Ocean Engineering 142 (2017) 217–223

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Short communication

Practical proportional integral sliding mode control for underactuated surface ships in the fields of marine practice

Zhijian Sun[a], Guoqing Zhang[a,b], Bowen Yi[a], Weidong Zhang[a,*]

[a] *Department of Automation, Shanghai Jiao Tong University, and Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240, PR China*

[b] *Navigation College, Dalian Maritime University, Liaoning 116026, PR China*

ARTICLE INFO

Keywords:
Sliding mode control
Ship motion control
Model uncertainties
Environmental disturbances

ABSTRACT

Environmental disturbances and systematical uncertainties are the main obstacles for ship motion control. This paper devotes to enhancing the control system robustness of underactuated surface ships with model uncertainties and environmental disturbances. A novel nonlinear robust adaptive scheme with sliding mode control is proposed for underactuated ships to track the desired path generated by the logical virtual ship in the presence of unknown plant parameters and environmental disturbances. Compared with the existing results, the proposed controller is designed based on the combination of PI sliding mode control and the upper bound estimation of disturbances. With the proposed design, the control scheme could not only obtain a better performance of the control system, the continuous scheme also reduce the chattering of system by a special construction of the sliding manifolds. Numerical simulations are given to demonstrate the effectiveness of the proposed method.

1. Introduction

Underactuated surface ships have played an important role in the marine exploration and research, such as dynamic positioning for offshore oil drilling (Tannuri et al., 2010), underwater pipe-laying (Fossen, 1994) and so on. Over the last few years, ship motion control has attracted lots of attention due to its practical applications and theoretical challenges (Zhang et al., 2015). It is well known that underactuated surface ships are equipped with propellers and rudders for surge and yaw motions only, meaning that no actuator is used for the control of sway motion directly (Zhang and Zhang, 2014), and it is a challenge for the ship motion control. Control in the presence of uncertainty is one of the main topics in modern control theory (Shtessel et al., 2014), as well as in the marine control community. In the ship motion dynamics, there always exist discrepancies between the actual dynamics and its models. These discrepancies are mostly caused by the environment disturbances, unknown plant parameters and systematical uncertainties. In addition, when the number of actuators are less than the degree of freedom (Dong and Guo, 2005; Reyhanoglu, 1997; Pettersen and Egeland, 1997), it also generates the non-integral constraints in the controller design.

With the above mentioned challenges in the field of ship motion control, the so-called robust control algorithms have arose in the control system community, such as robust adaptive control (Ioannou and Sun, 2012; Lavretsky and Wise, 2013; Du and Shi, 2016), robust neural damping (Zhang et al., 2015; Zhang and Zhang, 2015, H_∞ control (Cheng et al, 2015; Zou et al., 2016; Chang et al., 2015, backstepping techniques (Li et al., 2015; Liu et al., 2016; Wang et al., 2015, Network-based technology (Wang and Han, 2016a, 2016b; Wang and Xiong, 2015), multi-time scale methods (Yi et al., 2016), and sliding mode control (Xu et al., 2015; Zhang and Chu, 2012; Fossen, 2002), probably, all those algorithms are successful to handle bounded disturbances and uncertainties.

With the consideration of robustness, sliding mode control has been widely applied in the field of ship motion control. In Xu et al. (2015), a novel adaptive dynamic sliding mode control for the trajectory tracking of underactuated unmanned underwater vehicles is proposed to handle with environmental disturbances and systematical uncertainties. However, the assumptions that the first-order derivatives of environmental disturbances and the existence of thruster are too restrictive. In addition, Li et al. (2008) develops a point-to-point navigation for underactuated ships. Although such algorithm can guarantee the closed-loop system to be uniformly ultimately bounded, that is the trajectories converge to an invariant set rather than the

* Corresponding author.
E-mail addresses: sunzhijianfuzhou@sjtu.edu.cn (Z. Sun), zgq_dlmu@163.com (G. Zhang), yibowen@sjtu.edu.cn (B. Yi), support@elsevier.com, wdzhang@sjtu.edu.cn (W. Zhang).

http://dx.doi.org/10.1016/j.oceaneng.2017.07.010
Received 8 November 2016; Received in revised form 17 February 2017; Accepted 2 July 2017

equilibrium. Furthermore, in Li and Sun (2009), an adaptive sliding mode control law combined with backstepping technique is proposed to solve the trajectory tracking problem, but a simplified simple system model is investigated. Under the design with rudder angle, it makes tracking insensitive, especially for small-scale change yaw angle.

Motivated by the above research line, due to the high performance of sliding mode control in dealing with parameter perturbations and dynamic uncertainties (Zhang et al., 2014, 2013), a novel design is developed to solve the problem of the trajectory tracking for under-actuated surface ships based on the combination of sliding mode control and backstepping technique. Unlike Li et al. (2008), a continuous adaptive sliding mode surface term is derived to reduce the chatting of the closed-loop system. With this proposed design, the controller can not only guarantee the convergence of states to equilibrium, but also reduce the chatting of the system. Furthermore, the algorithm also enhances the robustness to the system uncertainty, such as systematic uncertainties and unknown parameters. The main contributions of this paper are twofold.

1). By combination of a novel nonlinear robust adaptive PI sliding mode scheme and the upper bound estimation of the disturbances, the proposed algorithm is developed to implement the trajectory tracking task of underactuated vehicles.

2). An adaptive continuous PI sliding mode scheme is constructed to stabilize the control system. By using this special property and structure, the control method could not only eliminate the chattering of the closed-loop system, but also relax some assumptions in Xu et al. (2015) (Table 1).

2. Problem formulation

According to Fossen (2002); Li et al. (2008), the kinematic and dynamical equations of underactuated surface ship can be described as Eq. (1). It has two control inputs: the force in surge degree and the control torque in the yaw degree (Jiang, 2002).

$$\begin{cases} \dot{x} = cos(\psi)u - sin(\psi)v \\ \dot{y} = sin(\psi)u + cos(\psi)v \\ \dot{\psi} = r \\ \dot{u} = \Theta_u^T f_u(\dot{\eta}, \eta) + \zeta_u \tau_u + \tau_{w1} \\ \dot{v} = \Theta_v^T f_v(\dot{\eta}, \eta) + \tau_{w2} \\ \dot{r} = \Theta_r^T f_r(\dot{\eta}, \eta) + \zeta_r \tau_r + \tau_{w3} \end{cases} \tag{1}$$

where (x,y) denotes the position coordinates of the underactuated surface vessel model in the earth-fixed frame and ψ is the yaw angle. $\dot{\eta} = R(\psi)(u, v, r)^{\mathrm{T}}$ and (u, v, r) are the velocities in surge, sway and yaw directions. The surge force τ_u and the yaw moment τ_r are considered as two available control inputs with the known nonzero constant control coefficients ζ_u and ζ_r. $\Theta_u \in \Re^{n_u}$, $\Theta_v \in \Re^{n_v}$ and $\Theta_r \in \Re^{n_r}$ are unknown constant vectors with known dimensions n_u,n_v and n_r. $f_u(\dot{\eta}, \eta) \in \Re^{n_u}$, $f_v(\dot{\eta}, \eta) \in \Re^{n_v}$ and $f_r(\dot{\eta}, \eta) \in \Re^{n_r}$ are all known smooth vector fields. τ_{w1}, τ_{w2} and τ_{w3} are the environmental disturbance acting on the surge, sway and yaw axes, respectively.

