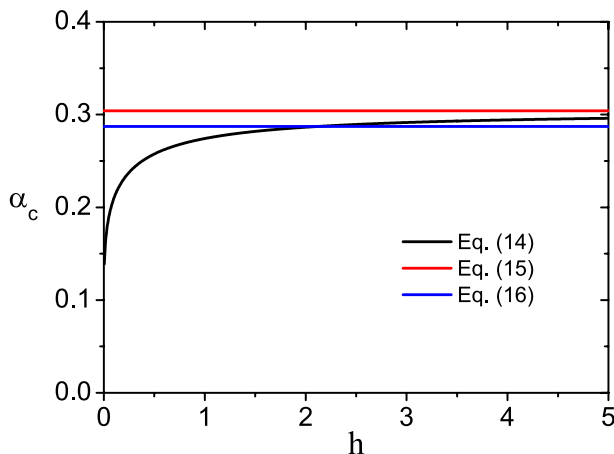


Fig. 4. Dependence of α_c on h based on Eq. (14) - Eq. (16).

coordinates (q, φ, Δ) can not describe. The most common remedy is to introduce more collective coordinates. For 360DWs, the relative tilting angle of the two constituent 180DWs ($\delta\varphi = \varphi_{\text{left}} - \varphi_{\text{right}}$) is an interesting extra collective coordinate. It should be of importance since the coupling between the two constituent 180DWs is the relatively weak magnetostatic interaction thus under not-too-high current densities φ_{left} and φ_{right} should be driven by FL-SOT to differ a little bit either permanently or cyclically. The former should induce a distortion from our simple ansatz and the latter would generate low-energy excitation of spin waves. However, the direct inclusion of a constant $\delta\varphi$ (i.e. step-like φ -distribution across 360DWs) would induce a discontinuity at the 360DW center. Alternatively, one may propose a continuous twisted φ -distribution, but this would greatly complicate the analytics thus beyond the scope of this work.

At last, 1D 360DWs in narrow MHs under investigation here look similar to the 1D DW skyrmions which are internal topological structure of DWs in wide MHs with PMA [72]. Both magnetic solitons carry integer 1D topological charges ($\mathcal{W}_{1D} = \pm 1$) and definite chirality determined by iDMI, however they are different in the following aspects: (i) 1D DW skyrmions evolve from internal DW defects (i.e. vertical Bloch lines) in wide MHs with PMA while 360DWs originate from the combination of two 180DWs in not-too-wide MHs; (ii) iDMI is crucial for the formation and stabilization of 1D DW skyrmions, however stable 1D 360DWs can exist without iDMI but need an external magnetic field along easy axis; (iii) the corresponding analytical profiles of these two are then different: the former depends on the iDMI parameter, while the latter relies on the magnetic field strength; (iv) 1D DW skyrmions (360DWs) move along the transverse (longitudinal) direction of wide (narrow) MHs with PMA, thus the current-driven results here may provide insights for exploring dynamical behaviors of 1D DW skyrmions under external stimuli. On the other hand, 1D 360DWs are also closely related to higher dimensional magnetic topological solitons. The ansatz for successful investigations on radius distributions for 2D Néel or Bloch skyrmions [73] as well as 3D hopfions [74] are all variations of our Eq. (14). This provides us further confidence that our ansatz is reasonable, meantime the resulting statics and dynamics are solid.

5. Conclusion

In this work, the topology, chirality and current-driven dynamics of 360DWs in Q1D MHs are systematically investigated. On one hand, the iDMI uniquely select the chirality of static 360DWs. On the other hand, the “full-circle” topology of 360DWs makes them completely different from the traditional 180DWs. For 360DWs, effective fields which are linear to the magnetization have been fully canceled out and disappear in the dynamical equations. In particular, the full cancellation of torques from external currents in $\mathbf{e}_\varphi$ -direction directly results in the absence of “Walker breakdown”-type process under increasing currents. In a certain MH, 360DWs will take either steady-flow or precessional-flow mode, depending on the strength of effective Gilbert damping constant therein. In MHs with normal substrates, the wall mobility of both modes are the same as that in the steady-flow mode of STT-driven propagation of 180DWs. While in MHs with HM substrates, the mobility will be modified by the ADL-SOT and iDMI. These results should deepen our understanding of topological solitons in low-dimensional magnetic systems, meanwhile provide necessary theoretical basis for expanding the application of 360DWs in the field of magnetic nanodevices.

