



# A distributed passivity approach to AUV teams control in cooperating potential games<sup>☆</sup>

Filippo Fabiani<sup>a,\*</sup>, Davide Fenucci<sup>a,b,c</sup>, Andrea Caiti<sup>a,b,c</sup>

<sup>a</sup> Department of Information Engineering, University of Pisa, Largo L. Lazzarino, 1, 56122, Pisa, Italy

<sup>b</sup> Research Center “E. Piaggio”, University of Pisa, Largo L. Lazzarino, 1, 56122, Pisa, Italy

<sup>c</sup> Interuniversity Center for Integrated Systems for Marine Environment (ISME), Genova, Italy

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## ABSTRACT

The paper proposes a general framework to manage a team of Autonomous Underwater Vehicles (AUVs), while keeping the communication constraints, during missions execution. Virtual spring-damper couplings (passive by definition) define the distributed interaction forces between neighbouring vehicles. In this way, through passivity theory, a suitable Lyapunov function for the closed loop system is built to ensure stable convergence of the network vehicles to an equilibrium point, also providing robustness in presence of communication fading and delays, very common in the marine environment. Simulations of typical missions show the effectiveness of the proposed approach. An equivalence between this typical port-Hamiltonian framework and a specific class of potential games, the Bilateral Symmetric Interaction (BSI) one, is also established. Hence, modelling the network with passive elements, it is possible to shape the transient behaviour of the players and the reached equilibria at the end of the game.

## 1. Introduction

A renewal of interest has been focused recently on the analysis and control of networked systems, and particularly on distributed systems of mobile agents. Such systems provide significant benefits in efficiency, scalability, and robustness when compared to classical centralized solutions. Applications of mobile agent networks are multi-disciplinary and highly diversified. In particular, recent advances in marine robotics have made AUVs more reliable and affordable allowing the execution of tasks that are dangerous, expensive and time consuming when performed by humans. There are a lot of practical applications that can benefit from the use of a team of AUVs: these include the defence field, patrolling, surveillance of an asset or of a predefined geographical area, coverage tasks, exploration, oceanographic surveying and mapping (Curtin et al., 1993). All these application scenarios involve communication among multiple agents. In the underwater domain, due to the well known limitations of the acoustic channel (Caiti et al., 2009, 2013a; Stojanovic, 2007), it is of paramount importance to maintain desired communication performances in order to achieve the mission objectives.

Basing on recent preliminary works developed by the authors (Fabiani et al., 2016a, 2016b), this paper presents a general framework

for coordinating a team of agents, applied to a group of AUVs. The *passivity* theory is exploited for guaranteeing the stable and robust convergence of the network to an equilibrium configuration, even in presence of communication delays. The term “stability” here is used to indicate that, for any initial condition which ensures the fulfilment of the communication constraints, such constraints are satisfied for the whole transient, and the proposed algorithm leads the vehicles to assume a stable, in classic Lyapunov sense, configuration. Moreover, the degrees of freedom offered by such an approach allow to tune the desired motion of the group in terms of transient behaviour and reached equilibria. In particular, it is possible to determine in advance which task has higher priority than the others without any consequence about stability. Furthermore, the behaviour of the group is made more flexible, with arbitrary split and join events, using an “energy tank” able to store and supply energy whenever required (Secchi et al., 2006): exploiting this further passive element the network topology may change depending on the emerging needs of the mission.

Downstream to what just said, an equally relevant motivation behind this work arises from the possibility to capture an ambitious and innovative point of contact between two different ways to model and control a network of agents: the one proposed here and BSI, a *potential game* that

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\* Corresponding author.

E-mail address: [filippo.fabiani@ing.unipi.it](mailto:filippo.fabiani@ing.unipi.it) (F. Fabiani).

exhibits symmetries across the variables. In fact, a BSI game is characterized by the notion of *pairwise* or *bilateral* strategic interaction, in which the utility function for each player depends only on its own action and that of another connected with him, whatever the behaviour of the rest of the network. If that happens for all players, the observations are said to be symmetric for each combination of connected agents. Hence, modelling the network with passive elements, which are purely design parameters, it may be possible to shape the transient behaviour of the players and, ideally, the reached equilibrium at the end of the game.

The proposed approach is based on the positive influences of several pioneering works, such as the one in (Leonard and Fiorelli, 2001), refined and extended in (Ogren et al., 2004) and (Zhang and Leonard, 2010). Here the artificial potentials and virtual leaders allowed to manage a group of multiple autonomous vehicles, also manipulating the team geometry and its direction of motion. Motivated by biological inspiration, such works focused also on gradient climbing missions in which the mobile sensor network sought out local maxima or minima in the environmental field; moreover, a convergent cooperative Kalman filter for exploration missions provided the estimates to drive the centre of the formation to move along level curves of the environmental field. In (Olfati-Saber and Murray, 2002), natural potential functions were obtained from structural constraints of a desired formation: in this way, the synthesized controller for each vehicle was able to steer and move agents exploiting only local informations, also avoiding collisions. With the same concepts in mind, the author of (Olfati-Saber, 2006) proposed a theoretical framework for the design and analysis of several distributed flocking algorithms, in presence or lack of obstacles. Another pioneering work that exploited artificial potentials to solve the constrained coverage Problem is in (Poduri and Sukhatme, 2004). In particular, the deployment of the mobile sensor network was addressed: each node was treated as a virtual charged particle, in order to synthesize an algorithm able to maximize the covered area and minimize the number of nodes of the network itself. Recently, in (Williams and Sukhatme, 2013a, 2013b), a mobility control that switch between a set of smooth, constraint-enforcing potential fields, satisfying local and non-local constraints composition was proposed. That potential-based control also drove the agents to maximize connectivity and maintain established links; the constraint satisfaction was achieved using a switched model of interaction which regulated link addition through repulsive potentials between constraint violators.

In the recent years, the authors proposed cooperative control algorithms based on the behavioural approach paradigm (Caiti et al., 2012) and its adaptation as potential BSI game (Caiti et al., 2013b), to maintain desired communication performance and fulfil each agent task. The main drawback of such algorithms were the absence of stability guarantees along the whole motion of the agents: as a matter of fact, they were able to provide only the local stability of equilibria points. In this context, the port-Hamiltonian framework allows to model the sensors network in a suitable, passive fashion, e.g. in (Fiaz et al., 2013; Vos et al., 2014, 2015). Passivity techniques have been widely studied in the domain of bilateral teleoperations for the control of a traditional single-master/single-slave system (Hokayem and Spong, 2006; Secchi et al., 2008), or for a more complex single-master/multiple-slaves system (Franchi et al., 2012). In spite of take advantage of the operator's intelligence for solving complex tasks as in bilateral teleoperations, the proposed framework seeks to provide full autonomy to the agents in order to accomplish the cooperative mission.

The paper is organized as follows: Section 2 presents the essentials mathematical and theoretical tools implied in the framework, i.e. graph theory, port-Hamiltonian systems and game theory; Section 3 outlines the implementation details of the cooperative algorithm and demonstrates the stability of the proposed solution including delays on communication links. After that, Section 4 provides several consideration about the relationship between the proposed approach and BSI games. Section 5 exploits the energy tanks approach proposed in (Franchi et al.,

2012) to enlarge the set of possible stable manoeuvres within the network, while Section 6 illustrates and discusses the effectiveness of the proposed framework in several application scenarios. Finally, Section 7 summarizes the work and draws the main conclusions.

Notation.  $\mathbb{R}$ ,  $\mathbb{R}_{>0}$  and  $\mathbb{R}_{\geq 0}$  respectively denote the set of real, positive real, non-negative real numbers. Vectors and matrices are denoted by bold characters, while scalars and sets by italics. Unless otherwise specified,  $0$  and  $I$  denotes the zero and identity matrices of suitable dimensions (context-dependent).  $A^T$  denotes the transpose of  $A$ ;  $\mathbb{S}^n$  denotes the set of symmetric  $n \times n$  matrices; for a given  $Q \in \mathbb{R}^{n \times n}$ , the notations  $Q \succ 0$  ( $Q \geq 0$ ) and  $Q \in \mathbb{S}_{>0}^n$  ( $Q \in \mathbb{S}_{\geq 0}^n$ ) denote that  $Q$  is symmetric and has positive (non-negative) eigenvalues.  $|\cdot|$  denotes the cardinality of a set,  $\|\cdot\|$  the Euclidean norm,  $\otimes$  and  $\times$  the Kronecker and Cartesian product, respectively.

## 2. Background and preliminaries

This section provides the essentials from the main analytical and theoretical tools required for the upcoming mathematical treatment. In particular, some reminders on graph theory, passivity, port-Hamiltonian systems and game theory will be very useful for a proper understanding. We remark here that we are using the jargon of different, though converging, research fields. Therefore, in the rest of the paper we may refer to the AUVs with the terms agents, vehicles or players, indistinctly.