For path-following control of underactuated surface ships, we define the control objectives in Fig. 1. The error variables have been define as follows (Zhang et al., 2015):

Table 1
Notations.

$\lvert\cdot\rvert$ is the norm of a scalar
$\lVert\cdot\rVert$ is the norm of a vector
$\lVert\cdot\rVert^2 = \Sigma_{i,j}(\cdot)_{i,j}^2 \cdot (\cdot)_{i,j}$ the element of $(\cdot)$ in row i and column j
$\widetilde{(\cdot)} = \widehat{(\cdot)} - (\cdot)$; $\widehat{(\cdot)}$ is the estimate of $(\cdot)$; $\widetilde{(\cdot)}$ is the estimation error

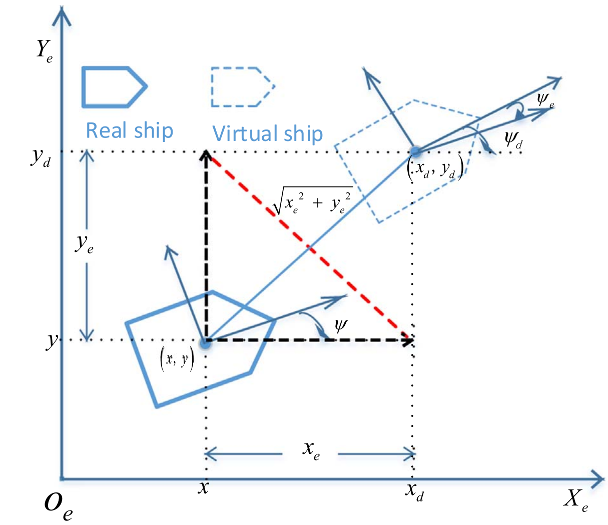

Fig. 1. General framework for path-following control of underactuated surface ship.

$$\begin{aligned} &x_e = x_d - x, \quad y_e = y_d - y \\ &z_e = \sqrt{x_e^2 + y_e^2}, \quad \psi_e = \psi_r - \psi \end{aligned} \tag{2}$$

where (x_d, y_d, ψ_d) denotes the desired position and orientation of an underactuated surface vessel model in earth-fixed frame, z_e denotes the position error. ψ_d is the ships's azimuth angle, which is defined as follows (Li et al., 2008):

$$\psi_r = \begin{cases} 0.5[1 - \mathrm{sign}(x_e)]\mathrm{sign}(y_e)\pi + \arctan(y_e/x_e), & \text{when } z_e \neq 0 \\ \psi_d, \text{ when } z_e = 0 \end{cases} \tag{3}$$

where $sign(\cdot)$ is a sign function, which is defined as follows:

$$\begin{cases} \mathrm{sign}(x) = -1, \ x < 0 \\ \mathrm{sign}(x) = 0, \ x = 0 \\ \mathrm{sign}(x) = 1, \ x > 0 \end{cases} \tag{4}$$

Assumption 1.

1) The environmental disturbances are bounded satisfying $|\tau_{w1}| \le \tau_{w1max}$, $|\tau_{w2}| \le \tau_{w2max}$, $|\tau_{w3}| \le \tau_{w3max}$.
2) The states of the reference model x_d, $\dot{x}_d$, $\ddot{x}_d$, y_d, $\dot{y}_d$, $\ddot{y}_d$ and $\dot{\psi}_d$ are all bounded.

Assumption 2. From the Fig. 1, it can be seen that the control objective is to develop a sliding mode control scheme to let the underactuated ship track the reference path, which is generated by a virtual ship as (5): Zhang et al. (2015)

$$\begin{cases} \dot{x}_d = u_d cos(\psi_d) \\ \dot{y}_d = u_d sin(\psi_d) \\ \dot{\psi}_d = r_d \end{cases} \tag{5}$$

Remark 1. Assumption 2 is introduced in the existing reference (Zhang et al., 2015), different from given the reference path by x_d and y_d. The advantage of this design not only satisfies the condition 2 of Assumption 1, but also obtains the reference path just by the variables u_d and r_d of the virtual ship, as Fig. 1.

3. Controller design

In this section, a practical integral sliding mode controller for path-following control of underactuated surface ships is proposed. Through the description of kinematic and dynamic expressed as (1) with Assumptions 1–2, all the states are guaranteed to be uniformly ultimately bounded in the closed-loop system. In order to prove the main results of the design of the controller, the Lyapunov Candidate Function (*LCF*) is given as follows:

$$V_1 = \frac{1}{2}(z_e - \delta_\Delta)^2 \tag{6}$$

Remark 2. By choosing the LCF of Eq. (6), the design is to let $(z_e - \delta_\Delta)$ respect the error of z_e, that means when $t \to \infty$, $z_e \to \delta_\Delta$. According to Eq. (3), if we use the z_e to design the Eq. (1) directly, it cannot guarantee the $|\psi_e| \le \pi/2$ (Yi et al., 2016). The proposed design is effective in the field of engineering.Based on the description of Eq. (3), it is obvious to get the derivative of V_1 with respect to time as:

$$\begin{aligned}\dot{V}_1 &= (z_e - \delta_\Delta)\dot{z}_e \\ &= (z_e - \delta_\Delta)(x_e\dot{x}_e + y_e\dot{y}_e)/z_e \\ &= \frac{z_e - \delta_\Delta}{z_e}(z_e\cos\psi_d(\dot{x}_d - u\cos\psi + v\sin\psi) + z_e\sin\psi_d(\dot{y}_d \\ &\quad - u\sin\psi - v\cos\psi)) \\ &= (z_e - \delta_\Delta)(\dot{x}_d\cos\psi_d + \dot{y}_d\sin\psi_d - u\cos\psi_e - v\sin\psi_e)\end{aligned} \tag{7}$$

If we define $u_e = \alpha_u - u$, and select the α_u as the form:

$$\alpha_u = (\dot{x}_d\cos\psi_d + \dot{y}_d\sin\psi_d - v\sin\psi_e + k_1(z_e - \delta_\Delta))(\cos\psi_e)^{-1} \tag{8}$$

where k_1 is a positive constant, and needs to be chosen later.

