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Classical dynamics of free electromagnetic laser pulses

S. Goto [a], R.W. Tucker [b], T.J. Walton [c,*]

[a] Institute for Molecular Science, 38 Nishigo-Naka, Myodaiji, Okazaki 444-8585, Japan
[b] Department of Physics, University of Lancaster and Cockcroft Institute, Daresbury Laboratory, Warrington, UK
[c] Department of Mathematics, University of Bolton, Deane Campus, Bolton, UK

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ABSTRACT

We discuss a class of exact finite energy solutions to the vacuum source-free Maxwell field equations as models for multi- and single cycle laser pulses in classical interaction with relativistic charged test particles. These solutions are classified in terms of their *chiral* content based on their influence on particular charge configurations in space. Such solutions offer a computationally efficient parameterization of compact laser pulses used in laser-matter simulations and provide a potential means for experimentally bounding the fundamental length scale in the generalized electrodynamics of Bopp, Landé and Podolsky.

Advances in laser technology have made possible the exploration of physical processes on unprecedented temporal and spatial scales. They have also opened up new possibilities for accelerating charged particles using laser-matter interactions. Multi- and single cycle high intensity ($10^{10} - 10^{15}\,\mathrm{W\,cm^{-2}}$) laser pulses can be produced using Q-switching or mode-locking techniques [1]. Such pulses can accelerate charged particles such as electrons to relativistic speeds where radiation reaction and quantum effects may influence their dynamics. Lower intensity pulses have also been used as diagnostic tools for exploring the structure of plasmas in various states [2,3]. In order to interpret experimental data involving classical laser interactions with both charged and neutral matter, theoretical models [4–7] rely crucially on parameterizations of the electromagnetic fields in laser pulses, particularly in situations where traditional formulations using monochromatic or paraxial-beam approximations have limitations [8–10].

In this Letter we discuss a viable methodology for parameterizing a particular class of propagating solutions to the source free *classical* Maxwell equations in vacuo that offers an efficient means to explore the classical effects of compact laser pulses on free electrons in dynamical regimes where quantum effects are absent. The parameterization is based on a remarkable class of explicit solutions of the scalar wave equation found by Ziolkowski [11–15] following pioneering work by Brittingham [16]. Such solutions can be used to construct classical Maxwell solutions with bounded total electromagnetic energy and fields bounded in all three spatial directions. With simple analytic structures their diffractive properties can be readily determined together with the behavior of charged particle-pulse interactions over a broad parameter range without recourse to expensive numerical computation. Finally, we argue that such parameterizations can be used to find compact finite energy solutions to other linear wave equations. This is illustrated by showing that the generalized theory of Bopp [17], Landé [18] and Podolsky [19] admits such particular solutions that reduce to the Maxwell solutions when a fundamental length parameter in their theory tends to zero. Compact laser pulses in this theory might be used to explore properties of the theory by searching experimentally for bounds on this parameter.

If a complex scalar field α satisfies $\Box\alpha = 0$ on spacetime and $\Pi_{\mu\nu}$ is any covariantly constant (degree 2) anti-symmetric tensor field on spacetime (i.e. $\Pi_{\mu\nu;\delta} = 0$) for all $\mu, \nu, \delta = 0, 1, 2, 3$, then the complex tensor field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ satisfies the source free Maxwell equations in vacuo with:

$$A_\nu = \partial_\gamma(\alpha\,\Pi_{\mu\beta})\,\epsilon_\nu^{\gamma\mu\beta}\sqrt{|\,g\,|} \qquad (1)$$

where $|\,g\,|$ is the determinant of the spacetime metric and $\epsilon_\nu^{\gamma\mu\beta}$ denotes the Levi–Civita alternating symbol. In the following g refers to the Minkowski metric tensor field, in which case the components

* Corresponding author.
E-mail addresses: sgoto@ims.ac.jp (S. Goto), r.tucker@lancaster.ac.uk (R.W. Tucker), t.walton@bolton.ac.uk (T.J. Walton).

