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Theoretical and phenomenological aspects of CPT violation

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Abstract

I review briefly various models and ways of quantum-gravity induced CPT violation, and discuss in some detail their phenomenology, in particular precision CPT tests in neutral mesons, and hydrogen/antihydrogen spectroscopy. As I shall argue, severe constraints can be placed in CPT violating parameters, with sensitivities that can safely exclude models with effects suppressed by a single power of Planck mass.

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1. Introduction: CPT theorem and its violation

Any complete theory of quantum gravity is bound to address fundamental issues, directly related to the emergence of space–time and its structure at energies beyond the Planck energy scale $M_P \sim 10^{19}$ GeV. From our relatively low energy experience so far, we are lead to expect that a theory of quantum gravity should respect most of the fundamental symmetries of particle physics, that govern the standard model of electroweak and strong interactions: Lorentz symmetry and CPT invariance, that is invariance under the combined action of charge conjugation (C), parity (reflection P) and time reversal symmetry (T). Actually the latter invariance is a theorem of any local quantum field theory that we can use to describe the standard phenomenology of particle physics to date. The *CPT theorem* can be stated as follows [1]: Any quantum theory, formulated on *flat space time* is symmetric under the combined action of CPT transformations, provided the theory respects (i) *locality*, (ii) *unitarity* (i.e. conservation of probability) and (iii) *Lorentz invariance*.

If such a theorem exists, then why do we have to bother to test CPT invariance, given that all our phenomenology up to now has been based on such quantum theories? The answer to this question is intimately linked with our understanding of *quantum gravity*. First of all, the theorem is not valid (at least in its strong form) in highly curved (*singular*) space times, such as black holes, or in general in space–time backgrounds of some quantum gravity theories involving the so-called *quantum space–time foam* backgrounds [2], that is *singular* quantum fluctuations of space time geometry, such as black holes etc., with event horizons of microscopic Planckian size (10^{-35} m). Such backgrounds result in *apparent* violations of *unitarity* in the following sense: there is part of

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information (quantum numbers of incoming matter) "disappearing" inside the microscopic event horizons, so that an observer at asymptotic infinity will have to trace over such "trapped" degrees of freedom. Thus, one faces a situation in which an initially pure state evolves in time to get mixed: the asymptotic states are described by density matrices, defined as follows:

$$\rho_{\rm out} = {\rm Tr}_M |\psi\rangle\langle\psi|,$$

where the trace is over trapped (unobserved) quantum states, that disappeared inside the microscopic event horizons in the foam. Such a non-unitary evolution results in the impossibility of defining a standard quantum-mechanical scattering matrix, connecting asymptotic states in a scattering process: $|{\rm out}\rangle = S|{\rm in}\rangle$, $S = {\rm e}^{{\rm i}H(t_{\rm f}-t_{\rm i})}$, where $t_{\rm f} - t_{\rm i}$ is the duration of the scattering (assumed much longer than other time scales in the problem). Instead, in foamy situations, one can define an operator that connects asymptotic density matrices [3]:

$$\rho_{\rm out} \equiv {\rm Tr}_M|{\rm out}\rangle\langle{\rm out}| = \$\rho_{\rm in}, \quad \$ \neq S\, S^\dagger,$$

where the lack of factorization is attributed to the apparent loss of unitarity of the effective low-energy theory, defined as the part of the theory accessible to low-energy observers who perform scattering experiments. This defines what we mean by *particle phenomenology* in such situations.

The \$ matrix is *not invertible*, and this reflects the effective unitarity loss. It is this property, actually, that leads to a *violation of CPT invariance* (at least in its strong form) in such a situation [4], since one of the requirements of CPT theorem (unitarity) is violated: In an open (effective) quantum theory, interacting with an environment, e.g. quantum gravitational, where $\$ \neq SS^\dagger$, CPT invariance is violated, at least in its strong form. The proof is based on elementary quantum mechanical concepts and the above-mentioned non-invertibility of \$, but will be omitted here due to lack of space [4]. Another reason for CPT violation (CPTV) in quantum gravity is *spontaneous breaking of Lorentz symmetry*, without necessarily implying decoherence. This may also occur in string theory and other models. In certain circumstances one may also violate locality, e.g. of the type advocated in [5] to explain observed neutrino physics "anomalies", but we shall not discuss this case here.