Substituting Eq. (8) into Eq. (7), Eq. (7) can be rewritten as:

$$\dot{V}_1 = -k_1(z_e - \delta_\Delta)^2 + (z_e - \delta_\Delta)u_e cos\psi_e \tag{9}$$

Consider the following LCF as:

$$V_2 = V_1 + \frac{1}{2}\psi_e^2 \tag{10}$$

Then the time derivative of V_2 can be expressed as:

$$\dot{V}_2 = \dot{V}_1 + \psi_e\dot{\psi}_e \tag{11}$$

In order to stabilize of r_e, the stabilizing function r_d is introduced. While, according to Eq. (2), the state variables are defined as follows:

$$\begin{cases}\dot{\psi}_e = \dot{\psi}_d - r \\ r_e = \alpha_r - r \\ \alpha_r = k_2\psi_e + \dot{\psi}_d\end{cases} \tag{12}$$

Thus, Eq. (11) can be rewritten as:

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \psi_e(-k_2\psi_e + r_e) \\ &= -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 + (z_e - \delta_\Delta)u_e cos\psi_e + r_e\psi_e\end{aligned} \tag{13}$$

Then, consider the LCF as follows:

$$V_3 = V_2 + \frac{1}{2}u_e^2 \tag{14}$$

Based on Eq. (1) and (8), the time derivative of u_e can be expressed as:

$$\begin{aligned}\dot{u}_e &= \dot{\alpha}_u - \dot{u} \\ &= \dot{\alpha}_u - \Theta_u^T f_u - \zeta_u\tau_u - \tau_{w_1}\end{aligned} \tag{15}$$

To this end, a practical proportional integral sliding mode control law can be designed in terms of tracking errors and also can stabilize the error variable u_e. Therefore, the sliding manifold S_1 can be designed with the form:

$$S_1 = u_e + k_3\int_0^t u_e d\tau + \int_0^t (z_e - \delta_\Delta)cos\psi_e d\tau \tag{16}$$

According to Eq. (15), its derivative becomes:

$$\begin{aligned}\dot{S}_1 &= \dot{u}_e + k_3u_e + (z_e - \delta_\Delta)\cos\psi_e \\ &= \dot{\alpha}_u - \Theta_u^T f_u - \zeta_u\tau_u - \tau_{w_1} + k_3u_e + (z_e - \delta_\Delta)\cos\psi_e\end{aligned} \tag{17}$$

Remark 3. It can be observed that the sliding manifold S_1 is made up of the trajectory tracking error z_e and the surge velocity error u_e. Therefore, when the sliding manifold converges to zero, with the design of control law, the trajectory tracking error z_e and the surge velocity error u_e all converge to zero. This novel design is effective to stabilize the state variables.

According to Eq. (17), it is obvious that $\dot{u}_e = \dot{S}_1 - k_3u_e - (z_e - \delta_\Delta)cos\psi_e$, therefore, the derivative of V_3 in (14) can be expressed as:

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + u_e\dot{u}_e \\ &= -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 + (z_e - \delta_\Delta)u_e\cos\psi_e + r_e\psi_e \\ &\quad + u_e(\dot{S}_1 - k_3u_e - (z_e - \delta_\Delta)\cos\psi_e) \\ &= -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - k_3u_e^2 + r_e\psi_e + u_e\dot{S}_1\end{aligned} \tag{18}$$

In order to stabilize S_1, the error variables of the unknown constant vectors Θ_i^T $(i = u, v, r)$ and the error variables of unstructured uncertainties τ_j $(j = w_1, w_2, w_3)$, consider the LCF as follows:

$$V_4 = V_3 + \frac{1}{2}S_1^2 + \frac{1}{2}\widetilde{\Theta}_u^T\Gamma_u^{-1}\widetilde{\Theta}_u + \frac{1}{2\gamma_1}\tilde{\tau}_{w_1max}^2 \tag{19}$$

Thus, based on Eq. (17), the derivative of V_4 with respect to time is

$$\begin{aligned}\dot{V}_4 &= \dot{V}_3 + S_1\dot{S}_1 + \widetilde{\Theta}_u^T\Gamma_u^{-1}\dot{\hat{\Theta}}_u + \tfrac{1}{\gamma_1}\tilde{\tau}_{w_1max}\dot{\hat{\tau}}_{w_1max} \\ &= -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - k_3u_e^2 + r_e\psi_e + (u_e + S_1)\dot{S}_1 \\ &\quad + \widetilde{\Theta}_u^T\Gamma_u^{-1}\dot{\hat{\Theta}}_u + \tfrac{1}{\gamma_1}\tilde{\tau}_{w_1max}\dot{\hat{\tau}}_{w_1max} \\ &= -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - k_3u_e^2 + r_e\psi_e \\ &\quad + (u_e + S_1)(\dot{\alpha}_u - \Theta_u^T f_u - \zeta_u\tau_u - \tau_{w_1} + k_3u_e + (z_e - \delta_\Delta)\cos\psi_e) \\ &\quad + \widetilde{\Theta}_u^T\Gamma_u^{-1}\dot{\hat{\Theta}}_u + \tfrac{1}{\gamma_1}\tilde{\tau}_{w_1max}\dot{\hat{\tau}}_{w_1max}\end{aligned} \tag{20}$$

In this part, the control law of standard sliding mode control can be divided into two parts. An equivalent control law τ_{nom} is used to deal with the tracking errors, and a disturbance control law τ_d to solves the problem of environment disturbances.

$$\tau_u = \tau_{nom} + \tau_d \tag{21}$$

Based on Eq. (20) and (21), we choose the τ_{nom} as follows:

$$\tau_{nom} = [k_3u_e + (z_e - \delta_\Delta)cos\psi_e - \widehat{\Theta}_u^T f_u - u_e + S_1 + \dot{\alpha}_u]\zeta_u^{-1} \tag{22}$$

Thus, Eq. (20) can be rewritten as:

$$\begin{aligned}\dot{V}_4 &= -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - k_3u_e^2 + r_e\psi_e \\ &\quad + (u_e + S_1)\widetilde{\Theta}_u^T f_u + u_e^2 - S_1^2 \\ &\quad + (u_e + S_1)(-\zeta_u\tau_d - \tau_{w1}) \\ &\quad + \widetilde{\Theta}_u^T\Gamma_u^{-1}\dot{\hat{\Theta}}_u + \frac{1}{\gamma_1}\tilde{\tau}_{w_1max}\dot{\hat{\tau}}_{w_1max} \\ &= -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 + r_e\psi_e - S_1^2 \\ &\quad + (u_e + S_1)(-\zeta_u\tau_d - \tau_{w1}) + (u_e + S_1)\widetilde{\Theta}_u^T f_u \\ &\quad + \widetilde{\Theta}_u^T\Gamma_u^{-1}\dot{\hat{\Theta}}_u + \frac{1}{\gamma_1}\tilde{\tau}_{w_1max}\dot{\hat{\tau}}_{w_1max}\end{aligned} \tag{23}$$

In Eq. (23), based on Assumption 1, we obtain that $|\tau_{w_1}| \le \tau_{w_1max}$, so the equation $(u_e + S_1)(-\zeta_u\tau_d - \tau_{w_1})$ can be transferred into the form below:

$$\begin{aligned}&(u_e + S_1)(-\zeta_u\tau_d - \tau_{w1}) \\ &\le |u_e + S_1|\tau_{w_1max} - (u_e + S_1)\zeta_u\tau_d \\ &\le \tfrac{1}{4\delta}\tau_{w_1max}|u_e + s_1|^2 + \delta\tau_{w_1max} - (u_e + S_1)\zeta_u\tau_d\end{aligned} \tag{24}$$

where δ is a small positive constant gain.

