



Pressure induced superconductor quantum critical point in multi-band systems

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ABSTRACT

In multi-band superconductors as inter-metallic systems and heavy fermions, external pressure can reduce the critical temperature and eventually destroy superconductivity driving these systems to the normal state. In many cases this transition is continuous and is associated with a superconducting quantum critical point (SQCP). In this work we study a two-band superconductor in the presence of hybridization V . This one-body mixing term is due to the overlap of the different wave-functions. It can be tuned by external pressure and turns out as an important control parameter to study the phase diagram and the nature of the phase transitions. We use a BCS approximation and include both inter- and intra-band attractive interactions. For negligible inter-band interactions, as hybridization (pressure) increases we find a SQCP separating a superconductor from a normal state at a critical value of the hybridization V_c . We obtain the behavior of the electronic specific heat close to the SQCP and the shape of the critical line as V approaches V_c .

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1. Introduction

The study of asymmetric superconductivity, i.e., of superconductivity in systems where different types of quasi-particles coexist at a common Fermi surface has raised a lot of interest in the last years. This in part is due to the relevance of this problem for many different areas in physics. It arises in cold atomic systems with superfluid phases [1], in color superconductivity in the core of neutron stars [2–4] and in condensed matter physics [5]. Furthermore it is closely related to inhomogeneous superconductivity, as FFLO phases [6], since this is a possible ground state for asymmetric systems. In condensed matter, as inter-metallic materials, there are electrons from different orbitals at the Fermi surface. If these quasi-particles have different effective masses, or occur in distinct numbers per atom there is a natural mismatch of their Fermi wave-vectors.

Then, in multi-band systems, even in the absence of external magnetic fields, one has to consider the possibility of inhomogeneous superconductivity or other types of exotic ground states as gapless superconducting phases [7,8] or phase separation [9].

In this paper we focus on the problem of driving a multi-band superconductor to the normal state by applying external pressure. When this occurs continuously, this transition is associated with a superconducting quantum critical point (SQCP). The theories which have been proposed for the SQCP rely in general on the presence of disorder or magnetic impurities [11]. However, there

is no reason to expect that this should play a role in clean systems driven to the normal state by external pressure. Here we discuss a model in which mixing transfers electrons that participate in Cooper pairing to a normal band eventually destroying superconductivity. Since there is no dissipation in the normal band, superconductivity disappears due to a loss of coherence in the system. Our model considers two hybridized bands in the presence of inter- and intra-band attractive interactions. These interactions are competing and determine the nature of the zero temperature phase transitions to the normal state as hybridization (pressure) increases. We show that only when intra-band interactions are dominant this transition is continuous. Otherwise, it is of first order and accompanied by phase separation as usual in this case [5].

The problem of superconductivity in systems with overlapping bands has been treated originally by Suhl et al. [13]. The relevance of the different interactions [14] has been discussed in terms of an energy associated with the Fermi surface mismatch, $\Delta(\delta_{k_f}) = v_F \delta_{k_f}$ and the critical temperature, $k_B T_c$. Only when the former is much smaller than the latter, inter-band interactions become important. In our approach, the mismatch δ_{k_f} depends on hybridization and can be controlled by pressure.

2. Model and formalism

We consider a model with two types of quasi-particles, a and b , with an attractive inter-band interaction [14] $\tilde{g}$, an attractive intra-band interaction $\tilde{U}$ and a hybridization term $\tilde{V}$ that mixes

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different quasi-particles states [5]. This one-body mixing term V is related to the overlap of the wave-functions and can be tuned by external parameters, like pressure, allowing to explore the phase diagram and quantum phase transitions of the model. The Hamiltonian is given by

$$H = \sum_{k\sigma} \tilde{\epsilon}_k^a a_{k\sigma}^\dagger a_{k\sigma} + \sum_{k\sigma} \tilde{\epsilon}_k^b b_{k\sigma}^\dagger b_{k\sigma} + \tilde{g} \sum_{kk'\sigma} a_{k'\sigma}^\dagger b_{-k-\sigma}^\dagger b_{-k-\sigma} a_{k\sigma} + \tilde{U} \sum_{kk'\sigma} b_{k'\sigma}^\dagger b_{-k'-\sigma}^\dagger b_{-k-\sigma} b_{k\sigma} + \sum_{k\sigma} \tilde{V}_k (a_{k\sigma}^\dagger b_{k\sigma} + b_{k\sigma}^\dagger a_{k\sigma}) \quad (1)$$

