

# Influence of self-affine roughness on the adhesive friction coefficient of a rubber body sliding on a solid substrate

George Palasantzas \*

*Department of Applied Physics, Materials Science Centre, University of Groningen Nijenborgh 4, 9747 AG Groningen, The Netherlands*

Received 21 April 2004; accepted for publication 7 July 2004

Available online 25 July 2004

## Abstract

In this work we investigate characteristics of the adhesive friction during sliding of a rubber body on a rough self-affined surface. The latter is characterized by the rms roughness amplitude  $w$ , the in-plane correlation length  $\xi$ , and the roughness exponent  $H$  ( $0 < H < 1$ ). The friction coefficient is shown to be proportional to the roughness amplitude  $w$ . Moreover, the friction coefficient is shown to depend strongly on the roughness exponent  $H$ . The influence of the latter is more prominent at intermediate sliding velocities, where the friction coefficient is independent of the sliding velocity. Similar, but weaker in magnitude, is the influence of correlation length  $\xi$  on  $\mu_{\text{ad}}$ . In any case, our work shows that understanding of the adhesive friction should take properly into account of the precise roughness nature both at all lateral roughness wavelengths.

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**Keywords:** Friction; Adhesion; Tribology; Surface structure, morphology, roughness, and topography

## 1. Introduction

The friction which develops between a rubber body sliding onto a hard solid surface is important from the fundamental and technological point of view in car industry (tire construction, wiper rubber blades), cosmetic industry etc. [1–4]. The major

difference in the frictional properties of rubbers with respect to other solids arise from their low elastic modulus  $E$ , and the high internal friction that is present over a wide frequency range [5]. The friction force between a rubber body and a hard rough solid substrate has two major contributions which are the hysteric and the adhesive ones [1]. The hysteric component arise from the oscillating forces that the surface asperities exert onto the rubber surface leading effectively to cyclic deformations and energy dissipation due to internal frictional damping [5]. As a result the hysteric

\* Tel.: +31 50 363 4272; fax: +31 50 363 4881.

E-mail address: [g.palasantzas@phys.rug.nl](mailto:g.palasantzas@phys.rug.nl).

contribution will have the same temperature dependence as that of an elastic complex modulus  $E(\omega)$  [5]. On the other hand, the adhesive component is important for clean and relative smoother surfaces [5].

In actual situations besides adhesive and hysteric friction, the rubber produces traction forces through tearing and wear. As deformation stresses and sliding speeds increase (e.g., tires in racing cars), the local stress can exceed the tensile strength of the rubber especially near the point of a sharp irregularity. The high local stress can deform the internal rubber structure beyond the point of elastic recovery. Indeed, when polymer bonds and cross-links are stressed to failure the material can no longer recover completely leading to tearing. The latter absorbs energy and results in additional friction forces within the contact surface. The wear processes are the ultimate result of tearing.

For rubbers and other elastically soft solids a weak adhesive junction due to van der Waals interactions between the surfaces may be well elongated before it breaks at a distance that is larger than the size of the surface asperities [6]. Thus, during the block sliding a large fraction of the junctions will be simultaneously (elastically) elongated and exert a force on the moving body in contact with the rough substrate. Furthermore, sliding onto real solid surfaces occurs in many cases onto rough surfaces with a significant degree of randomness [7–10]. The latter implies that these surfaces possess roughness over various length scales rather than a single one. This is a fact that has to be taken carefully into account in contact related phenomena (i.e., friction and adhesion) [5,6].

Up to now, it has been shown that for self-affine random rough surfaces the coefficient of hysteric friction depends significantly on the roughness exponent  $H$  ( $0 \leq H \leq 1$ ), which characterizes the degree of surface irregularity at short length scales [5,7]. Nevertheless, the previous studies did not considered how self-affine roughness influence the adhesive component of friction. This will be investigated in this article by inclusion of contributions from all lateral roughness wavelengths in terms of an analytic roughness model in Fourier space.

