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Comment about the article *Interpretation of the results of the experiment on generation of parametric X-radiation by relativistic electrons in a single-crystal target*, by S.V. Blazhevich and A.V. Noskov

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ABSTRACT

The strong dependence of the PXR yield on the orientation of the crystal surface, claimed in a recent article by S. Blazhevich and A. Noskov, is questioned. It is argued that, if absorption is neglected, the sum of the ordinary and forward PXR yields does not depend of the surface orientation.

In the article entitled *Interpretation of the results of the experiment on generation of parametric X-radiation by relativistic electrons in a single-crystal target* [1], S. Blazhevich and A. Noskov assert that the experimental results reported in [2] can be understood only if one takes into account the asymmetric orientation of the crystal surface with respect to the diffracting atomic planes. Their basic formula (6) involves the asymmetry parameter $\varepsilon = \hat{\mathbf{n}}\cdot\hat{\mathbf{k}}/\hat{\mathbf{n}}\cdot\hat{\mathbf{v}}$, where $\hat{\mathbf{n}}$ is the unit vector normal to the entrance and exit surfaces, $\mathbf{k} = k\hat{\mathbf{k}}$ the exit photon momentum and $\mathbf{v} = v\hat{\mathbf{v}}$ the electron velocity. Neglecting the X-ray absorption in the crystal, one can deduce from formula (6) that the intensity of the PXR angular distribution in the Laue geometry is roughly proportional to ε, for fixed angle $\theta_{\rm B}$ between and the diffracting planes and fixed length T of the electron path in the crystal. This is also stated in Ref. [3] on which formula (6) is based. Fig. 3 of [1] shows indeed a ratio about 2/3 between the predicted PXR yields for the asymmetric case of [2] ($\varepsilon = 0.66$) and the symmetric case ($\varepsilon = 1$).

This linear dependence on ε is surprising. ε can be theoretically as large as one wants and, taking $\varepsilon \gg 1$, one could greatly enhance the PXR yield "for free". However it is generally admitted that PXR is produced in the *bulk* of the crystal, therefore, if absorption is negligible, the total flux of X-ray leaving the crystal should not depend on the orientation of the surface. This flux includes the *PXR along the electron velocity*, in short *forward* PXR. Furthermore, the rough proportionality to ε implied by Eq. (6) of [1] persists in the limit $\chi_g \to 0$. In this limit the kinematical theory, which predicts no ε dependence, should be valid. Indeed the dynamical theory takes into account the iteration of Bragg scattering, the amplitude being of the form $\chi_{\mathbf{g}}$ times a series in $\chi_{\mathbf{g}}\chi_{-\mathbf{g}} = |\chi_{\mathbf{g}}|^2$, while the kinematical one keeps only the $0^{\rm th}$ order in $\chi_{\mathbf{g}}\chi_{-\mathbf{g}}$.

A result different from Eq. (6) of [1], but fulfilling the above conditions, was obtained in Ref.[4] for the Laue geometry:

$$\frac{dN_s^{\rm PXR}}{d\Omega} = \frac{\alpha\,\omega_B T}{4\pi\sin^2\theta_B}\frac{|C_s\chi_{\mathbf{g}}|^2\,\theta_s^2}{(\gamma^{-2}+\theta^2-\chi_0)^2+\varepsilon^{-1}|C_s\chi_{\mathbf{g}}|^2}. \tag{1}$$

$s = x$ or y labels the polarization, the y-axis being along $\mathbf{v}^R\times\mathbf{g}$, the x-axis along $\hat{\mathbf{y}}\times\mathbf{v}^R$ and $\mathbf{g}$ being the reciprocal lattice vector; $\theta = (\theta_x, \theta_y) \simeq \hat{\mathbf{k}} - \hat{\mathbf{v}}^R$, where $\hat{\mathbf{v}}^R$ is the symmetric of $\hat{\mathbf{v}}$ with respect to the diffracting planes; $C_x = \cos(2\theta_B)$ and $C_y = 1$; $\alpha \simeq 1/137$; $\omega_{\rm B} = g/(2\sin\theta_{\rm B})$; $\varepsilon = (\hat{\mathbf{n}}\cdot\mathbf{q})/(\hat{\mathbf{n}}\cdot\mathbf{q}')$ of Ref. [5] $\simeq \varepsilon$ of Ref. [1]. Formula (1) does not possesses an overall factor ε and converges to the kinematical one in the limit $\chi_{\mathbf{g}} \to 0$.

We found independently Eq. (1), starting from Eqs. (7) and (33) of [5], which give the spectral-angular distribution $dN_{\hat{\mathbf{e}}}/(d\omega\,d\Omega)$ of PXR (unseparated from Diffracted Transition Radiation) and integrating over ω. The large T behavior of is dominated by the $1/h_1$ pole of the amplitude through the "golden rule" $|(1-e^{-ih_1T})/h_1|^2 \to 2\pi\,T\,\delta(h_1)$.

We also (partly) verify our assertion that total flux of X-ray leaving the crystal does not depend on $\hat{\mathbf{n}}$. For the *forward* PXR angular distribution in the Laue geometry, we took Eq. (24) of Ref. [6], which we rewrite as

$$\frac{dN_s^{\rm FPXR}}{d\Omega'} = \frac{\alpha\,\omega_B T}{4^{(?)}\pi\sin^2\theta_B}\frac{\varepsilon^{-1}|C_s\chi_{\mathbf{g}}|^2\,\theta_s'^2}{(\gamma^{-2}+\theta'^2-\chi_0)^2[(\gamma^{-2}+\theta'^2-\chi_0)^2+\varepsilon^{-1}|C_s\chi_{\mathbf{g}}|^2]}, \tag{2}$$

where θ_x' and θ_y' are defined like θ_x and θ_y but replacing $\mathbf{v}^R$ by . We inserted a factor 4 (with a question mark) in the first denominator. It is not present in Ref. [6] but in Eq. (16) of Kubankin et al. [7] which treat the case $\varepsilon = 1$. Assuming that this correction is justified, one deduces for

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$\theta' = \theta$

$$\frac{dN^{\rm PXR}(\theta)}{d\Omega} + \frac{dN^{\rm FPXR}(\theta')}{d\Omega'} = \frac{\alpha\,\omega_B T}{4\pi \sin^2\theta_B} \sum_{s=x,y} \frac{|C_s \chi_{\mathbf{g}}|^2\,\theta_s^2}{(\gamma^{-2} + \theta^2 - \chi_0)^2}. \quad (3)$$

Thus the sum of the ordinary PXR and forward PXR yields does not depend on the orientation of $\hat{\mathbf{n}}$, as expected for a radiation produced in the bulk.

Remark. The methods used in [4,6,5] are in fact the same, although presented differently. They involve scattering states which are solutions of *homogeneous* Maxwell equations in the crystal. Ref. [5] invokes the *reciprocity principle*: the amplitude is the work made on the electron by the electric field of a scattering state *coming from the detector* in the time-reversed process. This method, presented in [8], is suitable for various types of radiations emitted by relativistic electrons in polarizable media, like PXR and Transition Radiation. Refs. [3,7] use a quite different method. They start from a solution of the *inhomogeneous* Maxwell equations in an infinite crystal, the source being an electron in infinite uniform motion. Internal and external homogeneous solutions are added to fit the continuity on the crystal surface.

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