Thin Solid Films 519 (2011) 2668–2673

Contents lists available at ScienceDirect

Thin Solid Films

journal homepage: www.elsevier.com/locate/tsf

Mueller matrices for anisotropic metamaterials generated using 4×4 matrix formalism

P.D. Rogers [a,*], T.D. Kang [a], T. Zhou [a], M. Kotelyanskii [a,b], A.A. Sirenko [a]

[a] Department of Physics, New Jersey Institute of Technology, Newark, NJ 07102, United States
[b] Rudolph Technologies Inc., Flanders, NJ 07836, United States

ARTICLE INFO

Available online 21 December 2010

Keywords:
Mueller matrix
4×4 matrix formalism
Dielectric–magnetic
Metamaterial
Anisotropic
Negative refractive index

ABSTRACT

Forward models for the Mueller Matrix (MM) components of materials with relative magnetic permeability tensor $\mu \neq 1$ are studied. 4×4 matrix formalism can be used to calculate the complex reflection coefficients and the MMs of dielectric–magnetic materials having arbitrary crystal symmetry. For materials with simultaneously diagonalizable ε and μ tensors (with coincident principal axes), analytic solutions to the Berreman equation are available. For the single layer thin film configuration, analytic formulas for the complex reflection and transmission coefficients are derived for orthorhombic symmetry or higher. The separation of the magnetic and dielectric contributions to the optical properties as well as the ability to distinguish materials exhibiting negative index of refraction are demonstrated using simulations of the MM at varying angles of incidence.

1. Introduction

Magnetically active materials in general and metamaterials in particular comprise important classes of materials both from a theoretical perspective as well as for possible device applications. The study of metamaterials has been of interest since the late 1960s when Veselago first explored the properties of *isotropic* materials having simultaneous negative values of ε and μ [1]. In this paper, we have used the Mueller Matrix (MM) formalism for theoretical study of the optical properties of *anisotropic* metamaterials in the frequency range close to the magnetic resonances where $\mu(\omega) \neq 1$. Forward MM models that match the symmetry of planar metamaterials are calculated by treating their behavior as a continuous anisotropic thin film. Our results focus on recently published studies pertaining to artificially created planar metamaterials [2] which use oscillator models for the diagonal components of the ε and μ tensors [3,4]. It will be shown that the MM formalism is useful in the analysis of the separation of the dielectric and magnetic contributions to the optical properties of a material including the important case of the negative index of refraction.

The calculation of a forward model for the MM components of a dielectric–magnetic material is critical to the analysis of the experimental data obtained from full MM spectroscopic ellipsometry. Through an iterative numerical comparison of the forward model against experimental data, the optical properties of a dielectric–magnetic material can be analyzed. Specifically, dispersion models for the relative dielectric permittivity tensor ε and the relative magnetic permeability tensor μ can be developed. 4×4 matrix formalism [5] provides a powerful and systematic method to calculate the complex reflection coefficients and the MMs of dielectric–magnetic materials having both arbitrary crystal symmetry and magnetic permeability tensor $\mu \neq 1$. For a sample whose principal axes are coincident with the laboratory system, that has simultaneously diagonalizable ε and μ tensors (with coincident principal axes), and is characterized by orthorhombic crystal symmetry or higher, exact analytical solutions for allowed electromagnetic wave propagation in a dielectric–magnetic medium are produced. For a non-depolarizing medium, forward MM models are determined directly from the complex reflection coefficients. Although the optical properties of a non-depolarizing medium can be also analyzed using the Jones matrices (JM), the MM approach has an advantage for experimental systems with imperfect, and hence, depolarizing optical elements. In addition, the investigated sample itself may introduce depolarization, as in the case of surface plasmon propagation in metal hole arrays [6]. In this paper, we demonstrate how the angle of incidence dependence of the off-diagonal elements M_{12} and M_{34} of the MM exhibit asymmetric results when materials having negative index of refraction are simulated. The MM approach can be used to determine these effects experimentally. Alternatively, measurements at variable angles of incidence of the ellipsometry parameters Ψ and Δ (in which the sign of Δ is resolved) [7,8] may be applicable to non-depolarizing anisotropic metamaterials.

* Corresponding author. Tel.: +1 914 395 3779.
E-mail address: paul.d.rogers@verizon.net (P.D. Rogers).