where $a_{k\sigma}^\dagger$ and $b_{k\sigma}^\dagger$ are creation operators for the light a and the heavy b quasi-particles, respectively. The index $l = a, b$. The dispersion relations $\tilde{\epsilon}_k^l = (\hbar^2 k^2 / 2m_l) - \mu_l$ and the ratio between effective masses is taken as $\alpha = m_a / m_b < 1$. For simplicity, we renormalize all the energies in the problem by the chemical potential μ_a of band a (the *non-tilde* quantities). Furthermore, we take $\hbar^2 / (2m_a \mu_a) = 1$. In this case, the dispersion relations can be written as, $\epsilon_k^a = \tilde{\epsilon}_k^a / \mu_a = k^2 - 1$ and $\epsilon_k^b = \tilde{\epsilon}_k^b / \mu_a = \alpha k^2 - b$, with $b = \mu_b / \mu_a$.

The V -term is responsible for the transmutation among the quasi-particles. In metallic systems, as transition metals [15], inter-metallic compounds and heavy fermions [16], it is due to the mixing of the wave-functions of the quasi-particles in different orbitals through the crystalline potential. In the problem of color superconductivity, it is the weak interaction that allows the transformation between up- and down-quarks [2,4,12]. For a system of cold fermionic atoms in an optical lattice with two atomic states (a and b), the V -term is due to Raman transitions with an effective Rabi frequency which is directly proportional to V [10]. Then, the physical origin of the V -term is different for each case. The main point is that at least in inter-metallic systems, hybridization can be easily controlled by pressure or doping [17] allowing to explore their phase diagram using these quantities as external parameters. Notice that since hybridization transforms a quasi-particle into another, in its presence only the total number of particles is conserved.

There are two order parameters that characterize the different superconducting phases of the system described by the above Hamiltonian. They are, $\Delta_{ab} = -g \sum_{k\sigma} \langle b_{-k-\sigma} a_{k\sigma} \rangle$ and $\Delta = -U \sum_{k\sigma} \langle b_{-k-\sigma} b_{k\sigma} \rangle$. These are related to inter-band and intra-band superconductivity, respectively. The anomalous correlation functions $\langle b_{-k-\sigma} a_{k\sigma} \rangle$ and $\langle b_{-k-\sigma} b_{k\sigma} \rangle$ can be obtained from the corresponding Greens functions which also yield the spectrum of excitations in the superconducting phases. We use the equation of motion method to calculate the anomalous Greens functions [18]. Excitonic types of correlations that just renormalize the hybridization [19] are neglected. The relevant anomalous, frequency dependent Greens functions are, $\langle \langle a_{k\sigma}; b_{-k-\sigma} \rangle \rangle_\omega$ and $\langle \langle b_{k\sigma}; b_{-k-\sigma} \rangle \rangle_\omega$. Here we use the notation of Ref. [20]. When we write the equations of motion for these Greens functions, new Greens functions are generated [18]. Some of these are of higher order, as they contain a larger number of creation and annihilation operators than just the two of the initials Greens functions. For these, we apply a BCS type of decoupling [18] to reduce them to the order of the originals propagators. Writing the equations of motion for the new Greens functions, we obtain a closed system of equations that can be solved [5]. Finally, the anomalous frequency dependent propagators, from which the order parameters can be self-consistently obtained, are given by [5]

$$\langle \langle a_{k\sigma}; b_{-k-\sigma} \rangle \rangle_\omega = \frac{\Delta_{ab} N(\omega)}{\omega^4 + C_2 \omega^2 + C_1 \omega + C_0} \quad (2)$$

with

$$N(\omega) = \Delta_{ab}^2 - V^2 - (\omega - \epsilon_k^b)(\omega - \epsilon_k^a) + \frac{\Delta}{\Delta_{ab}} (\omega + \epsilon_k^a)$$

and

$$\langle \langle b_{k\sigma}; b_{-k-\sigma} \rangle \rangle_\omega = \frac{\Delta \left[(\omega^2 - \epsilon_k^{a2}) + 2 \frac{\Delta_{ab}}{\Delta} V \omega \right]}{\omega^4 + C_2 \omega^2 + C_1 \omega + C_0} \quad (3)$$

where

$$C_2 = -[\epsilon_k^{a2} + \epsilon_k^{b2} + \Delta^2 + 2(\Delta_{ab}^2 + V^2)]$$

$$C_1 = 4\Delta_{ab}\Delta$$

$$C_0 = [\epsilon_k^a \epsilon_k^b - (V^2 - \Delta_{ab}^2)]^2 + \Delta^2 \epsilon_k^{a2} \quad (4)$$