If we choose the τ_d as:

$$\tau_d = \frac{\zeta_u^{-1}}{4\delta}\hat{\tau}_{w_1max}(u_e + s_1) \tag{25}$$

The dynamical sliding mode control law τ_u has the following form:

$$\begin{aligned}\tau_u =&\ \tau_{nom} + \tau_d\\ =&\ [k_3 u_e + (z_e - \delta_\Delta)cos\psi_e - \widehat{\Theta}_u^T f_u - u_e + S_1 + \dot{\alpha}_u]\zeta_u^{-1}\\ &+ \frac{\zeta_u^{-1}}{4\delta}\hat{\tau}_{w1max}(u_e + s_1)\end{aligned} \tag{26}$$

Therefore, the derivatives of V_4 in (23) can be expressed as:

$$\begin{aligned}\dot{V}_4 \le&\ -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - S_1^2 + r_e\psi_e\\ &+ (u_e + S_1)\widetilde{\Theta}_u^T f_u + \widetilde{\Theta}_u^T \Gamma_u^{-1}\dot{\widehat{\Theta}}_u + \frac{1}{\gamma_1}\tilde{\tau}_{w1max}\dot{\hat{\tau}}_{w1max}\\ &- \frac{1}{4\delta}\tilde{\tau}_{w1max}(u_e + S_1)^2 + \delta\tau_{w1max}\end{aligned} \tag{27}$$

Based on the description above, the parameters' update laws are selected as:

$$\begin{cases}\dot{\widehat{\Theta}}_u = \Gamma_u[-f_u(u_e + S_1) - a_u(\widehat{\Theta}_u - \widehat{\Theta}_u(0))]\\ \dot{\hat{\tau}}_{w1max} = \gamma_1[\frac{1}{4\delta}(u_e + S_1)^2 - a_r(\hat{\tau}_{w1max} - \hat{\tau}_{w1max}(0))]\end{cases} \tag{28}$$

Substituting Eq. (28) into the derivatives of V_4, Eq. (27) can be rewritten as:

$$\begin{aligned}\dot{V}_4 \le&\ -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - S_1^2 + r_e\psi_e + \delta\tau_{w1max}\\ &- a_u\widetilde{\Theta}_u^T(\widehat{\Theta}_u - \widehat{\Theta}_u(0)) - a_r\tilde{\tau}_{w1max}(\hat{\tau}_{w1max} - \hat{\tau}_{w1max}(0))\\ =&\ -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - S_1^2 + r_e\psi_e + \delta\tau_{w1max}\\ &- a_u\widetilde{\Theta}_u^T(\widetilde{\Theta}_u + \Theta_u - \widehat{\Theta}_u(0))\\ &- a_r\tilde{\tau}_{w1max}(\tilde{\tau}_{w1max} + \tau_{w1max} - \hat{\tau}_{w1max}(0))\\ \le&\ -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - S_1^2\\ &+ r_e\psi_e + \delta\tau_{w1max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2\\ &+ \tfrac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0)) \parallel_2^2) - a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2\\ &+ \tfrac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2)\end{aligned} \tag{29}$$

In a similar way, consider the LCF as:

$$V_5 = V_4 + \frac{1}{2}r_e^2 \tag{30}$$

According to Eq. (1) and (12), it can be easy to obtained the derivative of r_e with the form:

$$\begin{aligned}\dot{r}_e =&\ \dot{\alpha}_r - \dot{r}\\ =&\ \dot{\alpha}_r - \Theta_r^T f_r - \zeta_r\tau_r - \tau_{w3}\end{aligned} \tag{31}$$

To this end, a practical proportional integral sliding mode control can stabilize the error variable r_e. Therefore, the sliding manifold S_2 can be designed with the form

$$S_2 = r_e + k_4\int_0^t r_e d\tau + \int_0^t \psi_e d\tau \tag{32}$$

According to Eq. (31), its derivative becomes:

$$\begin{aligned}\dot{S}_2 =&\ \dot{r}_e + k_4 r_e + \psi_e\\ =&\ \dot{\alpha}_r - \Theta_r^T f_r - \zeta_r\tau_r - \tau_{w3} + k_4 r_e + \psi_e\end{aligned} \tag{33}$$

Remark 4. It can be observed that the sliding manifold S_2 is made up of the yaw error r_e and the attitude error ψ_e. Therefore, when the sliding manifold converges to zero, with the design of control law, the trajectory tracking error z_e and the surge velocity error u_e all converge to zero.

Based on (33), the derivative of r_e in (31) can be rewritten as:

$$\dot{r}_e = \dot{S}_2 - k_4 r_e - \psi_e \tag{34}$$

Therefore, the derivative of V_5 in (30) can be expressed as:

$$\begin{aligned}\dot{V}_5 =&\ \dot{V}_4 + r_e\dot{r}_e\\ \le&\ -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - S_1^2 + r_e\psi_e + \delta\tau_{w1max}\\ &- a_u \parallel \widetilde{\Theta}_u \parallel_2^2 + \tfrac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0) \parallel_2^2)\\ &+ \tfrac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2)\\ &- a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2 + r_e(\dot{S}_2 - k_4 r_e - \psi_e)\\ =&\ -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - S_1^2 + \delta\tau_{w1max}\\ &- a_u \parallel \widetilde{\Theta}_u \parallel_2^2 + \tfrac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0) \parallel_2^2)\\ &+ \tfrac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2)\\ &+ r_e\dot{S}_2 - k_4 r_e^2 - a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2\end{aligned} \tag{35}$$

Consider the LCF as:

$$V_6 = V_5 + \frac{1}{2}S_2^2 + \frac{1}{2}\widetilde{\Theta}_r^T \Gamma_r^{-1}\widetilde{\Theta}_r + \frac{1}{2\gamma_2}\tilde{\tau}_{w3max}^2 \tag{36}$$

Therefore, based on (33) and (35), the derivative of V_6 in (36) can be rewritten as:

$$\begin{aligned}\dot{V}_6 =&\ \dot{V}_5 + S_2\dot{S}_2 + \widetilde{\Theta}_r^T \Gamma_r^{-1}\dot{\widehat{\Theta}}_r + \frac{1}{\gamma_2}\tilde{\tau}_{w3max}\dot{\hat{\tau}}_{3max}\\ &\le -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - k_4 r_e^2\\ &- S_1^2 + \delta\tau_{w1max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2 - a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2\\ &+ \frac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0) \parallel_2^2) + r_e\dot{S}_2\\ &+ \frac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2)\\ &+ S_2\dot{S}_2 + \widetilde{\Theta}_r^T \Gamma_r^{-1}\dot{\widehat{\Theta}}_r + \frac{1}{\gamma_2}\tilde{\tau}_{w3max}\dot{\hat{\tau}}_{w3max} + r_e\dot{S}_2\\ =&\ -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - k_4 r_e^2\\ &- S_1^2 + \delta\tau_{w1max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2 - a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2\\ &+ \frac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0) \parallel_2^2)\\ &+ \frac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2)\\ &+ \widetilde{\Theta}_r^T \Gamma_r^{-1}\dot{\widehat{\Theta}}_r + \frac{1}{\gamma_2}\tilde{\tau}_{w3max}\dot{\hat{\tau}}_{w3max}\\ &+ (r_e + S_2)(\dot{\alpha}_r - \Theta_r^T f_r - \zeta_r\tau_r - \tau_{w3} + k_4 r_e + \psi_e)\end{aligned} \tag{37}$$

In a similar way, the control law of standard sliding mode control can be divided into two parts. An equivalent control law τ_{rnom} to deal with the tracking errors, and a disturbance control law τ_{dr} to solve the problem of environmental disturbances.

$$\tau_r = \tau_{rnom} + \tau_{dr} \tag{38}$$

Based on (37) and (38), we choose the τ_{rnom} as follows:

$$\tau_{rnom} = [k_4 r_e + \psi_e - \widehat{\Theta}_r^T f_r + \dot{\alpha}_r - re + S_2]\zeta_r^{-1} \tag{39}$$

