



On the improvement of combined fuzzy topological and directional relations information

Nadeem Salamat*, El-hadi Zahzah

Laboratoire de Mathématiques, Images et Applications, Université de La Rochelle, France

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ABSTRACT

Concept of combined extraction of topological and directional relations information developed by Zahzah et al. [1] by employing the Allen's temporal relations in 1D spatial domain was improved by Matsakis and Nikitenko [2]. This latter algorithm has high computational complexity due to its limitations of object approximation and segment fuzzification.

In this paper, fuzzy Allen relations are used to define the fuzzy topological and directional relations information between different objects. Some extended results of Salamat and Zahzah [3] are discussed. Polygonal object approximation allows us to use fuzzy operators and this approach reduces computational complexity of the method for computing the combined topological and directional relations. To validate the method, some experiments are tested giving satisfactory and promising results. Affine transformation are depicted, these properties will be helpful for using the method in other areas of image analysis such as object tracking.

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1. Introduction

One of the fundamental tenet of modern sciences is that a phenomenon cannot be claimed to be well understood, until it is characterized in quantitative manners. Advent of digital computers have determined an expansion in the use of quantitative methods where as vagueness and imprecise knowledge information give rise to fuzzy methods. Spatial relations belong to these categories. Spatial relations are used for content based image retrieval (CBIR), pattern recognition, database management, artificial intelligence (AI), cognitive science, perceptual psychology, robotics, linguistics expressions, medical imaging, image and video analysis [4–8]. Reasoning on spatial objects needs support from representation of topological, directional, ordinal, distance relations, object size and shape.

Topological relations are derived from geometric descriptions, while an uncertain relation is a relation which exists with a certain probability. The well-known models for finding topological relations between spatial regions are the 9-intersections [9] and Region Connection Calculus (RCC) model [10].

These theories are extended to deal with fuzzy objects [11–13], in both theories, objects are modeled as fuzzy sets in $\mathbb{R}^2$. In 9-intersections method, 44 useful topological relations were developed

between simple fuzzy regions. This method is also extended to 16 intersections and a set of 152 useful topological configurations between simple fuzzy regions in $\mathbb{R}^2$ are realized [14]. This theory depends upon point set topology and simple set operators are used for modeling topological relations. This theory cannot be extended to deal fuzziness at relation's level. In RCC theory number of fuzzy relations are 46. All of these relations represent the fuzziness at object's level. Fuzziness may be present in relation's semantic, in this theory this effort is made by Schockaert et al. [15] where connection relation based on closeness is defined. Different other approaches are adopted for representing the positional information as distance and orientation.

The methods for directional relations represent such information and fuzziness in relation's semantic are discussed in [16–20]. Most of these methods work only for a particularly *disjoint* topological relation and stop working as soon as objects *meet* or in some cases *overlap*. In the methods based on force histograms [20], 2D areal objects are represented by union of 1D segments.

The 9-intersections model for topological relations and the RCC theory have a rich support for the topological relations [21,22] and provide information about a topological relation without providing information that where a topological relation exists in the space? Consider for instance that two objects overlap, both of theories provide information about the topological relation overlap, but they do not give any information about where in space the two objects overlap. As object *A* overlaps object *B* from north or north_west or west, or other topological relation, object *A* disjoint from object *B* and lies in south of object *B*. To know the relative

* Corresponding author.

E-mail addresses: nsalam01@univ-lr.fr (N. Salamat), ezahzah@univ-lr.fr (E.-h. Zahzah).

position of object A according to object B , we have to apply another method type, because qualitative methods provide information regarding the extended objects and they do not care about the topological relation.

Allen [23] introduced 13 interval relations in temporal domain. These relations are commonly used to represent the knowledge in artificial intelligence. Different approaches for fuzzification of Allen temporal relations are proposed such as in [24]. These temporal Allen relations are applied into the spatial domain due to homeomorphism between the temporal domain and 1D spatial domain. Allen relations represent the topological relations in $\mathbb{R}$. These relations are used to answer the question that where in space a certain topological relation holds?

Allen relations are applied in a 2D space by decomposition of a two-dimensional object into 1D parallel segments. This decomposition process is repeated in all directions and method is applied to get combined topological and directional relations data. Combined topological and directional relations were first introduced by Malki et al. [1] by using the 1D Allen relations. Allen relations divide the whole space into 13 partitions and each partition corresponds to an Allen relation. This method was further improved by Matsakis and Nikitenko [2] and fuzzification was introduced at relations level.

This latter method is costly regarding time constraint. The aim of this paper is to reduce time consuming for the computation of combined fuzzy Allen relations information developed in [2]. Time is reduced in two ways, reducing and simplifying the number of computations and suggest an alternative way for the treatment of longitudinal section. As time constraint depends upon number of segments to be treated, to reduce the number of segments a polygonal object approximation is used. Algorithm for fuzzification of longitudinal section is replaced by fuzzy connectors. This approach decreases its time complexity from $O(nM\sqrt{M})$ to $O(nN \log(N))$, where n represents number of directions, N number of vertices of polygons and M number of pixels to be processed.

This paper is arranged as follows, next section describes some preliminary definitions. Section 2 describes the terminology used, Allen relations in 1D space, fuzzification of Allen relations and finally the definition of a histogram of fuzzy Allen relations. Fuzzy logic connectors are discussed in Section 3, these fuzzy operators are used for the treatment of a longitudinal section. Different affine transformations are discussed in Section 4, experiments and interpretation of results are described in Section 5. Time complexity for both methods is compared in Sections 6 and 7 concludes the paper.

1.1. Preliminary definitions

In this section we recall some basic definitions which are frequently used throughout remainder of the paper.

Fuzzy membership function: A membership function μ in a set X is a function $\mu: X \rightarrow [0, 1]$. Different fuzzy membership functions are proposed according to the requirements of the applications. For instance, Trapezoidal membership function is defined as

$$\mu(x; \alpha, \beta, \gamma, \delta) = \max\left(\min\left(\frac{x-\alpha}{\beta-\alpha}, 1, \frac{\delta-x}{\delta-\gamma}\right), 0\right) \quad (1)$$

it is written as $\mu_{(\alpha, \beta, \gamma, \delta)}(x)$ where $x, \alpha, \beta, \gamma, \delta \in \mathbb{R}$ and $\alpha < \beta \leq \gamma < \delta$.

