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Fatigue crack growth in miniature specimens: The equivalence of ΔK-correlation and that based on the change in net-section strain energy density

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ABSTRACT

In fatigue crack growth experiments of miniature specimens of Ti-6Al-4V alloy, it is shown that the stress intensity factor range (ΔK) cannot uniquely correlate the crack growth rates based on the standard stress intensity factor solution, but can only on the basis of the solution for uniformly displaced ends. It also is shown that, remarkably, a similar and strong correlation exists between *the change in the net section strain energy amplitude* and the rate of fatigue crack growth. The proposed approach is direct and simpler and it throws a new light on the nature of driving force for fatigue crack growth in structural materials.

Fatigue crack growth (FCG) experiments with miniature specimens are of interest with respect to the characterization of small-volume materials. Miniature specimens are used to evaluate biomaterials [1], the variability of properties in small components [2], and to evaluate the effects of irradiation of materials in nuclear applications [3], [4]. Because of the size limitation, it is often not possible to strictly adhere to ASTM standard specimens [5] for fatigue crack growth testing. Often in such evaluations the available stress intensity factor (K) solutions [6], developed for large specimens, are used to calculate ΔK. An open question is whether these stress intensity factor solutions will properly correlate the fatigue crack growth rates in miniature specimens.

In this study, to evaluate the ΔK-correlation of fatigue crack growth behavior in miniature single-edge-notched-tension (SENT) specimens of Ti-6Al-4V alloy, crack growth tests were performed with a wide range of stress amplitudes. The normalized fatigue crack lengths spanned a wide range: $0.1 < a/W < 0.7$, where a is crack length and W is specimen width. It is shown that the use of standard stress intensity factor solution for uniform-stress end condition leads to a large variance in the ΔK-based correlation. This variance is completely eliminated with the use of stress intensity factor solution, presented here, for uniform-displacement end conditions. Additionally, a new approach to correlate the fatigue crack growth rates in the present samples, using the change in net-section strain energy density, is demonstrated. Interestingly, this new and simpler approach provides a crack growth correlation that is, quantitatively, as strong as that of ΔK-correlation obtained using the K solution for uniformly displaced ends.

The Ti-6Al-4V alloy bars, used in making the miniature SENT samples, were made by a well-known powder metallurgical (PM) approach for making near-net-shape titanium alloys. The complete processing, microstructure details and tensile properties are given elsewhere [7]. After the PM processing, the bars were subjected to pneumatic-isostatic-forging at 860 °C to eliminate porosity and ensure full density (4.43 g/cm^3). The alloy chemistry and properties either meet or exceed the requirements of ASTM standard B348-13 for Grade 5 Ti-6Al-4V alloy. The SENT specimen geometry is shown in Fig. 1(a). The specimens were machined by electro-discharge-machining (EDM) from cylindrical bars of about 14 mm in diameter. The microstructure is shown in Fig. 1(b). A particular reason for the choice of this material is the extremely fine and uniform $\alpha + \beta$ microstructure (alpha platelets of size ~1 μm). Starter notches (~0.2–2 mm in depth) were machined by EDM to initiate the pre-cracks. Fatigue crack growth data were generated under ΔK-increasing test condition at far-field maximum stress levels of 35–500 MPa. The tests were performed at room temperature, at a stress ratio of 0.1, and at 35 Hz. Crack lengths were measured using a travelling microscope. The ΔK values were calculated using the average of crack lengths of two successive measurements. Crack growth rates were determined by the secant method.

Fig. 2(a) illustrates the fatigue crack growth data of the Ti-6Al-4V alloy correlated on the basis of the uniform-stress K solution from the