0040-6090/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.tsf.2010.12.066

2. 4×4 matrix formalism

Berreman's 4×4 matrix formalism can accommodate materials with magnetic permeability tensor $\mu \neq 1$ [5]. The Berreman equation describing electromagnetic wave propagation in a crystal is:

$$\frac{d\psi}{dz} = i\frac{\omega}{c}\tilde{\Delta}\psi \tag{1}$$

where ψ is a an array of the transverse components of the electromagnetic wave $[E_x, H_y, E_y, -H_x]^T$ in the medium. Fig. 1 illustrates the refraction of light incident in the $x-z$ plane propagating forward in an anisotropic dielectric–magnetic material. For a crystal with orthorhombic symmetry having principal axes parallel to the x, y and z coordinate axes, $\tilde{\Delta}$ in Eq. (1) is a 4×4 matrix [5]:

$$\tilde{\Delta} = \begin{pmatrix} 0 & \mu_{yy} - \frac{N_0^2 \sin(\theta_0)^2}{\varepsilon_{zz}} & 0 & 0 \\ \varepsilon_{xx} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_{xx} \\ 0 & 0 & \varepsilon_{yy} - \frac{N_0^2 \sin(\theta_0)^2}{\mu_{zz}} & 0 \end{pmatrix}. \tag{2}$$

Inserting Eq. (2) into Eq. (1) returns four exact solutions of the form $\psi_l(z) = \psi_l(0)e^{iq_l z}$ with $l = 1, 2, 3$ or 4, two for each of the p and s polarization states. θ_0 is the angle of incidence while $p(s)$ refers to radiation parallel (perpendicular) to the plane of incidence. q_{zp} and q_{zs} are the eigenvalues associated with p and s polarizations, respectively and constitute the z components of the wave vectors in the medium. These are:

$$q_{zp} = \frac{\omega}{c}\sqrt{\varepsilon_{xx}}\sqrt{\mu_{yy} - \frac{N_0^2\, sin^2(\theta_0)}{\varepsilon_{zz}}} \tag{3}$$

$$q_{zs} = \frac{\omega}{c}\sqrt{\mu_{xx}}\sqrt{\varepsilon_{yy} - \frac{N_0^2\, sin^2(\theta_0)}{\mu_{zz}}} \tag{4}$$

Fig. 1 shows q_{zp} and q_{zs}. The x component of the wave vector is constant for all of the incident and refracted waves. It is through these equations (eigenvalues of the Berreman equation) that information about the anisotropic optical properties of the medium [8] enters into the calculation of the complex reflection coefficients and, in turn, MM elements. For example, the anisotropic ε and μ tensors and the consequent differences between q_{zp} and q_{zs} are responsible for the two refracted waves shown in Fig. 1.

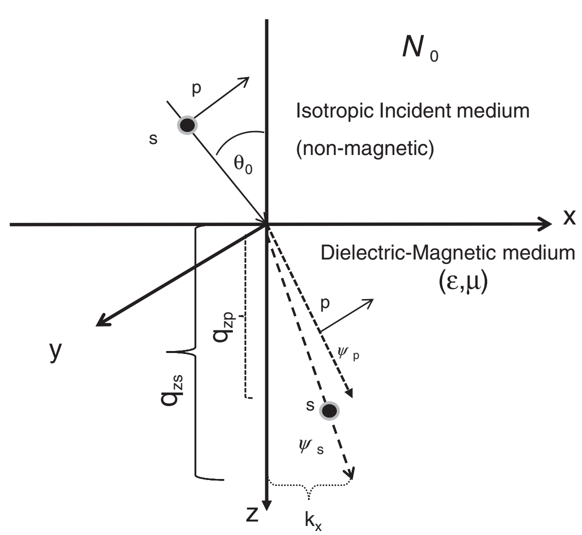

Fig. 1. Wave vector diagram of refracted waves propagating in an anisotropic dielectric–magnetic medium.

3. Analytic formulas

One of the key benefits of using 4×4 matrix formalism to calculate complex reflection and transmission coefficients is that procedures for matching electromagnetic boundary conditions are automatically built in to the method when both incident and, in the case of thin films, substrate media are isotropic and non-magnetic. For each polarization state there are two eigenvectors representing forward and backward propagating waves. In 4×4 matrix formalism, the complex reflection coefficients $r_{pp}(\omega)$ and $r_{ss}(\omega)$ and the complex transmission coefficients $t_{pp}(\omega)$ and $t_{ss}(\omega)$ are calculated from the eigenvectors of Eq. (1) via the solution of simultaneous boundary value equations relating to the continuity of the electric and magnetic fields at the media interface(s). For semi-infinite samples, backward propagating waves are not considered. For thin film samples, retention of the two backward propagating waves is essential to the proper calculation of the complex reflection and transmission coefficients as well as the MM elements. In this section, the cross polarization terms $r_{ps}(\omega)$, $r_{sp}(\omega)$, $t_{ps}(\omega)$ and $t_{sp}(\omega)$ vanish because the principal axes of the crystal correspond to the laboratory coordinate axes.