As mentioned before, the poles of these propagators give the energies (ω) of the excitations in the superconducting phase. Also from the discontinuity of the Greens functions on the real axis we can obtain the anomalous correlation functions characterizing the superconducting state. In general, the appearance of exotic superconducting phases is related to the existence of soft modes in the spectrum of excitations [7]. In the present case, for the energy of the excitations to vanish, it is required that $[\epsilon_k^a \epsilon_k^b - (V^2 - \Delta_{ab}^2)]^2 + \Delta^2 \epsilon_k^{a2} = 0$. This can occur by tuning the hybridization parameter, such that, $V = \Delta_{ab}$ in which case gapless excitations appear at $k = k_F^a$ where $\epsilon_k^a = 0$. Without this fine tuning there are no gapless modes. However, in case the intra-band interaction vanishes, there is a zero energy mode for the wave-vector k , such that, $\epsilon_k^a \epsilon_k^b - (V^2 - \Delta_{ab}^2) = 0$. We will see the effects of this behavior in the next section. If, for symmetry reasons, we neglect the term linear in ω ($C_1 = 0$), we obtain the energy of the excitations in the form

$$\omega_{1,2}(k) = \sqrt{A_k \pm \sqrt{B_k}} \quad (5)$$

with

$$A_k = \frac{\epsilon_k^{a2} + \epsilon_k^{b2}}{2} + \Delta_{ab}^2 + V^2 + \frac{\Delta^2}{2} \quad (6)$$

and

$$B_k = \left(\frac{\epsilon_k^{a2} - \epsilon_k^{b2}}{2} \right)^2 + V^2 (\epsilon_k^a + \epsilon_k^b)^2 + \Delta_{ab}^2 (\epsilon_k^a - \epsilon_k^b)^2 + 4V^2 \Delta_{ab}^2 + \frac{\Delta^4}{4} - \frac{\Delta^2}{2} (\epsilon_k^{a2} - \epsilon_k^{b2}) + \Delta^2 (V^2 + \Delta_{ab}^2) \quad (7)$$

The order parameters are determined self-consistently by a set of two coupled equations which for finite temperatures are given by [5]

$$\frac{1}{g\rho_a} = \sum_{j=1}^2 \int_{-\omega_D}^{\omega_D} \frac{(-1)^j d\epsilon}{2\sqrt{B(\epsilon)}} \left[\frac{\omega_j^2(\epsilon) - \lambda^2(\epsilon)}{2\omega_j(\epsilon)} \right] \tanh \frac{\beta\omega_j(\epsilon)}{2} \quad (8)$$

with

$$\lambda^2(\epsilon) = \left[\frac{\epsilon + (\alpha\epsilon - b)}{2} \right]^2 + (\Delta_{ab}^2 - V^2) + \frac{\Delta V}{4} \left[\Delta V + 4 \left(\frac{\epsilon + (\alpha\epsilon - b)}{2} \right) \right] - \left[\frac{\epsilon + (\alpha\epsilon - b)}{2} - \frac{\Delta V}{2} \right]^2 \quad (9)$$

where in B_k and $\omega_j(k)$, we substituted $\epsilon_k^a = \epsilon$ and $\epsilon_k^b = \alpha + (\alpha\epsilon - b)$.

$$\frac{1}{U\rho_b} = \sum_{j=1}^2 \int_{-\omega_D}^{\omega_D} \frac{(-1)^j d\epsilon}{2\sqrt{B(\epsilon)}} \left[\frac{\alpha^2 \omega_j^2(\epsilon) - (\epsilon + b - \alpha)^2}{2\alpha^2 \omega_j(\epsilon)} \right] \tanh \frac{\beta\omega_j(\epsilon)}{2} \quad (10)$$

In this equation, we substituted $\epsilon_k^b = \epsilon$ and $\epsilon_k^a = (\epsilon + b - \alpha)/\alpha$ in B_k and $\omega_j(k)$. The quantities ρ_a and ρ_b are the density of states at the

Fermi level of the a and b bands and ω_D is an energy cut-off. The right-hand sides of Eqs. (8) and (10) define the gap functions $f(\Delta_{ab}, \Delta)$ and $f_b(\Delta_{ab}, \Delta)$, respectively. In the next section we discuss the behavior of the dispersion relations, of the gap functions and obtain the free energy of the system. From these quantities we obtain the phase diagrams for finite and zero temperatures.