### 2.1. Graph theory

A graph  $\mathcal{G} := (V, E)$  is formally defined by a finite set of *nodes* (or *vertices*)  $V$  and a set of *edges*  $E \subset V \times V$ , connecting pairs of nodes. The node set  $V := \{v_1, v_2, \dots, v_l\}$  has  $l = |V|$  elements, while the edge set  $E := \{e_1, e_2, \dots, e_m\}$  contains  $m = |E|$  elements. Given  $e_j \in E$ , then there exist a pair  $v_i, v_j \in V$  such that  $e_j := (v_i, v_j)$ ; in this way,  $v_i$  and  $v_j$  are said to be *adjacent*, while  $(v_i, v_i)$  is called a *self-loop*. If the edges in graphs are to be interpreted as enabling information to flow between the vertices on the corresponding edge, these flows can be directed as well as undirected. Hence, *direct* and *indirect* graph can be distinguished. In the first case, edges have a fixed direction (i.e. the *tail* and the *head* of the edge are setted), while in the second case, if  $(v_i, v_j)$  belongs to  $E$ , then  $(v_j, v_i)$  belongs to  $E$  too. However, for indirect graph, one can arbitrarily assign an orientation to each edge. Any key feature of a graph can be described by means of matrices. In particular, the *incidence matrix*  $B(\mathcal{G})$  is a  $l \times m$  matrix defined as follows:

$$[B(\mathcal{G})]_{ij} := b_{ij} = \begin{cases} -1 & \text{if } v_i \text{ is the tail of } e_j, \\ 1 & \text{if } v_i \text{ is the head of } e_j, \\ 0 & \text{otherwise.} \end{cases}$$

The  $l$  rows of  $B(\mathcal{G})$  correspond to the nodes of  $\mathcal{G}$ , while the  $m$  columns denotes the edges of such graph. For further details on the graph theory, refer to (Mesbahi and Egerstedt, 2010).

### 2.2. Port-Hamiltonian systems and passivity

The port-Hamiltonian framework, introduced in (Maschke and Van der Schaft, 1993), allows to model complex (non-linear) systems as energy storing and energy dissipating components, connected via ports to power conserving transmissions and conversions. It is an energy-based framework in which each element interacts with the system via a port, that consists of a couple of dual effort and flow quantities, whose product gives the power flow in and out of the component. As well described in (Van der Schaft, 2006), let  $\mathbf{x} \in \mathbb{R}^n$  denotes the local coordinates for an  $n$ -dimensional state space manifold  $\mathcal{X}$ ,  $\mathbf{u} \in \mathbb{R}^m$  the control input and  $\mathbf{y} \in \mathbb{R}^m$  the output of the system. The generalized input-state-output dynamics expressed in terms of port-Hamiltonian framework is given by:

$$\begin{cases} \dot{\mathbf{x}} = [\mathbf{J}(\mathbf{x}) - \mathbf{R}(\mathbf{x})] \left( \frac{\partial H(\mathbf{x})}{\partial \mathbf{x}} \right)^\top + \mathbf{g}(\mathbf{x})\mathbf{u}, \\ \mathbf{y} = \mathbf{g}^\top(\mathbf{x}) \left( \frac{\partial H(\mathbf{x})}{\partial \mathbf{x}} \right)^\top, \end{cases} \quad (1)$$

where  $\mathbf{J}(\cdot) \in \mathbb{R}^{n \times n}$  is a skew-symmetric structure matrix,  $\mathbf{g}(\cdot) \in \mathbb{R}^{n \times m}$  is also a structure matrix,  $H: \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$  is the Hamiltonian function that represents the whole energy stored in the system and  $\mathbf{R}(\cdot) \in \mathbb{S}_{\geq 0}^n$  a dissipation matrix. The entries of both matrices  $\mathbf{J}(\cdot)$  and  $\mathbf{R}(\cdot)$  depend smoothly on  $\mathbf{x}$ . Modelling dynamical system as (1) provides several benefits: in particular, a basic property of a port-Hamiltonian system is related to its energy balance, tightly coupled with the notion of passivity. In fact, any port-Hamiltonian system is passive w.r.t. the supply rate and storage function  $H(\cdot)$  if  $H(\cdot)$  is bounded from below.

Let now consider a generic, affine, non-linear system

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}, \\ \mathbf{y} = \mathbf{h}(\mathbf{x}), \end{cases} \quad (2)$$

with the state vector  $\mathbf{x} \in \mathbb{R}^n$ , a control vector  $\mathbf{u} \in \mathbb{R}^m$  and an output vector  $\mathbf{y} \in \mathbb{R}^m$ .  $\mathbf{f}: \mathbb{R}^n \rightarrow \mathbb{R}^n$ ,  $\mathbf{g}: \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $\mathbf{h}: \mathbb{R}^n \rightarrow \mathbb{R}^m$  are non-linear functions of the state.

**Definition 1.** (from (Duindam et al., 2009)) The system in (2) is passive if there exists a continuous and differentiable lower bounded function of the state  $V: \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$  (called storage function) such that

$$\dot{V}(\mathbf{x}) \leq \mathbf{u}^\top \mathbf{y} \Leftrightarrow V(\mathbf{x}(t)) - V(\mathbf{x}(0)) \leq \int_0^t \mathbf{u}^\top(\tau) \mathbf{y}(\tau) d\tau.$$

$V(\cdot)$  represents the internal stored energy and  $\mathbf{u}^\top \mathbf{y}$  the energy flow exchanged with the external world (i.e. the *supply rate*). The pair  $(\mathbf{u}, \mathbf{y})$  is called power port, where  $\mathbf{u}$  and  $\mathbf{y}$  are power variables: these latter allow to control and interconnect passive systems.

Back to the port-Hamiltonian theory, due to the fact that  $H(\mathbf{x}) \geq 0$ , the passivity is always guaranteed and it is easy to show that  $\dot{H}(\mathbf{x}) \leq \mathbf{u}^\top \mathbf{y}$ . In this way, port-Hamiltonian framework provides a powerful tool for the stability analysis of dynamical systems, in order to achieve a feasible, stable and robust control.

### 2.3. Game theory and nash equilibria

A game  $\mathcal{G} := [\mathcal{N}, \mathbb{A}, \{U_i\}_{i \in \mathcal{N}}]$  in strategic, non-cooperative form is defined by a non-empty and finite set  $\mathcal{N}$  of  $N$  rational players, a *strategy space*  $\mathbb{A}$  and a set  $\{U_i\}_{i \in \mathcal{N}}$  of *utility functions*. A player is rational if he or she makes decisions which are consistent to the purpose of maximizing (minimizing) his/her utility function. For every  $i$ -th player in  $\mathcal{N}$ , his/her collection of all possible strategies forms a strategy set, expressed as:

$$\mathbf{A}_i := \{A_i \mid A_i \text{ is a valid strategy for the } i\text{-th player}\}.$$

In this way, the strategy space  $\mathbb{A}$  is defined as the Cartesian products of all individual strategy sets, i.e.:

$$\mathbb{A} := \mathbf{A}_1 \times \dots \times \mathbf{A}_N.$$

Each element  $A = (A_1, A_2, \dots, A_N) \in \mathbb{A}$  is said to be a *strategy profile*. Referring to the  $i$ -th player, then  $A$  can be rewritten as  $A := (A_i, A_{-i})$ , where  $A_{-i}$  denotes the joint strategy adopted by player  $i$ 's opponents. Hence, the domain of  $A_{-i}$  is defined by  $\mathbf{A}_{-i}$ . For each player  $i$ , his/her utility function  $U_i$  is a function that maps each strategy profile  $A$  to a real number,  $U_i: \mathbb{A} \rightarrow \mathbb{R}$ . Consequently,  $U_i(A)$  can be represented also as  $U_i(A_i, A_{-i})$ .

A Nash equilibrium (NE) is a crucial concept in predicting game outcome: it is a strategy profile such that if the opponents' strategies remain unaltered, no player would be tempted to move away from his/her current strategy. More precisely:

**Definition 2.** The strategy profile  $A^* \in \mathbb{A}$  is a NE if and only if

$$\forall A'_i \in \mathbf{A}_i, \forall i \in \mathcal{N}, U_i(A_i^*, A_{-i}^*) \geq U_i(A'_i, A_{-i}^*).$$

At a NE, no player is able to gain by deviating from the current point: it is a sort of “stable operating point”. Although a NE is often the ultimate objective of a game, such game in strategic/normal form may have either unique, multiple or no NE.

#### 2.3.1. Potential games

As stated in (Monderer and Shapley, 1996) a pure strategy, normal form, potential game has the strong property that admits at least one NE. By definition:

**Definition 3.** A game  $\mathcal{G}$  is an *exact potential game* if and only if a **potential function**  $V: \mathbb{A} \rightarrow \mathbb{R}$  exists such that,  $\forall i \in \mathcal{N}$ :

$$U_i(T_i, A_{-i}) - U_i(A_i, A_{-i}) = V(T_i, A_{-i}) - V(A_i, A_{-i}), \forall A_i, T_i \in \mathbf{A}_i, \forall A_{-i} \in \mathbf{A}_{-i}.$$

In an exact potential game, the change in a single player's utility due to his/her own strategy deviation results in exactly the same amount of change in the potential function. One interesting property of such games is that the global maximizers (minimizers) of the potential function  $V$  are NE.

### 3. The high-level cooperative algorithm

In this paper, the general issue addressed is the following:

**Problem.** Each AUV within a team has to accomplish its own task (or tasks), while keeping the communication connectivity with the other team members.

#### 3.1. Networked system model

Modelling the agents/targets network as an indirect graph without any self-loop, the aim is to design a high-level control policy that allows to maintain the predefined graph during the development of the whole mission.

From now on, we suppose that the following mild assumptions hold: i) the connectivity link is guaranteed if the agents lie within a fixed relative range; ii) each vehicle has to know its own absolute velocity and the relative position of another vehicle with respect to itself.

It is worth nothing that in order to fulfil the maintenance of the communication link it is assumed that the performance of the acoustic channel, potential packet loss issues or Signal-to-Noise Ratio (SNR) have been converted in a range constraint. This is always possible (i.e., underwater acoustic communication difficulties decreases as communication ranges get shorter), although sometime too conservative. However, how to exploit the channel characteristics to optimize communication ranges (Caiti et al., 2009, 2013a) is out of the scope of the paper.

