Nuclear Inst. and Methods in Physics Research B 421 (2018) 18–26

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

X-ray radiation generated by a beam of relativistic electrons in composite structure

S.V. Blazhevich*, A.V. Noskov

Belgorod State University, Pobedy Str., 85, Belgorod 308015, Russia

ARTICLE INFO

Keywords:
Relativistic electron
Coherent radiation
Composite structure

ABSTRACT

The dynamic theory of coherent X-ray radiation generated by a beam of relativistic electrons in the three-layer structure consisting of an amorphous layer, a vacuum (air) layer and a single crystal has been developed. The phenomenon description is based on two main radiation mechanisms, namely, parametric X-ray radiation (PXR) and diffracted transition radiation (DTR). The possibility to increase the spectral-angular density of DTR under the condition of constructive interference of the transition radiation waves from different boundaries of such a structure has been demonstrated. It is shown that little changes in the layers thicknesses should not cause a considerable change in the interference picture, for example, the transition of constructive interference into destructive one. It means that in the considered process the conditions of constructive interference are enough stable to use them for increasing the intensity of X-ray source that can be created based on the interaction of relativistic electrons with such a structure.

1. Introduction

When an electron moving rectilinearly with a steady speed crosses a boundary between two media the transition radiation (TR) arises along the electron velocity [1]. A great interest in transition radiation of relativistic electron is due to the possibility of its application as an alternative source of X-ray radiation [2]. When a charged particle crosses a single-crystal plate, TR photons emitted at the plate entrance surface might diffract on a system of parallel atomic planes of the crystal. Such radiation characterized by narrow spectral range propagating in the direction of Bragg scattering is known as diffracted transition radiation (DTR) [3,4]. TR from the entrance surface contributes to DTR, while TR from the outlet surface of the plate does not take part in the DTR formation. Hence, in the resulting DTR there is no interference of these two types of TR.

In addition to TR, when relativistic electron crosses single crystal, its interaction in a crystal bulk with parallel atomic planes results in generating parametric X-ray radiation (PXR) [5–7]. PXR photons are emitted in the direction of the Bragg scattering and propagate together with DTR photons. The dynamic theory of coherent X-ray radiation by relativistic electrons in a crystal is developed for general case of electron Coulomb field reflection asymmetric with respect to the target surface [8–10]. In that case the system of parallel reflecting layers in the target has an arbitrary non-zero angle with the target surface.

Traditionally, radiation of relativistic electrons is analyzed separately for amorphous, crystalline or multilayer targets. Till now coherent radiation of relativistic electrons in composite targets has not been theoretically evaluated, while experimental studies on generation of coherent X-ray radiation [11–15] in composite structures have revealed an essential growth of the intensity of DTR yield with the increase of the number of boundaries (layers). The influence of the electron beam divergence on the PXR and DTR spectral-angular density has been examined in [16,17].

In the present work, we present our analysis on the process of coherent radiation by a beam of relativistic electrons in a complex target formed by the two amorphous and one single-crystal layers in two-wave approximation for dynamic diffraction theory.

2. Geometry of radiation process

Let us consider a beam of relativistic electrons passing through a three-layer structure consisted of two layers of amorphous media and one single-crystal layer (Fig. 1a) with different thicknesses c, a and b respectively. We will use the Planck's system of units ($h/2\pi = c = 1$, where h is the Planck's constant, c is the light velocity).

Below we denote the dielectric susceptibility of amorphous media as χ_c and χ_a, the average dielectric susceptibility of the crystal as χ_0 and the coefficient of Fourier expansion of the crystal dielectric susceptibility over the reciprocal lattice vectors $\mathbf{g}$ as

* Corresponding author.
E-mail address: blazh@bsu.edu.ru (S.V. Blazhevich).

https://doi.org/10.1016/j.nimb.2018.02.013
Received 5 November 2017; Received in revised form 12 February 2018; Accepted 13 February 2018

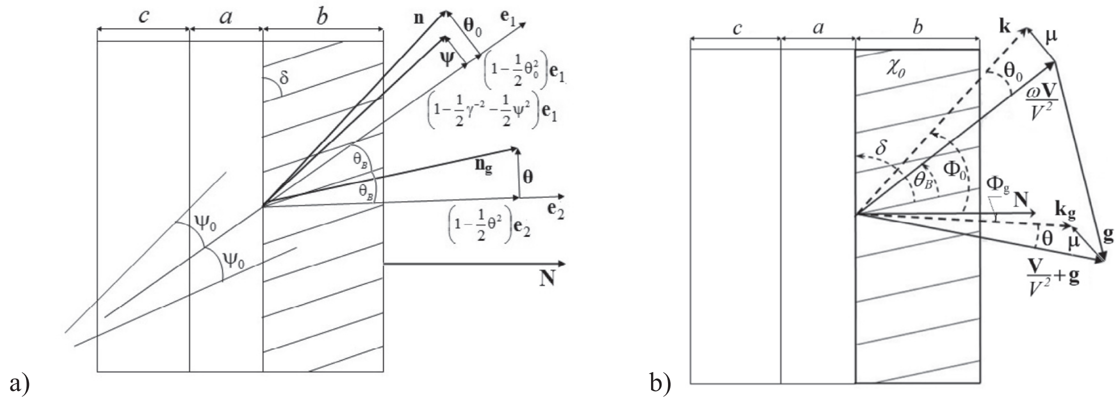

Fig. 1. Radiation process geometry (a). Geometry of the wave diffraction process in single-crystal layer (b).

$$\chi(\omega,\mathbf{r}) = \sum_{\mathbf{g}} \chi_{\mathbf{g}}(\omega)\exp(i\mathbf{gr}) = \sum_{\mathbf{g}} (\chi'_{\mathbf{g}}(\omega) + i\chi''_{\mathbf{g}}(\omega))\exp(i\mathbf{gr}), \tag{1}$$

while the angular variables ψ, θ и θ_0 as consistent with the definition for the velocity vector $\mathbf{V}$ of relativistic electron and unit vectors $\mathbf{n}$ (in the direction of the photon momentum emitted along the electron velocity) and $\mathbf{n_g}$ (in the Bragg scattering direction):

$$\mathbf{V} = \left(1-\tfrac{1}{2}\gamma^{-2}-\tfrac{1}{2}\psi^2\right)\mathbf{e}_1 + \psi, \quad \mathbf{e}_1\psi = 0$$
$$\mathbf{n} = \left(1-\tfrac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \theta_0, \quad \mathbf{e}_1\theta_0 = 0, \quad \mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B,$$
$$\mathbf{n_g} = \left(1-\tfrac{1}{2}\theta^2\right)\mathbf{e}_2 + \theta, \quad \mathbf{e}_2\theta = 0 \tag{2}$$

Here θ is the radiation angle counted from the detector axis $\mathbf{e}_2$, ψ is the angle of electron deviation from the original beam axis $\mathbf{e}_1$, θ_0 is the angle between the propagation direction of incident pseudo photon in electron Coulomb field and the beam axis $\mathbf{e}_1$, $\gamma = 1/\sqrt{1-V^2}$ is the Lorentz factor. The angular variables are considered as a sum of components parallel and perpendicular to the plane of the figure:

$$\theta = \theta_{||} + \theta_{\perp}, \quad \theta_0 = \theta_{0||} + \theta_0\perp, \quad \psi = \psi_{||} + \psi_{\perp}$$

At first, we will consider the radiation by single electron in the beam crossing three-layer structure at the angle $\psi(\psi_{\perp},\psi_{//})$ to electron beam axis $\mathbf{e}_1$.

3. Radiation amplitude

The X-ray wave propagation in the single-crystal medium will be considered within the scope of two-wave approximation of dynamical diffraction theory by analogy with the work [9].

As the electromagnetic field excited by relativistic electron is practically transverse in X-ray frequency band the incident $\mathbf{E}(\mathbf{k},\omega)$ and diffracted $\mathbf{E}(\mathbf{k}+\mathbf{g},\omega)$ in the crystal electromagnetic waves are characterized by two amplitudes with different values of polarization

$$\mathbf{E}(\mathbf{k},\omega) = E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)},$$
$$\mathbf{E}(\mathbf{k}+\mathbf{g},\omega) = E_{\mathbf{g}}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)} + E_{\mathbf{g}}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)}$$

where the vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to the vector $\mathbf{k} = k\mathbf{n}$ and vectors $\mathbf{e}_1^{(1)}$ and $\mathbf{e}_1^{(2)}$ are perpendicular to the vector $\mathbf{k_g} = \mathbf{k} + \mathbf{g} = k_{\mathbf{g}}\mathbf{n_g}$. The vectors $\mathbf{e}_0^{(2)}$ and $\mathbf{e}_1^{(2)}$ lie in the plane of $\mathbf{k}$ and $\mathbf{k_g}$ vectors ((π-polarization), but $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_1^{(1)}$ vectors are perpendicular to that plane (σ-polarization).

