

# Conformal field theory approach to the two-channel Anderson model

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## Abstract

The two-channel Anderson impurity model serves as a prototype for describing heavy-fermion materials with a possible mixed-valent regime with both quadrupolar and magnetic character. We report on the low-energy physics of the model, using a conformal field theory approach with exact Bethe Ansatz results as input.

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PACS: 71.27.+a; 75.20.Hr; 75.40.–s

Keywords: Heavy fermions; Anderson model; Two-channel Kondo physics

As is well-known, several uranium-based heavy fermion materials exhibit manifest non-Fermi liquid behavior at low temperatures. A particularly intriguing case is that of the  $\text{UBe}_{13}$  compound. It has been suggested that the electric quadrupolar degrees of freedom of the uranium ions (in  $5f^2$  configurations) in the  $\text{Be}_{13}$  host are overscreened by the local orbital motion of the conduction electrons, producing a two-channel Kondo response of this material, for a review, see Ref. [1]. Taking into account the possibility that the magnetic  $5f^3$  configurations may also be at play, one is led to consider the two-channel Anderson impurity model (for a review, see Ref. [1]):

$$H = H_0 + \varepsilon_s f_\sigma^\dagger f_\sigma + \varepsilon_q b_{\bar{\alpha}}^\dagger b_{\bar{\alpha}} + V(\psi_{\alpha\sigma}^\dagger(0) b_{\bar{\alpha}}^\dagger f_\sigma + f_\sigma^\dagger b_{\bar{\alpha}} \psi_{\alpha\sigma}(0)), \quad (1)$$

where  $H_0$  is the free part of the Hamiltonian. The conduction electrons  $\psi_{\alpha\sigma}^\dagger$  carry spin ( $\sigma = \uparrow, \downarrow$ ) and quadrupolar ( $\alpha = \pm$ ) quantum numbers, and hybridize with a local uranium ion via a matrix element  $V$ . The ion is modeled by a quadrupolar [magnetic] doublet of

energy  $\varepsilon_q$  [ $\varepsilon_s$ ], created by a boson [fermion] operator  $b_{\bar{\alpha}}^\dagger$  [ $f_\sigma^\dagger$ ]. Strong Coulomb repulsion implies single occupancy of the localized levels:  $f_\sigma^\dagger f_\sigma + b_{\bar{\alpha}}^\dagger b_{\bar{\alpha}} = 1$ .

We have carried out a non-perturbative analysis of the model, exploiting boundary conformal field theory [2] to trade the hybridization interaction in Eq. (1) for a scale invariant boundary condition on the conduction electrons. Identifying the proper boundary condition immediately identifies the critical theory to which the model belongs. By a numerical fit to the exact Bethe Ansatz solution [3] of Eq. (1) we can determine *all* parameters and scales of this theory, thus obtaining a complete description of the low-energy dynamics of the model. Our most important results can be summarized:

The model renormalizes to a line of fixed point Hamiltonians parameterized by the average charge  $n_c \equiv \langle f_\sigma^\dagger f_\sigma \rangle$  at the impurity site (Fig. 1), and exhibiting the same zero-temperature impurity entropy  $S_{\text{imp}} = k_B \ln \sqrt{2}$ , typical of two-channel Kondo physics [4]. The low-temperature specific heat induced by the impurity takes the form  $C_{\text{imp}} = \mu_s T \ln(T_s/T) + \mu_q T \ln(T_q/T) + O(T)$ , where  $\mu_{s[q]}$  and  $T_{s[q]}$  are amplitudes and temperature scales, respectively, measuring the participation of the spin [quadrupolar] degrees of freedom in the screening process. Similarly, the impurity response to a magnetic [quadrupolar] field is also of

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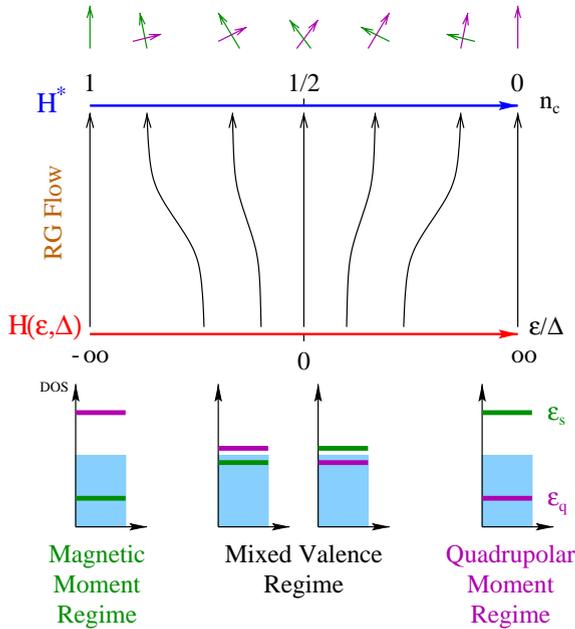


Fig. 1. Schematic renormalization group flow diagram of the two-channel Anderson impurity model. The flow connects the line of microscopic theories characterized by the ratio  $\epsilon/\Delta$  ( $\epsilon = \epsilon_s - \epsilon_q$ ,  $\Delta = \pi\rho V^2$ ) with the line of infrared fixed points parameterized by  $n_c$ .

two-channel Kondo type,  $\chi_{\text{imp}}^{s[q]} \sim \mu_{s[q]} \ln(T_{s[q]}/T)$ , with the amplitudes  $\mu_{s[q]}$  decreasing exponentially with  $\epsilon$  [ $-\epsilon$ ]. We have also calculated the Fermi edge singularities

caused by *time-dependent* hybridization between conduction electrons and impurity. These are coded by the scaling dimensions of the pseudo-particles  $b_{\alpha}^{\dagger}$  and  $f_{\sigma}^{\dagger}$  in Eq. (1), and we find that  $x_b = (3 + 2n_c^2)/16$  and  $x_f = (5 - 4n_c + 2n_c^2)/16$ , respectively. We point out that the values of the pseudo-particle scaling dimensions as obtained by various approximation schemes, e.g. NCA and large- $N$  calculations [5], deviate from our exact result.

The conformal field theory formalism allows us to determine also the asymptotic dynamical properties of the model, including Green's functions and resistivities. We have calculated the self-energy of the electrons and find that it leads to a zero-temperature resistivity *independent* of the level separation  $\epsilon$ :  $\rho(T=0) = 3n_i/4\pi(\epsilon v_F)^2$ , with  $n_i$  the (dilute) impurity concentration,  $v$  the density of states at the Fermi level, and  $v_F$  the Fermi velocity. The finite-temperature calculation is in progress.

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