The information required with ii) is measured or estimated by the available navigation system of the vehicles (see (Paull et al., 2014) for a recent review). Depending on the specific situation, or the specific set of vehicles, some sensors or some approaches can be more appropriate than another: for instance, in shallow water or for sea-bed inspection missions the presence of Doppler Velocity Log (DVL) sensor on board each vehicle is sufficient to measure the velocity. Acoustic modems with Ultra-Short Base Line (USBL) capability can not only deliver measurements of relative position, but can also concur to the establishment of a cooperative navigation system (Allotta et al., 2016) which exploits range and bearing information as communication among the vehicles take place. Additional refinements for deep water and to compensate current disturbance can be found in (Allotta et al., 2017). A complete treatment of the navigation issue is clearly out of the scope of this paper; here it is assumed that a navigation system is in place, and that the required information is

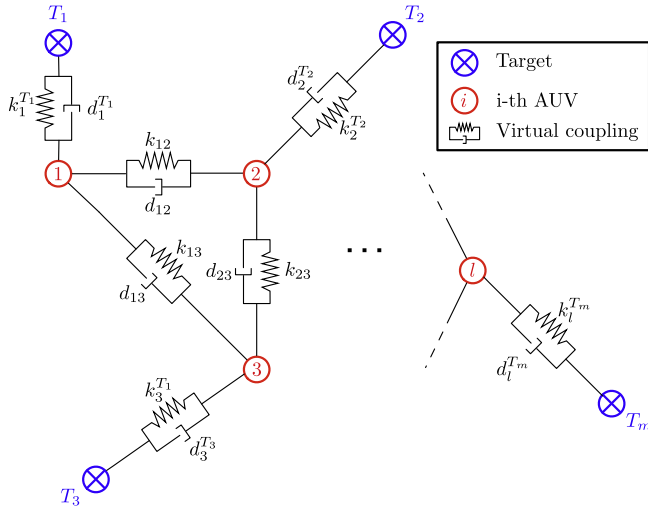


Fig. 1. A team of  $l$ -AUVs (numbered red circles) in a generic distribution in  $\mathbb{R}^n$ . The blue circled crosses suggest the target of the agents, while the spring-damper couples represent the virtual couplings between agents and/or targets.

available to the vehicles.

Let now consider the generic network configuration in Fig. 1, with  $l$ -vehicles and  $m$ -targets. Each link, either a communication connection or not, is represented by a pair of spring-damper element (virtual coupling, passive by definition). The graph  $\mathcal{G} := (V, E)$  associated to the concerned network is defined by:

$$V := \{1, 2, \dots, l, T_1, \dots, T_m\} \rightarrow |V| = l + m, E := \{(1, 2), \dots, (l, T_1), \dots\} \rightarrow |E| = a + t.$$

Note the separation between agent-agent and agent-target edges, with cardinality  $a$  and  $t$ , respectively. Considering the  $(i, j)$  edge, here the vertex  $i$  is always seen as the tail and  $j$  as the head of the link. Then, the incidence matrix  $\mathbf{B} \in \mathbb{R}^{|V| \times |E|}$  is given by:

$$\mathbf{B} = \begin{bmatrix} -1 & -1 & \dots & -1 & \dots \\ 1 & 0 & \dots & 0 & \dots \\ 0 & 1 & \dots & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & \dots \\ 0 & 0 & \dots & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \dots \end{bmatrix}$$

Conventionally, the rows of  $\mathbf{B}$  are sorted as the elements in  $V$ , while the columns as those in  $E$ . In this way, the first  $a$  columns represent the links between agents and the second  $t$  stand for the agent-target connections. For sake of clarity, assuming a simple damped double-integrator dynamics in  $n$ -dimensional space for each node of the network, the  $i$ -th AUV and  $j$ -th target equations of motion are given by:

$$m_i \ddot{\mathbf{q}}_i = -d \dot{\mathbf{q}}_i + \mathbf{u}_i, m_{T_j} \ddot{\mathbf{q}}_{T_j} = -d \dot{\mathbf{q}}_{T_j} + \mathbf{u}_{T_j},$$

where  $m_i, m_{T_j}$  are the respective masses,  $\mathbf{q}_i, \mathbf{q}_{T_j} \in \mathbb{R}^n$  are their positions (generalized coordinates) and  $\mathbf{u}_i, \mathbf{u}_{T_j} \in \mathbb{R}^n$  are the control vectors. The  $d$  element model a viscous friction acting on the moving nodes. Hence, the corresponding linear momenta  $\mathbf{p}_i, \mathbf{p}_{T_j} \in \mathbb{R}^n$  are:

$$\mathbf{p}_i = \bar{\mathbf{m}}_i \dot{\mathbf{q}}_i, \mathbf{p}_{T_j} = \bar{\mathbf{m}}_{T_j} \dot{\mathbf{q}}_{T_j},$$

with  $\bar{\mathbf{m}}_i := \mathbf{I} m_i, \bar{\mathbf{m}}_{T_j} := \mathbf{I} m_{T_j}$  and  $\mathbf{I}$  is the  $n \times n$  identity matrix. Now, consider the group of  $l + m$  vertices: to compactly denote the agent dynamics, the vector form is introduced. Thus,  $\mathbf{q} = [\mathbf{q}_1^\top, \mathbf{q}_2^\top, \dots, \mathbf{q}_{T_1}^\top, \dots]^\top \in \mathbb{R}^{n|V|}$  stands for the generalized coordinates vector,  $\mathbf{D}^A := \text{diag}(\mathbf{I} d, \dots,$

$\mathbf{I} d) \in \mathbb{R}^{n|V| \times n|V|}$  is the damping matrix and  $\mathbf{u} = [\mathbf{u}_1^\top, \mathbf{u}_2^\top, \dots, \mathbf{u}_{T_1}^\top, \dots]^\top \in \mathbb{R}^{n|V|}$  is the control vector. The linear momenta vector  $\mathbf{p} \in \mathbb{R}^{n|V|}$  is:

$$\mathbf{p} = \mathbf{M} \dot{\mathbf{q}},$$

where  $\mathbf{M} := \text{diag}(\bar{\mathbf{m}}_1, \dots, \bar{\mathbf{m}}_{T_m}) \in \mathbb{S}_{>0}^{n|V| \times n|V|}$  is the matrix of masses. Basing on the communication links, the relative distances vector  $\mathbf{z} \in \mathbb{R}^{n|E|}$  is defined as:

$$\mathbf{z} := (\mathbf{B}^\top \otimes \mathbf{I}) \mathbf{q}.$$

### 3.2. Energetic description of the network

Including the virtual couplings, the energy behaviour of the network is now analysed. The global Hamiltonian function  $H : \mathbb{R}^{n|V|} \times \mathbb{R}^{n|E|} \rightarrow \mathbb{R}_{\geq 0}$ :

$$H(\mathbf{p}, \mathbf{z}) = H^k(\mathbf{p}) + H^t(\mathbf{z}) + H^a(\mathbf{z}), \quad (3)$$

is divided as the sum of three different terms, where the potential contribution of the springs is split between AUV-target and AUV-AUV contributions. In details:

a) Kinematic contribution  $H^k : \mathbb{R}^{n|V|} \rightarrow \mathbb{R}_{\geq 0}$

$$H^k(\mathbf{p}) := \frac{1}{2} \mathbf{p}^\top \mathbf{M}^{-1} \mathbf{p}.$$

b) Agent-Target contribution  $H^t : \mathbb{R}^{n|E|} \rightarrow \mathbb{R}_{\geq 0}$

$$H^t(\mathbf{z}) := \frac{1}{2} \mathbf{z}^\top \mathbf{K}_T \mathbf{z},$$

c) with  $\mathbf{K}_T := \text{diag}(0, \bar{\mathbf{K}}_T) \in \mathbb{R}^{n|E| \times n|E|}$ ,  $\bar{\mathbf{K}}_T := \text{diag}(\bar{\mathbf{k}}_1, \dots, \bar{\mathbf{k}}_t)$ ,  $\bar{\mathbf{k}}_i := \mathbf{I} k_i^t$  and  $k_i^t$  is the elastic constant of the  $i$ -th vehicle-target coupling in  $E$  (for example,  $k_1^t = k_1^{T_1}$  and  $k_t^t = k_l^{T_m}$  in Fig. 1, for  $i = 1, \dots, t$ ).

d) Agent-Agent contribution  $H^a : \mathbb{R}^{na} \rightarrow \mathbb{R}_{\geq 0}$

Defining  $R_c$  as the maximum distance at which the AUV can still communicate with its neighbours, and  $R_d \leq R_c$  as the desired distance to be maintained between two vehicles, the last term of  $H(\mathbf{p}, \mathbf{z})$  is given by:

$$H^a(\mathbf{z}) := \frac{1}{2} \sum_{i=1}^a k_i^a (\|\mathbf{z}_i\|) \cdot (\|\mathbf{z}_i\| - R_d)^2,$$

where the elastic constant  $k_i^a(\cdot)$  models a non-linear spring over each agent-agent link.