Within the scope of two-wave approximation of diffraction dynamic theory the system of equations for the amplitude of the wave field strength can be expressed in the following form

$$\begin{cases}(\omega^2(1+\chi_0)-k^2)E_0^{(s)} + \omega^2\chi_{-\mathbf{g}}C^{(s)}E_{\mathbf{g}}^{(s)} = 8\pi^2 ie\omega\mathbf{e}_0^{(s)}\mathbf{V}\delta(\omega-\mathbf{kV}),\\ \omega^2\chi_{\mathbf{g}}C^{(s)}E_0^{(s)} + (\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)} = 0,\end{cases} \tag{3}$$

where

$$C^{(s)} = \mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)}, \quad C^{(1)} = 1, \quad C^{(2)} = \cos 2\theta_B$$
$$\mathbf{e}_0^{(1)}\mathbf{V} = \theta_{\perp}-\psi_{\perp} \equiv \Omega^{(1)}, \quad \mathbf{e}_0^{(2)}\mathbf{V} = \theta_{//} + \psi_{//} \equiv \Omega^{(2)},$$
$$\chi_{\mathbf{g}} = \chi'_{\mathbf{g}} + \chi''_{\mathbf{g}}, \quad \chi'_{\mathbf{g}} = \chi'_0(F(g)/Z)(S(\mathbf{g})/N_0)\exp(-g^2u_\tau^2/2), \quad \chi''_{\mathbf{g}} = \chi''_0\exp\left(-\tfrac{1}{2}g^2u_\tau^2\right)$$

where $\chi_0 = \chi'_0 + i\chi''_0$ is average dielectric susceptibility, F(g) is form-factor of an atom contained Z electrons, $S(\mathbf{g})$ is structural factor of unit cell in single-crystal layer contained N_0 atoms, u_τ is the mean-square amplitude of thermal atomic oscillations in the crystal. In the present work, the X-radiation frequency region is considered: $(\chi'_{\mathbf{g}} < 0)$, $(\chi'_0 < 0)$. The system of Eq. (3) under $s = 1$ describes the fields of σ- polarization, and under s = 2 the fields of π-polarization.

It should be noted, that the system (3) describes the fields of X-ray waves and Coulomb fields of relativistic electrons both in vacuum (under $\chi_{\mathbf{g}} = 0$, $\chi_0 = 0$) and in amorphous media (under $\chi_{\mathbf{g}} = 0$ and $\chi_0 = \chi_a$ or $\chi_0 = \chi_c$).

In Fig. 1b a scheme of the wave diffraction in single crystal is shown, where $\boldsymbol{\mu} = \mathbf{k}-\omega\mathbf{V}/V^2$ is the component of virtual photon momentum perpendicular to particle velocity $\mathbf{V}$ ($\mu = \omega\theta_0/V$, where $\theta_0 \ll 1$ is the angle between vectors $\mathbf{k}$ and $\mathbf{V}$), θ_B is Bragg angle the modulus of vector $\mathbf{g}$ can be expressed by the Bragg angle θ_B and Bragg frequency ω_B: $g = 2\omega_B\sin\theta_B/V$. The angle between the vector $\omega\mathbf{V}/V^2$ and the wave vector $\mathbf{k}$ of incident wave is denoted as θ_0 and the angle between the vector $\omega\mathbf{V}/V^2 + \mathbf{g}$ and wave vector $\mathbf{k_g}$ of diffracted wave as θ.

The magnitude of the wave vector of free photons in amorphous media $k_a = \omega\sqrt{1+\chi_a}$ and $k_c = \omega\sqrt{1+\chi_c}$ can be presented in following form:

$$k_a = \omega\left(1+\frac{\chi_0}{2}\right) + \frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda'_{\mathbf{g}a}-\frac{\omega\beta}{2}\right), \quad k_c = \omega\left(1+\frac{\chi_0}{2}\right) + \frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda'_{\mathbf{g}c}-\frac{\omega\beta}{2}\right),$$

where

$$\lambda'_{\mathbf{g}a} = \lambda^*_{\mathbf{g}}-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\omega\left(\frac{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi_a}{2}\right), \quad \lambda'_{\mathbf{g}c} = \lambda^*_{\mathbf{g}}-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\omega\left(\frac{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi_c}{2}\right),$$

$$\lambda^*_{\mathbf{g}} = \frac{\omega\beta}{2} + \frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda^*_0, \quad \lambda^*_0 = \omega\left(\frac{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi_0}{2}\right), \quad \beta = \frac{1}{\omega^2}(k_{\mathbf{g}}^2-k^2)-\chi_0\left(1-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\right)$$

$\gamma_0 = \cos\Phi_0$, $\gamma_{\mathbf{g}} = \cos\Phi_{\mathbf{g}}$, Φ_0 is the angle between the wave vector $\mathbf{k}$ of the incident wave and the normal $\mathbf{N}$ to crystal layer, $\Phi_{\mathbf{g}}$ is the angle between wave vector $\mathbf{k_g}$ and vector $\mathbf{N}$ (see in Fig. 1b), γ is of the electron Lorentz factor. The magnitude of the wave vector of free photon radiated in the direction of Bragg scattering will be expressed as $k_0 = \omega\left(1+\frac{\chi_0}{2}\right)+\lambda''_{\mathbf{g}}$, where $\lambda''_{\mathbf{g}} = -\omega\frac{\chi_0}{2}$.

The expression for the Coulomb field of a relativistic electron in front of the target in vacuum can be obtained from (1):

$$E_0^{(s)vac\,I} = \frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*). \tag{4a}$$

In the amorphous media the field consists of the electron Coulomb field and of the free photons field $E_a^{(s)}$ or $E_c^{(s)}$ of transition radiation:

$$E_0^{(s)sr} = \frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0+\chi_c-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*)+E_c^{(s)}\delta(\lambda_{\mathbf{g}}-\lambda'_{\mathbf{g}}), \tag{4b}$$

$$E_{a0}^{(s)sr} = \frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0+\chi_a-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*) + E_a^{(s)}\delta(\lambda_{\mathbf{g}}-\lambda'_{\mathbf{g}a}). \tag{4c}$$

The expressions for the modulus of the wave vector of the incident and diffracted photons in the crystal we can present as $k=\omega\sqrt{1+\chi_0}+\lambda_0$, $k_{\mathbf{g}}=\omega\sqrt{1+\chi_0}+\lambda_{\mathbf{g}}$, respectively, where λ_0 and $\lambda_{\mathbf{g}}$ are the dynamic addition agents. The dynamic addition agents can be found by solving the dispersion equation, which follows from system (3) taking into account the relation between λ_0 and $\lambda_{\mathbf{g}} = \omega\beta/2+\lambda_0(\gamma_{\mathbf{g}}/\gamma_0)$ [18]:

$$(\omega^2(1+\chi_0)-k^2)(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)-\omega^4\chi_{-\mathbf{g}}\chi_{\mathbf{g}}C^{(s)2}=0.$$

The expressions derived are as follows:

$$\lambda_{\mathbf{g}}^{(1,2)} = \frac{\omega|\chi'_{\mathbf{g}}|C^{(s)}}{2}\left(\xi^{(s)}-\frac{i\rho^{(s)}(1-\varepsilon)}{2} \pm\sqrt{\xi^{(s)2}+\varepsilon-2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)}+\kappa^{(s)}\varepsilon\right)-\rho^{(s)2}\left(\frac{(1-\varepsilon)^2}{4}+\kappa^{(s)2}\varepsilon\right)}\right), \tag{5a}$$

$$\lambda_0^{(1,2)} = \frac{\omega|\chi'_{\mathbf{g}}|C^{(s)}}{2\varepsilon}\left(-\xi^{(s)}+\frac{i\rho^{(s)}(1-\varepsilon)}{2} \pm\sqrt{\xi^{(s)2}+\varepsilon-2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)}+\kappa^{(s)}\varepsilon\right)-\rho^{(s)2}\left(\frac{(1-\varepsilon)^2}{4}+\kappa^{(s)2}\varepsilon\right)}\right), \tag{5b}$$

where

$$\xi^{(s)}(\omega)=\eta^{(s)}(\omega)+\frac{1-\varepsilon}{2\nu^{(s)}},\quad \eta^{(s)}(\omega)=\frac{1}{2\,|\chi'_{\mathbf{g}}\,|\,C^{(s)}}\frac{1}{\omega^2}(k_{\mathbf{g}}^2-k^2)$$
$$=\frac{2\sin^2\theta_B}{V^2\,|\chi'_{\mathbf{g}}\,|\,C^{(s)}}\left(\frac{\omega(1-\theta_{//}\cot\theta_B)}{\omega_B}-1\right)$$
$$\varepsilon=\frac{\gamma_{\mathbf{g}}}{\gamma_0}=\frac{\cos\Phi_{\mathbf{g}}}{\cos\Phi_0},\quad \rho^{(s)}=\frac{\chi''_0}{|\chi'_{\mathbf{g}}\,|\,C^{(s)}},\quad \kappa^{(s)}=\frac{\chi''_{\mathbf{g}}C^{(s)}}{\chi''_0},\quad C^{(1)}=1,\quad C^{(2)}=\cos2\theta_B \tag{6}$$