3.1. Semi-infinite sample

For a semi-infinite material, the two eigenvectors representing the forward propagating waves are used to calculate the complex reflection coefficients for p and s polarized radiation. The procedure for calculating the complex reflection coefficients involves matching the tangential components of the incident and reflected E and H fields to a linear combination of the two eigenvectors calculated at the common interface located at $z = 0$ [5,8]. The complex reflection coefficients are:

$$r_{pp} = \frac{\varepsilon_{xx}k_{z0} - N_0^2 q_{zp}}{\varepsilon_{xx}k_{z0} + N_0^2 q_{zp}} \tag{5}$$

$$r_{ss} = \frac{\mu_{xx}k_{z0} - q_{zs}}{\mu_{xx}k_{z0} + q_{zs}}. \tag{6}$$

In Eq. (5) and Eq. (6), the complex reflection coefficients are expressed as functions of the z components of the incident and refracted wave vectors which themselves take into account the anisotropic characteristics of the medium. Complex reflection coefficients stated in this formalism have been used in the study of media with indefinite permittivity and permeability tensors [9]. These results, obtained from 4×4 matrix formalism, also allow for the immediate analysis of the intriguing property of impedance matching. Consider an isotropic medium. From Eq. (5), at normal incidence, r_{pp} is zero when $N_0 = \sqrt{\varepsilon/\mu}$. A similar result can be obtained for s polarization from Eq. (6). These relationships are known as the impedance matching condition. It provides the condition for zero reflection at normal incidence even though the indices of refraction of the incident medium (N_0) and the index of refraction of the material ($\sqrt{\varepsilon\mu}$) are completely different. With incidence from vacuum, this condition is satisfied if $\varepsilon = \mu$. Aside from a trivial case for vacuum, when both ε and μ are equal to 1, this is only possible if the material is magnetic and provides confirmation that the material has magnetic permeability $\mu \neq 1$. In practice, it is difficult to achieve impedance matching because both the real and imaginary parts of the dielectric and magnetic tensors must be identical. Evidence of impedance matching in metamaterials was found by Grigorenko et al. in 2005 [10].

3.2. Thin film sample

For a single layer thin film material, all four eigenvectors and eigenvalues are used in the calculation of both the complex reflection and transmission coefficients. Both incident and substrate media are assumed to be isotropic, non-magnetic materials. The z components of the incident and substrate wave vectors are $k_{z0} = \frac{\omega}{c} N_0 \cos(\theta_0)$ and $k_{z2} = \frac{\omega}{c} N_2 \cos(\theta_2)$, respectively. The thin film has thickness d and is described by ε and μ tensors each having orthorhombic symmetry. We assume that the ε and μ tensors can be simultaneously diagonalized and have coincident principal axes. Higher symmetries can easily be derived from the orthorhombic case. The crystal is aligned such that its principal axes are coincident with the laboratory axes. Light is again incident in the $x-z$ plane (see Fig. 1). 4×4 matrix formalism matches the tangential components of the electric and magnetic field vectors at $z=0$ and $z=d$ to produce two generalized field vectors $\psi(0)$ and $\psi(d)$, respectively. A thin film layer matrix L is utilized to relate the fields inside the anisotropic film of thickness d at its two boundaries [8].

$$\psi(d) = L\psi(0) \tag{7}$$

L is a 4×4matrix calculated from the eigenvalues and eigenvectors of the $\tilde{\Delta}$ matrix according to:

$$L(d) = \tilde{\Psi} * K(d) * \tilde{\Psi}^{-1}. \tag{8}$$

In Eq. (8), $\tilde{\Psi}$ is composed of the four $\tilde{\Delta}$ eigenvectors as columns while K is a diagonal matrix given by $K_{ll} = e^{iq_l d}$ with q_l representing the four eigenvalues of $\tilde{\Delta}$. After some algebra relating the incident and reflected waves, the complex reflection coefficients for a thin film can be calculated. A similar process allows for the calculation of the complex transmission coefficients [5,8]. Using these procedures, we derived analytic expressions for both p and s polarizations.