## 2. Friction theory in the presence of adhesion

As it was mentioned earlier, for rubbers an adhesive junction due to van der Waals interactions between the surfaces may be well elongated before it breaks at a size larger than that of the surface asperities. In general, the energy dissipated in a viscoelastic medium is given by [6]

$$E = \int \sigma_{ij} \left( \frac{d\varepsilon_{ij}}{dt} \right) d^3x dt \quad (1)$$

with  $\sigma_{ij}$  the stress tensor and  $\varepsilon_{ij}$  the strain tensor. If we assume uniaxial deformations of a cylindrical bar, then in the frequency domain we obtain [6]

$$E = \frac{V}{2\pi} \int \omega \operatorname{Im} \left[ \frac{1}{E(\omega)} \right] |\sigma(\omega)|^2 d\omega \quad (2)$$

where  $V = \int d^3x$ . If a rubber body slides with velocity  $V$  over a sinusoidal rough surface with period  $L$ , then it will feel fluctuating forces with a characteristic frequency  $\sim V/L$ . In addition, if the surface has a wider distribution of length scales  $L$ , then it will be present a wider distribution of frequency components in the Fourier decomposition of the surface stresses acting on the sliding rubber [5]. The main energy dissipation will occur within a volume  $V \approx L^3$  (asperity volume) where if we denote by  $\sigma = \sigma_0 \cos(\omega_0 t)$  the fluctuating stress ( $\sigma_0 = F_N/L^2$  and  $F_N$  the normal force) [6], we set  $E = VT F_{\text{friction}}$  with  $T$  an oscillation period, we obtain an expression for the friction coefficient [6]

$$\mu = \frac{F_{\text{friction}}}{F_N} \simeq \sigma_0 \operatorname{Im} \left[ \frac{1}{E(\omega_0)} \right] \quad (3)$$

where  $\operatorname{Im}[\cdot \cdot \cdot]$  denotes the imaginary part of a complex number.

In order to take into account the dependence of the stress factor  $\sigma_0$  in Eq. (3) on contact details over a distribution of lateral length scales that is present for random rough surfaces, we proceed as follows. We will assume complete contact of the rubber body with the solid substrate up to macroscopic dimensions. We denote by  $C(q)$  the Fourier transform of the auto correlation function  $C(r) = \langle h(\vec{r})h(0) \rangle$  with  $h(\vec{r})$  the surface roughness height ( $\langle h \rangle = 0$ ).  $\langle \cdot \cdot \cdot \rangle$  is an ensemble average over possible roughness configurations. If we denote by  $U_{\text{el}}$  the energy that is spent to push the rubber

body into contact with the rough substrate then we have [11,12]

$$U_{\text{el}} = A_{\text{flat}} \frac{|E(\omega_0)|}{4(1-v^2)} \int_{Q_\lambda}^{Q_c} q C(q) d^2 q \quad (4)$$

where  $A_{\text{flat}}$  is macroscopic flat surface area, and  $Q_c = \pi/a_0$  with  $a_0$  of microscopic dimensions. The characteristic frequency in  $\omega_0$  in Eq. (4) (and in the following formulas) is given by  $\omega_0 = V/\xi$  with  $\xi$  the in-plane roughness correlation length. Moreover,  $Q_\lambda = 2\pi/\lambda$  and  $\lambda$  the system size of macroscopic dimensions so that  $\lambda \gg \xi \gg a_0$ . If we denote by  $w = \sqrt{\langle h^2 \rangle}$  the rms roughness amplitude then the average normal force per unit area ( $F_N/L^2$ ) in Eq. (3) can be obtained by differentiating  $U_c$  from Eq. (4) by the rms roughness amplitude ‘ $w$ ’. The latter is the magnitude of effective depth that the rubber will have to be pressed in order to stay in contact with the rough substrate surface. Therefore, we have for the average normal stress  $\langle \sigma_0 \rangle$

$$\langle \sigma_0 \rangle = \frac{1}{A_{\text{flat}}} \frac{\partial U_{\text{el}}}{\partial w} \quad (5)$$

where we ignore any weak frequency dependence of the Poisson ratio ‘ $v$ ’. If we combine Eqs. (3)–(5) we obtain the adhesive friction coefficient

$$\mu_{\text{ad}} \simeq \frac{|E(\omega_0)|}{4(1-v^2)} \left\{ \int_{Q_\lambda}^{Q_c} q \frac{\partial C(q)}{\partial w} d^2 q \right\} \text{Im} \left[ \frac{1}{E(\omega_0)} \right]. \quad (6)$$

Although sliding is assumed to take place in one direction over a two dimensional isotropic rough surface, the calculation of the average stress  $\langle \sigma_0 \rangle$  is performed on a two dimensional isotropic surface, which is the reason to consider also a two-dimensional roughness model for  $C(q)$  in Eqs. (4)–(6).