As the dynamic addition agents are small $\lambda_g^{(1,2)} \ll \omega$ we will use the approximate equality $\theta_0 \approx \theta$ (see in Fig. 1b). The electromagnetic field in the single-crystal layer consist of Coulomb field of the relativistic electron and the fields of two free propagating waves (incident and diffracted):

$$E_0^{(s)cr} = \frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\frac{-\omega^2\beta-2\omega\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0}{4\frac{\gamma_{\mathbf{g}}}{\gamma_0}(\lambda_0-\lambda_0^{(1)})(\lambda_0-\lambda_0^{(2)})}\delta(\lambda_0-\lambda_0^*)+E_0^{(s)^{(1)}}\delta(\lambda_0-\lambda_0^{(1)}) + E_0^{(s)^{(2)}}\delta(\lambda_0-\lambda_0^{(2)}), \tag{7a}$$

$$E_{\mathbf{g}}^{(s)cr} = -\frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{4\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)})(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)})}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*)+E_{\mathbf{g}}^{(s)^{(1)}}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)}) + E_{\mathbf{g}}^{(s)^{(2)}}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)}) \tag{7b}$$

Incident and diffracted fields in the crystal are coupled by the relation $E_0^{(s)cr} = 2\omega\lambda_{\mathbf{g}}(\omega^2\chi_{\mathbf{g}}C^{(s)})^{-1}E_{\mathbf{g}}^{(s)cr}$. The radiated field in vacuum behind the target can be presented as $E_{\mathbf{g}}^{(s)vacII} = E_{\mathbf{g}}^{(s)Rad}\delta(\lambda_{\mathbf{g}}-\lambda''_{\mathbf{g}})$.

To definite the field of the radiation $E_{\mathbf{g}}^{(s)Rad}$ in direction of Bragg scattering $\mathbf{k_g} = k_{\mathbf{g}}\mathbf{n_g}$ (see in Fig. 1) we will use the boundary condition for the fields on four boundaries of the considered three-layer target:

$$\begin{aligned}
&\int E_0^{(s)vacI}d\lambda_{\mathbf{g}} = \int E_0^{(s)sr}d\lambda_{\mathbf{g}},\int E_0^{(s)sr}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}c}d\lambda_{\mathbf{g}} = \int E_{a0}^{(s)sr}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}c}d\lambda_{\mathbf{g}},\\
&\int E_{a0}^{(s)sr}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(+a)}d\lambda_{\mathbf{g}} = \int E_0^{(s)cr}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(c+a)}d\lambda_{\mathbf{g}},\\
&\int E_{\mathbf{g}}^{(s)cr}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(a+c)}d\lambda_{\mathbf{g}} = 0,\\
&\int E_{\mathbf{g}}^{(s)cr}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(c+a+b)}d\lambda_{\mathbf{g}} = \int E_{\mathbf{g}}^{(s)vacII}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(c+a+b)}d\lambda_{\mathbf{g}}
\end{aligned} \tag{8}$$

Let us represent the obtained expressions describing the amplitude $E_{\mathbf{g}}^{(s)Rad}$ of the radiation in the direction of the Bragg scattering as sum of the amplitudes of the electric field of DTR and PXR:

$$E_{\mathbf{g}}^{(s)Rad} = E_{DTR}^{(s)} + E_{PXR}^{(s)}, \tag{9a}$$

$$\begin{aligned}
E_{DTR}^{(s)} = {}& \frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\exp\left[i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{(+a+b)}{\gamma_g}\right]\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)})}\\
&\times\left[\exp\left(i\frac{\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b\right)-\exp\left(i\frac{\lambda_{\mathbf{g}}^{(2)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b\right)\right]\\
&\times\left[\left(\frac{1}{\Delta_c}-\frac{1}{\Delta}\right)\exp\left(-i\frac{\omega c}{2\gamma_0}\Delta_c-i\frac{\omega a}{2\gamma_0}\Delta_a\right)+\left(\frac{1}{\Delta_a}-\frac{1}{\Delta_c}\right)\exp\left(-i\frac{\omega a}{2\gamma_0}\Delta_a\right)\right.\\
&\left.+\frac{1}{\Delta_0}-\frac{1}{\Delta_a}\right],
\end{aligned} \tag{9b}$$

$$\begin{aligned}
E_{PXR}^{(s)} = {}& \frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\exp\left[i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{(c+a+b)}{\gamma_g}\right]\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)})}\\
&\left[\left(\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)})}-\frac{1}{\Delta_0}\right)\times\left(\exp\left(i\frac{\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b\right)-1\right)\right.\\
&\left.-\left(\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)})}-\frac{1}{\Delta_0}\right)\left(\exp\left(i\frac{\lambda_{\mathbf{g}}^{(2)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b\right)-1\right)\right]
\end{aligned} \tag{9c}$$

where

$$\begin{aligned}
\Delta_c &= \gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2-\chi_c,\\
\Delta_a &= \gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2-\chi_a,\\
\Delta_0 &= \gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2-\chi_0,\\
\Delta &= \gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2
\end{aligned} \tag{10}$$

The explicit separation of expressions for PXR and DTR amplitudes allowed us to consider the interference of these mechanisms of radiation.

Let us consider the DTR amplitude for the case when the second layer of the target is vacuum ($\chi = 0$) (see in Fig. 1). To reveal and to study the effects in radiation which are not connected with the absorption let us consider a simple case of the thin non-absorptive target $\chi_0'' = \chi_c'' = 0$. In this case the expression (9b) has the following form:

$$E_{DTR}^{(s)} = \frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\exp\left[i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{(c+a+b)}{\gamma_g}\right]\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)})}$$
$$\times\left[\exp\left(i\frac{\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b\right)-\exp\left(i\frac{\lambda_{\mathbf{g}}^{(2)}-\lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}b\right)\right]$$
$$\times\left[\left(\frac{1}{\Delta_c}-\frac{1}{\Delta}\right)\exp\left(-i\frac{\omega a}{2\gamma_0}\Delta\right)\left(\exp\left(-i\frac{\omega c}{2\gamma_0}\Delta_c\right)-1\right)+\left(\frac{1}{\Delta_0}-\frac{1}{\Delta}\right)\right] \tag{11}$$

In the expression (11) describing the field strength amplitude of DTR waves in the direction of the Bragg scattering, the first term in the last factor is according to TR generated in the amorphous layer, and the second term is according to TR generated on the entrance surface of single-crystal layer.

The explicit separation of these terms gives us the possibility for studying of the interference of these waves of transition radiations.