3. Results and discussions

3.1. Zero temperature phase diagrams for pure inter- or intra-band interactions

For completeness we discuss briefly the behavior of the system at zero temperature [5]. For purely inter-band interactions the transitions are discontinuous and there is no SQCP in the system (see Figs. 1 and 2). From the ground state energy we can identify three characteristic values of the hybridization. Starting from the superconducting ground state, as hybridization increases at a value $V = V_1$ appears a minimum in the ground state energy at the origin ($\Delta = 0$) that coexists with an absolute minimum at finite Δ_{ab} associated with the superconducting state. Further increasing V there is a first order phase transition to the normal state at $V = V_2$, for which the energies of the normal and superconducting

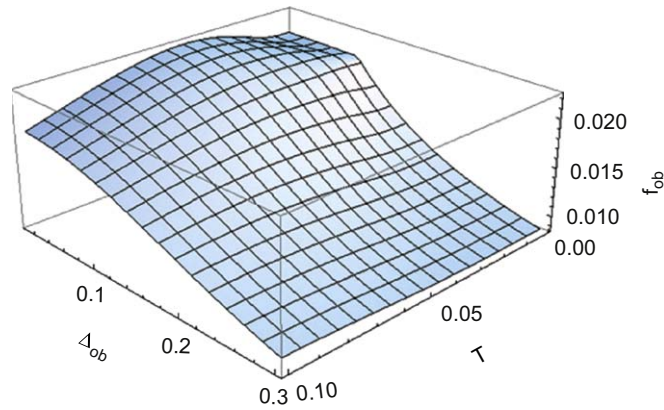


Fig. 1. (Color online) 3D graph of the asymmetrical gap function $f(\Delta_{ab}, \Delta = 0)$ versus the inter-band superconducting order parameter Δ_{ab} and temperature. See text for parameters used in the graph.

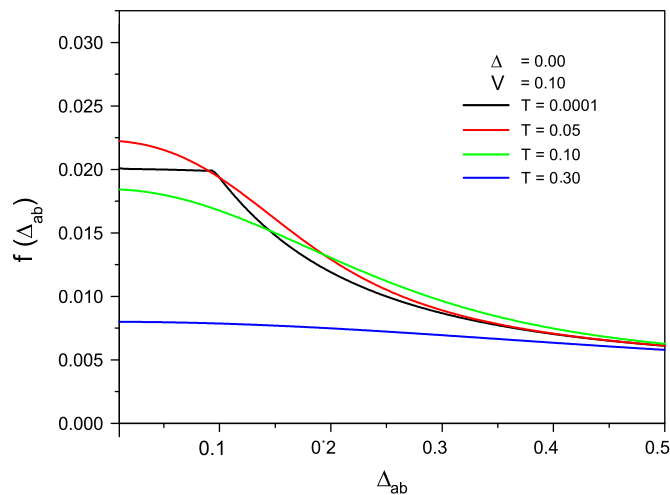


Fig. 2. (Color online) Graph of the asymmetrical gap function versus the order parameter Δ_{ab} for several values of temperature and $\Delta = 0$. These curves are intersections of the surface in Fig. 1 with planes of constant temperature. The parameters are the same used in Fig. 1 (see text).

states are degenerate. For still larger V , the superconducting state remains as a metastable state until $V = V_3$ where it stops being a minimum of the energy. The values $V = V_1$, $V = V_2$ and $V = V_3$ for a fixed set of parameters (ω_D , ρ_a , g) yield the zero temperature phase diagrams shown in Ref. [5]. It is interesting to point out that in the inter-band case sufficiently large values of V can give rise to gapless excitations which are associated with the existence of fully three dimensional Fermi surfaces in the superconducting state [5].