### 3. Results and discussion

A model for the modulus  $E(\omega)$  that will be used for the calculations is given by [5]

$$E(\omega) = \frac{E_1[(1+\alpha) + (\omega\tau)^2]}{(1+\alpha)^2 + (\omega\tau)^2} - j \frac{\alpha\omega\tau E_1}{(1+\alpha)^2 + (\omega\tau)^2} \quad (7)$$

with  $E_1 = E(\infty)$ ,  $E(\infty)/E(0) = 1 + \alpha$  (typically  $\alpha = 10^3$ ), and  $1/\tau$  the flip rate of molecular segments that are responsible for the viscoelastic properties of the rubber body.

As Eq. (6) indicates, in order to calculate the coefficient of friction  $\mu_{\text{ad}}$  the knowledge of the spectrum  $C(q)$  is necessary. A wide variety of surfaces/interfaces are well described by a kind of roughness associated with self-affine fractal scaling [7], for which  $C(q)$  scales as a power-law  $C(q) \propto q^{-2-2H}$  if  $q\xi \gg 1$ , and  $C(q) \propto \text{const}$  if  $q\xi \ll 1$  [7]. The roughness exponent  $H$  is a measure of the degree of surface irregularity [7,8], such that small values of  $H$  characterise more jagged or irregular surfaces at short length scales ( $< \xi$ ). The self-affine scaling behaviour is satisfied by the simple model [8]

$$C(q) = \frac{1}{2\pi} \frac{w^2 \xi^2}{(1 + a q^2 \xi^2)^{1+H}} \quad (8)$$

with  $a = (1/2H)[1 - (1 + a Q_c^2 \xi^2)^{-H}]$  if  $0 < H < 1$  (power-law roughness), and  $a = (1/2) \ln[1 + a Q_c^2 \xi^2]$  if  $H = 0$  [8]. For other correlation models see also Refs. [9,10].

Upon substitution of Eq. (8) into Eq. (6) we obtain for the coefficient of friction

$$\mu_{\text{ad}} \simeq \frac{|E(\omega_0)|}{2(1-v^2)} w \left\{ \int_{Q_\lambda}^{Q_c} \frac{\xi^2 q^2}{(1 + a q^2 \xi^2)^{1+H}} dq \right\} \times \text{Im} \left[ \frac{1}{E(\omega_0)} \right] \quad (9)$$

Analytic calculations of  $\mu_{\text{ad}}$  are possible for  $H = 0$ , 0.5, and 1. Therefore, we have

$$\mu_{\text{ad}}|_{H=0} = \frac{2w}{w_0^2} \left\{ \frac{1}{a} (Q_c - Q_\lambda) - \frac{1}{a^{3/2} \xi} [\tan^{-1}(X_c) - \tan^{-1}(X_\lambda)] \right\} \times \text{Im} \left[ \frac{1}{E(\omega_0)} \right] \quad (10)$$

$$\mu_{\text{ad}}|_{H=0.5} = \frac{2w}{w_0^2} \left\{ \frac{1}{a^{3/2} \xi} [\sinh^{-1}(X_c) - \sinh^{-1}(X_\lambda)] - \frac{1}{a} \left[ \frac{Q_c}{\sqrt{T_c}} - \frac{Q_\lambda}{\sqrt{T_\lambda}} \right] \right\} \text{Im} \left[ \frac{1}{E(\omega_0)} \right] \quad (11)$$

$$\mu_{\text{ad}}|_{H=1} = \frac{2w}{w_0^2} \left\{ \frac{1}{a^{3/2}\xi} [\tan^{-1}(X_c) - \tan^{-1}(X_\lambda)] - \frac{1}{2a} \left[ \frac{Q_c}{T_c} - \frac{Q_\lambda}{T_\lambda} \right] \right\} \text{Im} \left[ \frac{1}{E(\omega_0)} \right] \quad (12)$$

with  $X_c = \sqrt{a}\xi Q_c$ ,  $X_\lambda = \sqrt{a}\xi Q_\lambda$ , and  $w_0 = \sqrt{4(1-v^2)/|E(\omega_0)|}$ .

Fig. 1 shows the dependence of the friction coefficient for various relaxation times  $\tau$ . The inverse dependence of  $\mu_{\text{ad}}$  on  $\tau$  is observed before and after the plateau regime, where the coefficient of friction is independent of the sliding velocity. At low velocities the friction coefficient decreases with decreasing relaxation time, while after the plateau region at relatively high velocities it increases with decreasing  $\tau$ . The plateau is rather wide with width that depends also on the particular model for  $E(\omega)$ . With decreasing relaxation time  $\tau$  the plateau location shifts to higher velocities since the characteristic velocity  $V \approx \xi/\tau$  also increases with decreasing  $\tau$ .

