

Radiation from a particle in flight through a plate, the parameters of which vary according to an arbitrary periodical law



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ARTICLE INFO

Article history:

Received 24 November 2012

Received in revised form 9 February 2013

Accepted 19 February 2013

Available online 22 March 2013

Keywords:

Relativistic electron

Cherenkov radiation

Layered medium

ABSTRACT

The radiation from a relativistic charged particle in flight through a layered finite-length medium is investigated. Expressions for the spectral-angular distribution of radiation in forward and backward hemispheres are derived with no limitations on the amplitude and variation profile of the layered medium parameters. It is shown that the periodicity of layered medium strongly influences the radiation from the particle in the wavelength range of the order of layered medium period: the Cherenkov radiation emitted in the forward hemisphere is redirected by the periodical structure of medium in the backward direction. The visual explanation of this effect is given and a possible application is discussed.

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1. Introduction and formulation of the problem

The presence of matter may essentially influence the characteristics of high energy electromagnetic processes that give rise to production of Cherenkov Radiation (CR), transition radiation etc. The effects of interest arise in periodical structures of different configurations [1,2].

The present paper deals with radiation from a particle in flight through a layered medium of finite length L (Fig. 1). Below we have investigated some features of spectral-angular distribution $I_{\pm}(\omega, \theta_{\pm}, L)$ of the energy

$$W_{\pm} = \int I_{\pm}(\omega, \theta_{\pm}, L) d\omega d\theta_{\pm} \quad (1)$$

of radiation propagating in vacuum in the forward (+) and backward (−) directions (with respect to the direction of particle motion) during the whole time of particle motion. We shall confine to treatment of a problem of radiation with wavelength of the order of layered medium period l with l ranging from a fraction of micron to millimeter. In this wavelength range the permittivity ε_l and magnetic permeability μ_l of a layered medium (e.g., photonic crystal) may change in wide limits. Respectively, the effect of layered medium periodicity on radiation from relativistic particles may prove large.

It is peculiar that in periodic structures (layered media) there are forbidden bands on dispersion curves for electromagnetic waves. The presence of such bands may influence the radiation

from a charged particle in periodic structures and for this reason these may serve as additional means for controlling the particle radiation parameters. This mechanism was first investigated in case of infinite layered medium composed of two alternating plates having different values of permittivity ε [3]. In [4,5] an infinite stack of plates was replaced with a laminated medium, the permittivity of which is an arbitrary one-dimensional periodic function (the problem of a stack of the finite number of plates has been investigated in [6,7], see also [8]). However, the calculation methods used in [4,5] (quasi-classical approximation, perturbation theory) failed to allow for the influence of forbidden bands on the emitted electromagnetic waves. In the present paper we aim at the solution of this problem.

The analysis of angular distribution of radiation energy is complicated by the fact that propagating inside the layered medium are the Bloch waves (traveling waves modulated with the periodicity of medium), not the plane ones:

$$B_m(z+l) = \delta^{\mu_m} B_m(z), \quad \text{where } |\delta| \leq 1 \text{ and } \mu_m = \begin{cases} -1 & \text{when } m=1 \\ +1 & \text{when } m=2 \end{cases} \quad (2)$$

For this reason we consider the radiation in vacuum outside the plate (Fig. 1). The formulas required for calculation of $I_{\pm}(\omega, \theta_{\pm}, L)$ have been derived in [9] for the case of semi-infinite layered medium ($L = \infty$). In the present paper a more realistic problem of periodic medium of finite length L is solved.

