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The Hofstadter energy spectrum for an interacting 2DEG

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Abstract

We studied the effects of the Coulomb interactions between electrons on the Hofstadter butterfly, which characterizes the subband structure of the Landau levels of a two-dimensional electron gas in a perpendicular homogeneous magnetic field and a periodic lateral superlattice potential. The interactions essentially preserve the intricate gap structure of the Hofstadter spectra, but with a lower symmetry that depends on the filling of the Landau bands. For short enough periods and strong enough modulation the miniband structure can be resolved in the thermodynamic density of states.

Keywords: Electron density, excitation spectra calculations; Gallium arsenide; Heterojunctions; Interface states; Magnetic phenomena; Many body and quasi-particle theories; Quantum effects; Superlattices

Measurements of the magnetotransport in superlattices weakly modulated in one or two directions have revealed novel oscillations in the conductivity, the Weiss oscillations, reflecting the commensurability of the cyclotron radius $R_c = l^2 k_F = v_F/\omega_c$ of the electrons at the Fermi energy $E_F = \hbar^2 k_F^2/(2m^*)$ and the modulation period L; $l = (c\hbar/eB)^{1/2}$ is the magnetic length, and $\omega_c = eB/(mc)$ the cyclotron frequency [1]. The Weiss oscillations are superimposed on the Shubnikov–de Haas oscillations, which are well known in a homogeneous two-dimensional electron gas (2DEG) and caused by the commensurability of the Fermi wavelength λ_F and l. The single-particle energy spectrum for this problem has been investigated by several researchers, culminating in Hofstadter's butterfly, a graph showing the complicated self-similar splitting of a Landau band (LB) into minibands as a function of the magnetic flux through a unit cell of the lattice (for a review see Ref. [2]). The effects of the splitting of LBs into minibands has, to the best of our knowledge, not been observed directly in the currently common superlattices ($L \sim 300$ nm $n_s = 3 \times 10^{11}$ cm^{-2}) since usually several LBs are occupied, which may overlap near E_F. The energy spectrum has been investigated in the intermediate region in the absence of collision broadening. The coupling of the Landau levels (LLs) by the external periodic potential strongly reduces the original high symmetry of the Hofstadter spectrum but retains a very complicated subband structure [3,4]. All the above-mentioned theoretical investigations of the energy spectrum of a 2DEG in a periodic potential and a homogeneous perpendicular magnetic field have neglected the effects of the electron–electron interaction. Here we study, within the

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Hartree approximation (HA), these effects on the electronic energy spectrum and the thermodynamic density of states.

To describe the electrons in the conduction band in the presence of a lateral superlattice at the AlGaAs–GaAs interface in a constant perpendicular magnetic field $\boldsymbol{B}=B\hat{\boldsymbol{z}}$, we employ a model of a 2DEG, with the three-dimensional charge density given by $-en_s(\boldsymbol{r})\delta(z)$, and $\boldsymbol{r}=(x,y)$. The square superlattice is spanned by orthogonal lattice vectors of length L, the primitive translations of the Bravais lattice, B. The corresponding reciprocal lattice R is spanned by $\boldsymbol{G}=G_1\boldsymbol{g}_1+G_2\boldsymbol{g}_2$, with $G_1,G_2 \in Z$, $\boldsymbol{g}_1=2\pi\hat{\boldsymbol{x}}/L$, and $\boldsymbol{g}_2=2\pi\hat{\boldsymbol{y}}/L$. The external periodic potential the electrons are moving in is taken to be of the simple form $V(\boldsymbol{r})=V_0\{\cos(g_1x)+\cos(g_2y)\}$. The electron–electron interaction is included in the HA, leading to an effective single-electron Hamiltonian, $H=H_0+V_H(\boldsymbol{r})+V(\boldsymbol{r})$, where $V_H(\boldsymbol{r})$ is the effective potential in a medium with a dielectric constant κ, felt by each electron and caused by the total charge density of the 2DEG, $-en_s(\boldsymbol{r})$, and the neutralizing background charge density, $+en_b=+e\langle n_s(\boldsymbol{r})\rangle$. The periodic external potential $V(\boldsymbol{r})$ and the constant external magnetic field imply that all physical quantities of the non-interacting system are periodic with respect to translations of $\boldsymbol{R} \in B$. The periodicity of the Hartree potential follows from that of $n_s(\boldsymbol{r})$. In the following calculations we use the symmetric basis functions constructed by Ferrari [5] and used by Silberbauer [6]. The set of Hartree equations has to be solved iteratively together with the condition that the average electron density $n_s=N_s/A$ is constant, which determines the chemical potential μ. N_s is the number of electrons per unit cell with area A in B.