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$\Pi_{\mu\nu}$ can be used to encode three independent Hertz vector fields and their duals[1].

General solutions to $\Box\alpha = 0$ can be constructed by Fourier analysis. In cylindrical polar Minkowski coordinates $\{t, r, z, \theta\}$, axially symmetric solutions propagating along the z-axis have, for $z \geqslant 0$, the double integral representation $\alpha(t, r, z) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\,\widetilde{\alpha}(\omega, r, z)$ where:

$$\widetilde{\alpha}(\omega, r, z) = \int_0^{\frac{\omega}{c}} k\, f_\omega(k)\, J_0(kr)\, \exp\left(\pm iz\sqrt{\left(\frac{\omega}{c}\right)^2 - k^2}\right) dk + \int_{\frac{\omega}{c}}^{\infty} k\, f_\omega(k)\, J_0(kr)\, \exp\left(-z\sqrt{k^2 - \left(\frac{\omega}{c}\right)^2}\right) dk$$

in terms of the zero order Bessel function and the speed of light in vacuo c.

Conditions on the Fourier amplitudes $f_\omega(k)$ can be given so that the Hertz procedure above gives rise to real singularity free electromagnetic fields with finite total electromagnetic energy. A particularly simple class of pulses that can be generated in this way follows from the complex axi-symmetric scalar solution:

$$\alpha(t, r, z) = \frac{\ell_0^2}{r^2 + (\psi_1 + i(z - ct))(\psi_2 - i(z + ct))} \tag{2}$$

where ℓ_0, ψ_1, ψ_2 are strictly positive (real) parameters with physical dimensions of length. The relative sizes of ψ_1 and ψ_2 determine both the direction of propagation along the z-axis of the dominant maximum of the pulse profile and the number of spatial cycles in its peak magnitude. When $\psi_1 \gg \psi_2$, the dominant maximum propagates along the z-axis to the right. The parameter ℓ_0 determines the magnitude of such a maximum. The structure of such solutions has been extensively studied in [20,21] in conjunction with particular choices of $\Pi_{\mu\nu}$ together with generalizations discussed in [22,23].

In general the six anti-symmetric tensors with components $\delta^{\mu}_{[\gamma}\delta^{\nu}_{\sigma]}$ in a Minkowski Cartesian coordinate system are covariantly constant and can be used to construct a complex eigen-basis of antisymmetric *chiral* tensors $\Pi^{s,\kappa}$, with $s \in \{\mathrm{CE}, \mathrm{CM}\}$ and $\kappa \in \{-1, 0, 1\}$, satisfying

$$\mathcal{O}_z\, \Pi^{s,\kappa} = \kappa\, \Pi^{s,\kappa} \tag{3}$$

where the operator $\mathcal{O}_z$ represents θ rotations about the z-axis generated by $-i\partial_\theta$ on tensors[2]. These in turn can be used to construct a *complex basis* of chiral eigen-Maxwell tensor fields $F^{s,\kappa}$. The index s indicates that the CE (CM) chiral family contain electric (magnetic) fields that are orthogonal to the z-axis when $\kappa = 0$. The chiral eigen-fields $F^{s,0}$ inherit the axial symmetry of $\alpha(t, r, z)$ while those with $\kappa = \pm 1$ do not. The directions of electric and magnetic fields in any of these Maxwell solutions depend on their location in the pulse and the concept of a pulse polarization is not strictly applicable. The chiral content as defined here can be used in its place. Non-chiral pulse configurations can be constructed by superposition $\sum_s \sum_\kappa F^{s,\kappa}\mathcal{C}^{s,\kappa}$ with arbitrary complex coefficients $\mathcal{C}^{s,\kappa}$.