Fuzzy set: A fuzzy set A in a set X is a set of pairs $(X, \mu_A(x))$ such that $A = \{(x, \mu(x)) | x \in X\}$ where μ represents a membership function.

Force histogram: The force histogram attaches a weight to the argument object A that this lies *after* B in direction θ , it is defined as

$$\mathbf{F}^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta, A_\theta(v), B_\theta(v)) dv \quad (2)$$

The definition of Force histogram, $\mathbf{F}^{AB}(\theta)$, directly depends upon the definition of real valued functions ϕ, f and F used for the treatment

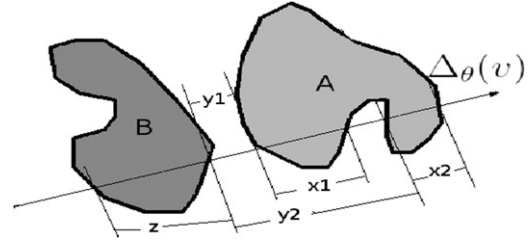


Fig. 1. Oriented line, segment and longitudinal section.

of points, segments and longitudinal sections, respectively [20]. These functions are defined as

$$\left. \begin{aligned} \phi_r(y) &= \begin{cases} \frac{1}{y^r} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases} \\ f(x_i, y_{ij}^0, z_j) &= \int_{x_i + y_{ij}^0}^{x_i + y_{ij}^0 + z_j} \int_0^{z_j} \phi(u-w) dw du \\ F(\theta, A_\theta(v), B_\theta(v)) &= \sum_{i=1, \dots, n, j=1, \dots, m} f(x_{ii}, y_{ij}^0, z_{jj}) \end{aligned} \right\} \quad (3)$$

where n, m represents the number of segments of object A and object B respectively and variables (x, y, z) are explained in Fig. 1.

2. Histograms of Fuzzy Allen Relations

In this section, certain terms used for computations are explained. Drawing of oriented lines, segments and longitudinal sections are explained in Section 2.1 and Allen relations in Section 2.2. Fuzzification is explained in Section 2.3, computation of histogram of fuzzy Allen relations are explained in Section 2.4 and treatment of longitudinal section is explained in Section 2.5.

2.1. Oriented lines, segments and longitudinal sections

Let A and B be two objects and $(v, \theta) \in \mathbb{R}$, where v is any real number and $\theta \in [-\pi, \pi]$. Let $A_\theta(v)$ be an oriented line at orientation angle θ and $A \cap A_\theta(v)$ is the intersection of object A and oriented line $A_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A . If there exist more than one segment then it is called longitudinal section as in case of Fig. 1 where $A \cap A_\theta(v)$ has two segments, length of its projection points on x -axis is x . Similarly for object B where $B \cap A_\theta(v) = B_\theta(v)$ is segment and z is length of its projection points. The variable y is the difference between the minimum of projection points of $A \cap A_\theta(v)$ and maximum of projection points of $B \cap A_\theta(v)$.

2.2. Allen relations in space

Allen [23] introduced 13 mutually exclusive and exhaustive interval relations. These relations are arranged as $\mathcal{A} = \{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$, where $\{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$ are the relations *before*, *meet*, *overlap*, *start*, *finish*, *during* (resp the inverse relations of the cited ones). The relation *eq* is the equality spatial relation. All the Allen relations in space are conceptually illustrated in Fig. 2. These relations have a rich support for the topological and directional relations.

2.3. Fuzzification of Allen relations

Different approaches are used for fuzzification of Allen temporal relations. Some of them use the fuzzification based on the human defined variables and fuzzification is described only for use in temporal domain, for the qualitative aspects of temporal knowledge and qualitative temporal reasoning processes. There is

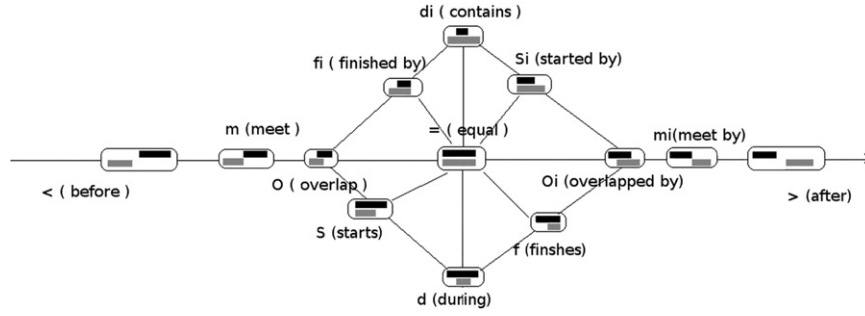


Fig. 2. Black segments represent the reference objects and gray segments represent the argument objects.

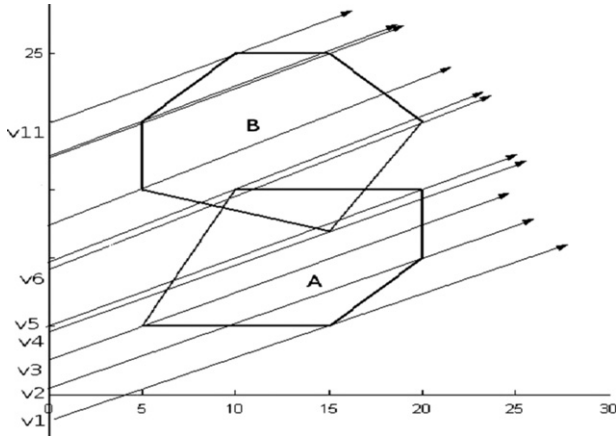


Fig. 3. Showing the computation of relations by an example where each line passes through the vertex of a polygon and inclination angle is fixed at 30° .

a homeomorphism between the Allen's temporal relations and 1D topological relations in spatial domain. Due to this homeomorphism, Allen relations are also used for extracting the combined topological and directional relations information [1,2]. Fuzzy Allen relations are used to represent the fuzzy topological relations where fuzziness is represented at the relation's level.