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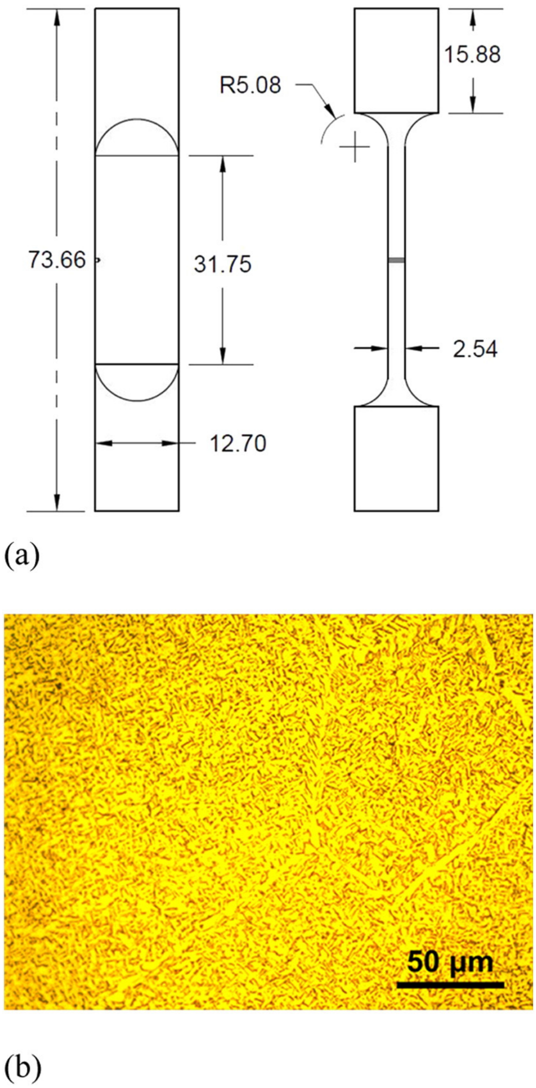

Fig. 1. (a) SENT specimen dimensions (mm) and (b) Microstructure of Ti-6Al-4V alloy.

compilation of stress intensity factors by Tada, Paris and Irwin [6]. It is quite surprising that the fatigue crack growth rates are indeed dependent on the remote maximum stress (or amplitude) level. The data seem to be stacked in layers, with higher applied stress levels producing higher growth rates, when compared at a given ΔK level of crack propagation. The crack growth rates of tests at the maximum stress levels of 35–100 MPa systematically increased with the applied stress level. The variance in growth rates is about a factor of ten at about 40 MPa√m, which is quite large. On the other hand, the crack growth rates of tests at maximum stress levels of 230–500 MPa nearly agreed with each other indicating that the ΔK-correlation, based on uniformly-stressed ends, works well at these stress levels. This is despite the fact that the K solution used does not have the effect of bending component included in it. The limitations of K solutions, arising from the boundary conditions are not addressed in fracture mechanics texts [8], [9] leading to erroneous application to characterize fatigue crack growth in miniature SENT specimens [10], [11]. The latter studies incorrectly attributed the variance in growth rates to crack closure or plane stress effects on crack growth, although there are studies [12], [13] showing the importance of specimen-end constraints on K. Further, the use of incorrect solutions could also be responsible for the apparent specimen geometry effect [14] on fatigue crack growth.

Fatigue fracture surfaces and plastic zone sizes were evaluated to assure that the present tests conformed to the linear elastic fracture mechanics (LEFM) conditions. The fatigue fracture surfaces (Fig. 2(b)) were all quite flat and most of the crack growth occurred under plane strain conditions. Table 1 summarizes the crack lengths at which FCG tests were terminated and the corresponding ΔK values, the maximum net-section stresses, and the sizes of the monotonic plane-strain plastic zones. The monotonic plastic zone sizes were calculated using Irwin's equation $((1/3\pi)(\Delta K/\sigma_{ys})^2)$. For the tests with σ_{max} values of 35–450 MPa, the plastic zones sizes at the test termination were between about one-fiftieth to about one-tenth of the specimen thickness. The maximum net-section stress levels at the test terminations, shown in Table 1, are much lower than the tensile yield stress level, which is 952 MPa. This is quite consistent with the flat fracture surfaces with negligible shear lips (Fig. 2(b)) over the crack length range for which the growth data were collected. It is therefore obvious that the tests conformed to the LEFM conditions.