For purely intra-bands interactions [22], as in the general case, the dispersion relations of the excitations in the superconductor do not vanish for any k or V , since the equation $[e_k^a e_k^b - V^2]^2 + \Delta^2 e_k^{a2} = 0$ does not have any non-trivial solution. For $T = 0$, differently from the inter-band case, as V increases, the order parameter Δ given by the minimum of the ground state energy vanishes continuously [5]. There are no metastable states in this case. We also observe from the gap equation that a minimum value of the interaction $g_b(V)$ is required to sustain superconductivity. At this value of the interaction there is a continuous second order phase transition and this mixing dependent critical interaction $g_b^c(V_c)$ (or interaction dependent critical hybridization) characterizes a superconducting quantum critical point. In practice this SQCP can be most easily reached by applying pressure to change the hybridization since the strength of the interaction is difficult to control. This is a common procedure, for example, in the study of HF materials [21].

3.2. General case at $T \neq 0$

In this section we consider both intra- and inter-band interactions and discuss the phase diagrams in the presence of controlled mixing and finite temperatures. For simplicity, and to show clearly the effect of each term we assume strong inter- or intra-band terms and in each case the other interaction is added as a perturbation or neglected.

For dominant inter-bands interactions, such that, $\Delta \rightarrow 0$, we see in Figs. 1 and 2 that the phase transitions that were initially discontinuous, of first order, become continuous as temperature increases.

Fig. 1 shows the inter-band gap function as a function of the order parameter Δ_{ab} and temperature. Notice that close to where

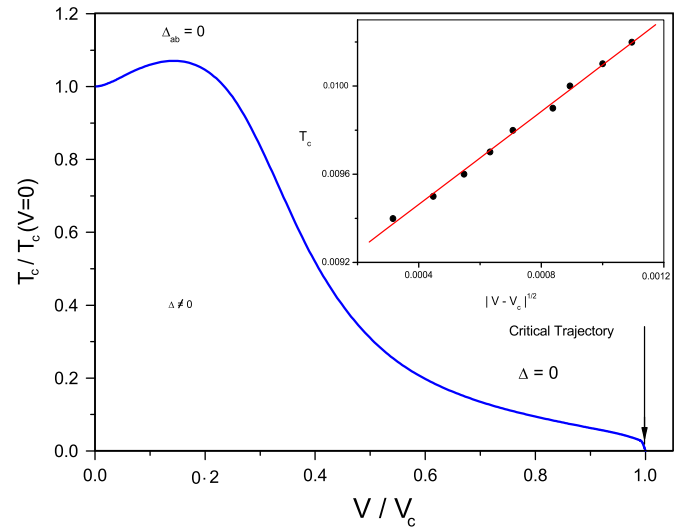


Fig. 3. (Color online) Renormalized phase diagram showing the superconducting critical temperatures as a function of hybridization, $T_c \propto V$. The transitions along the critical line are continuous and V_c is a superconducting quantum critical point. The inset shows the behavior of T_c close to the SQCP.

the transition changes from first to second order, there is a reentrant behavior which can also be seen in Fig. 3. In Figs. 1 and 2 we took $\Delta = 0$, $V = 0.1$, $\alpha = \frac{1}{7}$ and $b = 0.30$. The same values for these parameters were used in the other figures.

We now consider the case of dominant intra-band interactions, the inter-band term being taken as a small perturbation ($\Delta_{ab} \rightarrow 0$). In practice for inter-metallic compounds this is the most relevant case. Therefore, we study how the usual intra-band superconductivity changes under the influence of temperature and pressure (hybridization). We also obtain the variation of the electronic term of the specific heat in the normal phase as hybridization changes and the system goes through the SQCP.

Fig. 3 shows the phase diagram, with the critical temperature plotted as a function of hybridization. The critical line is a line of second order phase transitions. For the values of parameters used here (see above), we observe that hybridization initially increases the critical temperature before destroying superconductivity. Since the transitions are continuous there is a SQCP at a critical value of the hybridization V_c . For values of V close to V_c , the critical line vanishes at the SQCP as $T_c \propto |V - V_c|^\psi$ with a mean-field shift exponent $\psi = \frac{1}{2}$ as shown in the inset of Fig. 3.

In the next section, we calculate the linear term of the electronic specific heat in the normal phase as the system approaches the SQCP.