Defining  $\mathbf{f} \in \mathbb{R}^{n|E|}$  as the force vector acting on the  $a + t$  links:

$$\mathbf{f} = \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top + \mathbf{D}^c (\mathbf{B}^\top \otimes \mathbf{I}) \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top,$$

where  $\mathbf{D}^c := \text{diag}(\bar{\mathbf{d}}_1, \dots, \bar{\mathbf{d}}_a, \dots, \bar{\mathbf{d}}_t) \in \mathbb{S}_{>0}^{n|E| \times n|E|}$  contains the mutual damping elements  $\bar{\mathbf{d}}_i := \mathbf{I} d_i$  in  $E$ . In this way, assuming  $(\mathbf{p}, \mathbf{z})$  as the state variables, the input-state representation of the multi-agents open loop system in the port-Hamiltonian framework is:

$$\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} -\mathbf{D}^A & \mathbf{0} \\ \mathbf{B}^\top \otimes \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top \\ \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top \end{bmatrix} + \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \mathbf{u} \quad (4)$$

Now, Pfaffian constraints are introduced to generalize the proposed framework and then to consider fixed nodes (e.g. the targets). In this way, after choosing a suitable matrix  $\mathbf{A} \in \mathbb{R}^{n|V| \times nm}$  that satisfies the relation

$\mathbf{A}^\top \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top = 0$ , a sort of selection matrix  $\mathbf{S} \in \mathbb{R}^{n[V] \times n[V]}$  such that  $\mathbf{A}^\top \mathbf{S} = 0$  can be easily computed. Then, the open loop system in (4) assumes the following form:

$$\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} -\mathbf{S}^\top \mathbf{D}^A \mathbf{S} & \mathbf{0} \\ (\mathbf{B}^\top \otimes \mathbf{I}) \mathbf{S} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top \\ \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top \end{bmatrix} + \begin{bmatrix} \mathbf{S} \\ \mathbf{0} \end{bmatrix} \mathbf{u} \quad (5)$$

### 3.3. Control synthesis and stability

In order to synthesize a control law that stabilizes the system and allows to meet the communication constraints, the Hamiltonian function  $H(\mathbf{p}, \mathbf{z})$  is now chosen as the Lyapunov candidate for the multi-agents multi-targets network. Indeed, differentiating with respect to time:

$$\dot{H}(\mathbf{p}, \mathbf{z}) = \frac{\partial H^k}{\partial \mathbf{p}} \dot{\mathbf{p}} + \frac{\partial}{\partial \mathbf{z}} (H' + H^a) \dot{\mathbf{z}}, \quad (6)$$

and replacing the dynamic (5) in (6), hence:

$$\dot{H}(\mathbf{p}, \mathbf{z}) = \frac{\partial H}{\partial \mathbf{p}} \mathbf{S} \mathbf{u} - \frac{\partial H}{\partial \mathbf{p}} \mathbf{S}^\top \mathbf{D}^A \mathbf{S} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top + \frac{\partial H}{\partial \mathbf{p}} \mathbf{S}^\top (\mathbf{B}^\top \otimes \mathbf{I}) \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top.$$

Defining  $\bar{\mathbf{S}} := (\mathbf{B}^\top \otimes \mathbf{I}) \mathbf{S}$  and choosing the control vector  $\mathbf{u}$  as:

$$\mathbf{S} \mathbf{u} = -\bar{\mathbf{S}}^\top \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top - \bar{\mathbf{S}}^\top \mathbf{D}^c \bar{\mathbf{S}} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top, \quad (7)$$

the autonomous closed loop dynamics becomes:

$$\begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} -\mathbf{S}^\top \mathbf{D}^A \mathbf{S} + \bar{\mathbf{S}}^\top \mathbf{D}^c \bar{\mathbf{S}} & -\bar{\mathbf{S}}^\top \\ \bar{\mathbf{S}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top \\ \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top \end{bmatrix}. \quad (8)$$

**Proposition 1.** *The proposed control law in (7) leads the trajectories of the closed loop system (8) within the set given by  $(\partial H / \partial \mathbf{z})^\top \in \ker\{\bar{\mathbf{S}}^\top\}$ , i.e. at the equilibrium point between the elastic forces generated to the agent-agent, agent-target links.*

**Proof.** Just invoking the La Salle principle, applied to the obtained negative semi-definite  $\dot{H}(\mathbf{p}, \mathbf{z})$ , the trajectories of (8) converge to the largest invariant set where  $\dot{\mathbf{p}} = 0$ , that is:

$$-\bar{\mathbf{S}}^\top \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top = 0. \quad (9)$$

Note that the control law in (7) is distributed. In fact, each AUV can compute its control input knowing only the relative position and velocity of the agents connected with itself. Recalling the Assumption 2, the only information which requires to be shared within the team is hence the absolute velocity of each vehicle in the horizontal plane. Such an information can conveniently be coded in a data packet containing two floats and sent with a broadcast message by each vehicle on the communication channel. Note that such communication load, in addition to being small (8 bytes), it is also scalable w.r.t. the number of connected agents.

Moreover, as regards the choice of the control gains, the proposed control law offers several degrees of freedom, as well as stability guarantees. In fact, selecting different elastic constants, it is possible to determine in advance which task has higher priority than the other without any consequence about the stability property.

**Example 1.** The discussion just presented is now applied to the trivial case of two agents - two targets acting in  $\mathbb{R}^2$  (Fig. 2). The aim is to provide

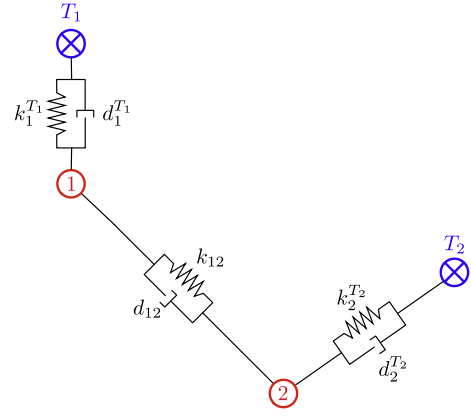


Fig. 2. A generic distribution in  $\mathbb{R}^2$  for the two AUVs - two targets case.

a clearer interpretation of the invariant set in (9) that contains the trajectories of the controlled system. So, the set  $E = \{(1, 2), (1, T_1), (2, T_2)\}$  contains the different connectivity links, the selection matrix  $\mathbf{S} = \text{diag}(\mathbf{I}^{4 \times 4}, \mathbf{0}^{4 \times 4})$  and the incidence matrix is:

$$\mathbf{B} = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Choosing a linear spring to model the agent-agent virtual coupling (i.e.  $k_1^a(\|\mathbf{z}_{12}\|) = k_1^a$ ), system (8) is stabilized to the set of  $\mathbf{z}$  that fulfil the following system of equations:

$$\begin{cases} k_1^a \left( 1 - \frac{R_d}{\|\mathbf{z}_{12}\|} \right) z_{12_x} + k_1^t z_{1T_{1x}} = 0 \\ k_1^a \left( 1 - \frac{R_d}{\|\mathbf{z}_{12}\|} \right) z_{12_y} + k_1^t z_{1T_{1y}} = 0 \\ -k_1^a \left( 1 - \frac{R_d}{\|\mathbf{z}_{12}\|} \right) z_{12_x} + k_2^t z_{2T_{2x}} = 0 \\ -k_1^a \left( 1 - \frac{R_d}{\|\mathbf{z}_{12}\|} \right) z_{12_y} + k_2^t z_{2T_{2y}} = 0, \end{cases}$$

where

$$\mathbf{z}_{12} = \begin{bmatrix} z_{12_x} \\ z_{12_y} \end{bmatrix}, \quad \mathbf{z}_{1T_1} = \begin{bmatrix} z_{1T_{1x}} \\ z_{1T_{1y}} \end{bmatrix}, \quad \mathbf{z}_{2T_2} = \begin{bmatrix} z_{2T_{2x}} \\ z_{2T_{2y}} \end{bmatrix},$$

represent the relative distances ordered in  $E$ , while  $k_1^a$ ,  $k_1^t$  and  $k_2^t$  the respective elastic constants.

### 3.4. Stability to communication delays

The required information to be exchanged among neighbouring vehicles (velocity) could be affected by communication delays on the connectivity links. This is particularly critical in marine acoustic communications, due to the speed of sound in the ocean (typical value 1500 m/s). Before stating the main result of this section, we make the following assumption, which will be clear later:

**Assumption 1.** The time varying delay acting on each connectivity link is represented by  $\tau_i(t)$ , a non-negative, bounded above,  $\varepsilon$ -Lipschitz function, with  $\varepsilon \in [0, 1)$ .

Thus, we are ready to state the following theorem, which is a generalization of the result of (Pasumathy and Kao, 2009):

**Theorem 1.** *Let Assumption 1 holds. If there exist  $|E|$  matrices  $\bar{\mathbf{P}}_i \in \mathbb{S}_{>0}^{n[V] \times n[V]}$  such that:*



$$-\mathbf{S}^\top \mathbf{D}^\top \mathbf{A} \mathbf{S} - \bar{\mathbf{D}} + \sum_{i=1}^{|E|} \bar{\mathbf{P}}_i < 0,$$

then Proposition 1 still holds, even in presence of communication delays.

**Proof.** The control law synthesized above can be restated as:

$$\mathbf{S} \mathbf{u} = -\bar{\mathbf{S}}^\top \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top - \left[ \bar{\mathbf{D}} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top - \sum_{i=1}^{|E|} \bar{\mathbf{T}}_i \bar{\mathbf{T}}_i \left( \frac{\partial H}{\partial \mathbf{p}} \right)_{\tau_i(t)}^\top \right],$$

where  $\bar{\mathbf{D}}$  contains the diagonal elements of  $\bar{\mathbf{S}}^\top \mathbf{D}^\top \mathbf{C} \bar{\mathbf{S}}$ , while  $\bar{\mathbf{T}}_i$  the out of diagonal elements of  $\bar{\mathbf{S}}^\top \mathbf{D}^\top \mathbf{C} \bar{\mathbf{S}}$  closely related with  $\tau_i(t)$ .