4. Spectral-angular radiation density

Using (9a), (9c), (11) and known expression for X-ray spectral-angular density [19]

$$\omega\frac{d^2N}{d\omega d\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}|E^{(s)Rad}|^2, \tag{12}$$

we will obtain the expressions describing the spectral-angular densities of PXR, DTR in considered three-layer target and their interference:

$$\omega\frac{d^2N^{(s)}}{d\omega d\Omega} \equiv T^{(s)} = T_{PXR}^{(s)} + T_{DTR}^{(s)} + T_{\mathrm{int(PXR,DTR)}}^{(s)},$$

$$\omega\frac{d^2N_{\mathrm{PXR}}^{(s)}}{d\omega d\Omega} \equiv T_{PXR}^{(s)} = \frac{e^2}{\pi^2}\frac{\Omega^{(s)2}}{\Lambda_0^2}\left(1-\frac{\xi}{\sqrt{\xi^2+\varepsilon}}\right)^2\frac{\sin^2\left(\frac{b^{(s)}}{2}\Sigma_1^{(s)}\right)}{\Sigma_1^{(s)2}}, \tag{13}$$

$$\omega\frac{d^2N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega} \equiv T_{DTR}^{(s)} = T_1^{(s)} + T_2^{(s)} + T_{\mathrm{int}}^{(s)}, \tag{14a}$$

$$T_1^{(s)} = \frac{e^2}{\pi^2}\Omega^{(s)2}\left(\frac{1}{\Lambda_c}-\frac{1}{\Lambda}\right)^2\sin^2\left(\frac{c\omega_B}{4\sin(\delta-\theta_B)}\cdot\Lambda_c\right)R_{DTR}^{(s)}, \tag{14b}$$

$$T_2^{(s)} = \frac{e^2}{4\pi^2}\Omega^{(s)2}\left(\frac{1}{\Lambda_0}-\frac{1}{\Lambda}\right)^2 R_{DTR}^{(s)}, \tag{14c}$$

$$T_{\mathrm{int}}^{(s)} = \frac{e^2}{2\pi^2}\Omega^{(s)2}\left(\frac{1}{\Lambda_0}-\frac{1}{\Lambda}\right)\left(\frac{1}{\Lambda}-\frac{1}{\Lambda_c}\right)$$
$$\times\left[\cos\left(\frac{a\omega_B}{2\sin(\delta-\theta_B)}\cdot\Lambda\right)-\cos\left(\frac{a\omega_B}{2\sin(\delta-\theta_B)}\cdot\Lambda\right.\right.$$
$$\left.\left.+\frac{c\omega_B}{2\sin(\delta-\theta_B)}\Lambda_c\right)\right]R_{DTR}^{(s)}, \tag{14d}$$

$$R_{DTR}^{(s)} = \frac{4\varepsilon^2}{\xi^{(s)2}+\varepsilon}\sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)2}+\varepsilon}}{\varepsilon}\right), \tag{14e}$$

In the expression for DTR spectral-angular density the term $T_1^{(s)}$ corresponds to the waves of TR from the amorphous layer and $T_2^{(s)}$ correspond to TR from the entrance boundary of a single-crystal layer, while the term $T_{\mathrm{int}}^{(s)}$ represents the result of their interference.

The expression for the term, which is the result of interference between DTR and PXR, can be derived in the form

$$\omega\frac{d^2N_{INT}^{(s)}}{d\omega d\Omega} \equiv T_{int(PXR,\mathrm{DTR})}^{(s)}, \tag{15a}$$

$$T_{int(PXR,\mathrm{DTR})}^{(s)} = \frac{e^2}{4\pi^2}\frac{\Omega^{(s)2}}{\Lambda_0}\left[\left(\frac{1}{\Lambda_c}-\frac{1}{\Lambda}\right)R_{INT}^{(s)(1)} + \left(\frac{1}{\Lambda_0}-\frac{1}{\Lambda}\right)R_{INT}^{(s)(2)}\right], \tag{15b}$$

$$R_{INT}^{(s)(1)}$$
$$= -16\varepsilon\frac{\xi^{(s)}-\sqrt{\xi^{(s)2}+\varepsilon}}{\xi^{(s)2}+\varepsilon}\frac{1}{\Sigma_1^{(s)}}\sin\left(\frac{b^{(s)}\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\sin\left(\frac{b^{(s)}}{2}\Sigma_1^{(s)}\right)\sin$$
$$\left(\frac{c\omega_B}{4\sin(\delta-\theta_B)}\cdot\Lambda_c\right)\times\sin\left(\frac{a\omega_B}{2\sin(\delta-\theta_B)}\Lambda+\frac{c\omega_B}{4\sin(\delta-\theta_B)}\Lambda_c\right.$$
$$\left.+\frac{b^{(s)}}{2}\sum_2^{(s)}\right), \tag{15c}$$

$$R_{INT}^{(s)(2)} = 8\varepsilon\frac{\xi^{(s)}-\sqrt{\xi^{(s)2}+\varepsilon}}{\xi^{(s)2}+\varepsilon}\frac{1}{\sum_1^{(s)}}\sin\left(b^{(s)}\frac{\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\sin\left(\frac{b^{(s)}}{2}\sum_1^{(s)}\right)\cos$$
$$\left(\frac{b^{(s)}}{2}\sum_2^{(s)}\right), \tag{15d}$$

where the following notations are introduced:

$$\Omega^{(1)} = \theta_\perp-\psi_\perp,\quad \Omega^{(2)} = \theta_{//}+\psi_{//},\quad C^1 = 1,\quad C^2 = \cos2\theta_b,\quad \Lambda_0$$
$$= (\theta_\perp-\psi_\perp)^2 + (\theta_{//}+\psi_{//})^2 + \gamma^{--2}-\chi_0',\quad \Lambda_c = (\theta_\perp-\psi_\perp)^2 + (\theta_{//}+\psi_{//})^2$$
$$+\gamma^{-2}-\chi_c',\quad \Lambda = (\theta_\perp-\psi_\perp)^2 + (\theta_{//}+\psi_{//})^2 + \gamma^{-2},\quad b^{(s)}$$
$$= \frac{1}{2\sin(\delta-\theta_B)}\frac{b}{L_{ext}^{(s)}},\quad \varepsilon = \frac{\sin(\delta+\theta_B)}{\sin(\delta-\theta_B)},\quad \sum_2^{(s)} = \sigma^{(s)}$$
$$+(\xi^{(s)}+\sqrt{\xi^{(s)2}+\varepsilon})/\varepsilon,\quad \sum_1^{(s)} = \sigma^{(s)} + (\xi^{(s)}-\sqrt{\xi^{(s)2}+\varepsilon})/\varepsilon,\quad \sigma^{(s)}$$
$$= \frac{1}{|\chi_{\mathbf{g}}'|C^{(s)}}\Lambda_0. \tag{16}$$

Here under $s = 1$ the expressions Eqs. (13)–(15) describe the σ -polarized waves, while at $s = 2$ the π - polarized ones.

The function $R_{DTR}^{(s)}$ describes the DTR spectrum. The parameter ε in Eqs. (13)–(15) describes the degree of asymmetry (relative to target surface) in the electrical field reflection for a single-crystal layer. The expression for ε parameter includes the variable δ – the angle between the target surface and the diffracting atomic planes of a single-crystal layer.

The constructive interference of TR waves emitted at different boundaries of the amorphous layer in vicinity of Bragg frequency can result in an essential increase of the DTR spectral-angular density. The conditions of constructive interference following from Eq. (14b) can be written in the following form

$$\frac{c\omega_B}{4\sin(\delta-\theta_B)}\cdot\Lambda_c = (2n+1)\frac{\pi}{2},\quad (n = 0,1,2\ldots). \tag{17a}$$

An additional increase in the DTR spectral-angular density can be reached due to the constructive interference of TR waves from the amorphous layer and the entrance surface of a crystalline layer that might take place under the conditions

$$\frac{a\omega_B}{2\sin(\delta-\theta_B)}\cdot\Lambda = (2m+1)\pi,\quad (m = 0,1,2\ldots), \tag{17b}$$

reducing from Eq. (14d).

To describe the coherent radiation generated by a divergent beam of relativistic electrons in the considered composite structure, the expressions (13)–(15) should be averaged over all possible directions of electron motion in the beam, which might be approximated, for example, by the 2D Gaussian distribution

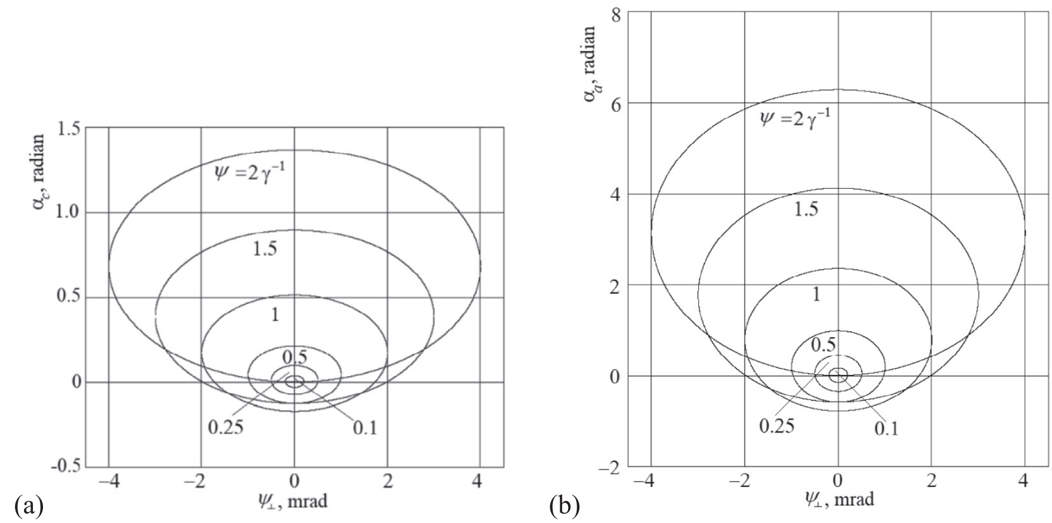

Fig. 2. The deviation of the condition of constructive interference in TR: (a) from the boundaries of the amorphous layer α_c (condition (17a)); (b) from amorphous layer and the entrance surface of the single-crystal layer α_a (condition (17b)).