The CPT violating effects can be estimated naively to be strongly suppressed, and thus inaccessible – for all practical purposes – to current, or immediate future, low-energy experiments. Indeed, naively, quantum gravity (QG) has a dimensionful constant: $G_{\rm N} \sim 1/M_{\rm P}^2$, where $M_{\rm P} = 10^{19}$ GeV is the Planck scale. Hence, CPT violating and decoherening effects may be expected to be suppressed by $E^3/M_{\rm P}^2$, where E is a typical energy scale of the low-energy probe. However, there may be cases where loop resummation and other effects in theoretical models may result in much larger CPT-violating effects of order: $\frac{E^2}{M_{\rm P}}$. This happens, for instance, in some loop gravity approaches to QG, or some non-equilibrium stringy models of space–time foam involving open string excitations. Such large effects can lie within the sensitivities of current or immediate future experimental facilities (terrestrial and astrophysical). Below we shall describe a few such sensitive probes, starting from neutral kaon decays.

2. Quantum gravity decoherence and CPT violation in neutral kaons

QG may induce decoherence and oscillations $K^0 \to \overline{K}^0$ [6,7]. The modified evolution equation for the respective density matrices of neutral kaon matter can be parametrized as follows [6]:

$$\partial_t \rho = {\rm i}[\rho, H] + \delta H\!\!/\rho,$$

where

$$H_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\frac{1}{2}\delta\Gamma & -{\rm Im}\Gamma_{12} & -{\rm Re}\Gamma_{12} \\ -\frac{1}{2}\delta\Gamma & -\Gamma & -2{\rm Re}M_{12} & -2{\rm Im}M_{12} \\ -{\rm Im}\Gamma_{12} & 2{\rm Re}M_{12} & -\Gamma & -\delta M \\ -{\rm Re}\Gamma_{12} & -2{\rm Im}M_{12} & \delta M & -\Gamma \end{pmatrix}$$

and

$$\delta H\!\!/_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix}.$$

Positivity of ρ requires: $\alpha, \gamma > 0$, $\alpha\gamma > \beta^2$. Notice that α, β, γ violate CPT, as they do not commute with a CPT operator Θ [7]: $\Theta = \sigma_3 \cos\theta + \sigma_2 \sin\theta$, $[\delta H/_{\alpha\beta}, \Theta] \neq 0$.

An important remark is now in order. We should distinguish two types of CPTV: (i) CPTV within quantum mechanics [8]: $\delta M = m_{K^0} - m_{\overline{K}^0}$, $\delta\Gamma = \Gamma_{K^0} - \Gamma_{\overline{K}^0}$. This could be due to (spontaneous) Lorentz violation (c.f. below). (ii) CPTV through decoherence α, β, γ (entanglement with QG 'environment', leading to modified evolution for ρ and $\$ \neq SS^\dagger$).

The important point is that the two types of CPTV can be *disentangled experimentally* [7]. The relevant observables are defined as $\langle O_i \rangle = \mathrm{Tr}\,[O_i\rho]$. For neutral kaons, one looks at decay asymmetries for $K^0, \overline{K}^0$, defined as

$$A(t) = \frac{R(\overline{K}^0_{t=0} \to \overline{f}) - R(K^0_{t=0} \to f)}{R(\overline{K}^0_{t=0} \to \overline{f}) + R(K^0_{t=0} \to f)},$$

where $R(K^0 \to f) \equiv \mathrm{Tr}\,[O_f \rho(t)] =$ denotes the decay rate into the final state f (starting from a pure K^0 state at $t = 0$).