It is evident from Table 1 that the crack growth data for stresses ≥230 MPa are from relatively small crack lengths (a/W < 0.5) whereas that for stresses ≤100 MPa are from relatively large crack lengths (a/W > 0.5). The finite-width correction factor for uniform-stress K solution increases rapidly for a/W > 0.5, even though it is supposed to represent the uniform stress state at the ends. A possible cause is the freedom of rotation of the ends of the specimen, which is mathematically present in this type of solution. To eliminate this problem, a K solution for uniformly displaced ends, for the present miniature specimen (height-to-width ratio, H/W ~3), was generated by the 2D boundary element method. This solution is given by

$$\Delta K = \Delta\sigma\sqrt{\pi a}\left[1.11+0.664\left(\frac{a}{W}\right)-3.52\left(\frac{a}{W}\right)^2+19.92\left(\frac{a}{W}\right)^3-33.26\left(\frac{a}{W}\right)^4+19.32\left(\frac{a}{W}\right)^5\right]. \quad (1)$$

Fig. 3(a) illustrates the fatigue crack growth data correlated using the ΔK values (ΔK_d) calculated using Eq. (1). An excellent correlation of fatigue crack growth data, at all stress levels, is seen. The success of this correlation points to the fact that the state of stress or strain energy in the net-section is perhaps controlling the crack growth. The difference between the correlations of Figs. 2(a) and 3(a) is essentially the extent of uniformity of stress in the crack plane.

In the following, we examine if a direct and simpler net-section-strain-energy approach will be able to correlate the fatigue crack growth data. Weibull [15] , [16] showed that if the net-section stress is maintained constant by continuous load decrement, the growth rates were nearly constant. McEvily and Illg concluded [17] that fatigue crack growth data could not be correlated on the basis of net-section stress alone. Weibull's' observations, however, suggest that to accelerate or to change the rate of growth of a fatigue crack, the net section stress should be changed. During fatigue crack growth, every time a new crack extension occurs, the stress in the remaining uncracked section of the crack plane increases. It is therefore logical to suppose that the *change* in the net section stress (or strain energy) that occurs with crack growth, and *not the absolute values* of net section stress (or strain energy) that is present at that instant of crack growth, is relevant to the fatigue crack growth.

We show here that there exists a strong correlation between the change in the net section strain energy and the crack growth rate, which is exactly similar to that exists in the ΔK-based correlation obtained by using Eq. (1). The partitioning of remotely applied cyclic stress amplitude, $\Delta\sigma_a$, between the cracked (a/W) and uncracked sections (1-a/W), before the formation of a crack, is

$$\Delta\sigma_a = \Delta\sigma_a\left(\frac{a}{W}\right)+\Delta\sigma_a\left(1-\frac{a}{W}\right) \quad (2)$$

where $\Delta\sigma_a$ is the remote cyclic stress range. When the fatigue crack is formed, the stress on the cracked area vanishes and that in the uncracked section will be higher than the initial stress amplitude, $\Delta\sigma_a$.