$$f(\boldsymbol{\psi}) = \frac{1}{\pi\psi_0^2} e^{-\frac{\psi_{//}^2+\psi_\perp^2}{\psi_0^2}}, \tag{18}$$

where ψ_0 is the "divergence" parameter of the electron beam (see Fig. 1). The angle ψ_0 defines a cone limiting the part of the beam where the beam density decreases less than 1/e times in comparison with the density on the axis of the beam. Hence, we have

$$\left\langle \omega \frac{d^2N^{(s)}}{d\omega d\Omega} \right\rangle = \langle T_{PXR}^{(s)} \rangle + \langle T_{DTR}^{(s)} \rangle + \langle T_{\mathrm{int(PXR,\,DTR)}}^{(s)} \rangle, \tag{19a}$$

$$\langle T_{DTR}^{(s)} \rangle = \langle T_1^{(s)} \rangle + \langle T_2^{(s)} \rangle + \langle T_{\mathrm{int}}^{(s)} \rangle, \tag{19b}$$

where

$$\langle T_j^{(s)} \rangle = \frac{1}{\pi\psi_0^2} \int\int d\psi_\perp d\psi_{//} e^{-\frac{\psi_{//}^2+\psi_\perp^2}{\psi_0^2}} T_j^{(s)}, \quad j = DTR, 1, 2, \mathrm{int}, PXR, \mathrm{int}(PXR, DTR) \tag{19c}$$

5. Estimations of the beam divergence influence on the interference effect in DTR

The expressions (17) allow us to estimate the influence of the beam divergence on the interference effect in the DTR generated by the relativistic electron beam in the composite target using the parameters Λ_c and Λ as the functions: $\Lambda_c(\theta_\perp,\psi_\perp,\theta_{//},\psi_{//}) = (\theta_\perp-\psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 + \gamma^{-2} - \chi_c'$ and $\Lambda(\theta_\perp,\psi_\perp,\theta_{//},\psi_{//}) = (\theta_\perp-\psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 + \gamma^{-2}$.

The width of the amorphous layer will be considered "optimal" if the waves of transition radiation generated on the enter and the output surfaces of the layer by the electrons moving along the beam axis ($\psi_\perp = 0, \psi_\parallel = 0$) are in strong constructive interference in the direction of the maximal angular density $\theta \equiv \sqrt{\theta_\perp^2 + \theta_\parallel^2} \approx \gamma^{-1}$:

$$c_{opt}(n) = \frac{(2\cdot n + 1)\cdot\frac{\pi}{2}\cdot(4\cdot sin(\delta-\theta_B))}{\omega_B\cdot(2\cdot\gamma^{-2}-\chi_c')}.$$

By substituting this expression to (17a) we obtain the deviation of the constructive interference condition as a function of the variables $\boldsymbol{\psi}(\psi_\perp,\psi_\parallel)$ and $\boldsymbol{\theta}$ $(\theta_\perp,\theta_\parallel)$:

$$\alpha_c(n,\theta_\perp,\theta_\parallel,\psi_\perp,\psi_\parallel) = (2n + 1)\cdot\frac{\pi}{2}\cdot\left(\frac{(\theta_\perp-\psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2 + \gamma^{-2}-\chi_c'}{(2\cdot\gamma^{-2}-\chi_c')}-1\right),$$

where ($n = 0,1,2...$).

As the interference conditions at the considered geometry are the same for all the directions of the radiation corresponding to the condition of the maximal angular density of the transition radiation by the electrons on the electron beam axis $\sqrt{\theta_\perp^2 + \theta_\parallel^2} = \gamma^{-1}$, then we can fix one of them, for instance, the one corresponding to the case of the radiation with σ-polarization: $\theta_\perp = \gamma^{-1}, \theta_\parallel = 0$). According to the determination of optimal layer width the deviation $\alpha_c(n,\gamma^{-1},0,0,0) = 0$.

If we fix the absolute value of the angle $\boldsymbol{\psi}(\psi_\perp,\psi_\parallel)$ namely $\psi = \mathrm{const}$, then $\psi_\parallel = \pm\sqrt{\psi^2-\psi_\perp^2}$ and we can investigate the interference condition deviation α_c as the function of $\psi_\perp$ for various values of the parameter ψ. Let us remember that the constructed interference in our case is characterized by the proximity of $sin^2\left(\frac{\pi}{2} + \alpha_c(n,\theta_\perp,\theta_\parallel,\psi_\perp,\psi_\parallel)\right)$ to one. For example, if $\alpha_c = 0.5$ then $\sin^2\left(\frac{\pi}{2} + 0.5\right) = 0.77 \approx 1$ and the correspondent interference will be of constructive character. In Fig. 2 the family of the curves demonstrate the dependences of the interference deviation parameter α_c as a function of $\psi_\perp$ for different values of the parameter ψ under the condition $\psi_\parallel = \pm\sqrt{\psi^2-\psi_\perp^2}$. In Fig. 2 one can see that for $\psi \leqslant \gamma^{-1}$ the angle of deviation of the interference condition not exceeds 0.5 rad so the interference in these conditions will have the pronounced constructive character.

Analogous, from condition (17b) we can express the optimal width of the vacuum layer between amorphous monocrystalline ones as:

$$a_{opt}(m) = \frac{(2\cdot m + 1)\cdot\pi\cdot(2\cdot sin(\delta-\theta_B))}{\omega_B\cdot 2\cdot\gamma^{-2}},$$

where ($m = 0,1,2...$).

The detuning of interference condition (17b) from the optimal can be expressed as the function of $\boldsymbol{\psi}(\psi_\perp,\psi_\parallel$ and $\boldsymbol{\theta}$ $(\theta_\perp,\theta_\parallel)$:

$$\alpha_a(n,\theta_\perp,\theta_\parallel,\psi_\perp,\psi_\parallel) = (2m + 1)\cdot\pi\cdot\left(\frac{(\theta_\perp-\psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2 + \gamma^{-2}}{(2\cdot\gamma^{-2})}-1\right)$$

For estimation of divergency influence on the character of the interference of transition radiation from the amorphous layer and the entrance surface of the monocrystalline layer we have fixed the direction of the radiation ($\theta_\perp = \gamma^{-1}, \theta_\parallel = 0$). In Fig. 2b the results of calculations of the dependence of the deviation α_a on the beam divergence for the same conditions as in Fig. 2a are shown. One can see that the deviation α_a of interference condition (17b) is more sensitive to divergence then the deviation α_c of the interference condition (17a. It is connected with the difference in χ_c' and χ_a' values. Since there is a vacuum in the layer a, then $\chi_a' = 0$.

The influence of the divergence on the constructive interference of

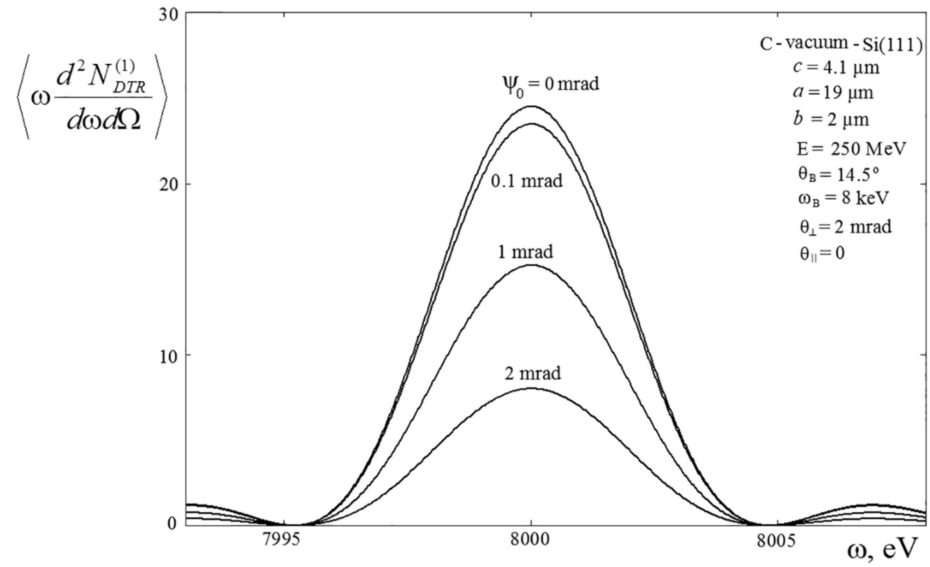

Fig. 3. DTR spectral-angular density under different values of the electron beam divergence ψ_0.

transition radiation depends on the relation between the divergence parameter ψ_0 and the characteristic angle of the TR $\theta \approx \gamma^{-1}$.