Furthermore, Eq. (9) yields for the friction coefficient (at high velocities) the simple dependence  $\mu_{\text{ad}} \propto w$ , while any more complex dependence will arise solely from the roughness parameters  $H$  and  $\xi$  as for example Eqs. (10)–(12) clearly indicate. Fig. 2 shows the dependence of  $\mu_{\text{ad}}$  on the roughness exponent  $H$ . It is shown that with decreasing roughness exponent  $H$  (or increasing roughness

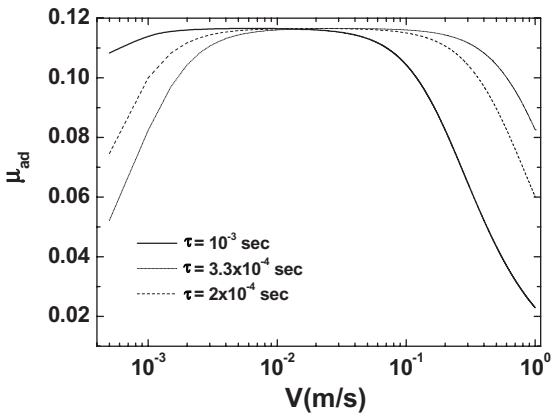


Fig. 1. Friction coefficient  $\mu_{\text{ad}}$  vs. sliding velocity  $V$  for  $H = 0.7$ ,  $w = 10$  nm,  $\xi = 200$  nm, and various relaxation times  $\tau$  as indicated.

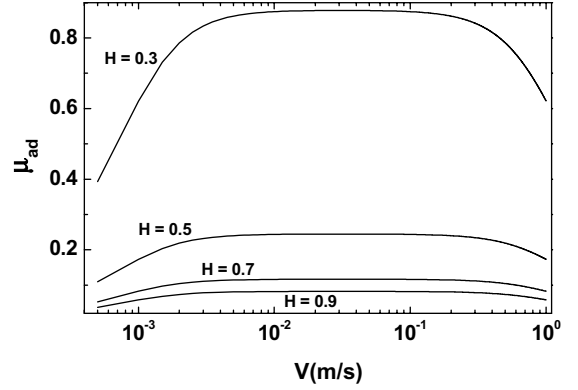


Fig. 2. Friction coefficient  $\mu_{\text{ad}}$  vs. sliding velocity  $V$  for  $w = 10$  nm,  $\xi = 200$  nm,  $\tau = 10^{-3}$ , and various roughness exponents  $H$  as indicated.

irregularity at short length scales  $< \xi$ ) the friction coefficient increases. The influence of the exponent  $H$  is more prominent at intermediate velocities within the plateau regime.

Similar is the behaviour of  $\mu_{\text{ad}}$  as a function of the lateral roughness correlation length  $\xi$  where with decreasing  $\xi$  (or equivalently for surface roughening at long roughness wavelengths)  $\mu_{\text{ad}}$  increases with highest effect observed within the plateau region. Notably with increasing correlation length  $\xi$  the plateau shifts to higher velocities since the characteristic velocity  $V \approx \xi/\tau$  also increases. Thus the correlation length  $\xi$  has the opposite effect on the location of the plateau region from that of the relaxation time  $\tau$ . This indicates that position of the plateau can be adjusted with a specific range of velocities as long as both parameters  $\tau$  and  $\xi$  are simultaneously increasing or decreasing so that the ratio  $\xi/\tau$  remains constant. If we compare Figs. 2 and 3 we can infer that the roughness exponent  $H$  has more dominant contribution on the friction coefficient  $\mu_{\text{ad}}$  than that of the lateral correlation length  $\xi$ . Therefore, any roughening procedure at short length scales ( $< \xi$ ) will lead to faster increment of frictional forces.