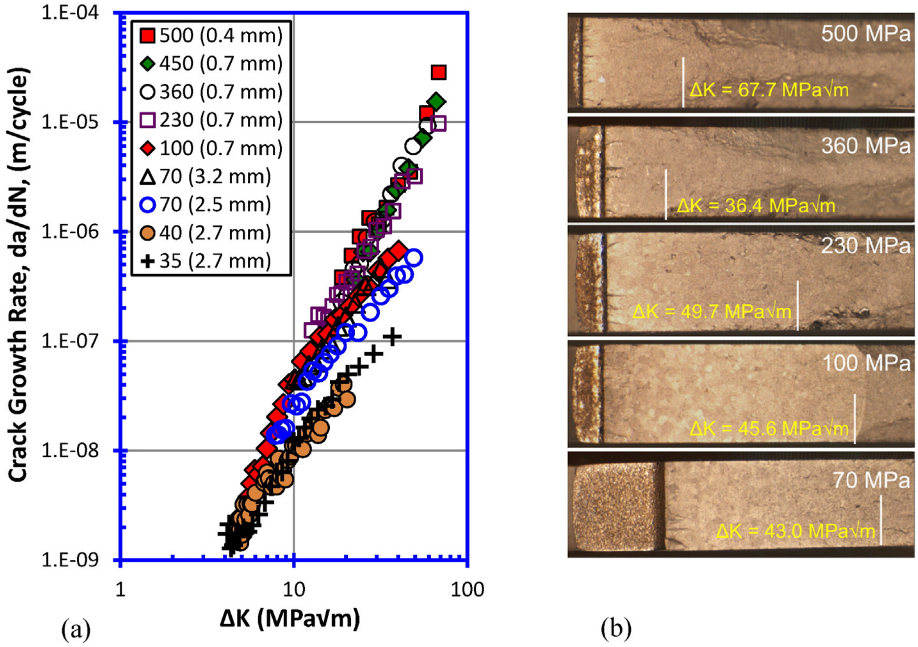

Fig. 2. (a) Fatigue crack growth rates as a function of ΔK for various far field maximum stress levels tested with R = 0.1. The numbers in parentheses indicate the pre-crack lengths. (b) Fatigue fracture surfaces at different maximum applied stress levels. The vertical bars correspond to the ΔK levels at the end of testing, as indicated.

The net stress amplitude in the uncracked section, $\Delta\sigma_l$, is given by

$$\Delta\sigma_l = \frac{\Delta\sigma_a}{\left(1-\frac{a}{W}\right)}. \tag{3}$$

After substituting for $\Delta\sigma_a$ from Eq. (2), Eq. (3) can be restructured as

$$\Delta\sigma_l = \Delta\sigma_a + \Delta\sigma_a\left(\frac{a/W}{1-a/W}\right) \tag{4}$$

At any time during the propagation of a fatigue crack, the *increase* in the net-section stress amplitude, $\Delta\sigma_{l,inc}$, from the initial stress amplitude that existed there before the formation of the crack, is just the second term on the right side of Eq. (4):

$$\Delta\sigma_{l,inc} = \Delta\sigma_a\left(\frac{a/W}{1-a/W}\right) \quad \text{or} \tag{5}$$

$$\Delta\sigma_{l,inc} = \Delta\sigma_a\left(\frac{a}{W-a}\right) \tag{6}$$

Table 1
Crack lengths and sizes of monotonic plastic zones at the end of testing.

Max. stress (MPa)	Crack length (mm)	ΔK/max. net-section stress at the end of test (MPa√m / MPa)	Plastic zone size (μm)
35	9.35	33.8/133	134
40	7.40	20.3/96	49
70	7.99	43.0/189	217
100	7.26	45.6/233	245
230	5.05	49.7/382	291
360	2.48	36.4/447	156
450	2.57	46.3/564	252
500	3.30	67.7/676	539

The correlation of the present fatigue crack growth data on the basis of the change in net-section stress, as given by Eq. (6), was not successful. We did find, however, that the corresponding change in net-section strain energy correlated with the fatigue crack growth data exceedingly well, as described below.

The elastic strain energy amplitude per unit volume of an uncracked solid is $\Delta\sigma_a^2/2E$ *(E is the elastic modulus)*. Hence, the *increase in elastic strain energy amplitude*, ΔC_a, in the uncracked net-section at the given crack length, following Eq. (6), is

$$\Delta C_a = \frac{\Delta\sigma_a^2}{2E}\left(\frac{a}{W-a}\right) \tag{7}$$

Equation (7) gives *the change in net section strain energy amplitude per unit volume* at the given crack length. Specifically, this quantity is the average change from the initial strain energy density ($\Delta\sigma_a^2/2E$) that existed in the net-section prior to the arrival of crack. The actual distribution of this strain energy density in the crack plane, however, is expected to be higher near the crack tip and diminishing with distance away from the crack tip. Nevertheless, it is logical to suppose that the extent of any new crack extension into this region should be determined by the change in the average net section strain energy density. This is because any change in crack tip stress/energy condition that would cause the actual crack extension, is related to the change in average net section strain energy.

Fig. 3(b) illustrates the present crack growth data plotted against the *average increase in net section strain energy density*, ΔC_a, as given by Eq. (7) with E = 110 GPa. It is quite remarkable to see that the fatigue crack growth data are uniquely and strongly correlated by the increase in net-section energy parameter, ΔC_a. Fig. 3(a) and (b) also show the equations that were used to fit the data. In both cases, the correlation coefficients (R^2 values) of the fits to the data are about 0.98. It is striking that the correlation on the basis of ΔC_a is very similar to and is as strong as the ΔK-based correlation obtained using Eq. (1). This perhaps suggests that crack growth rate increases linearly with an increase in net-section strain energy during fatigue crack growth. This is