The complex reflection and transmission coefficients for p polarized radiation are:

$$r_{pp} = \frac{q_{zp}\cos\left(q_{zp}d\right)\left(\frac{N_2}{N_0}k_{z0} - \frac{N_0}{N_2}k_{z2}\right) + i\left(\frac{N_0 N_2 q_{zp}^2}{\varepsilon_{xx}} - \frac{\varepsilon_{xx}k_{z0}k_{z2}}{N_0 N_2}\right)\sin\left(q_{zp}d\right)}{q_{zp}\cos\left(q_{zp}d\right)\left(\frac{N_2}{N_0}k_{z0} + \frac{N_0}{N_2}k_{z2}\right) - i\left(\frac{N_0 N_2 q_{zp}^2}{\varepsilon_{xx}} + \frac{\varepsilon_{xx}k_{z0}k_{z2}}{N_0 N_2}\right)\sin\left(q_{zp}d\right)}$$

$$t_{pp} = \frac{2k_{z0}q_{zp}}{q_{zp}\cos\left(q_{zp}d\right)\left(\frac{N_2}{N_0}k_{z0} + \frac{N_0}{N_2}k_{z2}\right) - i\left(\frac{N_0 N_2 q_{zp}^2}{\varepsilon_{xx}} + \frac{\varepsilon_{xx}k_{z0}k_{z2}}{N_0 N_2}\right)\sin\left(q_{zp}d\right)} \tag{9}$$

The complex reflection and transmission coefficients for s polarized radiation are:

$$r_{ss} = \frac{q_{zs}\cos(q_{zs}d)(k_{z0} - k_{z2}) + i\left(\frac{q_{zs}^2}{\mu_{xx}} - k_{z0}k_{z2}\mu_{xx}\right)\sin(q_{zs}d)}{q_{zs}\cos(q_{zs}d)(k_{z0} + k_{z2}) - i\left(\frac{q_{zs}^2}{\mu_{xx}} + k_{z0}k_{z2}\mu_{xx}\right)\sin(q_{zs}d)} \tag{10}$$

$$t_{ss} = \frac{2k_{z0}q_{zs}}{q_{zs}\cos(q_{zs}d)(k_{z0} + k_{z2}) - i\left(\frac{q_{zs}^2}{\mu_{xx}} + k_{z0}k_{z2}\mu_{xx}\right)\sin(q_{zs}d)}$$

The formulas are functions of the optical properties of the film material as well as the characteristics of both the incident and substrate media. For example, in a vacuum-thin film-vacuum configuration, the first terms in the numerator of each of the complex reflection coefficients become zero. This simpler form is applicable to many experimental configurations and will be used in the analysis of planar metamaterials below.

In order to verify the accuracy of our analytical expressions, we have calculated the complex reflection and transmission coefficients for the cases of the semi-infinite sample, and a single layer film on a semi-infinite substrate using both our numerical implementation of the 4×4 matrix algorithm and the analytical expressions in Eq. (5), Eq. (6), Eq. (9) and Eq. (10). We found that the results coincide within the rounding errors of the 4×4 matrix algorithm. This analysis was performed for a variety of conditions including negative permittivity and permeability values, which are expected to be observed in metamaterials.

4. Mueller matrices of a planar metamaterial

For the sample symmetry and the experimental configurations assumed in this paper, the off diagonal elements of the 2×2 Jones matrix are zero. For non-depolarizing materials, there are well established formulas to transform the Jones matrix to a full MM [8] and Eq. (11) is the transformation formula applicable when the off diagonal Jones matrix elements are both zero.

$$\begin{pmatrix} \frac{1}{2}\left(\left|r_{pp}\right|^2 + \left|r_{ss}\right|^2\right) & \frac{1}{2}\left(\left|r_{pp}\right|^2 - \left|r_{ss}\right|^2\right) & 0 & 0 \\ \frac{1}{2}\left(\left|r_{pp}\right|^2 - \left|r_{ss}\right|^2\right) & \frac{1}{2}\left(\left|r_{pp}\right|^2 + \left|r_{ss}\right|^2\right) & 0 & 0 \\ 0 & 0 & \Re\left(r_{pp}r_{ss}^*\right) & \Im\left(r_{pp}r_{ss}^*\right) \\ 0 & 0 & -\Im\left(r_{pp}r_{ss}^*\right) & \Re\left(r_{pp}r_{ss}^*\right) \end{pmatrix} \tag{11}$$

The MM of a dielectric–magnetic material is produced from its complex reflection coefficients which are, in turn, calculated from its frequency dependent ε and μ tensors. Accordingly, to produce a MM, accurate complex reflection formulas appropriate to the orientation of the crystal must be available. In addition, models for the dielectric and magnetic functions of the material are required for input into these reflection formulas. Eq. (11) illustrates that, for our configuration, there will be eight non-zero MM elements. However, only four of these terms are independent. Procedures for calculating the forward model of a MM for a planar metamaterial will now be discussed.

