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On the calculation of the righting lever curve for a damaged ship

Pekka Ruponen *, Teemu Manderbacka, Daniel Lindroth

NAPA, Finland

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ABSTRACT

The current damage stability criteria for ships are mainly based on the characteristics of the righting lever curve. The related calculations for different intermediate stages during the flooding process, and for the final equilibrium condition, are generally considered trivial. However, with the increased computing capacity the regulations are developing towards a more realistic assessment of the intermediate stages of flooding. Most notably, time-domain flooding simulation has become a viable option. Consequently, the practices and assumptions related to the calculation of the righting lever curve for a damaged ship need to be addressed. This paper presents these challenges from different perspectives, and reviews available numerical methods for assessment of damage stability. Sample calculation results with different methods are presented for various damage scenarios, and the results are thoroughly analyzed and discussed. Finally, some recommendations on using the different methods are given.

1. Background

Safety of life at sea has had an increasing priority in the maritime industry ever since the catastrophic RMS Titanic accident in 1912, and the development of the regulations has been mainly accident driven. In this publication, we are focusing on practical methods of assessing the safety of a ship after the hull has been breached, i.e. the residual or damage stability. The number of passengers in modern cruise vessels is of thousands, Levander (2011), and thus the society wants to ensure the safety of people in the case of a flooding accident. Regulatory or statutory requirements for the computational methods of assessment of the damage stability need to be clear and concise for the fair comparison of alternative designs. Consequently, the numerical methods for damage stability analyses are of special interest.

The righting lever curve, or simply stability curve, for an intact ship was introduced in the pioneering work of Atwood and de Clairbois (1798). Yet, the first criteria for intact ships were developed much later by Rahola (1939). Since then, the righting lever curve, and its characteristics, have been applied to determine the safety level of ships in various regulations. Initially this concerned only intact stability, but later the righting lever curve has been adopted also for damage stability regulations. A detailed overview of this development is presented in Francescutto and Papanikolaou (2011).

The first Safety of Life at Sea (SOLAS) regulation in 1914 concerned only the subdivision and ensuring sufficient reserve buoyancy after a breach in the hull, but the later upgrades of SOLAS in 1948 and 1960 introduced requirements for a minimum metacentric height (GM) and maximum heel angle in damaged conditions. Eventually the SOLAS 1990 introduced criteria for various properties of the righting lever curve. In the current SOLAS regulations, the s-factor that represents the survivability level is calculated from the properties of the righting lever (GZ) curve. In addition, alternative methods for measuring the survivability have been presented recently, e.g. within the GOALDS project, Papanikolaou et al. (2013) and by Cichowicz et al. (2016). Even these new approaches are based on the characteristics of the GZ curve, and consequently, the calculation procedure for obtaining this curve is of special interest.

The real sequence of flooding progression can only be calculated with a time-domain simulation of progressive flooding. A review of this development has been presented in Papanikolaou (2007). Since then, time-domain flooding simulation has proven to be a useful tool also for accident analyses, Krüger (2016). With the increased computing capacity, simulation has become a viable option for regulatory damage stability calculations, especially for cross-flooding analyses, Ruponen et al. (2012), but also for a more realistic assessment of progressive flooding inside the flooded compartments, Ruponen and Lindroth (2016). Recently, an advanced approach for combining time-domain simulation results and the traditional s-factor into a Survivability Performance Index (SPI) was introduced by Dafermos and Papanikolaou (2016). In addition, simulation can be used onboard a damaged ship for a rapid assessment of

* Corresponding author.
E-mail addresses: pekka.ruponen@napa.fi (P. Ruponen), teemu.manderbacka@napa.fi (T. Manderbacka), daniel.lindroth@napa.fi (D. Lindroth).