CRediT authorship contribution statement

Mei Li: Methodology, Investigation, Writing - original draft, Funding acquisition. **Jie Lu:** Conceptualization, Methodology, Investigation, Validation, Writing - review & editing, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

- [1] F.H. De Leeuw, R. Van Den Doel, U. Enz, Rep. Prog. Phys. 43 (1980) 689.
- [2] Y. Tserkovnyak, A. Brataas, G.E.W. Bauer, B.I. Halperin, Rev. Mod. Phys. 77 (2005) 1375.
- [3] M. Kläui, J. Phys.: Condens. Matter 20 (2008), 313001.
- [4] S. Mühlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, P. Böni, Science 323 (2009) 915.
- [5] X.Z. Yu, Y. Onose, N. Kanazawa, J.H. Park, J.H. Han, Y. Matsui, N. Nagaosa, Y. Tokura, Nature 465 (2010) 901.
- [6] W. Jiang, G. Chen, K. Liu, J. Zang, S.G. Te Velthuis, A. Hoffmann, Phys. Rep. 704 (2017) 1.
- [7] J.S. Chen, L.J. Wang, M. Zhang, L. Zhou, R.N. Zhang, L.P. Jin, X.S. Wang, H.L. Qin, Y. Qiu, J.W. Mei, et al., Nano Lett. 19 (2019) 6144.
- [8] X. Zhang, Y. Zhou, K.M. Song, T.-E. Park, J. Xia, M. Ezawa, X. Liu, W. Zhao, G. Zhao, S. Woo, J. Phys. Condens. Matter 32 (2020), 143001.
- [9] M. Ezawa, Phys. Rev. B 83 (2011), 100408.
- [10] S.-Z. Lin, A. Saxena, C.D. Batista, Phys. Rev. B 91 (2015), 224407.
- [11] B. Göbel, A. Mook, J. Henk, I. Mertig, O.A. Tretiakov, Phys. Rev. B 99 (2019) 060407(R).
- [12] L. Shen, J. Xia, X. Zhang, M. Ezawa, O.A. Tretiakov, X. Liu, G. Zhao, Y. Zhou, Phys. Rev. Lett. 124 (2020), 037202.
- [13] X. Zhang, J. Xia, L. Shen, M. Ezawa, O.A. Tretiakov, G. Zhao, X. Liu, Y. Zhou, Phys. Rev. B 101 (2020), 144435.
- [14] J.C. Slonczewski, A.I.P. Conf. Proc. 5 (1972) 170.
- [15] T. Ono, H. Miyajima, K. Shiget, K. Mibu, N. Hosoi, T. Shinjo, Science 284 (1999) 468.
- [16] X.R. Wang, P. Yan, J. Lu, EuroPhys. Lett. 86 (2009) 67001.
- [17] Z.Z. Sun, J. Schliemann, Phys. Rev. Lett. 104 (2010), 037206.
- [18] L. Berger, Phys. Rev. B 54 (1996) 9353.