3.3. Specific heat

The free energy of the system can be obtained in terms of the elementary excitations. It is given by

$$F = \frac{\Delta_{ab}^2}{g} + \frac{\Delta^2}{U} + \sum_k f(\omega_1^0(k), \omega_2^0(k)) - \frac{T}{2} \sum_k \left[\sum_{n=1}^2 \ln \left[2 \left(1 + \cosh \left(\frac{\omega_n}{k_B T} \right) \right) \right] \right] \quad (11)$$

where ω_1^0 and ω_2^0 are the dispersion relations for $\Delta = \Delta_{ab} = 0$, such that, the function $f(\omega_1^0, \omega_2^0)$ yields the normal contribution to the free energy.

Notice that instead of obtaining the gap functions from the Greens functions, they can be found by minimization of the free energy with respect to the order parameters. For illustration we consider the purely inter-band case. We get

$$\frac{\partial F}{\partial \Delta_{ab}} = \frac{2\Delta_{ab}}{g} + \sum_k \sum_{n=1}^2 \left\{ -\frac{T}{2} \left[\frac{1}{T} \frac{\partial \omega_n}{\partial \Delta_{ab}} \sinh \left(\frac{\omega_n}{k_B T} \right) \right] \right\} \quad (12)$$

However, $\sinh(x)/(1 + \cosh(x)) = \tanh(x/2)$ and making $\partial F/\partial \Delta_{ab} = 0$, we find

$$\frac{2\Delta_{ab}}{g} = \frac{1}{2} \sum_k \sum_{n=1}^2 \frac{\partial \omega_n}{\partial \Delta_{ab}} \tanh \left(\frac{\omega_n}{2k_B T} \right) \quad (13)$$

Since,

$$\frac{\partial \omega_{1,2}}{\partial \Delta_{ab}} = \frac{1}{2\omega_{1,2}} \left(\frac{\partial A_k}{\partial \Delta_{ab}} \pm \frac{1}{2\sqrt{B_k}} \frac{\partial B_k}{\partial \Delta_{ab}} \right) \quad (14)$$

and

$$\frac{\partial A_k}{\partial \Delta_{ab}} = 2\Delta_{ab} \quad (15)$$

also

$$\frac{\partial B_k}{\partial \Delta_{ab}} = 2\Delta_{ab}[(e_k^a - e_k^b)^2 + 4V^2] \quad (16)$$

we get

$$\frac{\partial \omega_n}{\partial \Delta_{ab}} = 4\Delta_{ab}(\omega_n^2 - E^2), \quad (17)$$

where

$$E^2 = e_k^a e_k^b + \Delta_{ab}^2 - V^2. \quad (18)$$

Then,

$$\omega_{1,2}^2 - E^2 = \frac{1}{2}[(e_k^a - e_k^b)^2 + 4V^2] \pm \sqrt{B_k} \quad (19)$$

Finally, we get

$$\frac{1}{g} = \sum_k \sum_{n=1}^2 \frac{(-1)^{n+1}}{2\sqrt{B_k}} \left[\left(\frac{\omega_n^2 - E^2}{2\omega_n} \right) \tanh \left(\frac{\omega_n}{2k_B T} \right) \right] \quad (20)$$

which is the gap equation obtained previously directly from the Greens function. The specific heat is given by

$$C_V(V, T) = -T \left(\frac{\partial^2 F}{\partial T^2} \right) \quad (21)$$

Using the equation for the free energy, we find

$$C_V(V, T) = \frac{1}{4k_B} \sum_k \sum_{n=1}^2 \frac{\omega_n^2}{T^2} \operatorname{sech}^2 \left(\frac{\omega_n}{2T} \right) \quad (22)$$

or

$$C_V = \frac{\pi}{k_B} \sum_{n=1}^2 \frac{1}{T^2} \int_0^{k_{F_n}} \omega_n^2(k) \operatorname{sech}^2 \left(\frac{\omega_n(k)}{2k_B T} \right) k^2 d^3 k. \quad (23)$$

This is shown in Fig. 4 along the critical trajectory ($V = V_c$, T).