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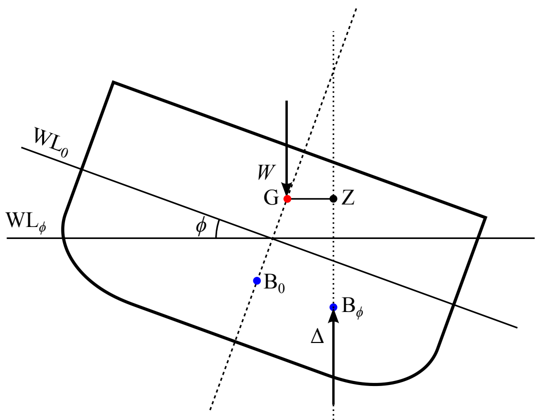

Fig. 1. Definition of the righting lever GZ when the ship is heeled to an angle ϕ.

progressive flooding and the development of stability, Ruponen et al. (2017).

Despite the fact that the recent development allows for a more realistic assessment of the flooding process, the stability criteria in the regulations still mainly rely on the characteristics of the stability curve. The calculation of this curve is generally considered trivial, but the treatment of floodwater, especially in the intermediate filling phases, leaves room for different interpretations. In this paper, the concept of the righting lever curve is revisited, with a review of alternative approaches for evaluating the progress of flooding in a damaged ship. Finally, case studies are presented with discussion and analyses of the results.

2. Calculation of the righting lever curve

The righting lever curve represents a ship's ability to withstand external heeling moments, e.g. due to wind and waves. When a ship heels to an angle ϕ, the center of buoyancy is shifted from the point B_0 to the point B_ϕ. The center of gravity G may also shift, if there are liquid loads. The lifting force of buoyancy Δ is equal to the weight of the ship W, but the directions of these forces are opposite. This pair of forces results in the righting moment, and the righting lever GZ is the lateral distance between the center of gravity and the center of buoyancy in the global coordinate system, as illustrated in Fig. 1. This is evaluated numerically by fixing the heel angle and balancing the trim angle (**free trim**) and the draft, so that the buoyancy equals the weight, and reaching an equilibrium between the trimming moments. In practice, iterative procedures are needed, but the required calculations are rapidly performed with modern computers. By repeating this procedure for a range of heel angles, the righting lever curve is obtained by fitting a smoothed curve to the set of evaluated points. For an intact ship, this procedure is trivial, but for a damaged ship with flooded compartments, the evaluation of the righting lever curve becomes more complex.

Especially for a damaged ship, it is essential that also the trim angle is balanced in the calculation of the GZ curve in order to avoid over-optimistic results, as pointed out by Pawlowski (2016). For ships the assumption of a constant heeling direction is quite realistic, but for floating offshore structures the evaluation of the GZ curve, even in intact condition, should allow for free twisting of the structure, as described in van Santen (2011).

In principle, the GZ curve for a damaged ship is evaluated with the same procedure, but the floodwater needs to be considered in the evaluation of the center of gravity and/or the center of buoyancy. In literature, two different methods for analysis of damage stability are presented, the method of **lost buoyancy** and the method of **added weight**. The basics of both approaches are well-known to naval architects, and are described in most of the distinguished text books, such as Nickum (1988), Tupper (2013) and Biran and López-Pulido (2014). For convenience, a short description of both approaches is given in the following.

In the lost buoyancy method, the flooded compartments are reduced from the buoyant hull with the permeability taken into account. The situation is illustrated in Fig. 2. The mass and the center of gravity of the ship are unchanged, unless there were liquid loads in flooded tanks that may have flown out. Thus the flooded compartments are in free communication with the sea, meaning that the floodwater can freely flow between the flooded compartments and the sea if the ship moves, e.g. due to an external heeling moment. This assumption implies that the time available for equalizing the water levels in flooded compartments is infinite, as the water levels are in hydrostatic balance with the sea. Furthermore, the method cannot account for accumulated water above the sea level, such as firefighting water or water on a ro-ro deck.