The energy, linear and angular momentum of the pulse in vacuo can be calculated from the components $T_{\mu\nu}$ of the Maxwell stress-energy tensor $T_{\mu\nu} = -\frac{1}{4} g_{\mu\nu}\mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta} - \mathcal{F}_{\mu\alpha}\mathcal{F}^{\alpha}_{\nu}$ where $\mathcal{F}_{\mu\nu} = \mathrm{Re}(F_{\mu\nu})$. If $\mathbf{e}$ and $\mathbf{b}$ denote time-dependent *real* electric and magnetic 3-vector fields associated with any pulse solution, its total electromagnetic energy $\mathcal{J}$, for a fixed set of parameters and any z, is calculated from

$$\mathcal{J} = \frac{1}{\mu_0}\int_{-\infty}^{\infty} dt \int_S (\mathbf{e}\times\mathbf{b})\cdot d\mathbf{S} \tag{4}$$

where $\mathbf{S}$ can be any plane with constant $z = z_0 > 0$. For *spatially compact* pulse fields in vacuo this coincides with the total pulse electromagnetic energy

$$\mathcal{E} = \int_{\mathcal{V}} \rho\, d\mathcal{V} = \int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta \int_0^{\infty} r dr\, \rho(t, r, z, \theta) \tag{5}$$

where $\rho \equiv \frac{1}{2}\left(\epsilon_0\,\mathbf{e}\cdot\mathbf{e} + \frac{\mathbf{b}\cdot\mathbf{b}}{\mu_0}\right)$ is integrated over all space $\mathcal{V}$. This follows since $\nabla\cdot(\mathbf{e}\times\mathbf{b}) = -\mu_0\,\partial_t\,\rho$. To correlate $\mathcal{J}$ with other laser pulse properties and the choice of parameters, we bring the pulse into classical interaction with one or more charged point particles. The world-line of a single particle, parameterized in arbitrary coordinate as $x^\mu = \xi^\mu(\tau)$ with a parameter τ, is taken as a solution of the coupled non-linear differential equations

$$\mathcal{A}_\mu(\tau) = \frac{q}{m_0 c^2}\,\mathcal{F}_{\mu\nu}(\xi(\tau))\, V^\nu(\tau) \tag{6}$$

in terms of the particle charge q and rest mass m_0, for some initial conditions $\xi(0)$, $V(0)$, where the particle 4-velocity satisfies $V^\nu V_\nu = -1$ and its 4-acceleration is expressed in terms of the Christoffel symbols $\Gamma^{\delta\beta}_{\mu}$ as $\mathcal{A}_\mu = \partial_\tau V_\mu(\tau) + V_\delta(\tau)\, V_\beta(\tau)\, \Gamma^{\delta\beta}_{\mu}(\xi(\tau))$. In the following, radiation reaction and inter-particle forces are assumed negligible. From the solution $\xi(\tau)$ one can determine the increase (or decrease) in the relativistic kinetic energy transferred from the electromagnetic pulse to any particle and the nature of its trajectory in the laboratory frame. This information can then be used to correlate the dynamical properties of the interaction with the laser pulse properties fixed by the parameters. To facilitate this exercise, it proves important to reduce the above equations of motion to dimensionless form and fix the physical dimensions of the fields involved. The Minkowski metric tensor field $g = g_{\mu\nu} dx^\mu\, dx^\nu$ (with $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$) in inertial coordinates $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$) has SI physical dimensions $[L]^2$. The SI dimension of electromagnetic quantities follows by assigning to $\epsilon_0 F_{\mu\nu}\, dx^{[\mu} dx^{\nu]}$ in any coordinate system the physical dimension of electric charge. Furthermore, in terms of Minkowski polar coordinates $\{t, r, z, \theta\}$, introduce (for ease of visualization) the dimensionless coordinates $\{R = \frac{r}{\Phi\ell_0}, T = \frac{ct}{\ell_0}, Z = \frac{z}{\Xi\ell_0}\}$ and dimensionless parameters $\Lambda, \Psi_j = \frac{\psi_j}{\ell_0}$ $(j = 1, 2)$ where $[\Psi_j] = [\Phi] = [\Xi] = 1, [\ell_0] = [L]$. Then with the dimensionless complex scalar field $\widehat{\alpha}(T, R, Z) = \alpha(t, r, z)$ and greek indices ranging over $\{T, R, Z, \theta\}$ with $\epsilon^{T,R,Z,\theta} = 1$, we write