Fuzzification of Allen relations doesn't depend upon particular choice of fuzzy membership function. Trapezoidal membership function (Eq. (1)) is used due to flexibility in shape. Let $r(I, J)$ be an Allen relation between segments I and J where $I \in A$ (argument object) $J \in B$ (reference object), r' is the distance between $r(I, J)$ and it's conceptional neighborhood. We consider a fuzzy membership function $\mu : r' \rightarrow [0, 1]$, where μ is a trapezoidal membership function defined in Eq. (1). The fuzzy Allen relations defined by Matsakis and Nikitenko [2] are

$$\left. \begin{aligned} f_{<}(I, J) &= \mu_{(-\infty, -\infty, -b-3a/2, -b-a)}(Y) \\ f_{>}(I, J) &= \mu_{(0, a/2, \infty, \infty)}(Y) \\ f_m(I, J) &= \mu_{(-b-3a/2, -b-a, -b-a, -b-a/2)}(Y) \\ f_{mi}(I, J) &= \mu_{(-a/2, 0, 0, a/2)}(Y) \\ f_o(I, J) &= \mu_{(-b-a, -b-a/2, -b-a/2, -b)}(Y) \\ f_{oi}(I, J) &= \mu_{(-a, -a/2, -a/2, 0)}(Y) \\ f_f(I, J) &= \min(\mu_{(-(b+a)/2, -a, -a, +\infty)}(Y), \mu_{(-3a/2, -a, -a, -a/2)}(Y), \mu_{(-\infty, -\infty, z/2, z)}(X)) \\ f_{fi}(I, J) &= \min(\mu_{(-b-a/2, -b, -b, -b+a/2)}(Y), \mu_{(-\infty, -\infty, -b, -(b+a)/2)}(Y), \mu_{(z, 2z, +\infty, +\infty)}(X)) \\ f_{si}(I, J) &= \min(\mu_{(-b-a/2, -b, -b, -b+a/2)}(Y), \mu_{(-\infty, -\infty, -b, -(b+a)/2)}(Y), \mu_{(-\infty, -\infty, z/2, z)}(X)) \\ f_s(I, J) &= \min(\mu_{(-(b+a)/2, -a, -a, +\infty)}(Y), \mu_{(-3a/2, -a, -a, -a/2)}(Y), \mu_{(z, 2z, +\infty, +\infty)}(X)) \\ f_d(I, J) &= \min(\mu_{(-b, -b+a/2, -3a/2, -a)}(Y), \mu_{(-\infty, -\infty, z/2, z)}(X)) \\ f_{di}(I, J) &= \min(\mu_{(-b, -b+a/2, -3a/2, -a)}(Y), \mu_{(z, 2z, +\infty, +\infty)}(X)) \end{aligned} \right\} \quad (4)$$

where $a = \min(x, z)$, $b = \max(x, z)$, x is the length of segment (I), z is the length of segment (J) and (x, y, z) are computed as given in Section 2.1.

Most of the relations are defined by a single membership function and some of them by minimum of multiple membership functions like $d(\text{during})$, $d_i(\text{during by})$, $f(\text{finish})$, $f_i(\text{finished by})$. Two relations are directly neighbors if the fuzzy Allen relations are shared between them. If $0 < r_1(I, J) < 1$ then $0 < r_2(I, J) < 1$ such that $r_1(I, J) + r_2(I, J) = 1$ and $r_1(I, J)$, $r_2(I, J)$ are neighbors expressed in the neighborhood graph (Fig. 2). This shows that sum of all the Allen relations is always one, from this $\text{equal}(f = (I, J))$ relation is defined.

Histogram of fuzzy Allen relations represent the total area of subregions of A and B that are facing each other in a given direction θ [2]. Formally, this definition is written as

$$F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta, A_\theta(v), B_\theta(v)) dv \quad (5)$$

$$F_r(\theta, A_\theta(v), B_\theta(v)) = \frac{x+z}{w} \sum_{k=1}^c \sum_{i=1}^{m_k} \sum_{j=1}^{n_k} [x_i^k z_j^k (a_k - a_{k-1})] r(I_i^k J_j^k) \quad (6)$$

where $F_r^{AB}(\theta)$ represents the histogram of a fuzzy Allen relation in direction θ and $F_r(\theta, A_\theta(v), B_\theta(v))$ is the representation of histogram for a given v where

$$w = \sum_{k=1}^c \sum_{i=1}^{m_k} \sum_{j=1}^{n_k} [x_i^k z_j^k (a_k - a_{k-1})], \quad x = \sum_{i=1}^{m_k} x_i^k \quad \text{and} \quad z = \sum_{j=1}^{n_k} z_j^k$$

In this case m_k and n_k represent the total number of segments of argument object A and reference object B and c represents number algorithm loop for fuzzification of a longitudinal section (algorithm is discussed in Section 6).

2.4. Histogram of fuzzy Allen relations

Fuzzy Allen relation for each segment is a fuzzy set and fuzzy aggregation operators are used to combine different values of fuzzy grades. In polygonal object approximation, fuzzy Allen relations are calculated for a limited number of segments and within this region spatial relations do not change, so simply generalize the given relations. This technique minimizes the number of segments to be treated, decreasing the temporal complexity of algorithm. This changes mathematical equation (Eq. (6)) for histogram of fuzzy Allen relations. Formally

$$F_r(\theta, A_\theta(v), B_\theta(v)) = r(I, J) \quad (7)$$

In discrete space, the integral can be written as a finite sum, thus above Eq. (5) for discrete space can be rewritten as

$$F_r^{AB}(\theta) = (X+Z) \sum_{k=1}^n r(I_k J_k) \quad (8)$$

where Z is the area of reference object and X is area of argument object in direction θ , n is total number of segments to be treated and $r(I_k J_k)$ is an Allen relation for segment pair $(I_k J_k)$ and $F_r(\theta, A_\theta(v), B_\theta(v))$ represents a histogram of a fuzzy Allen relation.

2.5. Treatment of longitudinal section

During the decomposition process of an object into segments, there can be multiple segments for a line $\Delta_\theta(v)$ depending on object shape and boundary which is called longitudinal section. Different segments of a longitudinal section are at a certain distance and these distances might affect final results. In this paper, we adopt the force histograms as in [20]. Here, we replace the sum operator in Eq. (3) by a fuzzy operator and use the logic connectors, i.e.

$$F_r(\theta, A_\theta(v), B_\theta(v)) = \odot (f_r(x_1, y_1^0, z), f_r(x_2, y_2^0, z), \dots, f_r(x_n, y_n^0, z))$$

where $\odot$ is a fuzzy operator and $r \in \mathcal{A}$, $\mathcal{A}$ is denoted in Section 2.2.