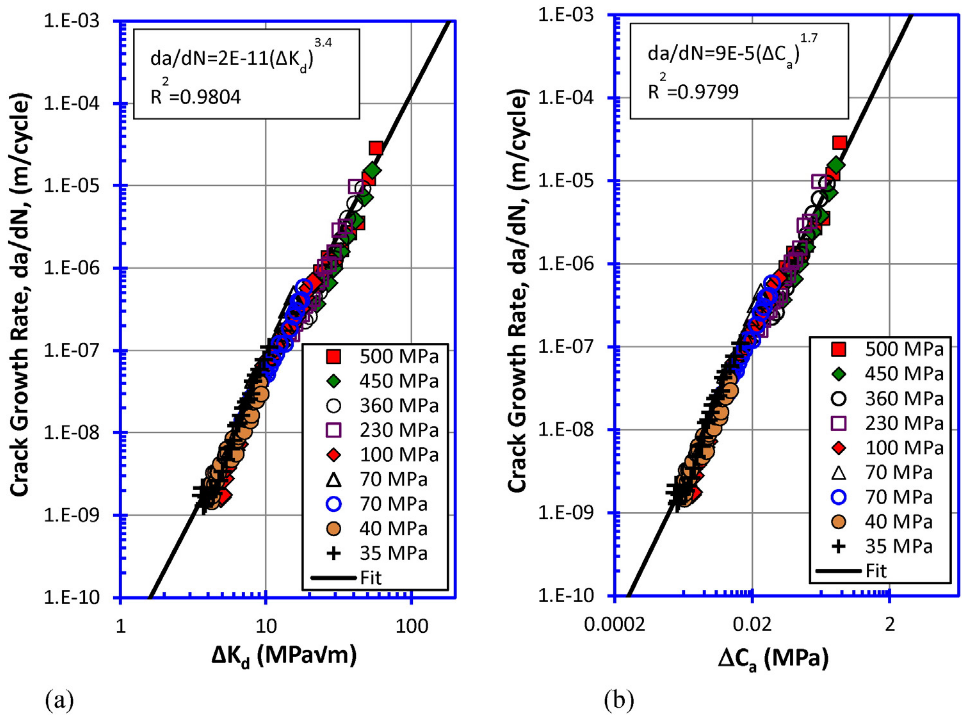

Fig. 3. The unique correlation of fatigue crack growth in terms of (a) ΔK calculated from stress intensity factor solution for uniform displacement (ΔK_d) and (b) the change in net section strain energy density amplitude, ΔC_a (Eq. 7). The equations fitted to the data and the correlation coefficients of the fit are given.

understandable because, as shown by Irwin, the fracture mechanics parameter K is related to Griffith's strain energy release rate, G. The change in net section strain energy here corresponds to the change in strain energy at the specimen-loading end. At zero crack length Eq. (7) reduces to zero, as it should, corresponding to the cyclic strain energy of a crack-free specimen. This raises the possibility that the net section strain energy density concept is applicable to short cracks—however, this needs to be experimentally validated. An extension of ΔK to zero crack length is not possible because K is inapplicable for a crack-free specimen.

It is instructive to note that the change in net section strain energy is determined without regard to the loading boundary condition—that is, it is independent of whether the specimen is uniformly stressed or uniformly displaced at the ends. However, fracture mechanics based solutions are dependent on the end conditions—the finite-width correction factors in essence provide the corrections for the K solution to account for the increase in stress due to the increase in crack length as well as to account for the nature of stress distribution (tension or bending or both). It is also important to note that the numerical K values for the uniformly displaced SENT specimens given by Bowie et al. [12] as well as others [13] show a dependence on the height-to-width (H/W) ratio. It would be very interesting to examine if there is indeed an effect of the specimen H/W ratio on the fatigue crack growth rates in miniature or large SENT specimens.

In summary, it is shown that a correct ΔK-correlation of fatigue crack growth rates in miniature SENT samples requires the correct stress intensity factor solution for uniformly displaced end conditions, which is common with miniature tests. It is also shown that an exactly equivalent correlation of crack growth rates can be obtained by a new and simpler approach, that is, by using the change in net section strain energy amplitude with crack extension. It would be extremely interesting to examine if the concept of the change in net section strain energy would work well for other specimen aspect ratios and geometries. Further work in this direction is in progress.

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