- [19] J. Slonczewski, J. Magn. Magn. Mater. 159 (1996) L1.
- [20] Z. Li, S. Zhang, Phys. Rev. Lett. 92 (2004), 207203.
- [21] P. Yan, X.S. Wang, X.R. Wang, Phys. Rev. Lett. 107 (2011), 177207.
- [22] X.S. Wang, P. Yan, Y.H. Shen, G.E.W. Bauer, X.R. Wang, Rev. Lett. 109 (2012), 167209.
- [23] F. Schlickeiser, U. Ritzmann, D. Hinzke, U. Nowak, Phys. Rev. Lett. 113 (2014), 097201.
- [24] X.S. Wang, X.R. Wang, Phys. Rev. B 90 (2014), 014414.
- [25] J. Lu, Phys. Rev. B 93 (2016), 224406.
- [26] M. Li, J.B. Wang, J. Lu, Sci. Rep. 7 (2017) 43065.
- [27] M. Yu, M. Li, J. Lu, Nanomaterials 9 (2019) 128.
- [28] M. Li, Z. An, J. Lu, Phys. Rev. B 100 (2019), 064406.
- [29] J. Lu, M. Li, X.R. Wang, Phys. Rev. B 101 (2020), 134431.
- [30] D.O. Smith, K.J. Harte, J. Appl. Phys. 33 (1962) 1399.
- [31] M.S. Cohen, J. Appl. Phys. 34 (1963) 1221.
- [32] R.H. Wade, Philosophical Magazine 103 (1964) 49.
- [33] L.J. Heyderman, H. Niedoba, H.O. Gupta, I.B. Puchalska, J. Magn. Magn. Mater. 96 (1991) 125.
- [34] M.O. Liedke, K. Potzger, A.H. Bothmer, J. Fassbender, B. Hillebrands, M. Rickart, P. P. Freitas, J. Appl. Phys. 100 (2006), 043918.
- [35] C.B. Muratov, V.V. Osipov, J. Appl. Phys. 104 (2008), 053908.
- [36] J. Dean, A. Kohn, A. Kovács, A. Zeltser, M.J. Carey, G. Hrkac, D.A. Allwood, T. Schref, J. Appl. Phys. 110 (2011), 073901.
- [37] K.J. O'Shea, K. Rode, H. Kurt, D. McGrouther, D.A. MacLaren, J. Phys. D: Appl. Phys. 48 (2015), 055001.
- [38] N. Chowdhury, W. Kleemann, O. Petravic, F. Kronast, A. Doran, A. Scholl, S. Cardoso, P. Freitas, S. Bedanta, Phys. Rev. B 98 (2018), 134440.
- [39] M.D. Mascaro, H.S. Körner, C. Nam, B.G. Ng, C.A. Ross, Appl. Phys. Lett. 98 (2011), 252506.
- [40] N.R. Pradhan, A.S. Licht, Y. Li, Y. Sun, M.T. Tuominen, K.E. Aidala, Nanotechnology 22 (2011), 485705.
- [41] A. Goldman, A.S. Licht, Y. Sun, Y. Li, N.R. Pradhan, T. Yang, M.T. Tuominen, K. E. Aidala, J. Appl. Phys. 111 (2012) 07D113.