The term  $(\partial H / \partial \mathbf{p})_{\tau_i(t)}$  represents the partial derivative of the Hamiltonian function w.r.t. the linear momenta, computed in according to the time varying delay on the  $i$ -th link. Omitting the time dependence of each  $\tau_i$  and defining:

$$\nabla H^\top := \begin{bmatrix} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top \\ \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top \end{bmatrix}, \quad \nabla H_{\tau_i}^\top := \begin{bmatrix} \left( \frac{\partial H}{\partial \mathbf{p}} \right)_{\tau_i}^\top \\ \left( \frac{\partial H}{\partial \mathbf{z}} \right)_{\tau_i}^\top \end{bmatrix},$$

the Lyapunov candidate  $V : \mathbb{R}^{n|V|} \times \mathbb{R}^{n|E|} \rightarrow \mathbb{R}_{\geq 0}$  is now chosen as in (Pasumarthu and Kao, 2009):

$$V(\mathbf{p}, \mathbf{z}) = H(\mathbf{p}, \mathbf{z}) + \sum_{i=1}^{|E|} \int_{t-\tau_i}^t \nabla H_{\tau_i} \mathbf{P}_i \nabla H_{\tau_i}^\top ds_i, \quad (10)$$

with each  $\mathbf{P}_i \in \mathbb{S}_{\geq 0}^{n(|V|+|E|) \times n(|V|+|E|)}$ . In view of the Assumption 1 the time derivative function of (10) is hence:

$$\dot{V}(\mathbf{p}, \mathbf{z}) = \dot{H}(\mathbf{p}, \mathbf{z}) + \sum_{i=1}^{|E|} \left[ \nabla H \mathbf{P}_i \nabla H^\top + -(1 - \dot{\tau}_i) \nabla H_{\tau_i} \mathbf{P}_i \nabla H_{\tau_i}^\top \right].$$

By noting that only the term  $(\partial H / \partial \mathbf{p})_{\tau_i}$  of  $\nabla H_{\tau_i}$  is really affected by lag, hence  $\mathbf{P}_i = \text{diag}(\bar{\mathbf{P}}_i, \mathbf{0})$ . Therefore, by compacting in vector form and after some algebraic manipulations, we have:

$$\dot{V}(\mathbf{p}, \mathbf{z}) = \begin{bmatrix} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top \\ \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top \end{bmatrix}^\top \mathbf{W} \begin{bmatrix} \left( \frac{\partial H}{\partial \mathbf{p}} \right)^\top \\ \left( \frac{\partial H}{\partial \mathbf{z}} \right)^\top \end{bmatrix}, \quad (11)$$

where  $\left( \frac{\partial H}{\partial \mathbf{p}} \right)_{\tau_i}^\top := \left[ \left( \frac{\partial H}{\partial \mathbf{p}} \right)_{\tau_i}^\top, \dots, \left( \frac{\partial H}{\partial \mathbf{p}} \right)_{\tau_{|E|}}^\top \right]$ , and the matrix  $\mathbf{W}$  is:

$$\begin{bmatrix} -\mathbf{S}^\top \mathbf{D}^\top \mathbf{A} \mathbf{S} - \bar{\mathbf{D}} + \sum_{i=1}^{|E|} \bar{\mathbf{P}}_i & \bar{\mathbf{T}}_1 & \dots & \bar{\mathbf{T}}_{|E|} \\ \mathbf{0} & -\kappa_1 \bar{\mathbf{P}}_1 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & -\kappa_{|E|} \bar{\mathbf{P}}_{|E|} \end{bmatrix}.$$

where  $\kappa_i := (1 - \dot{\tau}_i) > 0$  in view of Assumption 1. Each  $\tau_i$ , indeed, is an  $\varepsilon$ -Lipschitz function (with  $\varepsilon \in [0, 1]$ ), then there exists  $0 \leq \tilde{d} < 1$  such that  $|\dot{\tau}_i| \leq \tilde{d}$ , for all  $i = 1, \dots, |E|$ . Thus, the time derivative of the Lyapunov candidate in (11) is negative definite iff the upper block-triangular matrix  $\mathbf{W} < 0$ . Hence, by noticing that  $-\kappa_i \bar{\mathbf{P}}_i < 0$  is satisfied for all possible choices of  $\bar{\mathbf{P}}_i$ , the only condition that should be verified remains the one on the first entry.

**Remark 1.** Assuming a positive, bounded above,  $\varepsilon$ -Lipschitz delay function  $\tau_i$ , with  $\varepsilon \in [0, 1]$ , is quite realistic in marine environment. As a matter of fact, there are two main reasons why the time delay  $\tau_i$  may

increase: the two agents are increasing their relative distance, or because a packet is lost and needs to be retransmitted. In the first case,  $\tau_i$  is trivially bounded by the ratio  $\Delta v / c$ , where  $\Delta v$  is the relative velocity between the agents and  $c$  is the speed of sound in sea water (1500 m/s); for any kind of surface or submerged platform known to the Authors, either manned or unmanned, the ratio is indeed much less than one, hence the  $\varepsilon$ -Lipschitz condition is verified. As for the second case, this is studied in the following example.

**Example 2.** Consider two fixed nodes ( $\mathcal{A}$ ,  $\mathcal{B}$ ), at a predefined distance  $R_0$ : the node  $\mathcal{A}$  transmits messages, while  $\mathcal{B}$  receives them. If  $\mathcal{A}$  sends at time  $t_0$ , then  $\mathcal{B}$  successfully receives the message at time  $t_0 + \tau$ . If a second message, sent at time  $t_1$ , it is not properly received at  $t_1 + \tau$ , the node  $\mathcal{A}$  ideally re-sends such message at  $t_1 + \tau$ . In the unlikely event that it is not properly received again, the node  $\mathcal{A}$  sends the same message for the third time at  $t_1 + 2\tau$  (as many commercial acoustic modems work), that is correctly received by  $\mathcal{B}$  at  $t_1 + 3\tau$ . Thus, considering this unfortunate set of transmissions, the final delay is equal to  $3\tau$ , so  $\Delta\tau = 3\tau - \tau = 2\tau$ , while  $\Delta t \geq 3\tau$ . Then,  $\Delta\tau \leq \frac{2}{3} \Delta t$ . In general, if the packet is re-sent  $n$  times, we have  $\Delta\tau \leq \frac{n-1}{n} \Delta t$ . Therefore, even in the ideal case of immediate re-transmissions of the lost packet, Assumption 1 is still reasonable.

In practice, such assumption was always verified in the field tests reported in (Allotta et al., 2016).

#### 4. Relationship with potential BSI games

This section aims to introduce the BSI games and to show the close relation with the proposed way of model a network of interconnected agents.

##### 4.1. Symmetric observations and BSI games

BSI games are characterized by bilateral or pairwise strategic interactions. For the  $i$ -th player, the utility function  $U_i$  contains a term  $w_{ij}(A_i, A_j)$ , namely the observation seen by  $i$  due to the strategy of  $j$ , which takes place solely due to the pairwise interaction between him/her and another player  $j$ . A BSI game is a game where the observations are symmetric, i.e.  $w_{ij}(A_i, A_j) = w_{ji}(A_j, A_i)$ , across all pairs of players (Ui, 2000). That is,  $\forall i, j \in \mathcal{N}$ ,  $i \neq j$ , there exist functions  $w_{ij} : \mathbf{A}_i \times \mathbf{A}_j \rightarrow \mathbb{R}$  such that  $w_{ij}(A_i, A_j) = w_{ji}(A_j, A_i)$  for all  $A_i \in \mathbf{A}_i$  and  $A_j \in \mathbf{A}_j$ . Moreover, the utility function of player  $i$  is assumed to be of the form:

$$U_i(A_i, A_{-i}) = \sum_{j \neq i} w_{ij}(A_i, A_j), \quad \forall i \in \mathcal{N}.$$

As stated in (Ui, 2000), the next theorem shows that BSI games are exact potential games (the Proof is here omitted for the sake of brevity).