2.6. Example

We introduce an example to explain different steps of the method of combining topological and directional relations. This better explains difference between method developed by Matsakis and Nikitenko [2] and the method we proposed in this paper. Let us consider two simple objects for the computation of histogram of fuzzy Allen relations. Let these simple polygonal objects be A and B where $A = \{(5, 5), (15, 5), (20, 10), (20, 15), (10, 15)\}$ and $B = \{(5, 15), (15, 12), (20, 20), (15, 25), (10, 25), (5, 20)\}$ are the vertices of polygons, it is explained in Fig. (3). Computing the histograms of Allen relation for objects A and B by our method involves the following steps:

1. Compute the boundary of both objects, for object A , line segments joining the vertices of polygon A , in a similar way for object B .
2. Fix the angle, let it be 30° and draw lines passing through the vertices of polygons, in our case, total number of vertices are 11 there will be 11 lines, each line passes through a vertex of a polygon.
3. Compute the intersection of line with boundary of an object, each line has one or two intersecting points with one object due to simple concave objects. In case of convex or objects with holes there may be more than two intersecting points. Consider only those lines which intersect both objects, for all the other lines relations are zero. As in Fig. (3) only $\Delta_{30}(v4), \Delta_{30}(v5), \Delta_{30}(v6), \Delta_{30}(v7)$ lines intersect both objects, for other lines fuzzy Allen relations are zero. These intersecting points are:

$$\begin{aligned} \Delta_{30}(v4) \cap A &= \{(6, 6), (20, 14)\}, \Delta_{30}(v5) \cap A = \{(6, 7), (20, 15)\} \\ \Delta_{30}(v6) \cap A &= \{(9, 13), (12, 15)\}, \Delta_{30}(v7) \cap A = \{(10, 15)\} \\ \Delta_{30}(v4) \cap B &= \{(15, 12)\}, \Delta_{30}(v5) \cap B = \{(15, 12), (16, 13)\} \\ \Delta_{30}(v6) \cap B &= \{(9, 13), (20, 20)\}, \Delta_{30}(v7) \cap B \\ &= \{(8, 14), (19, 20)\} \end{aligned}$$

4. Take the projection (P) of these points on x -axis and calculate the values of (x, y, z) . Here, for line $\Delta_{30}(v4)$ these values are $P(\Delta_{30}(v4) \cap A) = \{6, 20\}$ and $P(\Delta_{30}(v4) \cap B) = \{15\}$, from this we get $(x, y, z) = (14, -9, 0)$.
5. Compute the fuzzy Allen relation for each segment by using Eq. (4). This system of equations will provide us the following results for the aforementioned (x, y, z) (segments obtained by taking intersection of objects with line $\Delta_{30}(v4)$).

$$\begin{aligned} f_{>}(I_4 J_4) &= 0, f_{<}(I_4 J_4) = 0, f_m(I_4 J_4) = 0, f_{mi}(I_4 J_4) = 0 \\ f_o(I_4 J_4) &= 0, f_{oi}(I_4 J_4) = 0, f_f(I_4 J_4) = 0, f_{fi}(I_4 J_4) = 0 \\ f_s(I_4 J_4) &= 0, f_{si}(I_4 J_4) = 0, f_d(I_4 J_4) = 0, f_{di}(I_4 J_4) = 1 \\ \text{and } f_{=}(I_4 J_4) &= 0 \end{aligned}$$

6. Repeat this process for lines $\Delta_{30}(v5), \Delta_{30}(v6)$ and $\Delta_{30}(v7)$ (for other lines relations are already zero), sum all the relations and multiply the resultant relation by the sum of surface

areas of two objects between line $\Delta_{30}(v4)$ and line $\Delta_{30}(v7)$. This will provide the number of pixels of both objects under a specific fuzzy Allen relation and for normalization, divide each fuzzy Allen by the sum of all the Allen relations for $\theta = 30^\circ$, this will give us the percentage area of two objects under a fuzzy Allen relation.

7. Increase the angle by 1° and repeat the above steps 2–6.

We calculate the values (x, y, z) for Matsakis and Nikitenko [2]. In this method objects are considered as regular closed sets. The difference between methods is the way to compute the triplet (x, y, z) and object approximation:

1. Fix the angle θ and draw the pencil of oriented lines $\Delta_\theta(v)$, in this case number of lines will be much greater than 11 due to pencil of lines.
2. Compute the intersection of a oriented line with both objects A and B . Let us consider the same line $\Delta_{30}(v4)$. Then $\Delta_{30}(v4) \cap A = \{(6, 6), (7, 7), (8, 7), (9, 8), (10, 8), (10, 9), (11, 9), (12, 10), (13, 10), (14, 11), (15, 11), (15, 12), (16, 12), (16, 13), (17, 13), (18, 13), (18, 14), (19, 14), (20, 14)\}$. Similarly for object B , $\Delta_{30}(v4) \cap B = \{(15, 12)\}$. Take projections on x -axis and compute the values of (x, y, z) , here the values will be the same as in above case and produce the same results, i.e., $x = 14, y = -9, z = 0$, x and z are the diameter of point sets on real line and y is the difference between the minimum of argument point set and maximum of reference point set. Remaining process of computation is identical to our method.
3. Compute Allen relations for every line that intersect the objects. Pencil of lines is created then same calculations are made for every line.

This example elaborates the difference between two methods, and how the new proposed method reduces the time for computation, by simplifying and limiting the number of intersections in one line and an object. Secondly, it decreases the number of lines to be treated for one direction.

3. Fuzzy logic connectors

In decision theory, there arise some situations where solution depends upon combination of different information provided by different sources. In such a situation, fuzzy connectors are used. In this section, different fuzzy aggregation operators (fuzzy logic connectors) are studied. Aggregation refers to combining values into the one aggregated value so that the final solution seems to be well addressed in a given fashion [25,26]. It is a mapping $\tau: [0, 1]^n \rightarrow [0, 1]$, which combines different fuzzy grades. In the literature of fuzzy set theory there exist a variety of operators such as fuzzy T -norms, T -conorms and so on, some commonly used operators are:

- Fuzzy OR: $\mu_{OR}(x) = \max(\mu_A(x), \mu_B(x))$;
- Fuzzy AND: $\mu_{AND}(x) = \min(\mu_A(x), \mu_B(x))$;
- Fuzzy Algebraic Product: $\mu_{PROD}(x) = \prod_{i=1}^n \mu_{i0}(x)$;
- Fuzzy Algebraic Sum: $\mu_{SUM}(x) = 1 - \prod_{i=1}^n (1 - \mu_i(x))$;
- Fuzzy γ operator: $\mu_\gamma(x) = [\mu_{SUM}(x)]^\gamma * [(\mu_{PROD}(x))]^{1-\gamma}$ where $\gamma \in [0, 1]$.