In the added weight method, the floodwater is treated as additional liquid cargo. For compartments that are connected to the sea, this method requires iterations for evaluation of the final equilibrium condition. For example, the accumulated water on the vehicle deck must be treated as an added weight since the method of lost buoyancy would result in an immediate draining of the water back to the sea if the floodwater level is above the sea level. The same applies also for firefighting water.

For the final equilibrium after flooding, both methods result in exactly the same floating position and static righting moment, but the actual GM in damaged condition and the righting lever curve are different. In principle, with the added weight method (subscript *aw*) the static righting moment at heel angle ϕ is:

$$M_{st}(\phi) = GZ_{aw}(\phi)\cdot(W + w) \tag{1}$$

where W is the total weight of the intact ship and w is the total weight of floodwater.

With the lost buoyancy method (subscript *lb*) the righting moment is:

$$M_{st}(\phi) = GZ_{lb}(\phi)\cdot W \tag{2}$$

Since the displacement is constant in this approach. Consequently, the following relation between the two calculation methods can be presented by combining the equations (1) and (2):

$$GZ_{lb} = GZ_{aw}\frac{W + w}{W} \tag{3}$$

For extensive flooding cases the difference between the methods is considerable since w is large. However, the treatment of the amount of floodwater with different heeling angles can even have a bigger impact. The lost buoyancy method limits the floodwater to the sea level in all flooded compartments, but with the added weight method such a limitation is not usually applied, and e.g. Vermeer et al. (1994) have used fixed amounts of floodwater at each intermediate phase (time step) of flooding.

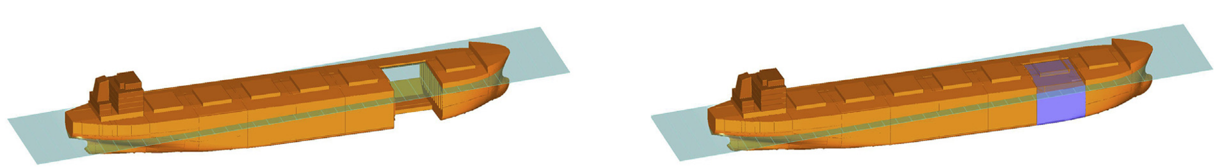

Fig. 2. 3D visualization of the lost buoyancy (left) and the added weight (right) methods.

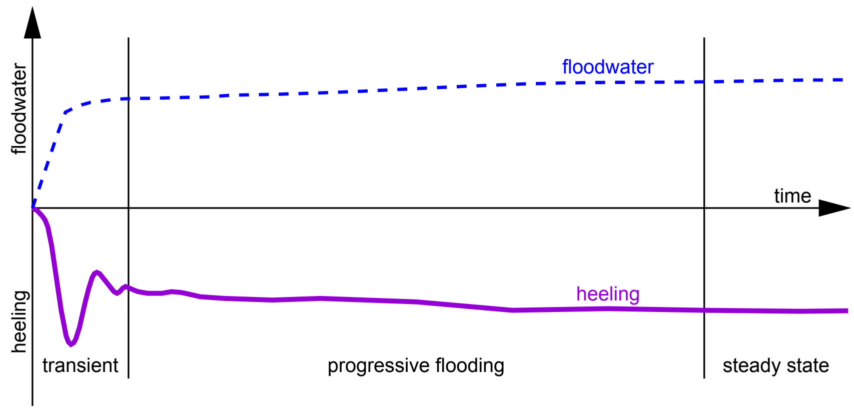

Fig. 3. Schematic representation of different stages of flooding in calm water, Ruponen (2014).

3. Time dependency

The flooding of a damaged ship is a time-dependent process, and subject to the case, the time scale can vary from a couple of minutes to even several days. This time dependency makes it difficult to provide a unique interpretation for the GZ curve during the flooding process.