To date, there have been relatively few spectroscopic studies of metamaterials which analyze their reflection properties using oblique angles of incidence. Driscoll et al. have done one such study using a planar array of split-ring resonators (SRRs) [2]. Reflection and transmission intensities were recorded for the single s polarization at varying angles of incidence. These results were fitted using the Fresnel equations to model the optical properties of the metamaterial as though it behaved as a continuous anisotropic thin film crystal.

These results are important to our study of MMs because the frequency dependent models of the material's ε and μ tensors together with our Eq. (9) and Eq. (10) enable the calculation of predictive MMs of this planar metamaterial. In the Driscoll experimental configuration, the ε and μ tensors have the following anisotropic symmetry:

$$\varepsilon(\omega) = \begin{pmatrix} \varepsilon_{xx}(\omega) & 0 & 0 \\ 0 & \varepsilon_{yy}(\omega) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mu(\omega) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mu_{zz}(\omega) \end{pmatrix}. \tag{12}$$

The tensors are described by the oscillator models given in Eq. (13).

$$\varepsilon_{xx}(\omega) = \varepsilon_s - \frac{A_e\omega_p^2}{\omega^2-\omega_{e0}^2+i\omega\gamma_e}$$
$$\mu_{zz}(\omega) = 1 - \frac{A_m\omega^2}{\omega^2-\omega_{m0}^2+i\omega\gamma_m} \tag{13}$$

In the formula for the ε tensor, ε_s is the static dielectric constant and ω_p is the plasma frequency. A_e and A_m are oscillator amplitudes. The formula for the μ tensor is modified from the traditional Lorentzian model in that the square of the frequency of incident radiation (ω) enters the numerator and $\mu(0)$ is forced to be equal to 1 [2–4]. The $\varepsilon_{yy}(\omega)$ response was not analyzed in the Driscoll paper.

The general formulas for thin films derived using 4×4 matrix formalism are used to calculate the complex reflection and transmission coefficients for this fabricated material. The experiment performed by Driscoll et al. is set up such that both incident and substrate medium are in vacuum with the x axis parallel to s polarized radiation. In this configuration, the complex reflection coefficients for p and s polarized radiation in Eq. (9) and Eq. (10) reduce to the following:

$$r_{pp} = \frac{\frac{i}{2}\left(\frac{q_{zp}}{k_{z0}\varepsilon_{yy}} - \frac{k_{z0}\varepsilon_{yy}}{q_{zp}}\right)\sin\left(q_{zp}d\right)}{\cos\left(q_{zp}d\right) - \frac{i}{2}\left(\frac{q_{zp}}{k_{z0}\varepsilon_{yy}} + \frac{k_{z0}\varepsilon_{yy}}{q_{zp}}\right)\sin\left(q_{zp}d\right)}$$
$$r_{ss} = \frac{\frac{i}{2}\left(\frac{q_{zs}}{k_{z0}\mu_{yy}} - \frac{k_{z0}\mu_{yy}}{q_{zs}}\right)\sin(q_{zs}d)}{\cos(q_{zs}d) - \frac{i}{2}\left(\frac{q_{zs}}{k_{z0}\mu_{yy}} + \frac{k_{z0}\mu_{yy}}{q_{zs}}\right)\sin(q_{zs}d).} \tag{14}$$

In Eq. (14), $q_{zp}(\omega)$ and $q_{zs}(\omega)$ have the same definitions as in Eq. (3) and Eq. (4) except for the interchange of the x and y axes to accommodate the experimental set up. k_{z0} is the z component of the free space wave vector.

Due to the complexity of the analysis using the Fresnel approach, Driscoll et al. [2] constrained themselves to study only the s polarization incident at the sample. 4×4 matrix formalism and full MM measurement should allow more complete analysis of the sample properties using the incident light of linear and elliptical polarizations. In order to develop a forward model and analyze the measurements of MMs at oblique angles of incidence, assumptions about the permittivity and permeability along other directions are required. Specifically, assumptions about the $\varepsilon_{yy}(\omega)$ response are necessary in order to illustrate how 4×4 matrix formalism could have been used to predict the MM for this metamaterial. Asymmetries in the SRR fabrication between the x and y axis suggest that $\varepsilon_{yy}(\omega) \neq \varepsilon_{xx}(\omega)$. For purposes of illustration only, we assume that the natural resonance of the $\varepsilon_{yy}(\omega)$ oscillation is 15 GHz as compared to 19.9 GHz for the $\varepsilon_{xx}(\omega)$ oscillation. We assume all other fitted parameters are identical. Using these parameters, the frequency dependent $\varepsilon_{xx}(\omega)$, $\varepsilon_{yy}(\omega)$ and $\mu_{zz}(\omega)$ values are calculated and are then input into Eq. (14) to produce the complex reflection coefficients. Eq. (11) is then used to transform the complex reflection coefficients into MM elements. Given the coincidence of the principal axes of the metamaterial with the laboratory system, the off diagonal Jones matrix elements will vanish and there will be only 8 non-zero elements of the predicted MM. These elements are illustrated in Fig. 2. 4×4 matrix formalism was used for the systematic calculation of the complex reflection coefficients. Driscoll et al. found that the r_{ss} coefficient, when calculated in conjunction with the fitted oscillator models, produced a good qualitative fit with s polarized experimental reflectivity data [2]. The simulated MM components, generated from the ε and μ tensors, contain additional critical information about the anisotropic dielectric and magnetic properties of the metamaterial. Actual experimental MM data should allow for the extraction of the anisotropic oscillator parameters through non-linear fitting procedures.