We can consider three cases:

1) $\psi_0 << \theta \approx \gamma^{-1}$, $\alpha_c(n,\theta_\perp,\theta_\|,\psi_\perp,\psi_\|) \approx \alpha_a(n,\theta_\perp,\theta_\|,\psi_\perp,\psi_\|) \approx 0$ – it means that for all electrons in the beam the interference will be maximally constructive;
2) $\psi_0 = \theta \approx \gamma^{-1}$, $\alpha_c(n,\theta_\perp,\theta_\|,\psi_\perp,\psi_\|) < 0.5rad$ - the interference of TR from the amorphous layer boundaries will have the constructive character, but for the same case $\alpha_a(n,\theta_\perp,\theta_\|,\psi_\perp,\psi_\|) < 2.2 \approx \pi/1.42$ and the constructive character of the interference will be destroyed for a significant part of the beam – the condition $\alpha_a(n,\theta_\perp,\theta_\|,\psi_\perp,\psi_\|) < 0.5$ will be satisfied only for $\psi_0 \leqslant 0.25\gamma^{-1}$.
3) $\psi_0 \gg \theta \approx \gamma^{-1}$, $\alpha_a(n,\theta_\perp,\theta_\|,\psi_\perp,\psi_\|) \gg \pi$ – the interference pattern in TR will be completely blurred.

The numerical calculations of the spectral-angular density of DTR presented in the following section confirm our estimations of the influence of the beam divergence on the constructive interference effect in transition radiation appearing on the boundaries of the amorphous layer and entrance surface of the single-crystal layer.

6. Numerical calculation

Numerical calculations have been performed based on the expressions (19) for the beam of relativistic electrons of the energy $E = 250\,\text{MeV}$ crossing the three-layer structure that consists of the amorphous carbon layer C, the vacuum layer and the single-crystal Si (111) layer. We assumed that the electron beam crosses the single-crystal layer under the conditions of symmetric reflection, i.e. a set of reflecting atomic planes in the crystal is perpendicular to the crystal layer surface ($\delta = \pi/2, \varepsilon = 1$) and the Bragg frequency $\omega_B = 8keV$ corresponds to the angle $\theta_B \approx 14.5^o$. The σ -polarized waves ($s = 1$, $\theta_{//} = 0$) were considered. Dependence of the DTR spectral-angular density on the electron beam divergence at aforementioned conditions were studied. We have chosen the thickness of the layers to be equal to c = 4.1 μm for the amorphous layer, a = 19 μm for the vacuum layer. These requirements satisfy the condition (17a (under $n = 0$) and the

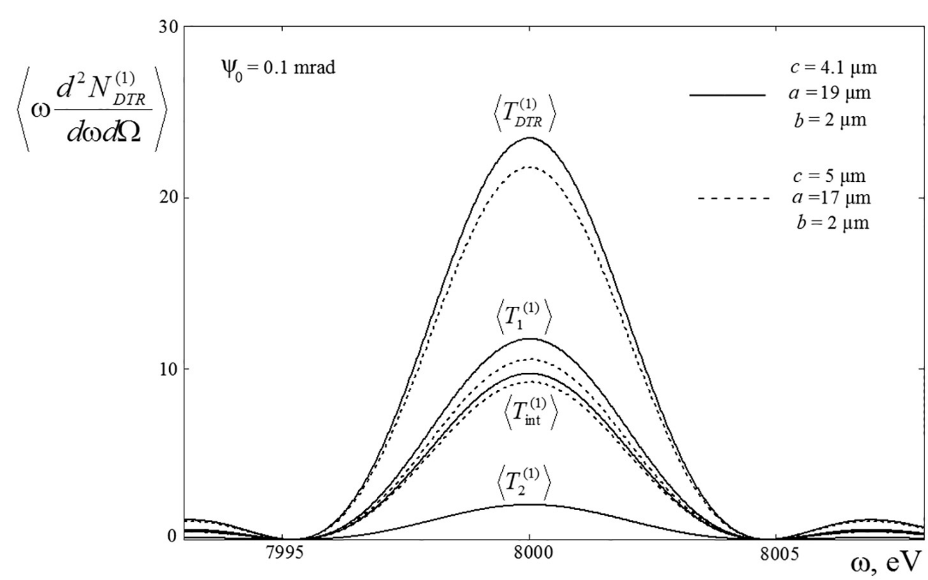

Fig. 4. The contributions of TR generated by relativistic electron beam on the amorphous layer $\langle T_1^{(1)} \rangle$ and on the front boundary of the single-crystal layer $\langle T_2^{(1)} \rangle$ and of their interference term $\langle T_{\text{int}}^{(1)} \rangle$ into DTR spectral-angular density $\langle T_{DTR}^{(1)} \rangle$.

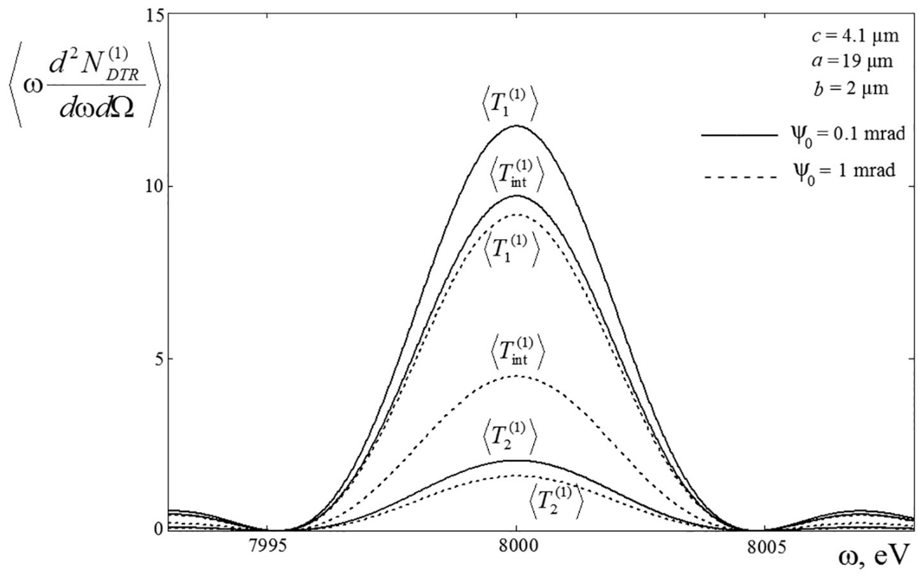

Fig. 5. The influence of the electron beam divergence on the diffracted transition radiation $\langle T^{(1)}_{DTR}\rangle = \langle T^{(1)}_1\rangle + \langle T^{(1)}_2\rangle + \langle T^{(1)}_{\mathrm{int}}\rangle$.

condition (17b) (under condition $m = 0$) for constructive interference of TR waves at the maximum of DTR angular density ($\theta = \gamma^{-1} \approx 2mrad$). The curves in Fig. 3 obtained by Eq. (19b) taking into account Eq. (14) demonstrate dependence of the spectral-angular density of DTR in three-layer structure on the value of the divergence angle ψ_0 .