$$A_\delta = \frac{m_0 c^2 \ell_0^3\, \Lambda}{q}\, \partial_\gamma\left(\widehat{\alpha}\, \widehat{\Pi}_{\mu\beta}\right)\epsilon^{\gamma\mu\beta}_{\delta}\sqrt{|\, g\,|} \tag{7}$$

for a choice of dimensionless covariantly constant tensor $\widehat{\Pi}_{\mu\beta}$ so that $[\epsilon_0 A_\mu\, dx^\mu]$ has the physical dimension of electric charge and

$$\mathcal{J} = \int_{-\infty}^{\infty} dT \int_0^{\infty} dR \int_0^{2\pi} d\theta\, P(T, R, Z, \theta)$$
$$\mathcal{E} = \int_{-\infty}^{\infty} dZ\, \widehat{\mathcal{E}}(T, Z).$$

The parameter Λ controls the strength of all electric and magnetic fields in $F_{\beta\delta}$ for fixed values of the parameters Ψ_1, Ψ_2, Φ, Ξ and the overall scale ℓ_0 will be fixed in terms of the total electromagnetic energy of the pulse. For a choice of such parameters the real fields $\mathbf{e}$ and $\mathbf{b}$ enable one to calculate a numerical value Γ such that $\mathcal{J} = \ell_0\Gamma$. The diffraction of the pulse peak along the z-axis can be used to define a pulse range relative to the maximum of the pulse peak at $z = 0$. To this end, the density $\widehat{\mathcal{E}}(T, Z)$ defines

[1] In the language of differential forms on Minkowski spacetime $A = \star d(\alpha\Pi)$, $F = dA$ where $d\star d\alpha = 0$, the 2-form Π satisfies $\nabla\Pi = 0$ and $\star$ denotes the Hodge map associated with g.

[2] In terms of the Lie derivative, $\mathcal{O}_z = -i\mathcal{L}_{\partial_\theta}$ and $\Pi^{\mathrm{CE},\pm1} = d(x \pm iy)\wedge dt$, $\Pi^{\mathrm{CE},0} = dz\wedge dt$, $\Pi^{\mathrm{CM},\kappa} = \star\Pi^{\mathrm{CE},\kappa}$ where $x = r\cos(\theta), y = r\sin(\theta)$

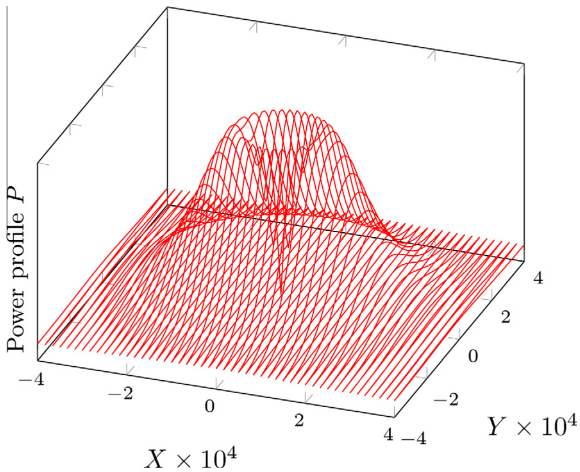

Fig. 1. Power profile P of the (CM, 1) laser pulse at $Z = 0$, $T = 0$ with parameters $\{\Lambda = 600,\ \Psi_1 = 1,\ \Psi_2 = 1000,\ \Phi = 0.001,\ \Xi = 1\}$.