The time-dependency of the flooding process has been noted already in the early research on damage stability, Welch (1916). However, in practice, the time dimension was introduced much later through the simplified method for estimation of the cross-flooding time by Solda (1961). Eventually a time-based sequential flooding analysis was presented by Sen and Konstantinidis (1987), but only recently, the increased computing capacity has enabled true time-domain flooding simulation also for ships with complex internal subdivision. During the past decades, several simulation tools have been developed, e.g. Spanos and Papanikolaou (2001), Jasionowski (2001), Santos et al. (2002), van't Veer et al. (2002, 2004), Ruponen (2007, 2014), Ypma and Turner (2010), Schreuder et al. (2011), Dankowski (2013), Lee (2015), Rodrigues and Guedes Soares (2015) and Kim et al. (2017). Recently focus has also been on the dynamics of transient flooding, Manderbacka et al. (2015a) and Acanfora and Cirillo (2017). In general, these methods use Bernoulli's equation for calculation of the water flow through the openings. Also computational fluid dynamics (CFD) has been applied to flooding analyses, e.g. Gao et al. (2011), Hashimoto et al. (2013, 2017) and Sadat-Hosseini et al. (2016), but the slow computation time makes them unsuitable for practical applications in ship design and operation, especially for larger numbers of cases.

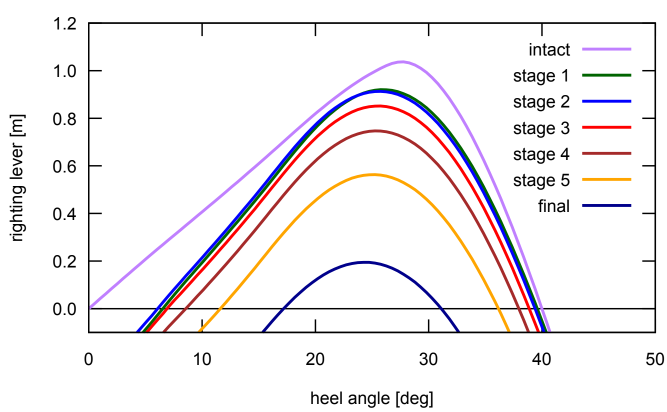

Fig. 4. Schematic presentation of GZ curves for various intermediate stages of flooding.

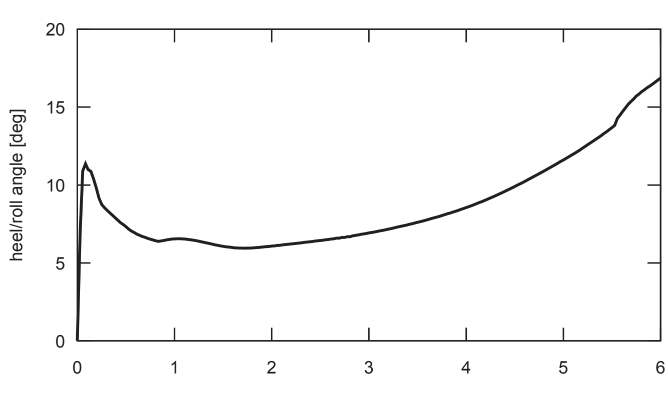

Fig. 5. Schematic presentation of flooding simulation results in calm water for the time history of the heel/roll angle.

In general, the flooding process can be divided into three separate stages with different characteristics, as presented e.g. in Ruponen (2014):

- transient flooding
- progressive flooding
- steady state

These stages are illustrated in Fig. 3. Naturally, the later stages can only occur if the ship survives the previous stage without capsizing or sinking. The transient flooding stage involves complex dynamics and fluid-structure interaction, Manderbacka et al. (2015a). Usually this stage lasts only a couple of roll cycles (about a minute), and it is followed by progressive flooding through internal openings to other rooms. This process can last from a couple of minutes to even several days, depending on the damage case. Especially the non-watertight doors inside the watertight (WT) compartments have a significant effect on this, Ruponen (2017). Eventually, if the ship does not sink or capsize, a steady equilibrium is reached. If the flooding does not take place in calm water, this is more a quasi-steady condition. This behavior is evident from results of various model tests in waves, e.g. Papanikolaou et al. (2000).