5. Separation of dielectric and magnetic contributions

For proper characterization of materials whose magnetic effects have non-negligible influence on their optical properties, it is important to be able to separate dielectric and magnetic contributions. Spectroscopic experiments usually provide values for the complex refractive index $n = \sqrt{\varepsilon\mu}$ at different frequencies, which do not contain any direct information as to whether it is ε or μ which is responsible for a particular feature observed in the spectrum. The

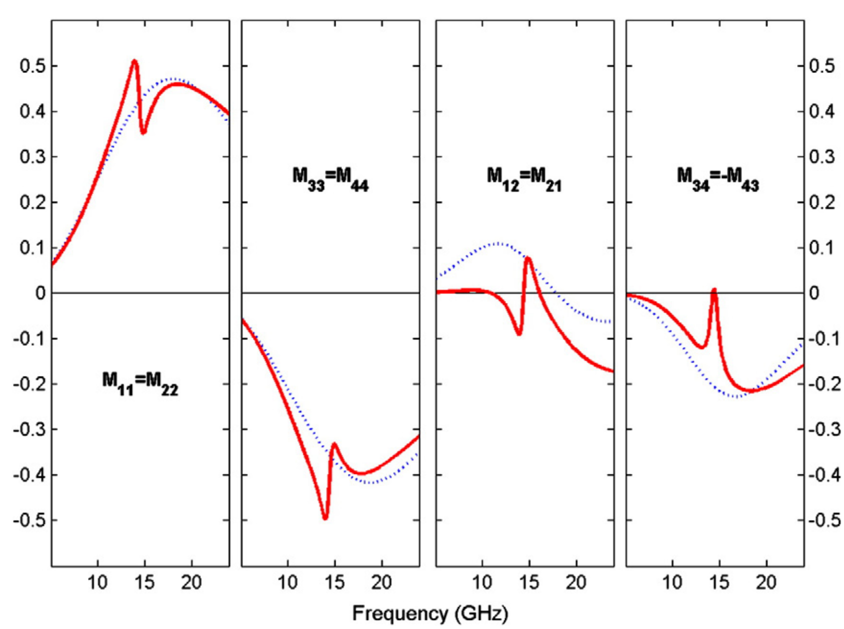

Fig. 2. The Mueller matrix components of a planar metamaterial in the proximity of the resonant feature at 14 GHz for two AOI. Dotted line $\theta_0 = 0^o$. Solid line $\theta_0 = 40^o$.