The OR operator is actually the *union* or *max* operator, while the AND is *intersection* or *min* operator. The contribution for resultant of OR (AND) fuzzy operator is a single input value, which is *maximum* (*minimum*). For other operators, both values contribute. The fuzzy algebraic sum (*product*) operator makes the set result larger than or equal to maximum (resp less than or equal to minimum) the

contributing values while Fuzzy γ operator changes the result value from minimum to maximum values depending on the choice of γ . Application of these operators is explained in Appendix A. In this example, t -norms and t -conorms are explained.

4. Affine properties of histograms of fuzzy Allen relations

Affine properties are important in the pattern recognition especially object matching in a scene analysis. These properties of histograms of fuzzy Allen relations are depicted below which are independent from fuzzy membership functions.

- Object commutativity: Pair (A, B) be assessable, let r be a fuzzy Allen relations and for all relations, except the relation *during*(d) and *during_by*(d_i), $F_r^A B(\theta) = F_r^B A(\theta + \pi)$. For relation *during* and *during_by* we have $F_d^{A,B}(\theta) = F_{d_i}^{B,A}(\theta)$.
- Orthogonal symmetry: Let orthogonal symmetry denoted by *sym* about the oriented line with slope α , then histogram of fuzzy Allen relations: $F_r^{sym(A),sym(B)}(\theta) = F_r^{A,B}(2\alpha - \theta)$.
- Central dilation: Let central dilation(scale) denoted by *dil* and λ is dilation ratio, then $F_r^{dil(A),dil(B)}(\theta) = \lambda^2 F_r^{A,B}(\theta)$.
- Stretch: Let stretch (*stre*) is orthogonal to x axis and k is stretch ratio, then histogram of fuzzy Allen relations satisfies. $F_r^{stre(A),stre(B)}(\theta) = k F_r^{A,B}(\theta)$.
- Translation: Let translation denoted by *trans* and $(trans(A), trans(B))$ is assessable then following relation holds for histogram of fuzzy Allen relations. $F_r^{trans(A),trans(B)}(\theta) = F_r^{A,B}(\theta)$.
- Rotation: Let rotation (*rot*) be a ρ -angle rotation and $(rot(A), rot(B))$ is assessable hence $F_r^{rot(A),rot(B)}(\theta) = F_r^{A,B}(\theta - \rho)$.

5. Comparison with others works and interpretation

In this section first we give the interpretation of results representation and then compare our approach with Matsakis and Nikitenko method [2]. For the experiment purpose, 360 directions are considered. Instead of drawing pencil of lines in a direction, only those lines are considered which passes through vertices of polygons. Fuzzy Allen relations are computed for each segment, if there exit longitudinal section, then fuzzy aggregation operator is applied to obtain the resultant fuzzy Allen relation for an oriented line.

Each relation is associated with the grey scale value like *before* with black, white color represents *after*, each relation has a

different boundary color for better visualization of relations while opposite relations have the same boundary color such as *m*(*meet*) and *mi*(*meet_by*) relations have the yellow boundary color. Interpretation of the grey level association to a relation and its boundary color is given in Fig. 4(a). Object *A* has the light grey color and object *B* is represented by dark grey. The 13 histograms for directional and topological relations are represented by layers and each vertical layer represents the total area of objects in specific direction.

These histograms are normalized as $A_j(\theta) = A_j(\theta) / \sum_{i=1}^{13} A_i(\theta)$ where $A_j \forall j = 1, \dots, 13$ is an Allen relation. This technique of normalization provides us the percentage area of both objects under a histogram of fuzzy Allen relation. In this example (figures in Table 1) first row represents the object pair and its histograms which are not normalized, computed by our method and method developed by Matsakis and Nikitenko (for Matsakis and Nikitenko method we copied the figure directly from [2]). The second row represents normalized histogram of fuzzy Allen relations with the respective method. Both of the histograms seem similar, a small difference in the shape represents the small change in the total surface area of two objects under the specific fuzzy Allen relation, which is less important.

In this set of examples (figures in Table 2), histograms for fuzzy Allen relations are compared with the method of Matsakis and Nikitenko [2]. Where the first column represents object pairs at different distances, second column represents computation of histograms of fuzzy Allen relations by our method and in third column histograms are represented computed by Matsakis and Nikitenko method. These histograms of fuzzy Allen relations are distance dependant. As the objects become closer, the new histogram for fuzzy Allen relation *meet* and *meet_by* emerges.

In both methods, similar histograms exist, there is a small difference in their shapes. Histogram shape represents the total area under the specific relation, as we consider object by the polygonal approximation and due to this approximation, there is a small change in object area under a particular histogram.

In this example (figures are shown in Table 3), polygonal objects are considered. First, objects are at certain distances, only *after* and *before* relation exist. Histogram resembles with the histogram computed by the Matsakis and Nikitenko method. Sharp and sudden changes in the histogram shape computed by our method are due to the problems of taking intersections in the 2D digital space. Empty intersection sometimes results a change in the object area (sharp ups and downs in histogram shape in first row) and sometimes it causes the existence of new spatial relation such as histogram in the *after* and *before* relations in third row.