the dimensionless range Z_{rg} by $\widehat{\mathcal{E}}(0,0)/\widehat{\mathcal{E}}(T_1, Z_{rg}) = 2$, where the peak at $Z = Z_{rg} > 0$ and $T = T_1 > 0$ is half the height of the peak at $Z = 0, T = 0$. If during the interval $[0, T_1]$ the pulse propagates with negligible deformation in Z, one may estimate its width Z_w at half height and the dimensionless pulse axial speed $\beta = Z_{rg}/T_1$. This yields the dimensionless pulse duration or temporal width $T_0 = Z_w/\beta$. From these dimensionless values one deduces the pulse SI characteristics in terms of ℓ_0 and hence $\mathcal{J}$. If the picosecond is used as a unit of time, the pulse duration becomes $t_0 = \ell_0 T_0/c = \ell_0 Z_w/(\beta c) = N\,10^{-12}$ s for some value N and hence $\ell_0 = (c\beta N/Z_w)\,10^{-12}$ m, $\mathcal{J} = (\Gamma \beta c N/Z_w)\,10^{-12}$ J, $z_w = \Xi c \beta n\,10^{-12}$ m and $z_{rg} = \ell_0 \Xi Z_{rg} = (\Xi \beta c N Z_{rg}/Z_w)\,10^{-12}$ m. A dimensionless spot-size of the pulse at $Z = Z_0 > 0, T = Z_0/\beta$ is then determined by the behavior of $P(R, Z_0/\beta, Z_0, \theta)$. At each value of Z_0 this function of R and θ has a clearly defined principal maximum. If one associates a circle of dimensionless radius $R_s(Z_0)$ with such a maximum locus it can be used to define a spot-size at $z = z_0$ with radius $r_s(z_0) = \ell_0 \Phi R_s(Z_0) = (c\beta N \Phi R_s(Z_0)/Z_w)10^{-12}$ m.

Fig. 1 displays a clearly pronounced principle maximum in the power density profile P as a function of $X = R\cos(\theta)$ and $Y = R\sin(\theta)$ at $Z = 0, T = 0$ for a specific choice of the parameters $(\Lambda, \Psi_1, \Psi_2, \Phi, \Xi)$. The same parameter set is used to numerically solve (6) for a collection of trajectories for charged particles, each arranged initially around the circumference of a circle in a plane orthogonal to the propagation axis of incident CM type laser pulses with different chirality. The resulting space curves in 3-dimensions, displayed in Fig. 2, clearly exhibit the different responses to CM pulses with distinct chirality values. The instantaneous *specific* relativistic kinetic energy of a particle with laboratory speed v is $\gamma - 1$ in terms of the Lorentz factor γ given by $\gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}}$. In Fig. 3, this quantity is displayed on the left for a charged particle accelerated by a fixed chirality (CM,−1) type pulse where the pulse energy is varied by changing Λ. On the right the energy transfer dependence on pulse chirality for both CE and CM type pulses with *fixed laser energy* is displayed. We deduce that the momentum and angular momentum [24] in the propagation direction can transfer an impulsive force and torque respectively to charges lying in an orthogonal plane. More generally, the classical