The spectral-angular distribution of radiation energy at large distances from a laminated plate is determined by means of well-known expression [4,5,7]

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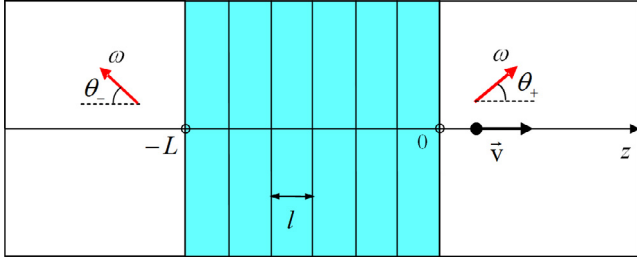


Fig. 1. Radiation from a relativistic charged particle in flight through a plate made of layered medium of finite length.

$$I_{\pm}(\omega, \theta_{\pm}) = \frac{2q^2}{\pi c} \frac{\cos^2 \theta_{\pm}}{\sin \theta_{\pm}} |a_{\pm}(\omega, \theta_{\pm})|^2, \quad (3)$$

where a_{\pm} is a dimensionless amplitude describing the free field (the radiation field) and q is the charge of particle. For calculation of a_{\pm} it is sufficient to avail of the complete solution of Maxwell equations that is valid for all $-\infty < z < \infty$.

Similar to [10] we shall represent the Bloch functions as sums of two arbitrary independent solutions $u_{(1)}(z)$, $u_{(2)}(z)$ of the same equation, to which the very Bloch functions also satisfy:

$$B_m(z) = p_m^{(1)} u_{(1)}(z) - p_m^{(2)} u_{(2)}(z). \quad (4)$$

For expansion coefficients in (4) the following expressions [10] will be used

$$\begin{aligned} p_m^{(1)} &= u_{(2)}(l+0) - \delta^{\mu_m} u_{(2)}(+0) \\ p_m^{(2)} &= u_{(1)}(l+0) - \delta^{\mu_m} u_{(1)}(+0). \end{aligned} \quad (5)$$

The calculations may be simplified using some arbitrariness in the choice of $u_{(1)}(z)$, $u_{(2)}(z)$. The steps of appropriate analytical calculations are described in [9] (see also [11]).

2. General formulae

Omitting the intermediate calculations we now give the resulting expressions

$$\begin{aligned} a_{+} &= \frac{\omega}{\sqrt{k\gamma}} \left[\eta_1^{(2)} \xi_{(2)} - \eta_1^{(1)} \xi_{(1)} + (\sigma_1 - \tau_1)(\varepsilon_1^{-1} - 1) + \frac{1}{2} \left(1 - \frac{v^2}{c^2} \right) \left(\frac{\gamma - 2\tau_1}{1 - kv/\omega} + \frac{\gamma_1}{1 + kv/\omega} \right) \right] \\ a_{-} \cdot \phi &= \frac{\omega}{\sqrt{k\gamma}} \left[\eta_2^{(1)} \xi_{(1)} - \eta_2^{(2)} \xi_{(2)} + (\sigma_2 - \tau_2)(\varepsilon_1^{-1} - 1) + \frac{1}{2} \left(1 - \frac{v^2}{c^2} \right) \left(\frac{\gamma - 2\tau_2}{1 + kv/\omega} + \frac{\gamma_2}{1 - kv/\omega} \right) \right] \end{aligned} \quad (6)$$

for the case of plate containing $N = 1; 2; 3 \dots$ periods of layered medium: $L = N \cdot l$. In (6)

$$\xi_{(m)} = i\omega/v \int_{+0}^{l+0} (\varepsilon_1^{-1} - \mu_l v^2/c^2 + i\nu \varepsilon_l/\omega \varepsilon_1^2) u_{(m)} \exp(i\omega z/v) dz \quad (7)$$