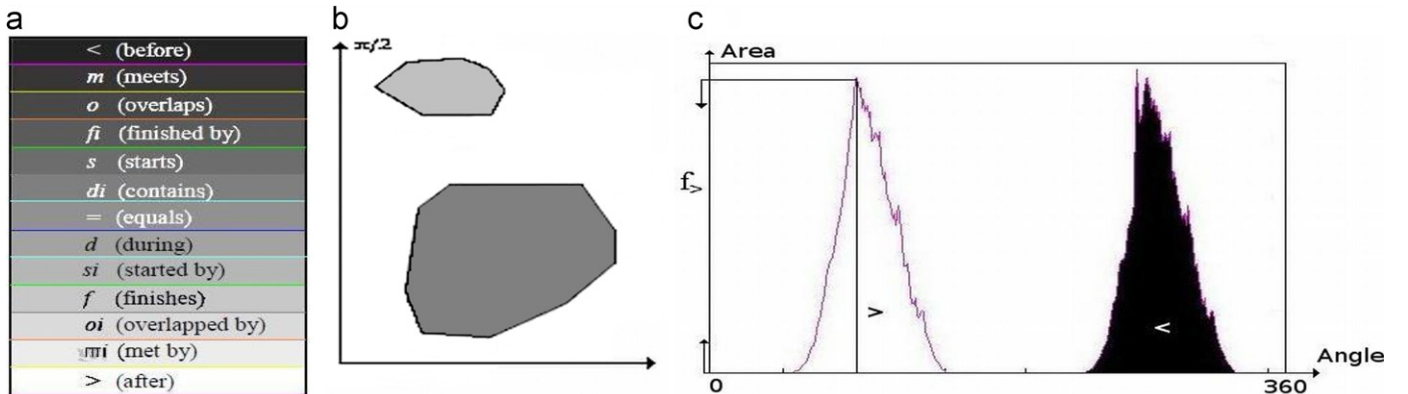


Fig. 4. (a) Histogram representation for fuzzy Allen relations, (b) object pair representation and (c) represents histogram of fuzzy Allen relations for object pair in (b). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1

Comparison of normalized and un-normalized histograms of fuzzy Allen relations with the Matsakis and Nikitenko and our method (axes are same as in Fig. 4(b) and (c)).

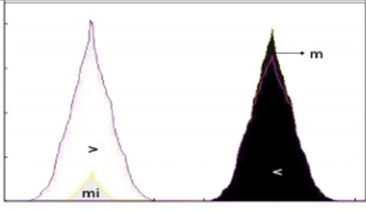
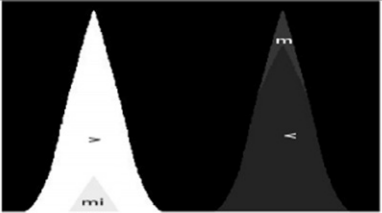
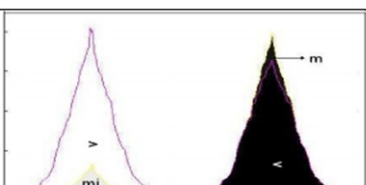
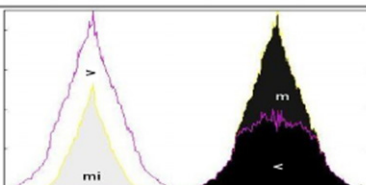
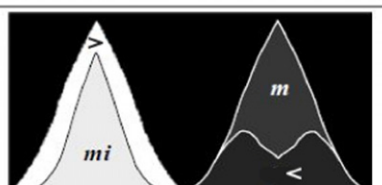
Obj. pairs	Our method	M & N method
		
		

Table 2

Comparison of histograms for rectangular objects (axes are same as in Fig. 4(b) and (c)).

Obj. pairs	Our method	M & N method
		
		
		

For the example Fig. 5(a)–(c), concave object is considered. Fig. 5(a) represents object pair, histogram in Fig. 5(b) is computed by our method and fuzzy operator OR is used. Histogram represented in Fig. 5(c) is computed by the method developed by Matsakis and Nikitenko. In certain direction object A is *before* as well as *after*, in the meanwhile there exist relation *meet* and *meet_by* due to closeness of two segments. In our method, sharp change in histogram is due to the change in surface area of objects due to the digitization process. This example shows that two segments of an object have the opposite fuzzy Allen relations (a case considered in Fig. 7(b)).

In this set of examples (from Fig. 6(a)–(g)), argument object is a concave object. As it changes its position, one segment changes its topological and directional relation with respect to argument segment. The other segment does not change its topological

relation. As a result both segments have opposite Allen relations with the reference segment and resulting histogram represents both relations at the same time.

Examples given above show that histograms of fuzzy Allen relations are approximately the same while they are computed either with the help of Matsakis and Nikitenko [2] method or polygonal object approximation. Our approach has a certain amount of decrease in the computation time. Computation time with our approach drops from $O(nM\sqrt{M})$ to $O(nN \log(N))$, where N represents the total number of vertices of polygons. For the treatment of longitudinal section, when there exist more than one segment, fuzzy Allen relations are computed between every pair of segments of reference and argument objects, then at the next stage fuzzy OR operator is used to integrate the available fuzzy Allen relations.

Table 3

Comparison of histograms of fuzzy Allen relations with the Matsakis and Nikitenko method and our method for the polygonal objects (axes are same as in Fig. 4(b) and (c)).

Obj. pairs	Our method	M & N method

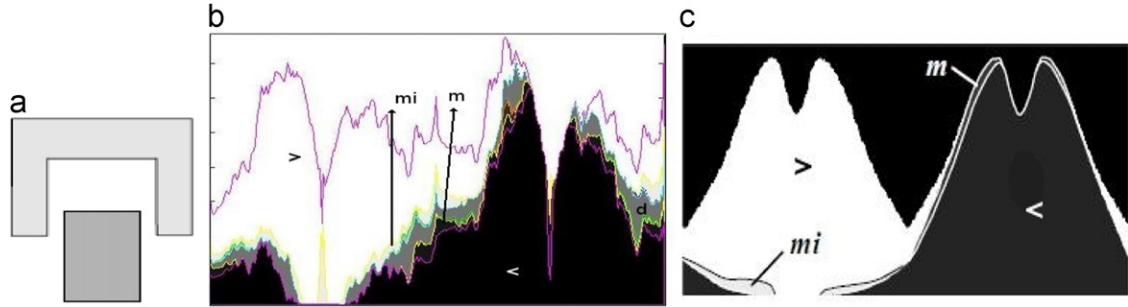


Fig. 5. (a) Object pair, histograms of fuzzy Allen relations with our method in (b) and Matsakis and Nikitenko method in (c) (axes are same as in Fig. 4(b) and (c)).

6. Complexity

Algorithm efficiency can be measured in terms of execution time. Method for finding the combined topological and directional relations information is to couple the force histograms with the fuzzy Allen relations. Time complexity for computing the combined topological and directional relations information depends upon the following three factors:

1. Object approximation.
2. Algorithm for treatment of longitudinal section.
3. Equation used for computation of histograms (Eqs. (6) and (7)).

These different aspects of time complexity are discussed separately here below. We compare time complexity of both algorithms and at the end we note that there is a sufficient decrease in the execution time of the modified algorithm.