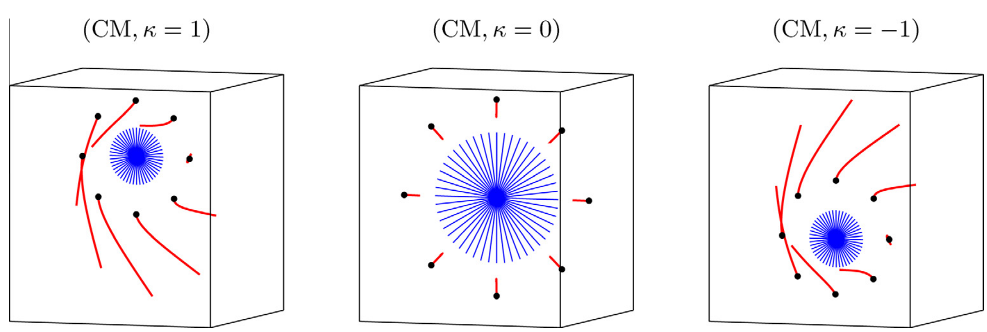

Fig. 2. Three-dimensional spacecurves for particles subject to an incident (CM, 1) laser pulse (left), (CM, 0) laser pulse (center) and (CM, −1) laser pulse (right) with parameters $\{\Lambda = 600,\ \Psi_1 = 1,\ \Psi_2 = 1000,\ \Phi = 0.001,\ \Xi = 1\}$. Each particle has initial velocity $\{\dot{R}(0) = 0,\ \dot{\theta}(0) = 0,\ \dot{Z}(0) = \frac{1}{200}\}$. The shaded circular disc region indicates the initial spot size ($R = 10{,}000$ for (CM, ±1) laser pulses and $R = 20{,}000$ for a (CM, 0) laser pulse) relative to the black markers on the spacecurves that denote the initial positions of the charged test particles.

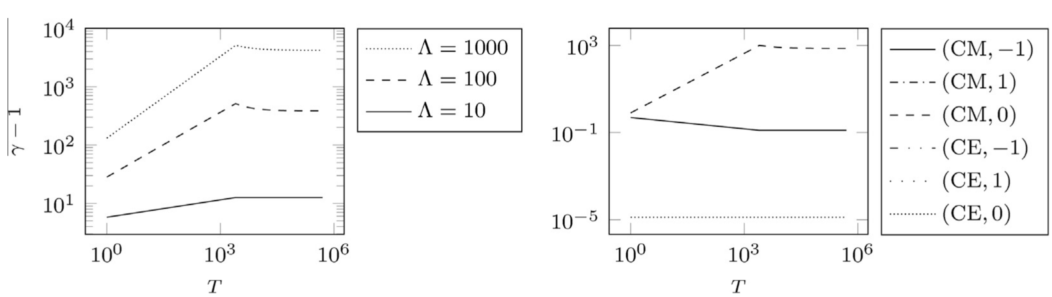

Fig. 3. Specific kinetic energy transfer to any charged test particle. On the left, the (CM, 1) pulse has parameters $\{\Psi_1 = 1,\ \Psi_2 = 1000,\ \Phi = 0.001,\ \Xi = 1\}$. On the right, relative values of $\gamma - 1$ are displayed for various (s, κ) pulses with parameters $\{\Lambda = 1,\ \Psi_1 = 1,\ \Psi_2 = 1000,\ \Phi = 0.001,\ \Xi = 1\}$. The differences between the energy transfers for some pulses appear indistinguishable relative to others owing to the logarithmic scales employed. In all cases, the charged particle has initial position $\{R(0) = 1,\ \theta(0) = \frac{\pi}{2},\ Z(0) = 1\}$ and initial velocity $\{\dot{R}(0) = 0,\ \dot{Z}(0) = \frac{1}{200},\ \dot{\theta}(0) = 0\}$.

Table 1
Table showing SI laser characteristics for various (s, κ) laser pulse configurations with parameters $\{\Psi_1 = 1, \Psi_2 = 1000, \Phi = 0.001, \Xi = 1\}$.

s	CE	CE	CM	CM	CM
κ	0	±1	0	±1	±1
$\mathcal{E}$ (J)	8.07	7.69	8.41	7.69	76903
Λ	40,000	1	0.05	1	100
z_{rg} (m)	1.44	2.43	0.899	1.80	1.80
t_0 (ps)	3	3	2	3	3
z_w (mm)	0.899	0.899	0.600	0.899	0.899
$r_s(0)$ (m)	0.018	0.009	0.012	0.009	0.009
I (TW cm^{-2})	2.646	1.009	0.930	1.009	10090

configurations of a high energy pulse labeled CE and CM can be distinguished by their interaction with different arrangements of charged matter.