and then

$$\begin{aligned} \gamma &= B_{1-} B_{2+} - \delta^{2N} B_{1+} B_{2-}, \quad \gamma_1 = (1 - \delta^{2N}) B_{1+} B_{2+} \\ \gamma_2 &= (1 - \delta^{2N}) B_{1-} B_{2-}, \quad \sigma_1 = B_{1-} B_{2+} - \delta^{2N} B_{1+} B_{2-} \\ \eta_1^{(m)} &= \alpha_2 p_2^{(m)} B_{1+} \delta^{2N} - \alpha_1 p_1^{(m)} B_{2+}, \quad \sigma_2 = B_{1-} B_{2-} - \delta^{2N} B_{1+} B_{2-} \\ \eta_2^{(m)} \cdot \Delta_2^{-N} &= \alpha_1 p_1^{(m)} B_{2-} - \alpha_2 p_2^{(m)} B_{1-}, \quad \tau_1 \cdot \Delta_1^N = \tau_2 \cdot \Delta_2^{-N} = B_{1-} B_{2+} - B_{1+} B_{2-} \\ B_{m\pm} &= B_m \pm \dot{B}_m / i k \varepsilon_l, \quad \alpha_m = (1 - \Delta_m^{-N}) / (1 - \Delta_m) \\ \Delta_m &= \delta^{\mu_m} \exp[i\omega l/v]. \end{aligned} \quad (8)$$

Functions $\varepsilon_l(z)$, $B_m(z)$ and $B_{m\pm}(z)$ in (6) and (8) shall be taken in $z = 0$ point, the point over the function implies the derivative with respect to its argument. In (6) there are no limitations on the amplitude and variation profile of $\varepsilon_l(z)$ and $\mu_l(z)$, $\phi = -\exp(i\omega N l/v)$

being an inessential phase factor and, eventually, $k(\theta) = \omega/c \cdot \cos \theta$, where $\theta = \theta_{+}$ for $a = a_{+}$ and $\theta = \theta_{-}$ for $a = a_{-}$.

Equations for a_{\pm} are simplified for the case of weakly absorbing semi-infinite layered medium ($L = \infty$), as $\delta^N \rightarrow 0$ when $N \rightarrow \infty$. An appropriate expression for a_{+} (in other equivalent form) is given in [9]. In other particular case, when $\varepsilon_l(z)$ and $\mu_l(z)$ are periodic step functions, expressions (6)–(8) describe the radiation from a charged particle traversing a stack of identical plates. In this case expressions (6)–(8) give the same numerical results, as those well known expressions used in [5–7].

3. Features of CR generated in a layered finite-length medium

The real part of the layered medium permittivity may be written down as follows:

$$\varepsilon_l'(z) = \overline{\varepsilon_l'(z)} + \Delta \varepsilon_l' \cdot \alpha(z), \quad -L \leq z \leq 0, \quad (9)$$

where $\overline{\varepsilon_l'(z)}$ and $\Delta \varepsilon_l'$ are the average value and the modulation depth of $\varepsilon_l'(z)$, and $\alpha(z)$ is the periodic function describing the variation profile of $\varepsilon_l'(z)$. Below we shall use the non-linear model of function $\alpha(z)$ given in [9] by confining ourselves to the following data:

$$\varepsilon_l'(0) = \overline{\varepsilon_l'(z)} = 1.5, \quad \Delta \varepsilon_l' = 0.5, \quad \varepsilon_l''/\varepsilon_l' = 0.02, \quad \mu_l = 1. \quad (10)$$

We shall consider the following range of frequencies

$$\omega \sim \pi v/l = \omega_0/2, \quad (11)$$

which is of special interest, and simplify the problem assuming that the dispersion of electromagnetic waves is negligible. In (11) $\omega_0 = 2\pi v/l$ is the cyclic frequency, at which the parameters of layered medium vary along the particle trajectory. The energy of a relativistic particle (electron) is taken to be equal to 10 MeV.

The results of numerical calculations of $I_{\pm}(\omega, \theta_{\pm}, L)$ using formulae (6)–(8) are given in cases of radiation in the forward hemisphere (Fig. 2), and that in the backward hemisphere (Fig. 3). As it follows from above data: (a) the forward radiation in vacuum is directional, (b) there are dips in radiation intensity in the frequency range of (11), and then: (c) the backward radiation in vacuum is directional and quasi-monochromatic, (d) the direction of radiation is the same as that in Fig. 2.