Time complexity due to object approximation: Objects are approximated as closed regular sets or by polygonal representation. Method

for raster data starts directly with the scan of lines. For polygonal approximation, the methods first determines the polygonal representation of object and polygons vertices. Computation time for the polygon representation depends upon the algorithm used for polygon representation. For example, we use the method discussed in [28], time complexity of this method is $O(MN_1^2)$ where M and N_1 represents respectively the number of dominant points and curve size. When we compute the force histogram, this time is also included in computation time for force histogram with polygonal object approximation.

In the method of computing the combined topological and directional relations information, force histograms are coupled with the fuzzy Allen relations. Time complexity of the method is directly related to the time complexity of force histograms. Time complexity of force histograms for polygonal object approximation is $O(nN \log(N))$ where N denotes the number of vertices of two polygons and n for number of directions to be computed. Computation time for raster data is $O(nM\sqrt{M})$ where M denotes the number of pixels of the processed image (see [27]).

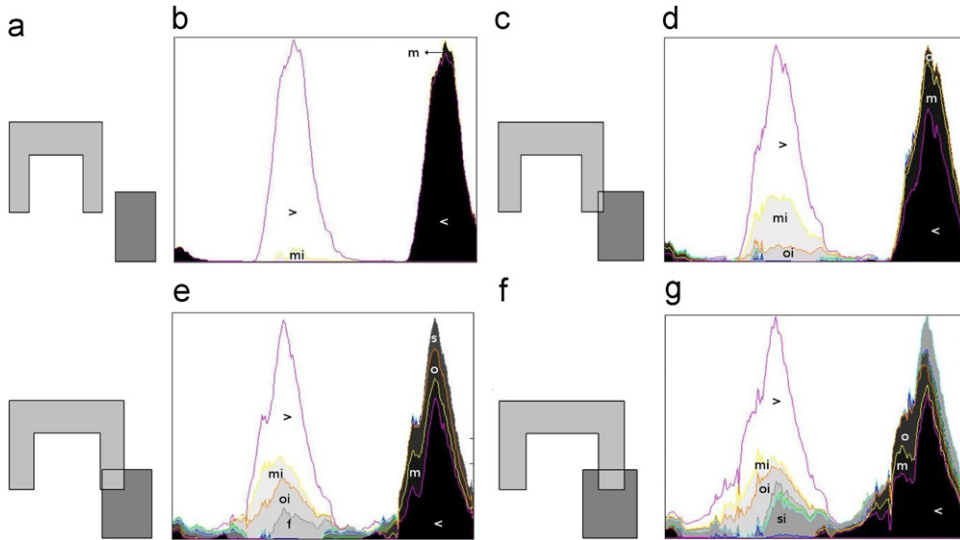


Fig. 6. Concave and convex object pairs and their histogram of fuzzy Allen relations (axes are same as in Fig. 4(b) and (c)).

Time complexity of Algorithm for treatment of longitudinal section: Method for computing the combined topological and directional relations information directly depends upon the force histograms. In this method force histograms are coupled with the fuzzy Allen relations, where algorithm developed by Matsakis and Nikitenko [2] (Algorithm 1) for fuzzification of segments of a longitudinal section. This algorithm imposes restrictions on the assumption of an object. Its time complexity is added to the time complexity of the force histograms. For better elaboration, the algorithm used by Matsakis and Nikitenko [2] is given below

Algorithm 1. Algorithm for the fuzzification of a longitudinal section I . The symbol $H(I_i^c \cup I_j^c)$ denotes the convex hull of $I_i^c \cup I_j^c$. Indexing is chosen in such a way that I_i, I_j are consecutive in I , c represents the number of iterations in while loop, its maximum value is n . The algorithm increases the degree of α_k associated with the open interval $J_k = H(I_i^c \cup I_j^c) - (I_i^c \cup I_j^c)$. Initially $\alpha_k = 0$ and α denotes the α -cut of a fuzzy membership function.

```

c ← 0;
α ← 1;
while α > 0 do
    ... There exists one set  $\{I_i^c\}_{i=1 \dots n_c}$  of mutually disjoint
    segments (and only one) such that:  $\alpha I = \cup_{i=1 \dots n_c} I_i^c$ . For any  $i$  and
    any  $j$  in  $1 \dots n_c$ , with  $i \neq j$  the length of  $I_i^c$  is denoted by  $x_i^c$  and
    the distance between  $I_i^c$  and  $I_j^c$  is denoted by  $d_{ij}$ .
    for any  $i$  in  $1 \dots n_c - 1$  do
        for any  $j$  in  $1 \dots n_c$  do
             $\beta \leftarrow \alpha \left( 1 - \frac{d_{ij}^c}{\min(x_i^c, x_j^c)} \right)$ 
            for any  $k$  in  $1 \dots n - 1$  do
                if  $J_k \subset H(I_i^c \cup I_j^c)$  then
                     $\alpha_k \leftarrow \max(\alpha_k, \beta)$ 
                end if
            end for
        end for
    end for
    α ← max{ $\alpha_k$ } $_{k=1 \dots n-1} \cap [0, \alpha]$ 
    c ← c + 1
end while

```

Algorithm 1 has the time complexity of order $O(n^3)$. Where n represents the number of segments for a longitudinal section of a line. With the proposed method, this algorithm is replaced by the

fuzzy logic connectors, these logic connectors have less time complexity as compared to this algorithm, these fuzzy logic connectors are only used for dealing with the longitudinal section.

Time complexity of Eqs. (6) and (7) Matsakis and Nikitenko used Eq. (6) for computing the combined topological and directional relation information. This equation has time complexity $O(n^3)$ where n represents the total number of segments exist in a longitudinal section. ($m+n$ represents the total number of segments for a longitudinal section and c represents the number of loops in Algorithm 1, its maximum value equals to number of disjoint segments). Proposed Eq. (7) has time complexity N where N represents the total number of vertices of polygons. This shows that the Eq. (6) has higher time complexity as compared to the time complexity of Eq. (7).

Algorithm for fuzzification of longitudinal section used by Matsakis and Nikitenko (Algorithm 1) put limits for the object approximation. Due to this algorithm, time complexity of method for combined topological and directional relations information is $O(nM\sqrt{M})$, where n represents the number of directions M number of pixels to be processed, in this paper we replace the algorithm for fuzzification of segments of a longitudinal section (Algorithm 1) with the fuzzy logic connectors. This substitution of algorithm for fuzzification of longitudinal section made it possible to consider objects by its polygon approximation, hence for the method we proposed, time complexity for computing the combined topological and directional relations information drops from $O(nM\sqrt{M})$ to $O(nN \log(N))$, where N represents the total number of vertices of polygons. Obviously $N \ll M$ and n is number of directions to be computed, it ranges from 32 to 360 directions.