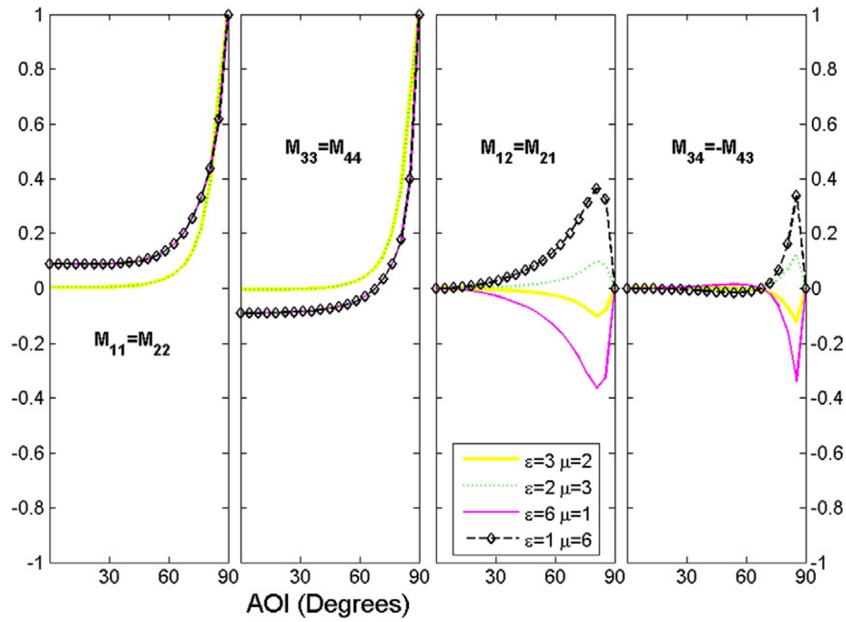

Fig. 3. Dielectric and magnetic contributions in the diagonal and off-diagonal MM components as functions of AOI. Different (ε,μ)combinations illustrate the difference in response of M_{12} and M_{34} compared to M_{11} and M_{33}. For example, the (2, 3) combination (black dotted line, online dotted green) and the (3, 2) combination (black squares, online solid yellow line) are degenerate for M_{11} and M_{33} but have opposite signs for M_{12} and M_{34}.

difference in the change of the various MM components in response to whether ε or μ is changing can separate dielectric and magnetic contributions. For metamaterials, this information is crucial for their design.

This discrimination is indeed possible by performing MM measurements made at varying angles of incidence. To illustrate this point, we model conditions where the index of refraction of a dielectric–magnetic material remains constant but its inputs (ε and μ) are varied. Specifically, we model a hypothetical case of isotropic ε and μ where each are allowed to vary between 1 and 6, but their product, $n^2 = \varepsilon\mu$, is held constant at 6. We simulate a given material composition (ε, μ) and compare it to another material whose values for ε and μ are interchanged. For example, Fig. 3 shows that the values of the diagonal MM elements are identical for both materials characterized by (3, 2) and (2, 3), respectively. However, this degeneracy is removed when the off-diagonal MM elements are analyzed over varying angles of incidence (AOI). It is evident in Fig. 3 that the MM response of the off-diagonal elements is the same in magnitude, but is either positive or negative depending on whether it is ε or μ that is changing. The (2,3) material has positive off-diagonal elements while the (3,2) material has negative off-diagonal elements. Moreover, as seen in Fig. 4, when we introduce the "left handed" [1]

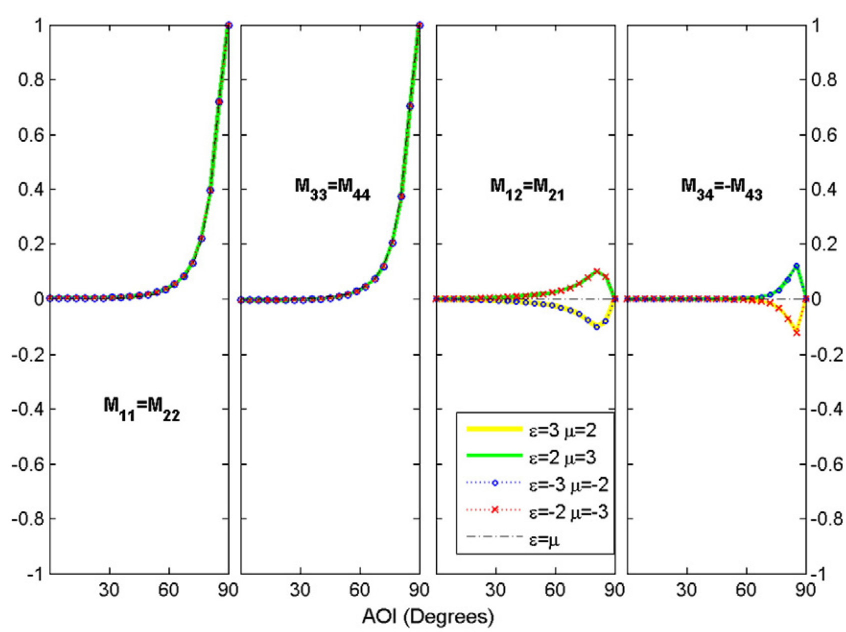

Fig. 4. Dielectric and magnetic contributions in the diagonal and off-diagonal MM components as functions of AOI. Different (ε,μ)combinations illustrate the difference in response of M_{12} compared to M_{34} when "left handedness" is introduced via negative values for ε and μ. The (−2,−3) combination (black "x", online red "x") and the (2, 3) combination (black dotted line, online solid green line) are degenerate for M_{11} and M_{33} but have opposite signs for M_{34}. In addition, the (−2,−3) combination and the (−3,−2) combination (black "o", online blue "o") are degenerate for M_{11} and M_{33} but have opposite signs for both M_{12} and M_{34}.