Substituting (39) into (37), the derivative V_6 can be rewritten:

$$\begin{aligned}\dot{V}_6 \le&\ -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - (k_4 - 1)r_e^2 - S_1^2 - S_2^2\\ &+ \delta\tau_{w1max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2 + \frac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0) \parallel_2^2)\\ &- a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2 + \frac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2)\\ &+ \widetilde{\Theta}_r^T \Gamma_r^{-1}\dot{\widehat{\Theta}}_r + \frac{1}{\gamma_2}\tilde{\tau}_{w3max}\dot{\hat{\tau}}_{rmax} + (r_e + S_2)\widetilde{\Theta}_u^T f_r\\ &+ (r_e + S_2)(-\zeta_r\tau_{dr} - \tau_{w3})\end{aligned} \tag{40}$$

In Eq. (40), based on the Assumption 1, we obtain that $|\tau_{w_3}| \le \tau_{w3max}$, then the equation $(r_e + S_2)(-\zeta_r\tau_{dr} - \tau_{w3})$ can be transferred into the form below:

$$
\begin{aligned}
&(r_e + S_2)(-\zeta_r \tau_{dr} - \tau_{w3}) \\
&\le |r_e + S_2|\tau_{w3max} - (r_e + S_2)\zeta_r \tau_{dr} \\
&\le \tfrac{1}{4\delta_2}\tau_{w3max}|r_e + S_2|^2 + \delta_2 \tau_{w3max} - (r_e + S_2)\zeta_r \tau_{dr}
\end{aligned}
\tag{41}
$$

if we choose the τ_{dr} as:

$$
\tau_{dr} = \frac{\zeta_r^{-1}}{4\delta_2}\hat{\tau}_{w3max}(r_e + S_2) \tag{42}
$$

Thus, the dynamical sliding mode control law τ_r has the following form:

$$
\begin{aligned}
\tau_r =\ & \tau_{rnom} + \tau_{dr} \\
=\ & [k_4 r_e + \psi_e - \widehat{\Theta}_r^T f_r + \dot{\alpha}_r - re + S_2]\zeta_r^{-1} \\
+\ & \tfrac{\zeta_r^{-1}}{4\delta_2}\hat{\tau}_{w3max}(r_e + S_2)
\end{aligned}
\tag{43}
$$

Substituting Eq. (43) into Eq. (40), the derivative V_6 can be rewritten:

$$
\begin{aligned}
\dot{V}_6 \le\ & -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - (k_4 - 1)r_e^2 - S_1^2 - S_2^2 \\
+\ & \delta\tau_{w1max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2 + \tfrac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0)\parallel_2^2) \\
-\ & a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2 + \tfrac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2) \\
+\ & \widetilde{\Theta}_r^T \Gamma_r^{-1}\dot{\widehat{\Theta}}_r + \tfrac{1}{\gamma_2}\tilde{\tau}_{w3max}\dot{\hat{\tau}}_{w3max} + (r_e + S_2)\widetilde{\Theta}_r^T f_r \\
-\ & \tfrac{1}{4\delta_2}\tilde{\tau}_{w3max}(r_e + S_2)^2 + \delta_2\tau_{w3max}
\end{aligned}
\tag{44}
$$

Therefore, the parameters' update laws are selected as:

$$
\begin{cases}
\dot{\widehat{\Theta}}_r = \Gamma_r[-f_r(r_e + S_2) - b_u(\widehat{\Theta}_r - \widehat{\Theta}_r(0))] \\
\dot{\hat{\tau}}_{w3max} = \gamma_2[\tfrac{1}{4\delta_2}(r_e + S_2)^2 - b_r(\hat{\tau}_{w3max} - \hat{\tau}_{w3max}(0))]
\end{cases}
\tag{45}
$$

Substituting Eq. (45) into Eq. (44), the derivative V_6 can be rewritten as

$$
\begin{aligned}
\dot{V}_6 \le\ & -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - (k_4 - 1)r_e^2 - S_1^2 - S_2^2 \\
& + \delta\tau_{w1max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2 + \frac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0)\parallel_2^2) \\
& - a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2 + \frac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2) \\
& + (r_e + S_2)\widetilde{\Theta}_r^T f_r - \frac{1}{4\delta_2}\tilde{\tau}_{w3max}(r_e + S_2)^2 + \delta_2\tau_{w3max} \\
& - \widetilde{\Theta}_r^T f_r(r_e + S_2) - b_u\widetilde{\Theta}_r^T(\widetilde{\Theta}_r + \Theta_r - \widehat{\Theta}_r(0)) \\
& + \frac{1}{4\delta_2}\tilde{\tau}_{w3max}(r_e + S_2)^2 - b_r\tilde{\tau}_{w3max}(\tilde{\tau}_{w3max} + \tau_{w3max} - \hat{\tau}_{w3max}(0)) \\
=\ & -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - (k_4 - 1)r_e^2 - S_1^2 - S_2^2 \\
& + \delta\tau_{w1max} + \delta_2\tau_{w3max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2 + \frac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0)\parallel_2^2) \\
& - a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2 + \frac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2) \\
& - b_u\widetilde{\Theta}_r^T(\widetilde{\Theta}_r + \Theta_r - \widehat{\Theta}_r(0)) - b_r\tilde{\tau}_{w3max}(\tilde{\tau}_{w3max} + \tau_{w3max} - \hat{\tau}_{w3max}(0)) \\
\le\ & -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - (k_4 - 1)r_e^2 - S_1^2 - S_2^2 \\
& + \delta\tau_{w1max} + \delta_2\tau_{w3max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2 + \frac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0)\parallel_2^2) \\
& - a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2 + \frac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2) \\
& - b_u \parallel \widetilde{\Theta}_r \parallel_2^2 - b_r \parallel \tilde{\tau}_{w3max} \parallel_2^2 + \frac{1}{2}b_u(\parallel \widetilde{\Theta}_r \parallel_2^2 + \parallel \Theta_r - \widehat{\Theta}_r(0)\parallel_2^2) \\
& + \frac{1}{2}b_r(\parallel \tilde{\tau}_{w3max} \parallel_2^2 + (\tau_{w3max} - \hat{\tau}_{w3max}(0))^2)
\end{aligned}
\tag{46}
$$

Theorem 1. *Consider the uncertain system (1) with Assumptions (1) – (2) and the control laws (26), (43), and the parameters updating laws (28), (45), therefore, all the variables are guaranteed to be uniformly ultimately bounded stable in the closed-loop system.*

Proof. Consider the LCF of the closed-loop system

$$
\begin{aligned}
V = V_6 =\ & \frac{1}{2}(z_e - \delta_\Delta)^2 + \frac{1}{2}\psi_e^2 + \frac{1}{2}u_e^2 + \frac{1}{2}S_1^2 \\
& + \frac{1}{2}\widetilde{\Theta}_u^T \Gamma^{-1}\widetilde{\Theta}_u + \frac{1}{2\gamma_1}\tilde{\tau}_{w1max}^2 + \frac{1}{2}r_e^2 \\
& + \frac{1}{2}S_2^2 + \frac{1}{2}\widetilde{\Theta}_r^T \Gamma_r^{-1}\widetilde{\Theta}_r + \frac{1}{2\gamma_2}\tilde{\tau}_{w3max}^2
\end{aligned}
\tag{47}
$$