**Theorem 2.** Assume  $\mathcal{G}$  is a BSI game. Then it is also an exact potential game with the following potential function:

$$V(A) = \sum_{i \in \mathcal{N}} \sum_{\substack{j \in \mathcal{N}, \\ j < i}} w_{ij}(A_i, A_j).$$

Note that the original utility function proposed in (Ui, 2000) is obtained combining BSI games and no-conflict games. Due to the linear combination property of utility functions (Lā et al., 2016), the resulting exact potential game has utility functions of the form  $U_i(A) = \sum_{j \neq i} w_{ij} - P_i(A_i)$ , where  $P_i : \mathbf{A}_i \rightarrow \mathbb{R}$  indicates the no-conflict term. Hence, the potential function can be rewritten as:

$$V(A) = \sum_{i \in \mathcal{N}} \sum_{\substack{j \in \mathcal{N}, \\ j < i}} w_{ij}(A_i, A_j) - \sum_{i \in \mathcal{N}} P_i(A_i). \quad (12)$$

#### 4.2. Equivalence relations

With the previous concepts in mind, if the strategy set for the  $i$ -th agent in Fig. 1 is  $A_i := (\mathbf{p}_i, \mathbf{q}_i)$ , consider its general Hamiltonian function:

$$h_i(A_i, A_{-i}) := \underbrace{\frac{1}{2} \sum_{k \in \mathcal{N}_i^a} h^{ak}(\mathbf{z}_k) + \frac{1}{2} \sum_{j \in \mathcal{N}_i^t} h^{bj}(\mathbf{z}_j)}_{\sum_{q \in \mathcal{N}_i} w_{iq}(\mathbf{z}_q)} + \underbrace{\frac{1}{2} \mathbf{p}_i^\top \bar{\mathbf{m}}_i^{-1} \mathbf{p}_i}_{-P_i(\mathbf{p}_i)},$$

where  $\mathcal{N}_i^a$  and  $\mathcal{N}_i^t$  are the sets of agents and targets connected with the  $i$ -th agent, while  $\mathcal{N}_i$  contains all that links, i.e.  $\mathcal{N}_i = \mathcal{N}_i^a \cup \mathcal{N}_i^t$ . Note that the strategies of the neighbouring agents  $A_{-i}$  are included in  $\{\mathbf{z}_q\}_{q \in \mathcal{N}_i}$ , coupled with  $A_i$ . In this way, the functions  $h^{ak}(\cdot)$  and  $h^{bj}(\cdot)$ , i.e. the springs contributions (possibly non-linear), depend only on the relative distance between two nodes; it is straightforward to observe that the action of the spring on the nodes is the same: from here, the symmetry of the observations in the proposed framework. The kinematic contribution instead  $\mathbf{p}_i^\top \bar{\mathbf{m}}_i^{-1} \mathbf{p}_i$  represents a sort of pure coordination term, because no agent connected with the  $i$ -th cares about which action is taken from it.

Now, assuming the  $i$ -th Hamiltonian function as the  $i$ -th player's utility function, it is quite easy to show that the global Hamiltonian in (3) can be rewritten as the potential function of the game in (12), just imposing without abuse  $w_{ij} = 0$  on non-existing links. In fact:

$$\begin{aligned} -\sum_{i \in \mathcal{I}'} P_i(A_i) &= \frac{1}{2} \sum_{i=1}^{|\mathcal{V}|} \mathbf{p}_i^\top \bar{\mathbf{m}}_i^{-1} \mathbf{p}_i = H^k(\mathbf{p}), \sum_{i \in \mathcal{I}'} \sum_{j \in \mathcal{I}', j < i} w_{ij}(A_i, A_j) = \sum_{i \in \mathcal{I}'} \sum_{q \in \mathcal{N}_i} w_{iq}(\mathbf{z}_q) \\ &= H'(\mathbf{z}) + H^a(\mathbf{z}). \end{aligned}$$

**Remark 2.** If there are some fixed nodes, the potential contribution of the springs depends only on the strategy of the considered agent. It also becomes a pure coordination term.

**Remark 3.** The Hamiltonian function in (3) is a convex function in the generalized coordinates  $(\mathbf{p}, \mathbf{z})$ . The resolution of the minimization Problem, that leads to a NE of the game, is not trivial due to the presence of the communication constraints and fixed nodes: these may induce several local minima in the potential function and make richer the invariant set (9). However, in the simple case where there are no constraints (i.e. any spring with equilibrium point different from 0) or fixed nodes, the set of strategies that minimizes the Hamiltonian function is the one that leads the agent in the NE  $(0, 0)$ .

In the authors' previous works (and here), as in (Caiti et al., 2012) and (Caiti et al., 2013b), the desired intra-agents behaviour was obtained by means of non-linear springs, whose elastic potential was given by:

$$h^{ai}(\mathbf{z}_i) := \begin{cases} K_1 (\|\mathbf{z}_i\| - R_d)^2 & \|\mathbf{z}_i\| \leq R_d \\ K_2 \frac{(\|\mathbf{z}_i\| - R_d)^2}{R_c - \|\mathbf{z}_i\|} & \text{otherwise} \end{cases} \quad (13)$$

where  $K_1, K_2 > 0$  were design parameters. As an example, Fig. 3 shows the function in (13) with a typical value for the maximum communication range of a medium frequency acoustic modem, e.g.  $R_c = 3000$  m, and  $R_d = 2000$  m.

Thus, the resulting gradient-based control rules were able to solve the general area coverage Problem, reducing the overlap of sonar detection and ensuring that each agent maintains at least one other vehicle in its communication range. However, there were several limitations to the application of such control law: although the possibility to tune the appropriate interest functions (i.e. utility functions), the myopic coordination of AUVs could neither provide any guarantee of stability of the networked system, nor ability to predict which NE of the game was

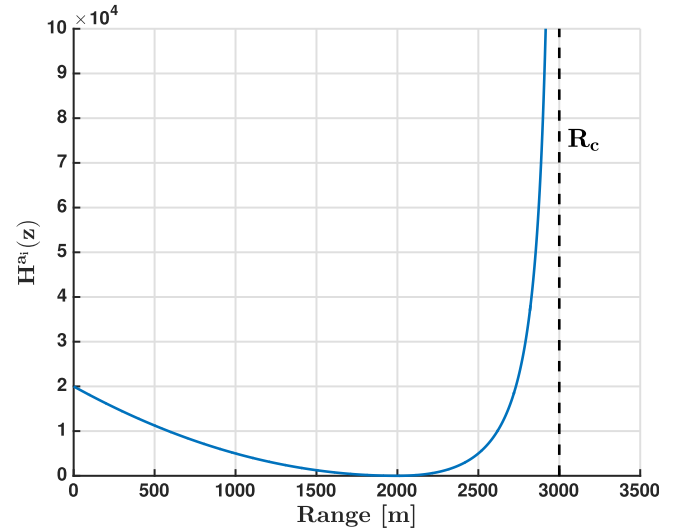


Fig. 3. Elastic potential associated to an agent-agent connection, with  $R_c = 3000$  m and  $R_d = 2000$  m.

reached.

At a price of a slightly more complex network model (couples of spring-damper, incidence matrix, port-Hamiltonian framework and so on), with the control algorithm (7) presented in this work, that is also gradient-based and offers the same degrees of freedom of the previous one, it is always possible to arbitrarily change the transient behaviour of the network, maintaining the robust stability guarantees previously demonstrated. Moreover, exploiting the equivalence relations with potential games, it is possible to shape and determine in advance the potential functions, its set of local minima and the set of NE related with game.

#### 5. Integrating the energy tanks

This section provides a modelling strategy for obtaining a flexible cohesive behaviour of the group of AUVs by applying the *artificial tank* approach introduced and demonstrated in (Franchi et al., 2012). As there stated, the passivity is guaranteed during a *split* manoeuvre, i.e. the cancellation of the coupling force between a pair of agents when they are outside the communication range. Not the same holds for a *join* manoeuvre, i.e. the re-establishment of the coupling: as a matter of fact, some extra energy can still be produced during the join procedure.

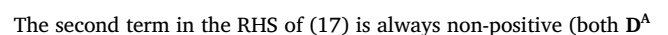
According to the scenario in Fig. 4, the relative distance between two agents at the join operation may induce to an higher elastic potential energy with respect to the one at the split operation, leading to an increase of the Hamiltonian function and consequently a loss of stability guarantees. This entails a new edge in the graph and the corresponding update of the incidence matrix  $\mathbf{B}$ . However, the passivity of the global system can be maintained by endowing each AUV with a container of dissipated energy, called *tank*.

##### 5.1. Mathematical model

Due to the natural damping of the marine environment, the amount of power dissipated by each of  $l$  mobile nodes is:

$$D_i = \frac{\partial H}{\partial \mathbf{p}_i} \bar{\mathbf{d}}^A \left( \frac{\partial H}{\partial \mathbf{p}_i} \right)^\top,$$

where  $\bar{\mathbf{d}}^A$  are the elements of the diagonal matrix  $\mathbf{D}^A$  (implicitly assuming that such natural damping is the same for all the AUVs). Thus, the dynamics of the  $i$ -th tank is:





and  $D^c$  are positive definite matrices); then, making explicit the first one:

$$\frac{\partial \mathcal{H}}{\partial \mathbf{t}} \Theta \mathbf{P} \mathbf{S}^T \mathbf{D}^A \mathbf{S} \left( \frac{\partial \mathcal{H}}{\partial \mathbf{p}} \right)^T = \mathbf{t}^T \Theta \begin{bmatrix} \frac{1}{t_1} D_1 \\ \vdots \\ \frac{1}{t_l} D_l \end{bmatrix} = \sum_{i=1}^l \theta_i D_i.$$

Noting that  $\theta_i \in \{0, 1\}$ , such term represents at most the energy dissipated by the AUVs; hence, the second term is always greater than the first one, and  $\dot{\mathcal{H}}(\mathbf{p}, \mathbf{z}, \mathbf{t}) \leq 0$ . Again, invoking the La Salle principle applied to the obtained negative semi-definite  $\dot{V}(\cdot)$ , the trajectories of (14) converge to the largest invariant set where  $\mathbf{p} = \mathbf{0}$ . ■

## 6. Simulation results and discussion

This section aims to show the simulation results obtained by applying the control algorithm presented in Section 3 to a team (or teams) of AUVs. Due to its flexibility, the proposed control law can be exploited to accomplish several cooperative missions, noting once again that the maintenance of the communication link among the vehicles belonging to a team (or a sub-team) is a fundamental requirement for the success of the collective task. Thus, after choosing a proper initial incidence matrix, the non-linear spring associated to each connection is modelled as reported in Section 4. In this way, defining an initial sub-graph between the agents of a team, the fulfilment of the communication constraints can be guaranteed: if two agents start within the communication range, they remain connected during the whole task, converging if possible to the desired distance  $R_d$ .