In (10) $\Delta \mu_l' = 0$ and the value of $\Delta \varepsilon_l/\overline{\varepsilon_l'} \cong 0.33$ is small enough. For this reason the layered medium may be approximately de-

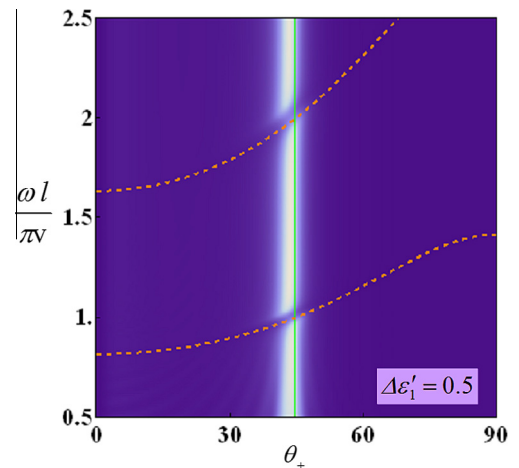


Fig. 2. The spectral-angular distribution of radiation energy in the forward direction $I_{+}(\omega, \theta_{+}, L)$, in vacuum at the departure of 10 MeV electron from a plate. The plate is made of layered medium with parameters (9) and (10), $L = 100 \cdot l$. The light-colored areas in the figure correspond to larger values of I_{+}/h (the lighter the area, the larger is the value of I_{+}/h). The vertical straight line corresponds to (12), the dotted curves – to (16) at $n = 1; 2$.

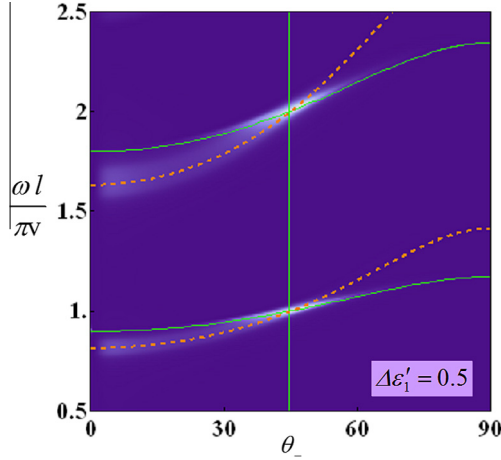


Fig. 3. The spectral-angular distribution of radiation energy $L(\omega, \theta_-, L)$ in the backward direction at the electron incidence from vacuum to the plate. The continuous curves correspond to (17) for θ_- and $s = 1; 2$. The remaining explanations are similar to those in Fig. 2

scribed by means of average values $\bar{\epsilon}'_l(z) = 1.5$ and $\bar{\mu}_l(z) = 1$. Here the Cherenkov condition is satisfied for 10 MeV electron:

$$\cos \bar{\theta}_l \cong c/v \sqrt{\bar{\epsilon}'_l \bar{\mu}'_l} \leq 1 \quad (12)$$

Here $\bar{\theta}_l$ is the angle showing the direction of CR. In case when CR emerges from the finite-length medium to vacuum, one may write down

$$\sqrt{\bar{\epsilon}'_l \bar{\mu}'_l} \sin \bar{\theta}_l \cong \sin \theta \quad (13)$$

(the refraction law). In (13) θ is the angle showing the propagation direction of CR in vacuum ($\theta = \theta_+$ for radiation in the forward and $\theta = \theta_-$ for radiation in the backward directions). Substituting the average values of $\bar{\epsilon}_l$ and $\bar{\mu}_l$ (see (10)) in (12) and (13) one finds

$$\theta_{\pm} \cong \arcsin \sqrt{\bar{\epsilon}'_l \bar{\mu}'_l - c^2/v^2} \cong 45^\circ \quad (14)$$

The vertical straight lines in Figs. 2 and 3 correspond to these values of θ_{\pm} . The central peaks in Figs. 2 and 3 are seen to correspond to (14) sufficiently well.