Based on Eq. (46), the derivative of V with respect to time is

$$
\begin{aligned}
\dot{V} \le\ & -k_1(z_e - \delta_\Delta)^2 - k_2\psi_e^2 - (k_3 - 1)u_e^2 - (k_4 - 1)r_e^2 - S_1^2 - S_2^2 \\
& + \delta\tau_{w1max} + \delta_2\tau_{w3max} - a_u \parallel \widetilde{\Theta}_u \parallel_2^2 + \frac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0)\parallel_2^2) \\
& - a_r \parallel \tilde{\tau}_{w1max} \parallel_2^2 + \frac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2) \\
& - b_u \parallel \widetilde{\Theta}_r \parallel_2^2 - b_r \parallel \tilde{\tau}_{w3max} \parallel_2^2 + \frac{1}{2}b_u(\parallel \widetilde{\Theta}_r \parallel_2^2 + \parallel \Theta_r - \widehat{\Theta}_r(0)\parallel_2^2) \\
& + \frac{1}{2}b_r(\parallel \tilde{\tau}_{w3max} \parallel_2^2 + (\tau_{w3max} - \hat{\tau}_{w3max}(0))^2)
\end{aligned}
\tag{48}
$$

Therefore, it is obvious that the derivative of V with respect to time can be rewritten as:

$$
\dot{V} \le -\lambda V + \varrho \tag{49}
$$

where λ and ϱ are positive constants satisfying

$$
\begin{cases}
\lambda = & \min\{k_1, k_2, (k_3 - 1), 1, a_u, a_r\gamma_1, (k_4 - 1), 1, b_u, b_r\gamma_2\} \\
\varrho = & \delta\tau_{w1max} + \delta_2\tau_{w3max} + \frac{1}{2}a_u(\parallel \widetilde{\Theta}_u \parallel_2^2 + \parallel \Theta_u - \widehat{\Theta}_u(0)\parallel_2^2) \\
& + \frac{1}{2}a_r(\parallel \tilde{\tau}_{w1max} \parallel_2^2 + (\tau_{w1max} - \hat{\tau}_{w1max}(0))^2) \\
& + \frac{1}{2}b_u(\parallel \widetilde{\Theta}_r \parallel_2^2 + \parallel \Theta_r - \widehat{\Theta}_r(0)\parallel_2^2) \\
& + \frac{1}{2}b_r(\parallel \tilde{\tau}_{w3max} \parallel_2^2 + (\tau_{w3max} - \hat{\tau}_{w3max}(0))^2) \\
\ge & 0
\end{cases}
\tag{50}
$$

□

4. Analysis the passive-boundedness of sway motion

In this part, we will discuss the boundedness of the velocity of sway velocity. As we known, the dynamical equations of the sway have been proposed in existing work (Zhang et al., 2015; Li et al., 2008). In this paper, based on our previous work (Zhang et al., 2015), we will discuss the boundness of the sway velocity with a similar procedure. Nevertheless, this paper and Zhang et al. (2015) had different background. In this paper, we focus on the system uncertainty and unknown parameters in the ship motion control, while Zhang et al. (2015) only considered known parameters.

Consider the dynamical equation of sway in (1). $\dot{v}$ can be rewritten as follows:

$$
\dot{v} = \Theta_v^T f_v(\dot{\eta}, \eta) + \tau_{w2} \tag{51}
$$

$\Theta_v \in \Re^{n_v}$ is an unknown constant vector with a known dimension n_v. $f_v(\dot{\eta}, \eta) \in \Re^{n_v}$ is a known smooth function vectors. τ_{w2} is the environmental disturbance, which has effects on sway.

Considering the LCF as:

$$
V_v = \frac{1}{2}v^2 \tag{52}
$$

According to Eq. (51), differentiate Eq. (52) as follows:

$$
\dot{V}_v = (\Theta_v^T f_v(\dot{\eta}, \eta) + \tau_{w2})v \tag{53}
$$

From the references (Fossen, 1994; Zhang et al., 2015; Zhang and Zhang, 2014), we know that $\Theta_v^T f_v(\dot{\eta}, \eta) = -\frac{m_{11}}{m_{22}}ur - \frac{d_{22}}{m_{22}}v$, where the inertia and damping parameters m_{11}, m_{22} and d_{22} are all positive constants. According to the kinematic and dynamical equations of the underactuated surface ship, the environment disturbance τ_{w_2} in Eq. (1) can be defined as $\frac{d_{w2}}{m_{22}}$.

Thus, Eq. (53) can be written as follows (Zhang et al., 2015).

$$\begin{aligned}\dot{V}_v &= -\frac{m_{11}}{m_{22}}urv - \frac{d_{22}}{m_{22}}v^2 + \frac{d_{w2}}{m_{22}}v \\ &= -\frac{d_{22}}{m_{22}}v^2 + \left(\frac{d_{w2}}{m_{22}} - \frac{m_{11}}{m_{22}}ur\right)v \qquad (54)\end{aligned}$$

Based on the design of the control system, the variables u and r are all uniformly ultimate bounded, and according to the Assumption 1, the environmental disturbances in the sway satisfy $|\tau_{w_2}| = |\frac{d_{w2}}{m_{22}}| \le \tau_{w2max}$. It is easy to get the upper bound $\xi \ge |d_{w_2} - m_{11}ur|$, where ξ is a positive constant.

Thus,

$$\left(\frac{d_{w2}}{m_{22}} - \frac{m_{11}}{m_{22}}ur\right)v \le \frac{\xi v}{m_{22}} \le \frac{v^2}{4m_{22}} + \frac{\xi^2}{m_{22}} \qquad (55)$$

Therefore, Eq. (54) can be written as

$$\begin{aligned}\dot{V}_v &= -\frac{m_{11}}{m_{22}}urv - \frac{d_{22}}{m_{22}}v^2 + \frac{d_{w2}}{m_{22}}v \\ &\le -\left(\frac{d_{22}}{m_{22}} - \frac{1}{4m_{22}}\right)v^2 + \frac{\xi^2}{m_{22}} \\ &= -2\left(\frac{d_{22}}{m_{22}} - \frac{1}{4m_{22}}\right)V_v + \frac{\xi^2}{m_{22}} \qquad (56)\end{aligned}$$

According to Eq. (56), if $|v| \ge \frac{|d_{w2} - m_{11}ur|}{(d_{22} - 0.25)^{\frac{1}{2}}}$, $\dot{V}_v \le 0$, therefore, v satisfies the passive-boundedness property, and when $t \to \infty$, the v can be guaranteed uniformly ultimately bounded.

Remark 5. Based on Assumption 1, we haveanalyze the passive-boundedness of sway motion. According to (1), $\Theta_v \in \Re^{n_v}$ is a unknown constant vector with known dimensions n_v, but in the ship motion control system, $\Theta_v^T f_v(\dot{\eta}, \eta) = -\frac{m_{11}}{m_{22}}ur - \frac{d_{22}}{m_{22}}v$, where the inertia and damping parameters m_{11}, m_{22} and d_{22} are all positive constants. Therefore, Based on this property, we have expanded in our previous work (Zhang et al., 2015) to analysis the passive-boundedness of sway motion.