**Remark 5.** Generally, at the beginning of each mission, a support ship or a docking station allow the deployment of vehicles within a restricted area; so it is quite realistic to assume the existence of a communication link among vehicles from the beginning.

In the following, the simulation results obtained with several types of typical tasks are presented, where velocities are supposed to be exchanged between the vehicles at a rate of 1 Hz. Moreover, all the simulations (except the last one in Section 6.3, which is delay-free) are performed considering a fixed targets and fixed communication delay of 1 s.

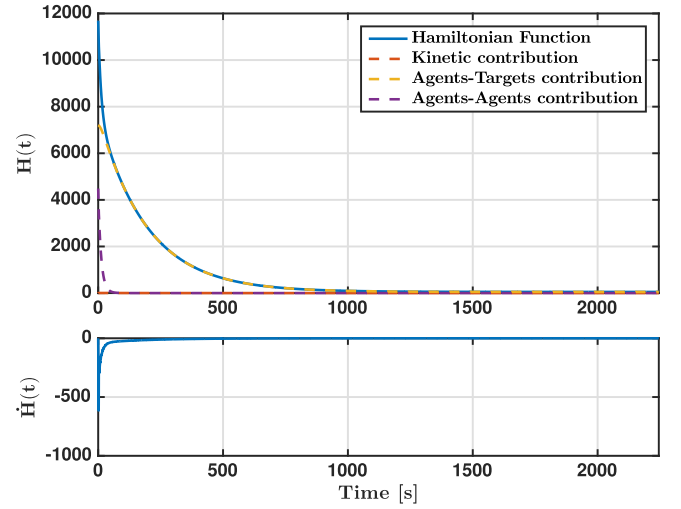


Fig. 6. Hamiltonian function with each single contribution of (3) (top) and its time-derivative (bottom).

### 6.1. Patrolling: single and multiple assets

In a surveillance or patrolling task, the vehicles of a team have to cover a certain area around an asset to be defended.

The first scenario in Fig. 5 involves only one target (the asset itself,  $m = 1$ ) and a team of three vehicles ( $l = 3$ ). Hence, the agent-target connections are modelled by a spring-damper couple from each vehicle to the asset, setting instead the desired intra-agent range  $R_d = 100$  m. As it can be noticed, the team reaches a stable configuration around the asset to be defended. Furthermore, each vehicle remains within the desired distance with respect to all the others. In Fig. 6 the Hamiltonian function (3) and its time derivative are represented. Note that the function  $\dot{H}(\cdot)$  is negative semi-definite, as expected with the choice of the control input defined in (7).

In Fig. 7 there is another example of such task with two sub-teams of AUVs while performing a patrolling of multiple assets. Here there are two main assets and two sub-teams consisting of three and four vehicles, respectively (numbers 1-2-3 and 4-5-6-7); each target is connected by a virtual spring-damper couple with each agent of the corresponding sub-team. The communication constraints are defined among the vehicles

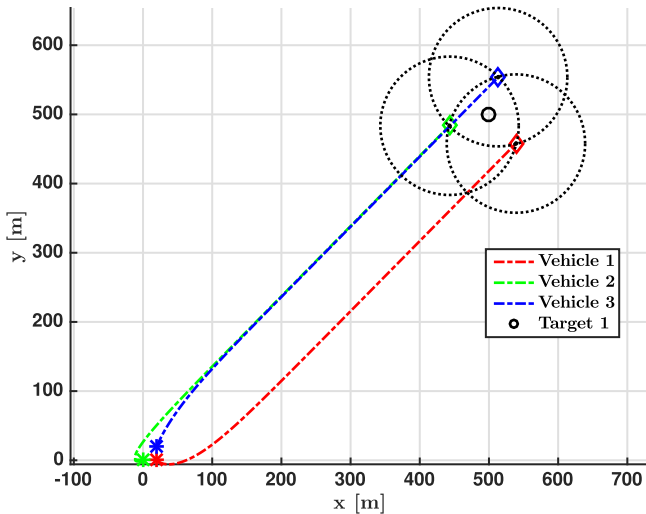


Fig. 5. Cooperative surveillance task. The vehicles start from the positions indicated by the coloured stars. At the end, the vehicles (coloured diamonds) are disposed around the asset (black circle) while satisfying the distance constraints, represented with the dotted circles.

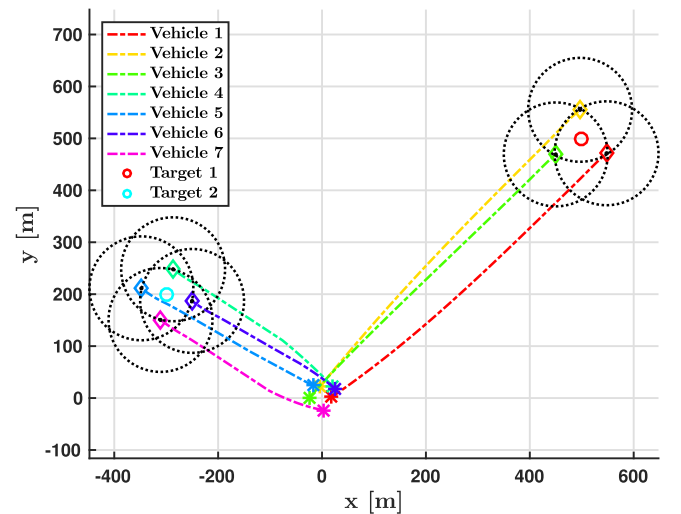


Fig. 7. Cooperative patrolling of multiple assets. The two sub-teams start from the positions indicated by the coloured stars. At the end, the vehicles (coloured diamonds) are disposed around the assets (red and cyan circles) while satisfying the distance constraints, represented with the dotted circles.

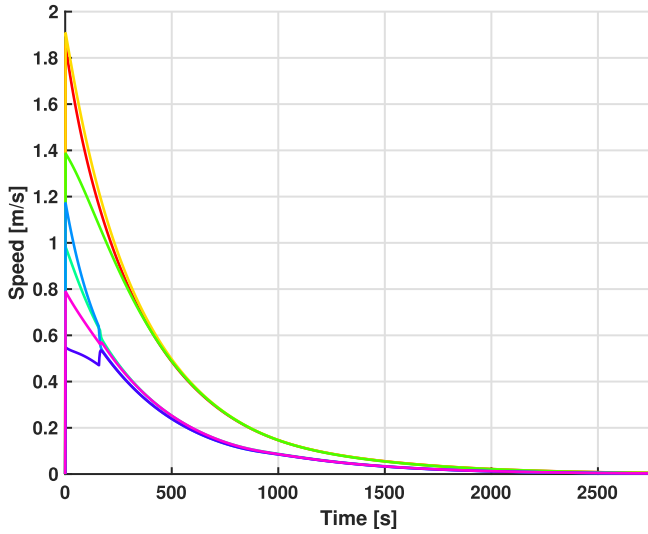


Fig. 8. Velocities of the AUVs of the two sub-teams involved in the mission.

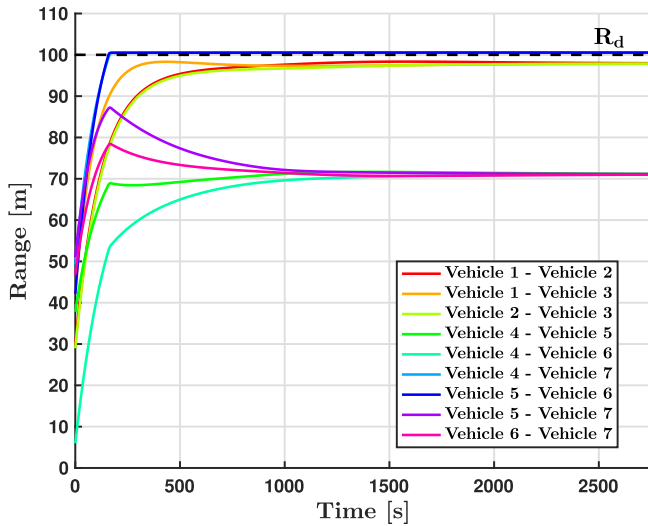


Fig. 9. Relative distances among agents belonging to the same sub-team. Note that the solid blue (5–6) and light blue (4–7) lines represent the virtual couplings between the agents placed on the two diagonal of the square: the communication constraint is still fulfilled.

belonging to the same sub-team. As it can be noticed in Fig. 8, each team reaches a stable configuration around the asset to be defended while the velocities decrease almost exponentially: this fact directly depends on the choice of the non-linear springs, which effects decrease as the AUVs get closer to the target. Due to the symmetry of the Problem, the sub-team with three vehicles shapes an equilateral triangle around its target, while the one with four agents shapes a square. Moreover, each vehicle remains within the desired distance with respect to all others within the team (Fig. 9).