The Coulomb interaction not only couples directly the subbands of a particular LL, but also the subbands of different LLs. If only the basis states in the same LL are taken into account, the modulation strength is irrelevant since it can be factored out of the Hamiltonian matrix. In this situation, corresponding to the usual discussion of Hofstadter's butterfly on the basis of Harper's equation, a highly symmetric energy spectrum is expected. Of course, this restricted model is only appropriate for describing a system with very weak modulation (as compared with the inter-Landau level energy spacing, $\hbar\omega_c$). The bandwidth of the subbands of the first LB is shown in Fig. 1, for the two cases of 3 or 6 magnetic flux quanta through a unit cell ($pq=3,6$), identifiable from the number of subbands appearing in each figure. The period is short, $L=50$ nm, the modulation strong, $V_0=4.0$ meV, and $T=1.0$ K. However, only one LL is used in the calculation, so that the results apply to the case of a weak superlattice potential. The electron–hole symmetry is exact, as is reflected in each case by the point symmetry of the subband structure around μ at half filling. The screening is strongly dependent on N_s and the average filling factor. It is, therefore, strongest for $pq=6$ when a maximum of six electrons can occupy the lowest Landau level in the unit cell and when the LB is half filled, $N_s=3$. The (dimensionlessly written) thermodynamic density of states $D_T=l^2\hbar\omega_c(\partial n_s/\partial\mu)_{TB}$, with $n_s=\langle n_s(\boldsymbol{r})\rangle$, is shown in Fig. 2. Clearly, the strong screening in the $pq=6$ case obscures the fine structure in D_T leading to only one pronounced peak. The subbands in the other cases turn up as separate peaks in D_T. The Hofstadter butterfly should be recovered when the calculation is performed for $N_s=0$ and only one LL, and if the width and location of the subbands are plotted as functions of the inverse of the number of magnetic flux quanta through a unit cell of the lattice, i.e., of $\Phi_0/\Phi=1/(pq)$. In addition, all energies have to be scaled according to: $E\rightarrow(E-\hbar\omega_c/2)/V_0$. For interacting electrons N_s, or the

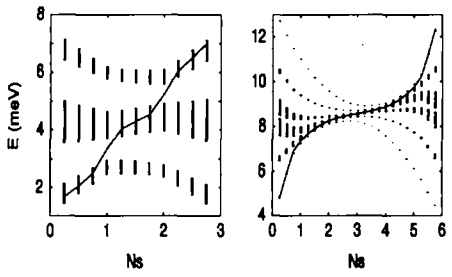


Fig. 1. The bandwidth of the minibands of the lowest Landau band as a function of N_s for a Hartree interacting 2DEG and ($p=1$, $q=3$) (left), ($p=2$, $q=3$) (right). The chemical potential μ is indicated by a continuous curve. $L=50$ nm, $T=1.0$ K, $V_0=4.0$ meV. One Landau level is included in the HA. $m^*=0.067m_0$, $\kappa=12.4$.

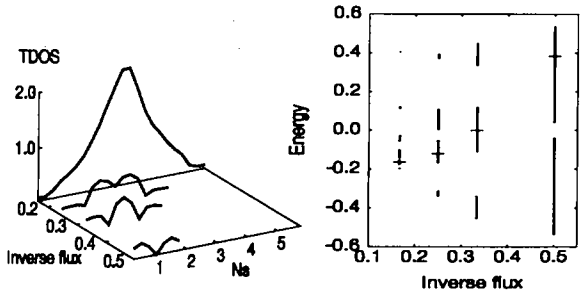


Fig. 2. The thermodynamic density of states (left) $D_T = l^2\hbar\omega_c(\partial n_s/\partial\mu)_{TB}$ as a function of $1/pq$ and N_s. The scaled bandwidth (right) of the subbands of the lowest Landau band, $(E-\hbar\omega_c/2)/V_0$, as a function of $1/pq$ for $N_s=1.5$. The chemical potential μ is indicated by crosses. $L=50$ nm, $T=1.0$ K, $V_0=4.0$ meV. One Landau level is included in the HA.