5. Numerical simulations

In this Section, the experiments of the control system are executed in MATLAB R2016a on the computer. Some numerical simulations are included to demonstrate the efficiency and effectiveness of the proposed scheme. Consider an underactuated surface vessel model with the model parameters (Zhang et al., 2015; Li et al., 2008; Do et al., 2004). The physical properties of the underactuated surface ship is 38 m length, and the mass is 118×10^3 kg, comparative example with the reference (Li et al., 2008), and following the design parameters in Li et al. (2008). The parameters of the ship are $m_{11} = 1.2 \times 10^5$ kg, $m_{22} = 1.779 \times 10^5$ kg, $m_{33} = 6.36 \times 10^7$ kg, $d_{11} = 2.15 \times 10^4$ kg/s, $d_{22} = 1.47 \times 10^5$ kg/s, and $d_{33} = 8.02 \times 10^6$ kg/s. The environmental disturbances in the simulations are choose as: $\tau_{w_1} = 1.1 \times 10^5[1 + 0.35sin(0.2t) + 0.15cos(0.5t)]$, $\tau_{w_2} = 2.6 \times 10^5[1 + 0.3cos(0.4t) + 0.2sin(0.1t)]$ and $\tau_{w_3} = 9.5 \times 10^7[1 + 0.3cos(0.3t) + 0.1sin(0.5t)]$ (Zhang et al., 2015). The desired reference trajectories are generated by the virtual ship (5), and we assume that the value of $u_d = 6$ m/s, and the value of r_d can be divided into three scenarios, which are shown below (Zhang et al., 2015):

$$r_d = \begin{cases} exp(0.005t/300) \quad \text{rad/s}, & t \le 30\text{s} \\ 0 \quad \text{rad/s}, & 30\text{s} < t \le 70\text{s} \\ 0.05 \quad \text{rad/s}, & t > 70\text{s} \end{cases} \qquad (57)$$

The initial conditions of the closed-loop system are similar to the one used in Li et al. (2008), which is defined as follows:

$$\begin{aligned}&[x(0), y(0), \psi(0), u(0), v(0), r(0)] \\ &\quad = [-80, 20, 0, 0, 0, 0] \\ &\hat{\Theta}_u(0) = 0.7\Theta_u, \quad \hat{\Theta}_r(0) = 0.7\Theta_r \\ &\hat{\tau}_{w1max}(0) = 0.7\tau_{w1max}, \quad \tau_{w1max} = 2 \\ &\hat{\tau}_{w3max}(0) = 0.7\tau_{w3max}, \quad \tau_{w3max} = 3 \qquad (58)\end{aligned}$$

and the design parameters are considered as:

$$\begin{aligned}&k_1 = 0.001, \quad k_2 = 15, \quad k_3 = 1.1, \quad k_4 = 1.1 \\ &\Gamma_u = 3 \times 10^{-6}I, \quad \Gamma_r = 5 \times 10^{-2}, \quad \gamma_1 = \gamma_2 = 1 \\ &a_u = 0.2, \quad a_r = 0.05, \quad b_u = 2 \times 10^5, \quad b_r = 1 \qquad (59)\end{aligned}$$

The simulation results are shown in Figs. 2–4. Fig. 2 shows the path-following trajectory in the comparative reference (Li et al., 2008) and the proposed algorithm, and we observe that the underactuated surface ship can follow the reference trajectories accurately and smoothly under the nonzero-mean time-varying disturbances and the systematical uncertainty. The desired reference paths consist of straight lines and circles can make the simulation experiment more suitable with the practice requirements. The desired reference trajectories can represent somewhat realistic performance in the problem of trajectory tracking or path following. Fig. 3 shows the tracking error between the actual and the desired vehicle, which is defined as $e = \sqrt{(x_e^2 + y_e^2)}$, it can be seen that the tracking error e can quickly converges to the origin in presence of the nonzero-mean time-varying disturbances. Furthermore it can be also observed that the tracking error in sway direction ψ_e converges to the equilibrium, and it reduces

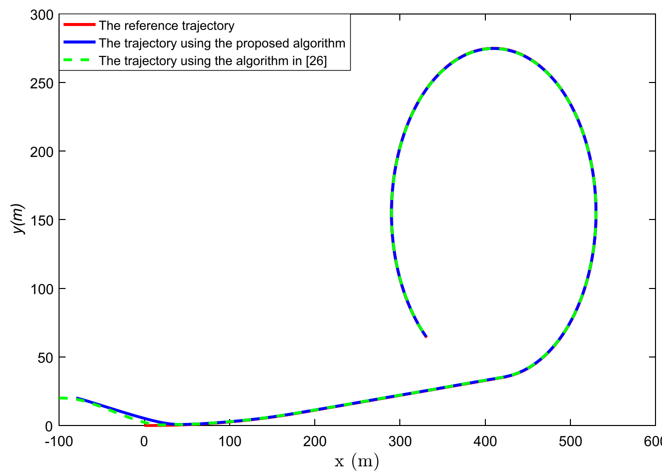

Fig. 2. The trajectory tracking path with perturbations effect.

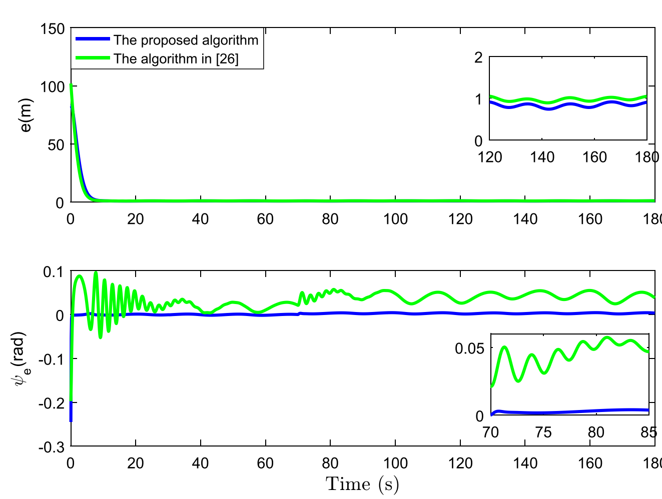

Fig. 3. Tracking error and attitude error with perturbations effect.

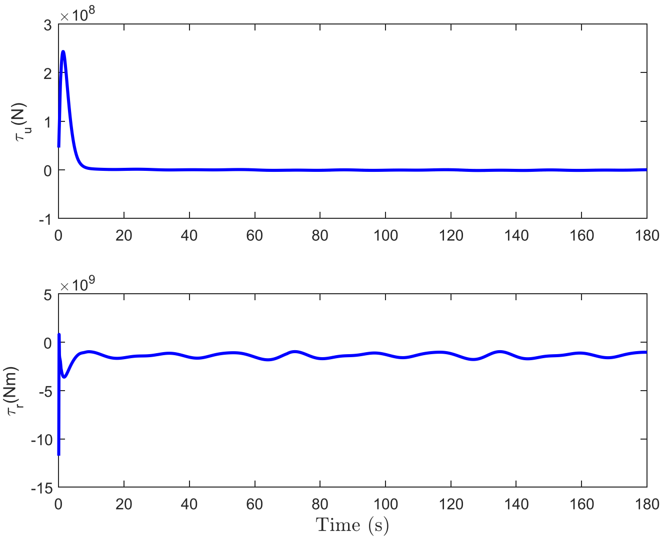

Fig. 4. Control inputs with perturbations effect.

the chattering of ψ_e than Li et al. (2008). The control inputs are shown in Fig. 4, simulation results show that the thruster outputs are smooth, it can reduce the consumption of thrust, and also reduce the damage of the propeller thrust through the continuous scheme in special construction of the sliding manifolds. All of the above simulation results show that the effective of the proposed method are sufficiently demonstrated, especially in the practice of marine engineering.