The conventional approach is to calculate the righting lever curves for certain predefined intermediate stages, based on an assumed progress of flooding. A schematic example of such results is shown in Fig. 4. Time-domain flooding simulation can be used to calculate the development of flooding more realistically. Usually the main result of such an analysis is the time history of heel/roll motion, as illustrated in Fig. 5. For the damage cases that end up in capsizing or foundering of the ship, the results will provide the time to sink. However, time-domain simulations do not provide information on the residual stability of the ship during the flooding process, unless the analysis is quasi-stationary and the GZ curve is calculated for the individual time steps.

The definition of the GZ curve during the flooding process is not obvious, and this problem has previously been discussed by Dankowski

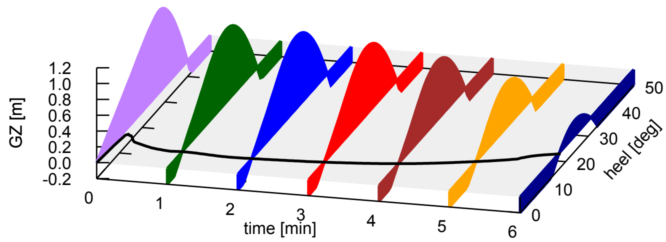

Fig. 6. Schematic presentation of the GZ curve for a flooded ship as a function of time.

(2013). The conventional approach is to consider various filling degrees for the flooded compartments in the intermediate stages. However, in order to evaluate the GZ curve along the flooding process, the time needs to be frozen, while the ship is heeled to different angles, and the righting moment lever is calculated based on the centers of buoyancy and mass. Thus during the progressive flooding, each GZ curve is associated to a frozen snapshot in time, whereas the real situation is a continuous process, Fig. 6. This assumption indicates that the heeling of the ship is done extremely fast when calculating the GZ values. Consequently, it would be reasonable to assume that there is no flow between the flooded compartments and the sea. It should be noted that Figs. 4–6 actually represent the same example results from different perspectives.

The internal structures in the flooded compartments will restrict the free flow of water. If the ship is heeled very rapidly, the volumes of water will remain practically unchanged since there is no time for water to flow to other rooms through the openings. On the other hand, if the heeling is an extremely slow process, the water levels in all flooded compartments will eventually be in hydrostatic balance with the sea, i.e. part of the lost buoyancy. These interpretations are illustrated in Fig. 7.

The new revised explanatory notes for SOLAS Chapter II-1, Reg. 7, IMO (2017), state that:

"For each phase of a flooding stage (except the final full phase), the instantaneous transverse moment of this floodwater is calculated by assuming a constant volume of water at each heeling angle. The GZ curve is calculated with a constant intact displacement at all stages of flooding. Only one free surface needs to be assumed for water in spaces flooded during the current stage."

This implies that the heeling of the ship is done extremely fast for the intermediate flooding phases and extremely slowly after the flooding process has finished. Consequently, the recommended interpretation is not consistent. Moreover, the explanatory notes do not contain any background information for the selected approaches. The present study and the calculation examples intend to clarify the differences between the alternative methods for treatment of floodwater in the GZ calculation, and their suitability for various flooding scenarios.

4. Calculation of intermediate stages of flooding

4.1. Background

A crucial part of a damage stability analysis is to define the intermediate stages of flooding that will be calculated. The Explanatory Notes for SOLAS Chapter II-1, IMO (2017), state that:

> "For each damage scenario, the damage extent and location determine the initial stage of flooding. Calculations should be performed in stages, each stage comprising of at least two intermediate filling phases in addition to the full phase per flooded space. Unrestricted spaces in way of damage should be considered as flooded immediately. Every subsequent stage involves all connected spaces being flooded simultaneously until an impermeable boundary or final equilibrium is reached."