An intuitive procedure to understand the role of the rubber-solid substrate adhesion at short lateral length scales where contact occurs is as follows. If  $w/\xi \approx \sigma^a/E$ , then the perpendicular uniform pressure  $\sigma^a$  applied on the rubber body will be large enough to deform the rubber making contact with

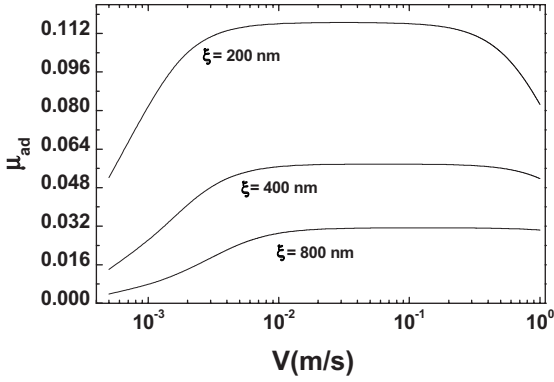


Fig. 3. Friction coefficient  $\mu_{\text{ad}}$  vs. sliding velocity  $V$  for  $H = 0.7$ ,  $w = 10$  nm,  $\tau = 10^{-3}$ , and various correlation lengths  $\xi$  as indicated.

the substrate [5,6]. If we equate the elastic energy  $U_{\text{el}} \approx E\xi w^2$  that is stored in the rubber with the gain in adhesion energy  $U_{\text{ad}} \approx -\Delta\gamma\xi^2$  (where  $-\Delta\gamma$  is the local change of surface free energy upon contact due to the rubber–substrate interaction), then we obtain  $\xi \approx (w/\xi)^{-2}(\Delta\gamma/E)$  [5,6]. For strong roughness or  $w/\xi \approx 1$  and typical parameters  $E \approx 1$  MPa and  $\Delta\gamma \approx 3$  meV/Å<sup>2</sup>, the adhesion will be able to deform the rubber and follow the substrate morphology for length scales  $\xi < 100$  nm. For smoother surfaces or  $w/\xi \approx 0.01$ , the rubber will follow the roughness profile up to a macroscopic length scale  $\approx 1$  mm. Note that ratio  $w/\xi$  represents effectively the local surface slope of the substrate roughness or  $|\nabla h| \approx w/\xi$ . This is effectively valid for large roughness exponents (e.g.,  $H \approx 1$ ) [13]. The local surface slope, for which an effective measure is the average value  $\rho_{\text{rms}} = \sqrt{\langle |\nabla h|^2 \rangle}$  depends strongly on the roughness exponent for  $H < 1$  [12,13], and it can be calculated analytically by Eq. (8) [13]

$$\begin{aligned} \rho_{\text{rms}} &= \sqrt{\int_{Q_c}^{Q_c} q^2 C(q) dq} \\ &= \left[ \frac{w}{\sqrt{2}a\xi} \right] \left[ \frac{1}{1-H} \{T_c^{1-H} - T_\lambda^{1-H}\} \right. \\ &\quad \left. + \frac{1}{H} \{T_c^{-H} - T_\lambda^{-H}\} \right]^{1/2} \end{aligned} \quad (13)$$

with  $T_c = (1 + aQ_c^2\xi^2)$ , and  $T_\lambda = (1 + aQ_\lambda^2\xi^2)$ . Therefore, a more precise estimation of the length

scales that can be followed by the rubber body should take into account the effect of the roughness exponent  $H$ .

#### 4. Conclusions

In conclusion, the adhesive coefficient of friction upon sliding onto rough self-affine surfaces strongly depends on the roughness exponent  $H$  or the degree of surface irregularity at short length scales. The effect of the latter becomes more prominent at intermediate sliding velocities where the coefficient of friction appears to be independent of the sliding velocity (plateau area). Similar, but weaker in magnitude, is the influence of correlation length  $\xi$  on the friction. At any rate, our work shows that any estimation of adhesive friction should take properly into account of the precise roughness characteristics (both at short and long lateral roughness wavelengths) of the involved substrate.

We should note that in the present work we assume that the hysteric and adhesive components of friction are separable. This is plausible approximation since the two components are based on two different mechanisms (internal energy dissipation for hysteric friction due to cyclic deformation from substrate roughness, and for adhesive friction bonding with the substrate that excerpt a force on the moving body) [6,11,14]. In addition, these two frictional components are significant for a different type of morphologies, namely, for rougher surfaces the hysteric component and for smoother surfaces the adhesive component. Nevertheless, we should keep in mind that due to the cross-linked macromolecular structure of the rubber, a blocking at the surface, due to a bond with the track, can excite the cross-linked framework [14], leading to further complications of the frictional phenomena which are not considered here.

#### Acknowledgments

I would like to acknowledge useful discussions with G. Backx, and J.Th.M. DeHosson on topics related to adhesion phenomena.

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