- [42] A.L. Gonzalez Oyarce, T. Trypiniotis, P.E. Roy, C.H.W. Barnes, *Phys. Rev. B* 87 (2013), 174408.
- [43] J.E. Bickel, S.A. Smith, K.E. Aidala, *J. Appl. Phys.* 115 (2014) 17D135.
- [44] C.B. Muratov, V.V. Osipov, E. Vanden-Eijnden, *J. Appl. Phys.* 117 (2015) 17D118.
- [45] Y. Jang, S.R. Bowden, M. Mascaro, J. Unguris, C.A. Ross, *Appl. Phys. Lett.* 100 (2012), 062407.
- [46] A.L. Gonzalez Oyarce, J. Llandro, C.H.W. Barnes, *Appl. Phys. Lett.* 103 (2013), 222404.
- [47] J. Zhang, S.A. Siddiqui, P. Ho, J.A. Currivan-Incorvia, L. Tryputen, E. Lage, D. C. Bono, M.A. Baldo, C.A. Ross, *New J. Phys.* 18 (2016), 053028.
- [48] A. Kubetzka, O. Pietzsch, M. Bode, R. Wiesendanger, *Phys. Rev. B* 67 (2003) 020401(R).
- [49] F.I. Kaya, A. Sarella, D. Wang, M. Tuominen, K.E. Aidala, *AIP Adv.* 6 (2016), 055025.
- [50] F.I. Kaya, A. Sarella, D. Wang, M. Tuominen, K.E. Aidala, *AIP Adv.* 6 (2016), 056408.
- [51] R.V. Kohn, V.V. Slastikov, *Proc. R. Soc. A* 461 (2005) 143.
- [52] C.B. Muratov, V.V. Osipov, *J. Comput. Phys.* 216 (2006) 637.
- [53] I. Dzyaloshinsky, *J. Phys. Chem. Solids* 4 (1958) 241.
- [54] T. Moriya, *Phys. Rev.* 120 (1960) 91.
- [55] Mark D. Mascaro, C.A. Ross, *Phys. Rev. B* 82 (2010), 214411.
- [56] Q. Zhu, C. Mu, H. Xia, W. Wang, J. Wang, Q. Liu, *Physica B* 407 (2012) 4584.
- [57] A.L. Gonzalez Oyarce, Y. Nakatani, C.H.W. Barnes, *Phys. Rev. B* 87 (2013), 214403.
- [58] J. Zhang, C.A. Ross, *Appl. Phys. Lett.* 103 (2013), 162411.
- [59] S. Zhang, Q. Zhu, C. Mu, Y. Zhang, Q. Liu, J. Wang, *J. Magn. Magn. Mater.* 347 (2013) 124.
- [60] W. Dong, Y. Su, H. Lei, J. Hu, *AIP Advances* 5 (2015), 117215.
- [61] Y. Su, L. Weng, W. Dong, B. Xi, R. Xiong, J. Hu, *Sci. Rep.* 7 (2017) 13416.
- [62] A. Bogdanov, A. Hubert, *J. Magn. Magn. Mater.* 138 (1994) 255.
- [63] H.-B. Braun, *Phys. Rev. B* 50 (1994) 16485.
- [64] N. Nagaosa, Y. Tokura, *Nat. Nanotech.* 8 (2013) 899.
- [65] W. Jiang, X. Zhang, G. Yu, W. Zhang, X. Wang, M.B. Jungfleisch, J.E. Pearson, X. Cheng, O. Heinonen, K.L. Wang, Y. Zhou, A. Hoffmann, S.G.E. te Velthuis, *Nat. Phys.* 13 (2017) 162.
- [66] X. Zhang, Y. Zhou, M. Ezawa, *Nat. Commun.* 7 (2016) 10293.
- [67] J. Barker, O.A. Tretiakov, *Phys. Rev. Lett.* 116 (2016), 147203.
- [68] T.H. Pham, J. Vogel, J. Sampaio, M. Vaňatka, J.-C. Rojas-Sánchez, M. Bonfim, D. S. Chaves, F. Choueikani, P. Ohresser, E. Otero, et al., *Europhys. Lett.* 113 (2016) 67001.
- [69] K. Shahbazi, A. Hrabec, S. Moretti, M.B. Ward, T.A. Moore, V. Jeudy, E. Martinez, C.H. Marrows, *Phys. Rev. B* 98 (2018), 214413.
- [70] Z.-D. Li, Q.-L. Bao, P.-B. He, T.-F. Xue, B. Wu, *J. Magn. Magn. Mater.* 512 (2020), 166981.
- [71] I. Lemes, G.S.D. Beach, *Phys. Rev. Appl.* 12 (2019), 044031.
- [72] R. Cheng, M. Li, A. Sapkota, A. Rai, A. Pokhrel, T. Mewes, C. Mewes, D. Xiao, M. De Graef, V. Sokalski, *Phys. Rev. B* 99 (2019), 184412.
- [73] X.S. Wang, H.Y. Yuan, X.R. Wang, *Commun. Phys.* 1 (2018) 31.
- [74] X.S. Wang, A. Qaiumzadeh, A. Brataas, *Phys. Rev. Lett.* 123 (2019), 147203.