Furthermore:

> "It is assumed that the non-watertight divisions considered in the calculations are limited to "A" class fire-rated bulkheads and decks, and do not apply to "B" class fire-rated bulkheads normally used in accommodation areas (e.g. cabins and corridors)."

The current industry practice is to calculate all possible combinations of progression through A-class fireproof boundaries, as presented in Ruponen and Lindroth (2016). The drawback with this approach is that with complex A-class arrangements the number of alternative intermediate stages can become enormous. For a large passenger ship there can be several hundreds of A-class stages in extensive damage cases, even if some simplifications are done in the modelling. The A-class structures are a challenge for designers of passenger ships, Spigno et al. (2015).

Lemoine et al. (2013) have presented an alternative sequential flooding analysis method without actual simulation. Instead, they have applied the simplified cross-flooding time analysis for all subsequent flooding stages. The main disadvantage of this approach is that it requires a lot of manual definitions, and it cannot handle simultaneous flooding to different compartments.

In the present study three alternative methods are used, conventional method (Conv.), time-domain flooding simulation with damaged compartments treated as lost buoyancy (Sim. OTS) and simulation with the breach modelled as openings (Sim. Breach). The details of these methods are presented in the following sections.

4.2. Conventional approach (Conv.)

In the conventional approach, the sequential flooding is divided into different stages and phases. A new stage includes flooding of a new room(s), and each stage is divided into a number of phases with different filling levels in the newly flooded room(s). A common free surface is applied for the set of fully flooded rooms. The lost buoyancy method is used for the final phase of each stage, and for all rooms that are flooded during the previous stages. This approach is fully in line with the recommendations in the revised Explanatory Notes for SOLAS Chapter II-1, IMO (2017).

The next intermediate phase is defined based on the difference between the sea level height and the water level height from the previous phase. This height difference is divided by the number of remaining intermediate phases for the flooding stage. If a stage with a total of n phases

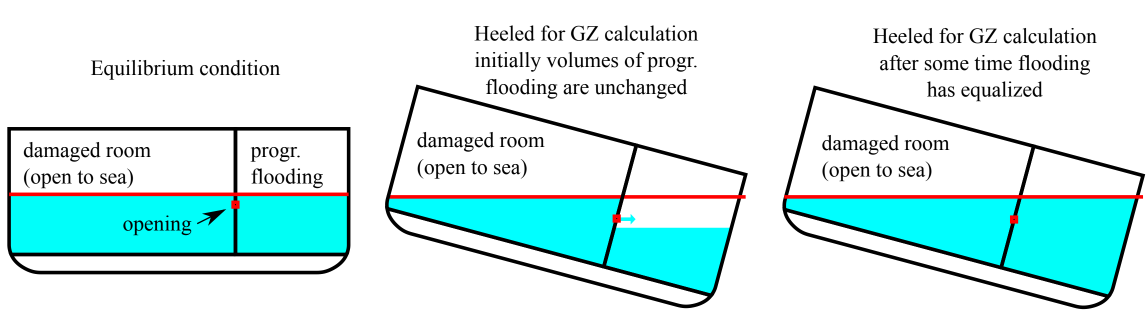

Fig. 7. Schematic presentation of the effect of an opening on the amount of floodwater in the calculation of GZ curve.

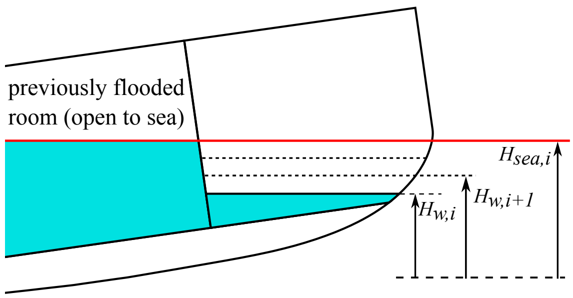

Fig. 8. Definition of the amount of floodwater for the next intermediate phase (i+1) on the basis of water level difference in the conventional approach.