## 6.2. Coverage task

In this case, each vehicle is assigned to a specific target ( $l = m$ ) with the overall objective of covering the maximum area in the targets' neighbourhood. Each coupling between an agent and the associated target is modelled by a spring-damper connection. Fig. 10 shows the trajectories followed by the team members in a scenario with three vehicles ( $l = m = 3$ ) and a relative distance among the agents  $R_d = 2500$  m. In this case, the team reaches a stable configuration in which no

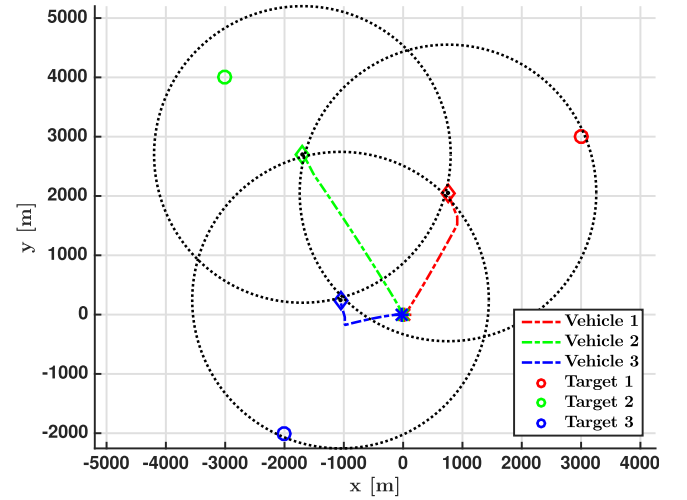


Fig. 10. Cooperative coverage task. The vehicles start to approach to the assigned targets until the communication limit (dotted circles) is attained.

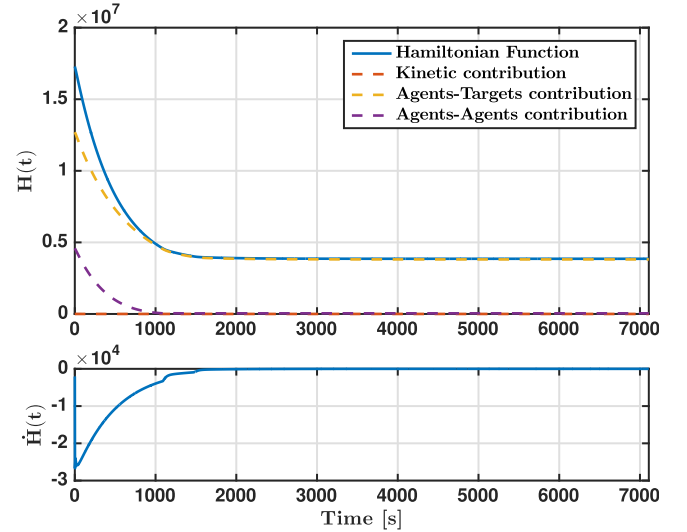


Fig. 11. Hamiltonian function with the different contributions (top) and its time-derivative (bottom).

vehicle can move closer to the corresponding target without breaking the imposed constraints due to the communication maintenance.

As it can be seen from Fig. 11, after an initial transient in which it decreases, the Hamiltonian function settles at a constant value, different from zero. This is the effect of the agent-target springs that do not reach their equilibrium point (the well-known invariant in (9)). As expected, the time-derivative of the Hamiltonian is still negative semi-definite.

## 6.3. Exploration task

Given a team of AUVs and a set of significant spots, the fulfilment of an exploration task implies that each site is visited (i.e. reached within a certain tolerance  $R_T$ ) at least once by a vehicle (see Fig. 12).

Consider the case with three agents, that have to maintain the connectivity one to each other, and three possible targets. Depending on the sites location, even if the elastic constants on the agent-target connections are not the same, the proposed control law (7) leads the vehicles in a static, equilateral triangle shaped configuration (deadlock) that does not allow the accomplishment of the exploration task (e.g. as in the case of the coverage task in Fig. 10).

In order to guarantee the success of the mission, one possible solution

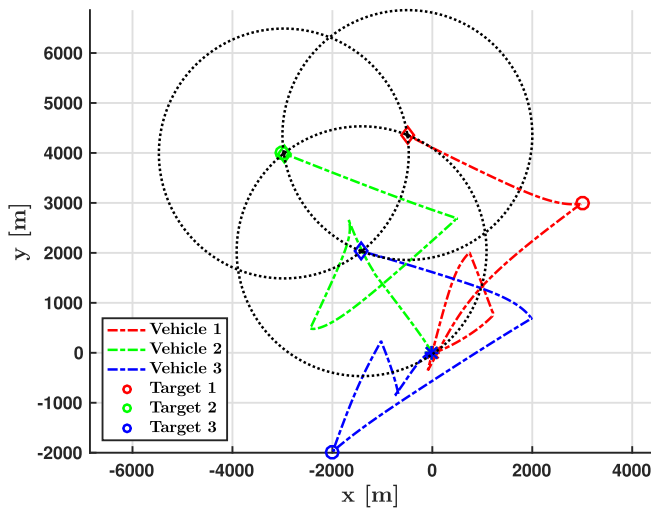


Fig. 12. Exploration task. The agents start from the positions indicated by the coloured stars and reach the corresponding target (coloured circles) one at a time. In this case, the priority is 3-1-2.

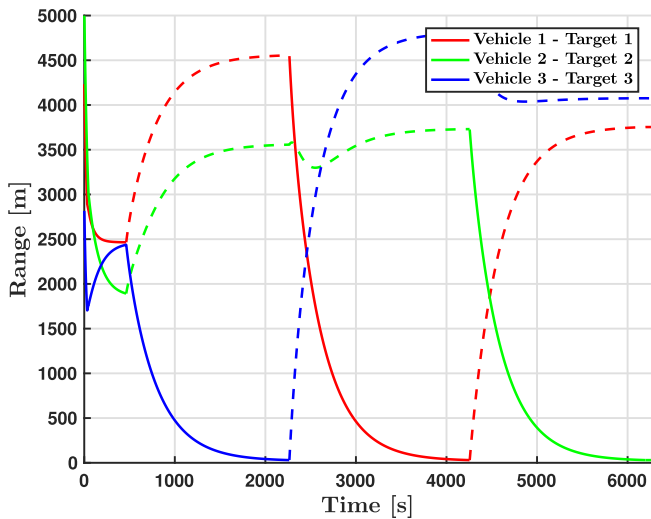


Fig. 13. Relative distances between agents and corresponding target over time. The solid intervals of each line suggest the phase of link activation, while the dashed one represents the “absence” of connection.

when a deadlock occur is to maintain only one agent-target link at time, breaking and regenerating the connections following a certain policy. In this case, the agents have assigned a predefined priority that schedule the spot exploration order: when a deadlock occurs, the agent with highest priority takes the “leadership” of the group, while the others “disconnect” themselves from their related targets (split manoeuvre) as long as the leader does not accomplish its sub-task. Now the story is trivial: the first leader breaks the connection with its target, a new leader is elected, e.g. the agent with immediately lower priority, and the corresponding connection is restored (join manoeuvre). Depending on the position reached by the new leader, this operation could require an injection of energy into the system (i.e. when the join manoeuvre happens at an higher distance from the target with respect to the split one).

This amount of extra energy is picked up from the tank of the moving agent (Figs. 13 and 14), implementing the algorithm described in Section 5. Note that the split and join manoeuvres can be executed also between agents, allowing to relax the assumption of the communication maintenance if possible or required. In this case, the required extra amount of energy would be extracted from both the tanks, accordingly with the

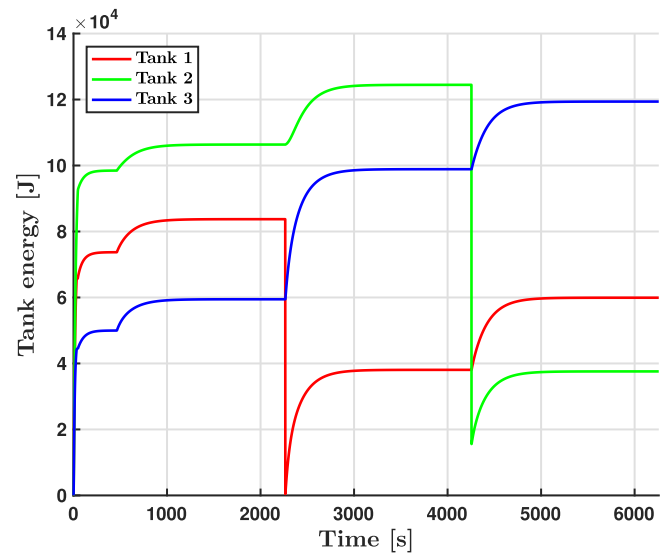


Fig. 14. Tanks dynamics. During the evolution of the mission, the tank is charged up until it reaches a saturation. The downward spikes characterize the join manoeuvres, where the involved vehicle pick up an extra amount of energy from its tank.

procedure reported in (Franchi et al., 2012).

## 7. Conclusions and future developments

Exploiting the port-Hamiltonian system modelling, jointly with the passivity theory, the proposed distributed control law allows to fulfil communication constraints between interconnected marine vehicles during the execution of several missions. Moreover, such framework suggests a way to draw quick conclusions about the soundness and stability of the whole system, providing robustness even in presence of time varying delays on the connectivity links, which are very common in marine environment. The large number of degrees of freedom, that can be exploited upstream of the mission, represents one of the main features of the algorithm: each passive virtual coupling introduces an artificial potential that can be arbitrarily shaped, without any consequence about the stability of the sensors network. In this way, selecting different elastic constants, it is possible to determine in advance which task has higher priority, “choosing” the set of reachable equilibria. The behaviour of the group is also made flexible, with arbitrary split and joint events: the passivity property of the whole network is kept safe using artificial tanks, able to store and supply energy whenever required.

With the setting of the present paper, the equivalence between the proposed framework and BSI games has been shown. Whether this equivalence can be formally extended to a larger class of systems it is an interesting challenge for future research.

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