material with negative permittivity and permeability, but keeping $\varepsilon\mu=6$, the M_{12} and M_{34} components respond in opposite directions. For example, while the $(-2,-3)$ material has diagonal and off-diagonal MM elements identical in magnitude to the $(+2,+3)$ material, the sign of M_{34} becomes negative. It is also interesting to note that the off-diagonal MM responses for two left handed materials can be distinguished. For example, the signs of the M_{12} and M_{34} components respond in opposite directions for the $(-2,-3)$ material as compared to the $(-3,-2)$ material. The difference in the angular response between M_{12} and M_{34} is an indication of the material being "left handed". This observation is extremely important as it is happening in the thin film sample where the study of such MM measurements at varying AOI may be the only way to identify the anomalous properties of the metamaterial comprising the film. In the above cases for both right handed and left handed materials, the ability to distinguish ε and μ vanishes at normal incidence. However, the contrast between the magnetic and electric contributions is at maximum for AOIs that are close or even exceed the Brewster angle of ~68° that corresponds to $n=\sqrt{6}$. Given that there are only 4 independent MM elements to measure, varying the AOI contributes a critical degree of freedom to the proper characterization of ε and μ tensors. Fig. 4 also shows the interesting impedance matching condition discussed in Section 3. When $\varepsilon=\mu$, there is zero reflection at normal incidence.

The simple examples considered above can, of course, be analyzed using the alternative approach of the Jones matrices. Switching between dielectric and magnetic contributions as well as between the positive and negative values of these contributions does naturally cause changes in Ψ and Δ dependencies. However, the behavior of these ellipsometric parameters is more complex, and not as illustrative, as compared to switching signs in the off-diagonal Mueller Matrix components.

Since real metamaterial samples are usually anisotropic, one should not always expect to see such well pronounced and easily understandable effects in real experimental data. However, the fact that the angular dependencies of the MM elements respond differently to dielectric and magnetic contributions, as well as to the positive and negative values of ε and μ, should allow for the ability to distinguish these different situations while extracting ε and μ by non-linear fitting of the experimental data.

6. Summary

We have presented an analytical approach for the study of dielectric–magnetic materials using 4×4 matrix formalism. Wave vectors in a dielectric–magnetic medium are derived directly from the eigenvalue solutions of the Berreman equation. We utilized the wave vector approach to derive analytic formulas for the complex reflection and transmission coefficients of thin films whose ε and μ tensors match to orthorhombic symmetry. Any other system that has simultaneously diagonalizable ε and μ tensors (with coincident principal axes) can be reduced to this case by rotations of the reference frame. We have demonstrated how these calculations can produce the full MM of a non-depolarizing material. Forward models for the active MM elements of a planar metamaterial were calculated. The separation of the magnetic and dielectric contributions to the optical properties of an anisotropic material, as well as identification of negative refractive index in a thin film, are possible using the MM approach at varying AOI.

Acknowledgements

This work is supported by NSF-MRI: DMR-0821224.

References

[1] V.G. Veselago, Sov. Phys. Usp. 10 (1968) 509.
[2] T. Driscoll, D.N. Basov, W.J. Padilla, J.J. Mock, D.R. Smith, Phys. Rev. B 75 (2007) 115114.
[3] T. Driscoll, G.O. Andreev, D.N. Basov, S. Palit, T. Ren, J. Mock, S.-Y. Cho, N.M. Jokerst, D.R. Smith, Appl. Phys. Lett. 90 (2007) 092508.
[4] J.B. Pendry, A.J. Holden, D.J. Robbins, W.J. Stewart, IEEE Trans. Microwave Theory Tech. 47 (1999) 2075.
[5] D.W. Berreman, J. Opt. Soc. Am. 62 (1972) 502.
[6] E. Altewischer, M.P. van Exter, J.P. Woerdman, J. Opt. Soc. Am. B 22 (2005) 1731.
[7] M. Schubert, B. Rheinländer, J.A. Woollam, B. Johs, C.M. Herzinger, J. Opt. Soc. Am. A 13 (1996) 875.
[8] R.M.A. Azzam, N.M. Bashara, Ellipsometry and Polarized Light, North-Holland, Amsterdam, 1977.
[9] D.R. Smith, D. Schurig, Phys. Rev. Lett. 90 (2003) 077405.
[10] A.N. Grigorenko, A.K. Geim, H.F. Gleeson, Y. Zhang, A.A. Firsov, I.Y. Khrushchev, J. Petrovic, Nature 438 (2005) 335.