In the case of neutral kaons, one may consider the following set of asymmetries: (i) *identical final states*: $f = \overline{f} = 2\pi$: $A_{2\pi}, A_{3\pi}$, (ii) *semileptonic*: A_{T} (final states $f = \pi^+ l^- \overline{\nu} \neq \overline{f} = \pi^- l^+ \nu$), A_{CPT} ($\overline{f} = \pi^+ l^- \overline{\nu}$, $f = \pi^- l^+ \nu$), $A_{\Delta m}$. Typically, for instance when final states are 2π, one has a time evolution of the decay rate $R_{2\pi}$: $R_{2\pi}(t) = c_{\mathrm{S}} e^{-\Gamma_{\mathrm{S}} t} + c_{\mathrm{L}} e^{-\Gamma_{\mathrm{L}} t} + 2c_{\mathrm{I}} e^{-\Gamma t} \cos(\Delta m t - \phi)$, where S = short-lived, L = long-lived, I = interference term, $\Delta m = m_{\mathrm{L}} - m_{\mathrm{S}}$, $\Gamma = \frac{1}{2}(\Gamma_{\mathrm{S}} + \Gamma_{\mathrm{L}})$. One may define the *decoherence parameter* $\zeta = 1 - \frac{c_{\mathrm{I}}}{\sqrt{c_{\mathrm{S}} c_{\mathrm{L}}}}$, as a measure of quantum decoherence induced in the system. For larger sensitivities one can look at this parameter in the presence of a regenerator [7]. In our decoherence scenario, ζ depends primarily on β, hence the best bounds on β can be placed by implementing a regenerator [7].

The experimental tests (decay asymmetries) that can be performed in order to disentangle decoherence from quantum mechanical CPT violating effects are summarized in Table 1. In Fig. 1 we give a typical profile of a decay asymmetry, that of A_{T} [7], from where bounds on QG decoherening parameters can be extracted. Experimentally, the best available bounds come from CPLEAR measurements [9] $\alpha < 4.0 \times 10^{-17}$ GeV, $|\beta| < 2.3. \times 10^{-19}$ GeV, $\gamma < 3.7 \times 10^{-21}$ GeV, which are not much different from theoretically expected values α, β, $\gamma = O\left(\xi \frac{E^2}{M_{\mathrm{P}}}\right)$.

Table 1
Qualitative comparison of predictions for various observables in CPT-violating theories beyond (QMV) and within (QM) quantum mechanics

Process	QMV	QM
$A_{2\pi}$	$\neq$	$\neq$
$A_{3\pi}$	$\neq$	$\neq$
A_{T}	$\neq$	$=$
A_{CPT}	$=$	$\neq$
$A_{\Delta m}$	$\neq$	$=$
ζ	$\neq$	$=$

Predictions either differ ($\neq$) or agree ($=$) with the results obtained in conventional quantum-mechanical CP violation. Note that these frameworks can be qualitatively distinguished via their predictions for A_{T}, A_{CPT}, $A_{\Delta m}$ and ζ.

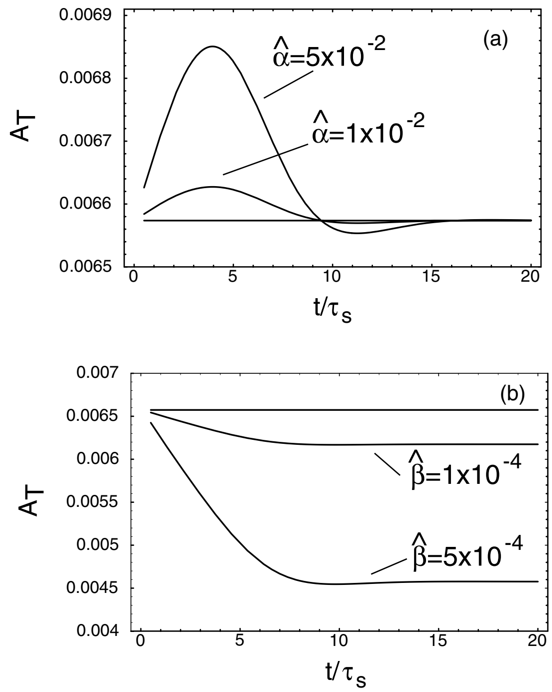

Fig. 1. A typical neutral kaon decay asymmetry A_{T} [7] indicating the effects of quantum-gravity induced decoherence.