In Fig. 4 the curves, Eq. (19b), demonstrate the contribution of TR by the amorphous layer $\langle T^{(1)}_1\rangle$ and the front surface of the single-crystal layer $\langle T^{(1)}_2\rangle$ while the contribution by the term describing their interference $\langle T^{(1)}_{\mathrm{int}}\rangle$ to the DTR angular density $\langle T^{(1)}_{DTR}\rangle$ under the beam divergence parameter $\psi_0 = 0.1$ mrad (see Fig. 3). Fig. 4 reveals an essential contribution of TR by the amorphous layer $\langle T^{(1)}_1\rangle$ to the spectral-angular density of DTR. The solid curves are plotted at the conditions (17a and (17b) corresponding to the case of constructive interference of TR waves. The dashed curves in Fig. 4 correspond to the conditions $c = 5\,\mu$m, $a = 17\,\mu$m. As illustrated in Fig. 4 these changes in the thicknesses of layers don't lead either to a dramatic change in the character of interference or to the transition of the constructive interference into the destructive one.

This fact points to the stability of the considered interference conditions and allows us to expect the realizability of experimental observation and investigation of interference effects in the coherent radiation generated in the considered three-layer target by beams of relativistic electrons.

In Fig. 5 the curves describing the terms $\langle T^{(1)}_1\rangle$, $\langle T^{(1)}_2\rangle$ and $\langle T^{(1)}_{\mathrm{int}}\rangle$ plotted for two different values of angular divergence ψ_0 of the electron beam are presented. These curves demonstrate considerable influence of the electron beam divergence on the contribution of the interference term in the spectral-angular density of DTR generated in the considered composite target.

In Fig. 6 the curves got by Eq. (12a) describe the contributions of PXR, DTR and their interference term to the spectral-angular density of coherent radiation under conditions of Fig. 3. The curves show that the contribution of PXR to the spectral-angular density of radiation is negligible in comparison with the contribution of DTR and their interference term. Moreover, the spectral-angular densities of PXR ($\langle T^{(1)}_{PXR}\rangle$), DTR ($\langle T^{(1)}_{DTR}\rangle$) and interference term ($\langle T^{(1)}_{\mathrm{int}(PXR,\mathrm{DTR})}\rangle$) as well as the contributions of transition radiation from amorphous layer $\langle T^{(1)}_1\rangle$,

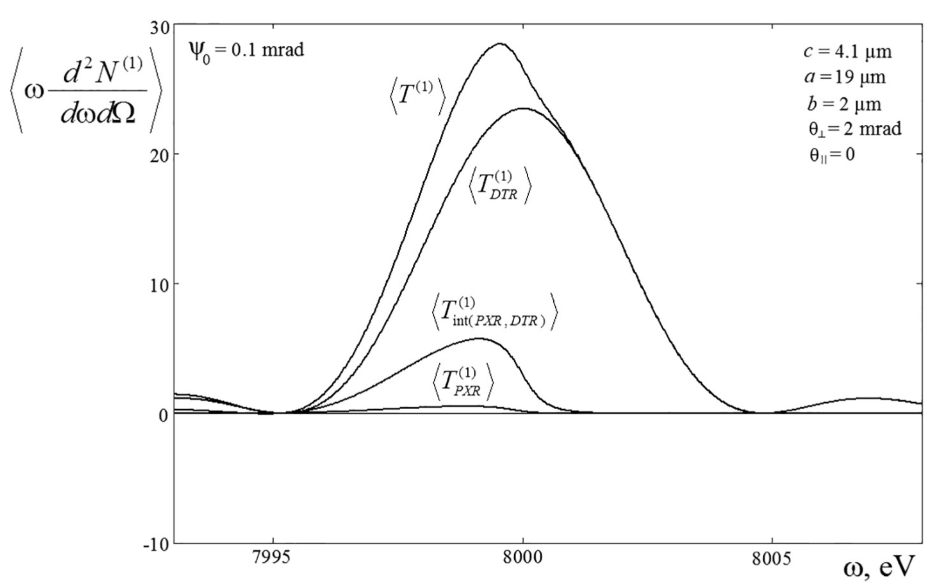

Fig. 6. Spectral-angular densities of PXR ($\langle T^{(1)}_{PXR}\rangle$), DTR ($\langle T^{(1)}_{DTR}\rangle$), their interference term $\langle T^{(s)}_{\mathrm{int(PXR,DTR)}}\rangle$ and the total spectral-angular density of coherent radiation $\langle T^{(1)}\rangle$.

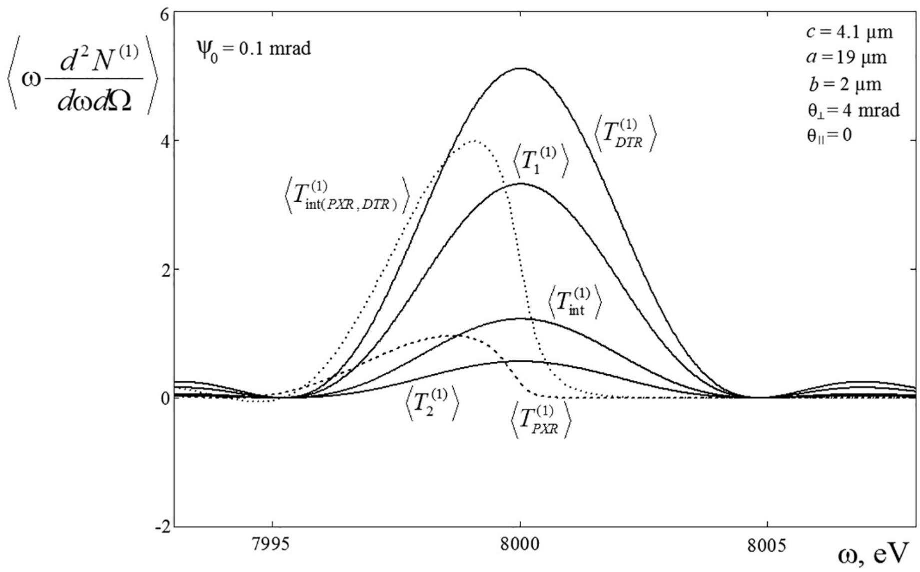

Fig. 7. Spectral-angular density of DTR $\langle T_{DTR}^{(1)} \rangle$, PXR $\langle T_{PXR}^{(1)} \rangle$ and their interference term $\langle T_{\mathrm{int(PXR,DTR)}}^{(1)} \rangle$. The other curves describe the terms forming the above-pointed term: $T_{DTR}^{(s)} = T_1^{(s)} + T_2^{(s)} + T_{\mathrm{int}}^{(s)}$.

the entrance surface of the single-crystal layer $\langle T_2^{(1)} \rangle$ and the interference term $\langle T_{\mathrm{int}}^{(1)} \rangle$ to the spectral-angular density of DTR ($\langle T_{DTR}^{(1)} \rangle$) have been studied for other fixed observation angle $\theta_\perp = 4\,mrad$.

For this case, in Fig. 7 the curves plotted at the conditions of Fig. 3 and constructive interference (Eq. (17)) are presented. One can see that under such conditions the term of PXR and DTR interference ($\langle F_{\mathrm{int(PXR,DTR)}}^{(s)} \rangle$) exceeds the term of PXR ($\langle T_{PXR}^{(1)} \rangle$), while the interference term $\langle T_{\mathrm{int}}^{(1)} \rangle$ exceeds $\langle T_2^{(1)} \rangle$.

The curves in Fig. 8 demonstrate the PXR, DTR and the contributions of the interference terms to the total spectral-angular density of the coherent radiation for the same conditions as in Fig. 7 except the other thicknesses of the layers in the composite target c and a. One can see that these deviations in layer thicknesses do not result in significant changes in the character of the interference process or, for example, the transition from constructive interference into destructive one.

As it is seen from (17a and (17b) the conditions of the constructive interference in DTR from the composite target also depend on the relativistic electron energy ($\sim\gamma$) and its dispersion ($\sim\Delta\gamma$). To estimate the influence of the electron energy dispersion on the interference effect we have made the calculation of spectral-angular density of DTR for two different values of γ. The results are presented in Fig. 9.

We can see that the change in the electron energy of 10 percent leads to the change in DTR spectral-angular density about of the same order but does not considerably influence the interference effect.