7. Conclusion

Fuzzy Allen relations can be used to detect and analyze object position in space and these relations have a rich support for defining the fuzzy topological relations. These relations represent fuzziness in relation's semantics, they can also answer the questions that where in space a certain topological relation exists.

Polygonal approximation of objects and application of fuzzy logic connectors simplifies the algorithm given by Matsakis and Nikitenko [2]. This approach decreases its time complexity, it drops down from $O(nM\sqrt{M})$ to $O(nN \log(N))$, where n represents number of directions, N number of vertices of polygons and M number of pixels to be processed, due to using fuzzy logic connectors



Fig. 7. Different positions of reference object's segments in the case of longitudinal sections.

in lieu of the fuzzification of segments of a longitudinal section developed by Matsakis.

This approach of using fuzzy operator will open new fields of applications for fuzzy logic connectors. This technique can further be developed for defining the dynamic spatial relations in a quantitative way. In this paper, computations for all the directions are calculated for experimental purpose and verification of affine properties will be helpful for affine invariant description of relative object positions in scene and image understanding applications by combined topological and directional relations information. The aim of this paper is to validate formally the method we propose, and for the future work we project to apply this method for real data obtained by video sequences of real applications after extracting objects and their polygonal approximation with the help of all the image processing techniques.

Appendix A

Let's consider that the following situations arise for the segments of a longitudinal section when the objects are concave or objects have disconnected boundary.

Fig. 7(a) We discuss here the cases arising in Fig. 7(a) with all aggregation operators. For the above cited examples, we use terms $f(x_1, y_1, z)$ and $f(x_2, y_2, z)$ to express histograms of fuzzy Allen relations for first and second segments, here x_1 is the length of first segment of the argument object and x_2 is the length of second segment, similarly y_1 is the difference between the first segment of argument object and the reference object and y_2 is the difference between the second segment of argument object and the reference object, in this case consider both $y_1 > a_1/2$ and $y_2 > a_2/2$ where $a_1 = \min(x_1, z)$ and $a_2 = \min(x_2, z)$.

$$f(x_1, y_1, z) = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^t$$

$$f(x_2, y_2, z) = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^t$$

It means $f_{>}(I_1, J) = 1$ and $f_{>}(I_2, J) = 1$ and all the other values of histogram are zero. The possible outcomes with the application of different fuzzy operators are

$$F_{OR}(\theta, A_\theta(v), B_\theta(v)) = \max(f(x_1, y_1, z), f(x_2, y_2, z)) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

$$F_{AND}(\theta, A_\theta(v), B_\theta(v)) = \min(f(x_1, y_1, z), f(x_2, y_2, z)) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

$$F_{PROD}(\theta, A_\theta(v), B_\theta(v)) = f(x_1, y_1, z) \times f(x_2, y_2, z) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

$$F_{SUM}(\theta, A_\theta(v), B_\theta(v)) = 1 - ((1 - f(x_1, y_1, z)) \times (1 - f(x_2, y_2, z))) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

In this case, the two segments of argument object have the same Allen relations with the reference segment. Both segments behave like the crisp Allen relation. In such a case, all fuzzy operator provide us similar information.

Fig. 7(b) The cases arising in Fig. 7(b) with all aggregation operators. In this case consider $y_1 < -b - 3a_1/2$ and $y_2 > a_2/2$ where $a_1 = \min(x_1, z)$, $b_1 = \max(x_1, z)$ and $a_2 = \min(x_2, z)$ then $f(x_1, y_1, z)$ and $f(x_2, y_2, z)$ are used to express the histograms of fuzzy Allen relations

for first and second segments.

$$f(x_1, y_1, z) = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1)^t$$

$$f(x_2, y_2, z) = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^t$$

The possible outcomes are

$$F_{OR}(\theta, A_\theta(v), B_\theta(v)) = \max(f(x_1, y_1, z), f(x_2, y_2, z)) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t$$

$$F_{AND}(\theta, A_\theta(v), B_\theta(v)) = \min(f(x_1, y_1, z), f(x_2, y_2, z)) \\ = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

$$F_{PROD}(\theta, A_\theta(v), B_\theta(v)) = f(x_1, y_1, z) \times f(x_2, y_2, z) \\ = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

$$F_{SUM}(\theta, A_\theta(v), B_\theta(v)) = 1 - ((1 - f(x_1, y_1, z)) \times (1 - f(x_2, y_2, z))) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t$$

The above cited examples explain that, in a particular situation, *AND* and *PROD* operators cannot be used for the decision making process, when both segments of one longitudinal section of argument object have the same Allen relation with the segment of reference object, all fuzzy operators have the same results. In real situation different cases may arise and segments may have opposite relation as in Fig. 7(b). In case of conjunction operators, all the information may be lost.

These results show that fuzzy conjunction operators give results counter intuitive (both *AND*, *PROD* represents the conjunction operators) and the disjunction operators better fits the human intuition and provide here a better fusion of available fuzzy information. The third type of fuzzy operators such as Fuzzy γ operator can also be used to make possible contributions of two fuzzy values, but in this case finding compensation values of γ is a problem and for each case we have to adjust γ .

Concave objects or objects with holes have several segments, called longitudinal section. In such a case, fuzzy Allen relations are calculated for each segment separately then fuzzy connectors are applied for combining information from both segments.

Appendix B. Supplementary materials

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.patcog.2011.09.005.

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Nadeem Salamat is currently enrolled in Ph.D. at the University of La Rochelle, France. He received his master's degree in Master Computer Science and Technology with Mathematics and their application, specialty computers, mathematics, applied mathematics to the economy, from University of La Rochelle in 2008. Since September, 2008. He is enrolled in Ph.D. at the Mathematics, image and its applications laboratory (MIA) at University of La Rochelle and working under the supervision of Prof. Dr. El-hadi zahzah.

El-hadi Zahzah is a senior lecturer at La Rochelle University at the Mathematics, Image and Applications Laboratory (MIA). He received his Ph.D on computer science on 1992 at Paul Sabatier Toulouse University, Research Institute on Computer Science of Toulouse (IRIT). He has worked on many fields related to image processing and analysis, he published several papers on Image classification using fuzzy set theory, Neural networks and Dempster Shafer Theory. His main recent research interests is image analysis on moving object using fuzzy spatial relations.