3. Spontaneous violation of Lorentz symmetry and (anti)hydrogen

A second possibility for CPTV effects arises if the Lorentz symmetry is violated *spontaneously*, but no quantum decoherence or unitarity loss necessarily occurs. Such a situation may be envisaged in some string theory (non supersymmetric) models, where some tensorial fields acquire vevs $\langle T_{\mu_1\ldots\mu_n}\rangle \neq 0$. This will result in a spontaneous breaking of Lorentz symmetry by (exotic) string vacua, implying a modified Dirac equation (MDE) for fermions in the so-called standard model extension (SME) [10,11]. In view of the recent 'massive' production of antihydrogen ($\overline{\mathrm{H}}$) at CERN [12], which implies that interesting direct tests of CPT invariance using $\overline{\mathrm{H}}$ are to be expected in the near future, we consider for our purposes here the specific case of MDE for Hydrogen H (anti-hydrogen $\overline{\mathrm{H}}$). Let the spinor ψ represent the electron (positron) with charge $q=-|e|(q=|e|)$ around a proton (antiproton) of charge $-q$. Then the MDE reads

$$\big(\mathrm{i}\gamma^\mu D^\mu - M - a_\mu\gamma^\mu - b_\mu\gamma_5\gamma^\mu \\ -\tfrac{1}{2}H_{\mu\nu}\sigma^{\mu\nu} + \mathrm{i}c_{\mu\nu}\gamma^\mu D^\nu + \mathrm{i}d_{\mu\nu}\gamma_5\gamma^\mu D^\nu\big)\psi = 0,$$

where $D_\mu = \partial_\mu - qA_\mu$, $A_\mu = (-q/4\pi r, 0)$ Coulomb potential. The parameters a_μ, b_μ induce CPT and Lorentz violation, while the parameters $c_{\mu\nu}$, $d_{\mu\nu}$, $H_{\mu\nu}$ induce Lorentz violation only.

In SME models there are energy shifts between states $|J,I;m_J,m_I\rangle$, with $J(I)$ denoting electronic (nuclear) angular momenta. Using perturbation theory, one finds [11]

$$\Delta E^H(m_J,m_I) \simeq a_0^{\mathrm{e}} + a_0^{\mathrm{p}} - c_{00}^{\mathrm{e}}m_{\mathrm{e}} - c_{00}^{\mathrm{p}}m_{\mathrm{p}} \\ + (-b_3^{\mathrm{e}} + d_{30}^{\mathrm{e}}m_{\mathrm{e}} + H_{12}^{\mathrm{e}})\frac{m_J}{|m_J|} \\ + (-b_3^{\mathrm{p}} + d_{30}^{\mathrm{p}}m_{\mathrm{p}} + H_{12}^{\mathrm{p}})\frac{m_I}{|m_I|},$$

where e, electron; p, proton. The corresponding results for antihydrogen ($\overline{\mathrm{H}}$) are obtained upon

$$a_\mu^{\mathrm{e,p}} \to -a_\mu^{\mathrm{e,p}}, b_\mu^{\mathrm{e,p}} \to -b_\mu^{\mathrm{e,p}}, d_{\mu\nu}^{\mathrm{e,p}} \to d_{\mu\nu}^{\mathrm{e,p}}, H_{\mu\nu}^{\mathrm{e,p}} \to H_{\mu\nu}^{\mathrm{e,p}}.$$

One may study the spectroscopy of forbidden transitions 1S–2S: If CPT and Lorentz violating parameters are constant they drop out to leading order energy shifts in free H ($\overline{\mathrm{H}}$). Subleading effects are then suppressed by the square of the fine structure constant: $\alpha^2 \sim 5\times 10^{-5}$, specifically: $\delta\nu_{1\mathrm{S}-2\mathrm{S}}^{\mathrm{H}} \simeq -\frac{\alpha^2 b_3^{\mathrm{e}}}{8\pi}$. This is too small to be seen.