6. Conclusion

In this paper, a practical proportional sliding mode controller is designed to solve the problem of trajectory tracking of underactuated surface vessels. By combining the adaptive continuous sliding mode control with backsteeping technique, the method can effectively solve the problem of system chattering and trajectory tracking. In the design of control scheme, in order to solve the unknown disturbances in the ship motion control, the method of estimating the upper bound of unknown disturbances is explored. In the design of controller, the sliding mode control is used to enhance the robustness of the control system to the systematical uncertainties, and the special construction of the continuous sliding manifold can reduce the chattering of the system. In order to reduce the complexity of the feedback law, the output of the controller is divided into two parts, which includes an equivalent control part to solve the systematical uncertainty and a disturbance control part to solve the unknown disturbances. The global stabilization of the overall system is discussed based on the Lyapunov stability theory. In comparison with the controller design in Li et al. (2008), the control algorithm can not only reduce the chattering of the error variable of ψ_e, but also make the ψ_e have a better performance and a small error, thus being more effective to be applied in the practical ship motion control.

Acknowledgment

This paper is partly supported by the National Science Foundation of China (61473183, 61521063, U1509211), National Postdoctoral innovative Talent Program (BX201600103), China Postdoctoral Science Foundation (2016M601600).

References

Chang, X.H., Park, J.H., Zhou, J., 2015. Robust static output feedback H control design for linear systems with polytopic uncertainties. Syst. Control Lett. 85, 23–32.
Cheng, J., Zhu, H., Zhong, S., et al., 2015. Finite-time H control for a class of discrete-time Markovian jump systems with partly unknown time-varying transition probabilities subject to average dwell time switching. Int. J. Syst. Sci. 46 (6), 1080–1093.
Do, K.D., Jiang, Z.P., Pan, J., 2004. Robust adaptive path following of underactuated ships. Automatica 40 (6), 929–944.
Dong, W., Guo, Y., 2005. Global time-varying stabilization of underactuated surface vessel. IEEE Trans. Autom. Control 50 (6), 859–864.
Du, H., Shi, P., 2016. A new robust adaptive control method for modified function projective synchronization with unknown bounded parametric uncertainties and external disturbances. Nonlinear Dyn., 1–9.
Fossen, T.I., 1994. Guidance and Control of Ocean Vehicles. John Wiley and Sons Inc.
Fossen, T.I. 2002. Marine control systems. Trondheim, Norway:Marine Cybernetics.
Ioannou, P.A., Sun, J., 2012. Robust adaptive control. Cour. Corp..
Jiang, Z.P., 2002. Global tracking control of underactuated ships by Lyapunov's direct method. Automatica 38 (2), 301–309.
Lavretsky, E., Wise, K.A., 2013. Robust Adaptive Control[M]//Robust and Adaptive Control. Springer, London, 317–353.
Li, J.H., Lee, P.M., Jun, B.H., et al., 2008. Point-to-point navigation of underactuated ships. Automatica 44 (12), 3201–3205.
Li, Y., Wei, C., Wu, Q., et al., 2015. Study of 3 dimension trajectory tracking of underactuated autonomous underwater vehicle. Ocean Eng. 105, 270–274.
Li, Z., Sun, J., Oh, S., 2009. Design, analysis and experimental validation of a robust nonlinear path following controller for marine surface vessels. Automatica 45 (7), 1649–1658.
Liu, Y.J., Gao, Y., Tong, S., et al., 2016. Fuzzy approximation-based adaptive backstepping optimal control for a class of nonlinear discrete-time systems with dead-zone. IEEE Trans. Fuzzy Syst. 24 (1), 16–28.
Pettersen, K.Y., Egeland, O., 1997. Exponential stabilization of an underactuated surface vessel. Model. Identif. Control 18 (3), 239.
Reyhanoglu, M., 1997. Exponential stabilization of an underactuated autonomous surface vessel. Automatica 33 (12), 2249–2254.
Shtessel, Y., Edwards, C., Fridman, L., Levant, A., 2014. Sliding Mode Control and Observation. Springer, New York.
Tannuri, E.A., Agostinho, A.C., Morishita, H.M., Moratelli, L., Jr, 2010. Dynamic positioning systems: An experimental analysis of sliding mode control. Control Eng. Pract. 18 (10), 1121–1132.
Wang, T., Zhang, Y., Qiu, J., et al., 2015. Adaptive fuzzy backstepping control for a class of nonlinear systems with sampled and delayed measurements. IEEE Trans. Fuzzy Syst. 23 (2), 302–312.
Wang, Y.L., Han, Q.L., 2016a. *Network − Based* heading control and rudder oscillation reduction for unmanned surface vehicles. IEEE Trans. Control Syst. Technol..
Wang, Y.L., Han, Q.L., 2016b. *Network − based* fault detection filter and controller coordinated design for unmanned surface vehicles in network environments. IEEE Trans. Ind. Inform. 12 (5), 1753–1765.
Wang, Y.L., Xiong, J., 2015. Fault detection filter design for networked control systems with *time − varying* sampling periods and packet dropouts. Int. J. Adapt. Control Signal Process..
Xu, J., Wang, M., Qiao, L., 2015. Dynamical sliding mode control for the trajectory tracking of underactuated unmanned underwater vehicles. Ocean Eng. 105, 54–63.
Yi, B., Qiao, L., Zhang, W., 2016. *Two − time* scale path following of underactuated marine surface vessels: Design and stability analysis using singular perturbation methods. Ocean Eng. 124, 287–297.
Zhang, B.L., Ma, L., Han, Q.L., 2013. Sliding mode H control for offshore steel jacket platforms subject to nonlinear self-excited wave force and external disturbance. Nonlinear Anal.: Real. World Appl. 14 (1), 163–178.
Zhang, B.L., Han, Q.L., Zhang, X.M., et al., 2014. Sliding mode control with mixed current and delayed states for offshore steel jacket platforms. IEEE Trans. Control Syst. Technol. 22 (5), 1769–1783.
Zhang, G., Zhang, X., 2014. Concise robust adaptive path-following control of underactuated ships using DSC and MLP. IEEE J. Ocean. Eng. 39 (4), 685–694.
Zhang, G., Zhang, X., 2015. A novel DVS guidance principle and robust adaptive path-following control for underactuated ships using low frequency gain-learning. ISA Trans. 56, 75–85.
Zhang, G., Zhang, X., Zheng, Y., 2015. Adaptive neural path-following control for underactuated ships in fields of marine practice. Ocean Eng. 104, 558–567.
Zhang, M., Chu, Z., 2012. Adaptive sliding mode control based on local recurrent neural networks for underwater robot. Ocean Eng. 45, 56–62.
Zou, L., Wang, Z., Gao, H., 2016. Observer-based H control of networked systems with stochastic communication protocol: The finite-horizon case. Automatica 63, 366–373.