The linear temperature dependent term of the electronic specific heat on the normal phase at low temperatures just above the superconducting transition can be obtained as a function of V all along the critical line and in particular for $V = V_c$. It is given by

$$C_V = \frac{\pi^2}{3} k_B^2 T \sum_{i=1,2} \rho_i(\mu_0) = \gamma T \quad (24)$$

where $\rho_i(\mu_0)$ is the density of states of the hybrid bands at the Fermi level. This is given by

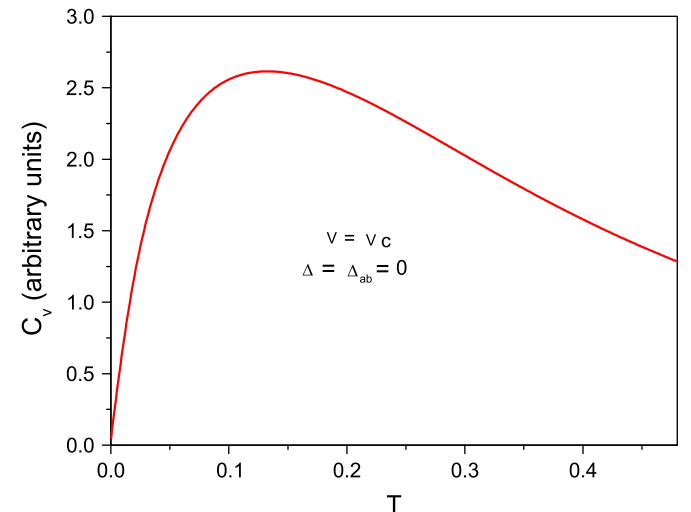


Fig. 4. (Color online) Electronic specific heat as a function of temperature along the critical trajectory ($V = V_c$). The parameters used are given in the text.

$$\rho_i(\omega) = \frac{\gamma}{(2\pi)^3} \left| \frac{\partial \omega_i}{\partial k} \right| \quad (25)$$

where

$$\frac{\partial \omega_{1,2}}{\partial k} = 2k \left\{ \alpha^\pm \pm \left[\frac{\alpha^-(\alpha^-k^2 - b^-)}{\sqrt{(\alpha^-k^2 - b^-)^2 + V^2}} \right] \right\} \quad (26)$$

with

$$\alpha^\pm = \frac{1 \pm \alpha}{2} b^- = \frac{1 - b}{2} \quad (27)$$

We wish to calculate $\rho_i(\omega)$ at the Fermi surface, i.e., for $k = k_{F_{1,2}}$, such that, $\omega_1(k_{F_1}) = 0$ and $\omega_2(k_{F_2}) = 0$. We finally get

$$\rho_{1,2}(\omega = \mu_0 = 0) = \frac{\gamma}{2\pi^2} \left| \alpha^\pm \pm \left[\frac{\alpha^-(\alpha^-k_{F_{1,2}}^2 - b^-)}{\sqrt{(\alpha^-k_{F_{1,2}}^2 - b^-)^2 + V^2}} \right] \right| \quad (28)$$

The values of $k_{F_{1,2}}$ can be easily found and from these we can obtain the values for the coefficient $\gamma(V)$ of the linear term of the specific heat in the normal phase as hybridization changes. Fig. 5 shows the values of γ just above the superconducting transition as a function of hybridization all along the critical line. We have used the same set of parameters which yield the phase diagram shown in Fig. 3. It is interesting to compare these figures. We notice that the maximum T_c occurs in a region of the phase diagram where γ is increasing with V . At the critical value of the hybridization, the coefficient of the linear term is passing through a broad maximum. Since γ is finite for $V > V_c$ the SQCP at V_c is associated with a superconducting-normal metal transition. Since γ is proportional to the total density of states at the Fermi level, Fig. 5 is helpful to understand the initial increase of T_c with hybridization as shown in Fig. 3. In the BCS approximation used here this is related to the density of states at the Fermi level and this as shown by the behavior of γ in Fig. 5 increases initially as V increases.

The results of this paper are obtained using a mean-field approximation which does not include fluctuations. The mean-field character of the theory is reflected, for example, in the shape of the critical line that vanishes as $T_c \propto \sqrt{|V - V_c|}$ close to the SQCP as shown in Fig. 3. The specific heat calculated above was obtained in the normal phase and is due solely to the contributions of unpaired quasi-particles in the hybridized bands.