Furthermore, by a suitable choice of parameters, (CE, κ) type modes can be constructed that yield the same physical properties $(\mathcal{J}, z_{rg}, z_w, \beta)$ for all κ. Similarly the (CM,κ) type modes yield a κ independent set with physical properties distinct from those determined by the (CE,κ) modes. The pulse group speed magnitudes (as defined above) of all these configurations are determined numerically and are bounded above by the value c. To illustrate some of these statements, Table 1 summarizes the SI laser pulse characteristics for a specific choice of $\{\Psi_1, \Psi_2, \Phi, \Xi\}$ and various values of Λ.

Motivated by the desire to ameliorate the divergences in perturbative QED a number of generalized theories of electromagnetism have been proposed. To date, there is little experimental evidence for testing their predicted departures from Maxwell's theory. However, with the increase in laser technology one may now be entering regimes that may discriminate between such theories. In particular, Bopp–Landé–Podolsky electrodynamics is linear in the electromagnetic fields, contains a fundamental length λ_0 and approaches Maxwell's theory when this length tends to zero. Unlike Maxwell's theory, it contains solutions describing static point charges with *finite* electromagnetic energy.

The classical source-free Bopp–Landé–Podolsky field equations in vacuo for the complex electromagnetic field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ are:

$$\partial^\mu F_{\mu\nu} - \lambda_0^2 \Box \partial^\mu F_{\mu\nu} = 0. \tag{8}$$

Clearly, all classical source-free vacuum Maxwell solutions satisfy these equations. However, there exist additional solutions to (8) that are *not* source-free vacuum Maxwell solutions. For example, such additional solutions of (8) follow from (1) provided:

$$\Box \alpha + \lambda_0^2 \Box^2 \alpha = 0. \tag{9}$$

Furthermore, it follows that as $\lambda_0 \to 0$, one recovers the class of Maxwell solutions discussed above. For non-zero λ_0, there exist particular non-Maxwellian solutions to (9) satisfying

$$\Box \alpha + \frac{1}{\lambda_0^2} \alpha = 0.$$

In Minkowski cylindrical coordinates $\{t, r, \theta, z\}$, a non-separable axi-symmetric compact laser solution is given for arbitrary real Λ_0 by:

$$\alpha(t, r, z) = \frac{\Lambda_0^2 K_1\left(\frac{\zeta(r,t,z)}{\lambda_0}\right)}{\zeta(t, r, z)}$$

where

$$\zeta(t, r, z) \equiv \sqrt{r^2 + [\psi_1 + i(z - ct)][\psi_2 - i(z + ct)]}$$

in terms of strictly positive (real) parameters ψ_1, ψ_2 that determine propagation along the positive z-axis and where $K_1(z)$ denotes the first order modified Bessel function of the second kind. The function $\zeta(t, r, z)$ is non-zero for all real t, r, z and hence both the real and imaginary parts of $\alpha(t, r, z)$ are bounded. To our knowledge, this constitutes for the first time a new source-free finite-energy solution to Bopp–Landé–Podolsky electrodynamics that does not arise in the classical Maxwell electrodynamics.

The interaction of the Bopp–Landé–Podolsky pulse with classical charged test particles follows from the divergence of the Bopp–Landé–Podolsky stress–energy–momentum tensor [25]. This in turn leads to the same equation of motion (6) for the test particle trajectories [25] and may offer an experimental means to discriminate between the Maxwell and Bopp–Landé–Podolsky descriptions of source-free electromagnetic fields in vacuo.

Further details of this approach using scalar field modulated Hertz potentials based on the construction (1) for finding spatially compact solutions to other linear tensor and spinor wave equations will be presented elsewhere.

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