But what about the case where atoms of H (or $\overline{\mathrm{H}}$) are in magnetic traps? Magnetic fields induce hyperfine Zeeman splittings in 1S, 2S states. There are four spin states, mixed under the the magnetic field B ($|m_J,m_I\rangle$ basis): $|d\rangle_n = |\frac{1}{2},\frac{1}{2}\rangle$, $|c\rangle_n = \sin\theta_n|-\frac{1}{2},\frac{1}{2}\rangle + \cos\theta_n|\frac{1}{2},-\frac{1}{2}\rangle$, $|b\rangle_n = |-\frac{1}{2},-\frac{1}{2}\rangle$, $|a\rangle_n = \cos\theta_n|-\frac{1}{2},\frac{1}{2}\rangle - \sin\theta_n|\frac{1}{2},-\frac{1}{2}\rangle$, where $\tan 2\theta_n = (51\ \mathrm{mT})/\mathrm{n}^3B$. The $|c\rangle_1 \to |c\rangle_2$ transitions yield dominant effects for CPTV [11]:

$$\delta\nu_c^H \simeq -\frac{\kappa(b_3^{\mathrm{e}} - b_3^{\mathrm{p}} - d_{30}^{\mathrm{e}}m_{\mathrm{e}} + d_{30}^{\mathrm{p}}m_{\mathrm{p}} - H_{12}^{\mathrm{e}} + H_{12}^{\mathrm{p}})}{2\pi},$$

$$\delta\nu_c^{\overline{\mathrm{H}}} \simeq -\frac{\kappa(-b_3^{\mathrm{e}} + b_3^{\mathrm{p}} - d_{30}^{\mathrm{e}}m_{\mathrm{e}} - d_{30}^{\mathrm{p}}m_{\mathrm{p}} - H_{12}^{\mathrm{e}} + H_{12}^{\mathrm{p}})}{2\pi},$$

$$\Delta\nu_{1\mathrm{S}-2\mathrm{S},c} \equiv \delta\nu_c^H - \delta\nu_c^{\overline{\mathrm{H}}} \simeq -\frac{\kappa(b_3^{\mathrm{e}} - b_3^{\mathrm{p}})}{\pi},$$

where $\kappa = \cos 2\theta_2 - \cos 2\theta_1$, $\kappa \simeq 0.67$ for $B = 0.011$ T. Notice that $\Delta\nu_{c\to d} \simeq -2b_3^{\mathrm{p}}/\pi$, and, if a frequency resolution of 1 mHz is attained, one may obtain a bound $|b_3| \leqslant 10^{-27}$ GeV. Other low energy atomic and nuclear physics experiments may place stringent bounds on spatial components of the CPTV parameters of the SME, and are summarized in Fig. 2 [13].

We next point out that, in some stringy models of space time foam, interaction of string matter with space–time solitonic defects results in a modified Dirac equation of SME type but only with (boost sensitive) temporal components of a_0 which, however, turn out to be energy dependent [14]. For instance, for protons, one has $a_0 \sim \xi\frac{E^3}{E-m_{\mathrm{p}}}\frac{1}{M_{\mathrm{P}}}$, where ξ depends on string interaction coupling and is model dependent. The model also predicts modified Dispersion relations [15]. The energy dependence of a_0 in this case implies that hyperfine Zeeman splittings due to external magnetic field B acquire shifts $\Delta E \sim a_0(E)$. Hence (say 1S level)

$$\delta\nu_{1\mathrm{S}}^H - \mu_{\mathrm{N}}B \sim \frac{\xi}{M_{\mathrm{P}}}\frac{m_{\mathrm{p}}^3}{\epsilon_{1\mathrm{S}}^2}\mu_{\mathrm{N}}B \sim \xi 10^{-21}\left(\frac{B}{\mathrm{mT}}\right)\mathrm{GeV},$$

where $\epsilon_{1\mathrm{S}}$ is the energy level, μ_{N} nuclear magneton. H, $\overline{\mathrm{H}}$ spectroscopic measurements may be devised