Due to the nature of the approximations we used, the superconducting transition temperatures reflect the variations of the density of states and consequently of γ as hybridization is changed. As pointed out before, the increase in T_c for small V is consistent with the behavior of γ shown in Fig. 5. It is remarkable however that superconductivity is destroyed while still both bands contribute to the density of states at the Fermi level. So we are dealing with a superconductor to normal metal transition and not to a semiconductor state.

It can be easily shown that $k_{F_2}^2 - k_{F_1}^2 = 2\sqrt{(\alpha - b/2\alpha)^2 + (V^2/\alpha)}$, such that, hybridization increases the mismatch $\delta k_F = |k_{F_1} - k_{F_2}|$ between the Fermi wave-vectors of the hybrid bands. As it is well known that [9], this can give rise to superconducting instabilities which, however, always occur discontinuously through first order transitions. In the present case the $T = 0$ superconductor-normal transition is continuous being associated with a SQCP. Since there is no dissipation in the electronic bands but a lack of coherence in one of them we may attribute to this the destruction of the superconducting phase.

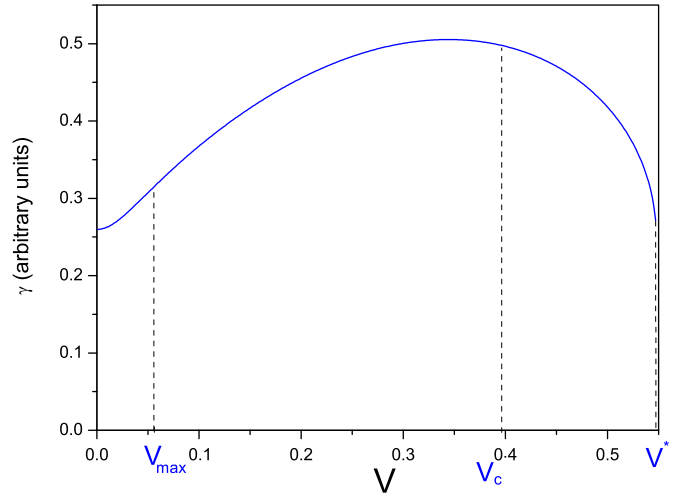


Fig. 5. (Color online) Coefficient of the linear temperature dependent term of the electronic contribution to the specific heat in the normal phase as a function of hybridization and at $V = V_c$ for the same parameters as in Fig. 3. It is also shown the value $V_{\max}$ for which T_c is a maximum in Fig. 3. For $V = V^* = \tilde{V}^*/\mu_a = \sqrt{\mu_b/\mu_a} = \sqrt{b}$, one of the hybrid bands is above the Fermi level and γ has a discontinuity.

It is clear that in multi-band systems hybridization occurs even at zero pressure. The point is that this depends on applied pressure and for this reason it can be used as a control parameter and give rise to the effects studied here.

A final remark is that, in fact, the control parameter is V/μ_a , i.e., the ratio of hybridization over the bandwidth of the light quasi-particles. The latter also depends on pressure and although we have naturally assumed that this ratio increases with pressure, it may also decrease depending on a particular system.

We have investigated a mechanism to drive a multi-band superconductor to a superconducting quantum critical point through the application of pressure or doping [17]. It does not rely on the presence of magnetic impurities or disorder, but on the sensitivity of hybridization to external parameters. The mechanism studied here should inevitably play a role in all multi-particle BCS superfluids if one disposes of a way to control the parameter V by external means. It may, however, act in competition with other effects.

It has been demonstrated in different multi-band systems that a pressure induced SQCP can be reached, as in some heavy fermion systems [23], or eventually will be reached, as in the Fe pnictides [24]. In the latter family many experiments show a decrease of the superconducting temperatures with pressure, specially in overdoped materials which are far away from magnetic instabilities. It remains an experimental challenge to increase the external pressure to higher values and eventually destroy superconductivity [24] in these systems. The model presented here provides a realistic mechanism for the description of this phenomenon. It makes several predictions, as the behavior of the specific heat just before the SQCP which allow it to be tested. Indeed, for superconductors with sufficiently small critical temperatures T_c , measurements of the coefficient of the linear term of the electronic specific heat just above the superconducting transition, as well as of T_c , as a function of pressure should be useful to put in evidence the mechanism proposed here. This mechanism can destroy superconductivity while all hybrid bands still contribute to the density of states, such that, the zero temperature transition is from the superconductor to a metallic state.

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