The dips in the spectral-angular distribution of energy of CR in Fig. 2 reflect the fact of presence of forbidden bands for electromagnetic waves that freely propagate in the layered medium. Really, in one-dimensional periodical (along OZ axis) structure, the edges of Brillouin zones are determined by equation

$$k_z = \pi n/l, \quad (15)$$

where n is an integer. In our case the wave number $k_z = \omega/c \cdot \sqrt{\bar{\epsilon}'_l \bar{\mu}'_l} \cos \bar{\theta}_l$ and (15) is equivalent to the condition of Bragg diffraction. If radiation emerges from the medium to vacuum, (15) transforms (see (13)) to

$$\omega = \pi n c/l \sqrt{\bar{\epsilon}'_l \bar{\mu}'_l - \sin^2 \theta_{\pm}}. \quad (16)$$

The dotted curves in Figs. 2 and 3 correspond to plots of function (16) at $n = 1; 2$. The dips in the spectral-angular distribution in Fig. 2 agree well with behavior of these curves. The peaks for backward radiation in Fig. 3 pass through the dotted curves (16) describing the edges of Brillouin zones. This fact testifies the relation of these peaks with the periodical structure of layered medium: some part of CR spectrum is redirected by this structure in the backward direction, giving the contribution in the range of angles determined by (14).

The resonance radiation from a relativistic particle in the layered medium also makes a contribution to the spectral-angular distribution of radiation energy. The contribution of this radiation is approximately described using the formula given in [4]. In case when the resonance radiation emerges from the layered medium to vacuum, one may write down

$$\omega \cong \omega_0 s / \left| 1 \mp \frac{v}{c} \sqrt{\bar{\epsilon}'_l \bar{\mu}'_l - \sin^2 \theta_{\pm}} \right| \quad (17)$$

using the refraction law (13) (s is a natural number). The plots of curve $\omega = \omega(\theta_-)$ determined by (17) at $s = 1; 2$ are given in Fig. 3 for comparison. The contribution of resonance radiation in Fig. 3 is seen as thin offshoots branching from the peaks. For radiation in the forward direction from (17) it follows $\omega l / \pi v \geq 6.8s$ and, therefore, in Fig. 2 the resonance radiation does not make contribution. As the transition radiation is weak, it is not observable in Figs. 2 and 3.

The data in Figs. 2 and 3 have other visual explanations, that are not given here for brevity.

4. Conclusions

In the present paper the radiation from a relativistic electron in flight through a layered medium of finite length is investigated. Expressions for spectral-angular distribution of radiation in forward and backward hemispheres are derived with no limitations on the values of amplitude and variation profile of the permittivity and magnetic permeability of layered medium.

The results of numerical calculations testify that the periodicity of layered medium strongly influences the CR from the relativistic charged particle in the wavelength range of the order of layered medium period: this part of CR emitted by the particle in the forward hemisphere is redirected by the periodical structure of medium in the backward direction (Fig. 3). The layered medium serves two purposes: it generates CR (owing to the influence of particle) and simultaneously partially redirects CR back.

The periodical structure with adjustable l and $\Delta \epsilon'_l$ may be induced in the medium by using, e.g., ultrasonic vibrations. In such a case one may easily control the radiation wavelength of diffracted CR in the millimeter and sub-millimeter ranges by attuning ultrasonic vibrations in the range of frequencies in excess of 10 MHz. One may also use the effect of CR diffraction for probing the periodical structure by an electronic bunch.

Acknowledgments

The authors are thankful to anonymous reviewers, the valuable comments of which helped to improve the statement of problem. One of the authors (L.Sh.G.) is thankful to A.A. Ponomarenko for valuable discussions.

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