7. Conclusion

The dynamic theory of coherent X-ray radiation generated by a beam of relativistic electrons in the three-layer structure consisted of an amorphous layer, a vacuum (air) layer and a single crystal has been developed. The expressions for spectral-angular density of diffracted transition radiation (DTR), parametric X-ray radiation (PXR) and their interference term have been derived within the framework of two-wave approximation of the dynamic theory of X-ray diffraction in a single-crystal. The derived expressions consist of the contributions of transition radiation (TR) from the amorphous layer, TR from the front boundary of the single-crystal layer and their interference term to the spectral-angular density of DTR. The dependence of DTR characteristics

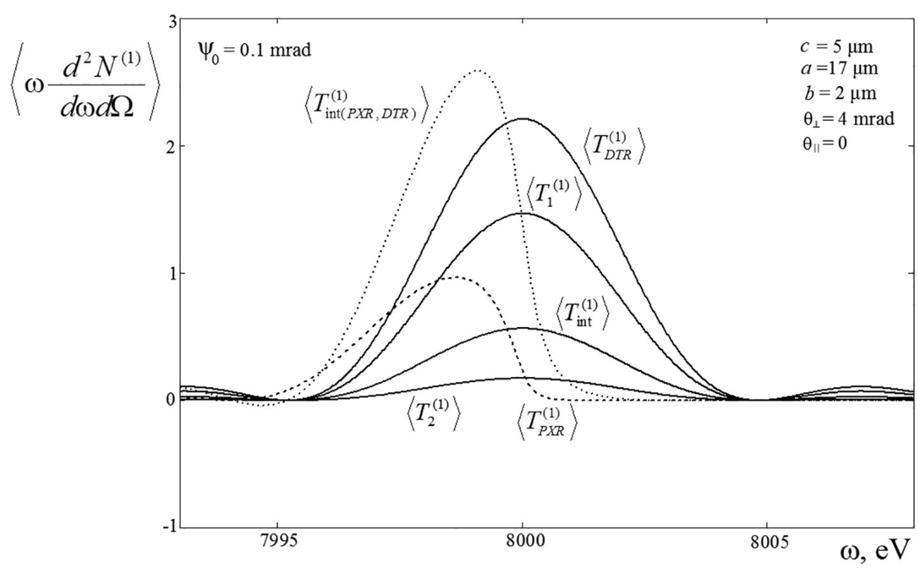

Fig. 8. The same as in Fig. 7, but for other thicknesses of the target layers (a and c).

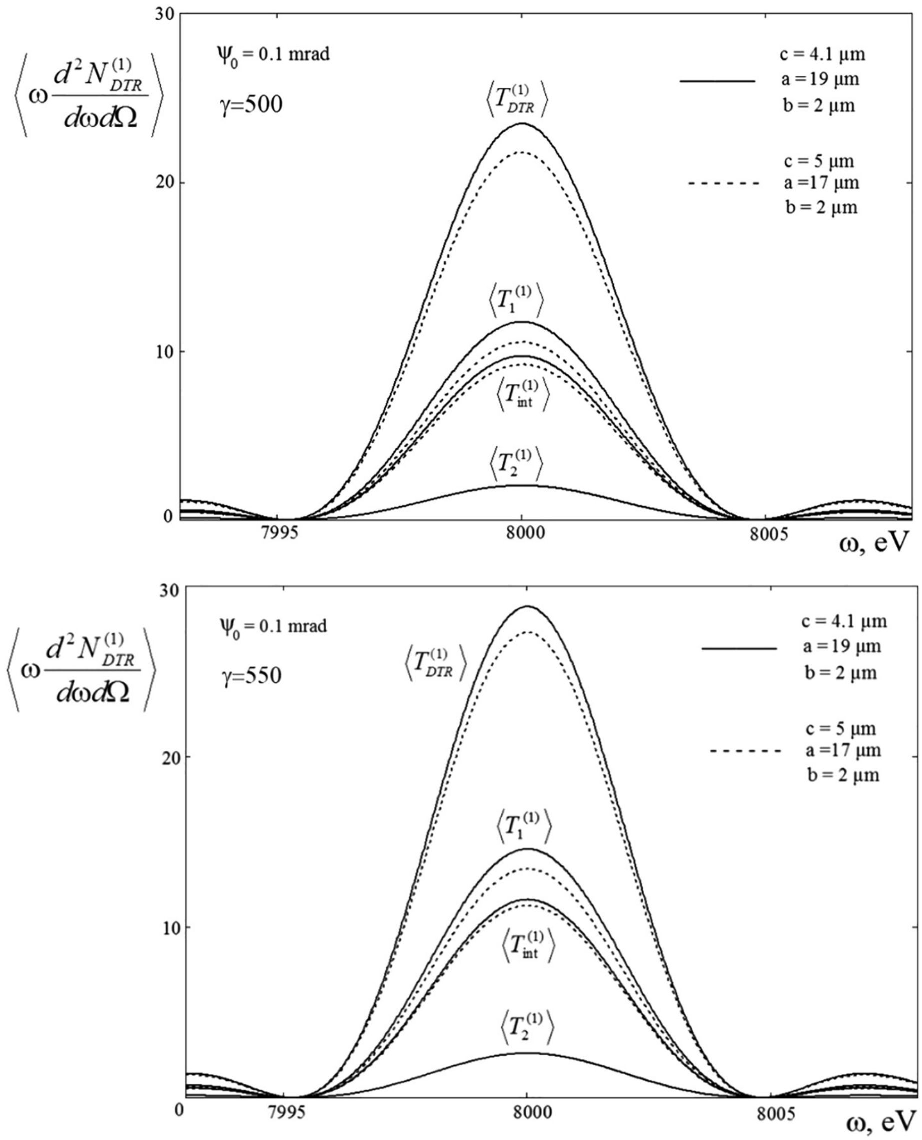

Fig. 9. DTR spectral-angular density for two different values of the relativistic electron energy. Solid lines correspond to optimal width of the target layers for $\gamma = 500$.

in such a structure on the beam divergence has been studied. The considerable influence of the beam divergence on the interference of TR from the amorphous layer and the front boundary of the single-crystal layer has been shown. It has also been demonstrated that small but perceptible changes in the thickness of the layers do not lead to the change of the character of interference in the vicinity of Bragg frequency (to the transition from constructive to destructive interference) i.e. these conditions are enough stable to use them for increasing the intensity of X-ray source. The considered composite target allows to increase the intensity of DTR up to 9 time in comparison with a single crystal target.

Acknowledgement

The present work is supported by the Ministry of Education and Science of the Russian Federation (project of state task No 3.4877.2017/ВУ).

References

[1] V.L. Ginzburg, V.N. Tsytovich, Transition Radiation and Transition Scattering, Nauka, Moscow, 1984.
[2] R. Rullhusen, X. Artru, P. Dhez, Novel Radiation Sources Using Relativistic Electrons, World Scientific, Singapore, 1999.
[3] A. Caticha, Phys. Rev. A. 40 (1989) 4322.
[4] V.G. Baryshevsky, Nucl. Instr. Meth. A. 122 (1997) 13.
[5] M. Ter-Mikaelian, High-Energy Electromagnetic Process in Condensed Media, Wiley, New York, 1972.
[6] G.M. Garibian, C. Yang, Sov. Phys. JETP 34 (1971) 495.
[7] V.G. Baryshevskii, I.D. Feranchuk, Sov. Phys. JETP 34 (1997) 502.
[8] S. Blazhevich, A. Noskov, Nucl. Instr. Meth. B 252 (2006) 69.
[9] S.V. Blazhevich, A.V. Noskov, Nucl. Instrum. Meth. B 266 (2008) 3770.
[10] S.V. Blazhevich, A.V. Noskov, J. Exp. Theor. Phys. 109 (2009) 901.
[11] M.Yu. Andreyashkin, V.N. Zabaev, K. Ioshida, V.V. Kaplin, E.I. Rozum, S.R. Uglov, I. Endo, JETP Lett. 62 (1995) 791.
[12] M.Yu. Andreyashkin, V.N. Zabaev, et al., JETP Lett. 65 (1997) 594.
[13] K. Nakayama, M. Sekimura, I. Yanase, I. Endo, Y. Takashima, V. Kaplin, A. Potylitsin, Nucl. Instr. Meth. B 145 (1998) 236.
[14] Y. Takashima, K. Aramitsu, I. Endo, A. Fukumi, et al., Nucl. Instr. Meth. B. 145 (1998) 25.
[15] V.V. Kaplin, S.I. Kuznetsov, N.A. Timchenko, S.R. Uglov, V.N. Zabaev, Nucl. Instr. Meth. B 173 (2001) (2001) 238.
[16] S.V. Blazhevich, A.V. Noskov, J. Exp. Theor. Phys. 120 (2015) 753.
[17] S.V. Blazhevich, A.V. Noskov, J. Exp. Theor. Phys. 123 (2016) 551.
[18] Z. Pinsker, Dynamical Scattering of X-rays in Crystals, Springer, Berlin, 1984.
[19] V.A. Bazylev, N.K. Zhevago, Radiation of High Energy Particles in the Substances and External Fields, Nauka, Moscow, 1987 [in Russian].