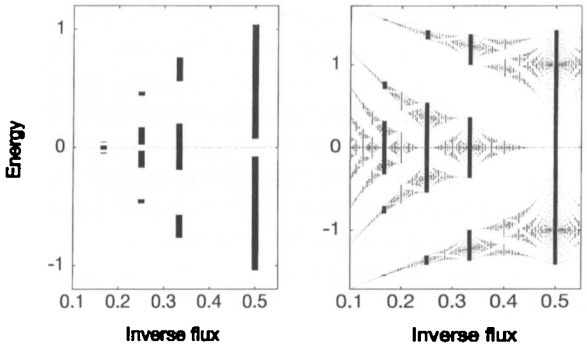


Fig. 3. The scaled bandwidth $(E-\hbar\omega_c/2)\exp\{(\pi l/L)^2\}/V_0$ of the subbands of the lowest Landau band as function of $1/pq$ for the Hartree interacting 2DEG with $\nu=1/2$ (left), and for the noninteracting 2DEG (right), the Hofstadter butterfly. One Landau level is included in the calculations. In the left subfigure the six minibands for the case $pq=6$ cannot all be resolved due to vanishing band gaps. The parameters used for the left subfigure are: $L=50$ nm, $T=1.0$ K, $V_0=4.0$ meV.

average filling factor of the LB turns out to be a new parameter controlling the bandstructure. We therefore present, in Fig. 2, the subband structure for four values of the inverse flux $1/(pq)$ in the case of $N_s=1.50$ together with the location of μ. For low density, the subband structure is almost symmetrical around the energy zero, as in the noninteracting case, but the subbands become quite asymmetric for a higher density of electrons. For the short-period superlattice studied here, and for a low density of electrons, the essential gap structure does survive in the presence of interaction. The presentation in Fig. 2 corresponds to the experimental procedure of keeping the density of electrons fixed but changing the magnetic field. Another way to investigate the screening is to keep the average filling factor, ν, constant but to vary the magnetic field and, thus, also the average density of the 2DEG. Fig. 3 compares the subband structures for $1/(pq)=1/2,1/3,1/4,1/6$ and $\nu=1/2$ with the complete Hofstadter butterfly. Here the energies have been scaled with the factor $(E-\hbar\omega_c/2)\exp\{(\pi l/L)^2\}/V_0$ so that the results can be directly compared with earlier calculations of the Hofstadter spectrum [2]. The energy spectrum in the interacting case shows an overall reduction in dispersion or bandwidth due to the strong screening that is most effective for large flux and large number of available states. The bandwidths of the subbands for the interacting case has been evaluated here on a discrete lattice in the magnetic Brillouin zone without attempting an interpolation between the lattice points; thus, the actual bandwidths can be larger by a small percentage of the widths shown.

In an interacting 2DEG subject to a superlattice potential and a homogeneous perpendicular magnetic field, not only the magnetic flux through a unit cell but also the density of electrons determines the complicated splitting of the LLs into subbands. We have shown that in the HA the essential gap structure of the energy spectrum remains, although the screening leads to a quenching of the Hofstadter butterfly at small values of the inverse flux. The symmetry of the butterfly is also lowered by the coupling to higher LLs due to increased strength of the periodic potential, as has already been discussed by other authors [3,4]. For periods around $L=200$ nm (currently attainable in experiments on superlattices), the 2DEG can effectively screen the periodic potential even for a very low density n_s when only one or two Landau bands are partly occupied. Only at shorter lattice constants ($L<100$ nm) and thus much higher magnetic fields we can, on the basis of our Hartree calculation, expect the subband structure to be resolvable in experiments, when n_s is maintained low enough. These predictions are made on the basis of the calculated energy spectrum and the structure observed in the thermodynamic density of states,

D_T. The transport coefficients do not depend on D_T in any simple way, and may be more sensitive to the subband structure of the energy spectrum [2]. In order to keep the computational efforts within reasonable limits, we have restricted our calculations to a few characteristic values of the magnetic field. Our results indicate the manner in which screening effects will change the overall appearance of the Hofstadter energy spectrum. Unfortunately, there is no simple way to extrapolate from these special values to arbitrary rational numbers of flux quanta per unit cell. From the experimental point of view, these details of the energy spectrum seem not to be accessible anyway. The challenge for the near future is to resolve the most prominent gaps of the energy spectrum experimentally.

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