LEADING ORDER BOUNDS

EXPER.	SECTOR	PARAMS. (J=X,Y)	BOUND (GeV)
Penning Trap	electron	$\bar{b}_J^e$	5×10^{-25}
Hg-Cs clock comparison	electron	$\bar{b}_J^e$	10^{-27}
	proton	$\bar{b}_J^p$	10^{-27}
	neutron	$\bar{b}_J^n$	10^{-30}
H Maser	electron	$\bar{b}_J^e$	10^{-27}
	proton	$\bar{b}_J^p$	10^{-27}
spin polarized matter	electron	$\bar{b}_J^e / \bar{b}_Z^e$	10^{-29} / 10^{-28}
He-Xe Maser	neutron	$\bar{b}_J^n$	10^{-31}
Muonium	muon	$\bar{b}_J^\mu$	2×10^{-23}
Muon g-2	muon	$\bar{b}_J^\mu$	5×10^{-25} (estimated)

X,Y.Z celestial equatorial coordinates $\bar{b}_J = b_3 - md_{30} - H_{12}$

(Bluhm, hep-ph/0111323)

Fig. 2. Table summarising recent bounds of CPT violating parameter b in the Standard Model extension from atomic and nuclear physics spectroscopic tests (from Bluhm hep-ph/0111323).

to constrain the parameter ξ in a_0. Also, one may envisage using relativistic beams of H, $\overline{\mathrm{H}}$ to enhance such CPTV effects.

A note is appropriate at this stage on the *frame dependence* of the above results on CPTV effects. If Lorentz symmetry is violated (LV) then the effects should be frame dependent. $\Delta\nu_c^H$ depends on spatial components of LV couplings, and so it is subject to sidereal variations due to Earth rotation (clock comparison experiments using H alone). Usually, in such situations there is a preferred frame, which might be taken to be the cosmic microwave background frame with velocity $w \sim 10^{-3}c$. High precision tests are then possible, if modified dispersion relations for matter probes exist; such tests proceed via quadrupole moment measurements [16], which exhibit sensitivities up to 10^{23}GeV $> M_P = 10^{19}$ GeV for minimally suppressed QG modified dispersion relations. Severe constraints on such models come also from astrophysics [17] (e.g. Crab Nebula magnetic field measurements imply sensitivity of some quantum gravity effects up to scales 10^{27} GeV $\gg M_P = 10^{19}$ GeV).

4. Conclusions

There are plenty of low energy nuclear and atomic physics experiments which yield stringent bounds in models with Lorentz and CPT violation. Frame dependence of Lorentz violating (LV) effects may be crucial in providing such stringent experimental constraints. Indeed, experiments from nuclear physics (via quadrupole moment measurements) can constrain some models of QG predicting LV modified dispersion relation of matter probes, by exploiting appropriately the frame dependence of such effects. It is worthy of stressing that such measurements exhibit sensitivity to energy scales that exceed the Planck scale by several orders of magnitude, thereby safely excluding models with minimal (linear) Planck scale suppression.

The recently "massive" production of Antihydrogen [12] will undoubtedly turn out to be very useful in providing physical systems appropriate for placing stringent bounds on some of these CPTV parameters (relevant to spontaneous violation of Lorentz symmetry) via spectroscopic measurements and comparison with hydrogen results, provided the frequency resolution improves. A natural question arises at this point, concerning the possibility of constraining CPT violating QG-induced decoherence parameters using H, $\overline{\mathrm{H}}$. This remains to be seen. In addition, such tests may be performed in other low-energy probes such as slow neutrons in the gravitational field of the Earth. Preliminary studies in this system reveal a striking formal similarity with that of neutral kaons, and the analysis can be easily carried through in this case. At present, however, stringent bounds on the decoherening parameters cannot be placed.

Certainly, more work needs to be done, both theoretical and experimental, before conclusions are reached, but we do think that the current and immediate future experimental situation looks very promising in providing important information about Planck scale Physics from low energy high precision data.

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