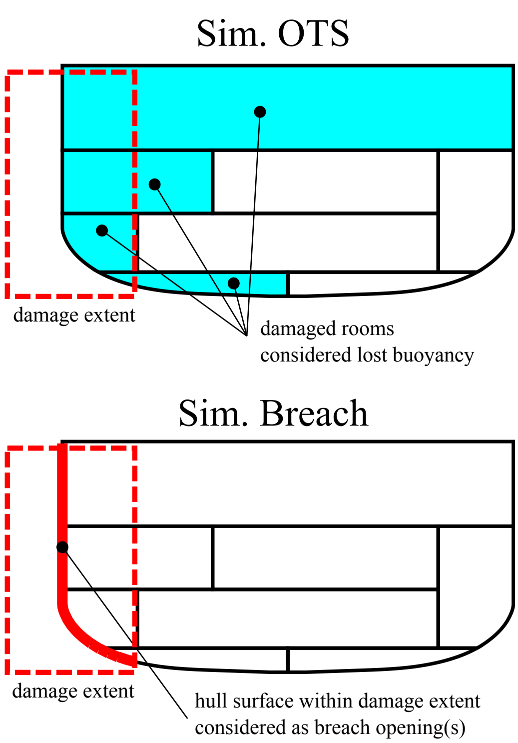

Fig. 9. Different methods for modelling the damage in flooding simulation.

is considered, the water level height in the i+1:th phase is evaluated from the following equation:

$$H_{w,i+1} = H_{w,i} + \frac{1}{n+1-i}(H_{sea,i} - H_{w,i}) \tag{4}$$

where $H_{w,i}$ is the floodwater level and $H_{sea,i}$ is the sea level at the floating position of the i:th phase. The principle idea is illustrated in Fig. 8. An alternative approach could be to apply steps with a constant volume change, but the height steps are considered to provide better coverage of the intermediate flooding conditions also for the rooms, where the free surface area changes significantly as a function of height. In addition, the height step approach is in line with the SOLAS Explanatory Notes.

Cross-flooding is considered to commence first before the A-class bulkheads collapse. The simplified method of IMO (2013) is used. This is followed by all alternative A-class stages (denoted by #n). In the final stage, all A-class bulkheads are considered collapsed and all rooms within the damaged zones are treated as lost buoyancy. The principle is described in Ruponen and Lindroth (2016).

4.3. Time-domain flooding simulation (Sim. OTS & Sim. Breach)

In the present study, the time-domain flooding simulation tool in the NAPA software is used. This method is based on the application of Bernoulli's equation and semi-empirical discharge coefficients for the openings. A detailed description of the numerical method is given in Ruponen (2007, 2014), but for convenience a short presentation is given in this section. The method has been successfully validated against both model tests, Ruponen et al. (2007), and full-scale flooding tests, Ruponen et al. (2010).

The applied method is based on implicit time integration with a pressure-correction algorithm. This has proven to be an efficient and accurate approach, especially for damage cases that involve extensive progressive flooding to several compartments.

At each time step the conservation of mass must be satisfied in each flooded room:

$$\int_{\Omega} \frac{\partial \rho}{\partial t} d\Omega = -\int_{S} \rho \mathbf{v} \cdot d\mathbf{S} \tag{5}$$

where ρ is density, **v** is the velocity vector and S is the surface that bounds the control volume Ω. The normal vector of the surface points outwards from the control volume.

The velocities in the openings are calculated by applying Bernoulli's equation for a streamline from point A that is in the middle of a flooded room to point B in the opening:

$$\int_{A}^{B} \frac{dp}{\rho} + \frac{1}{2}(u_B^2 - u_A^2) + g(h_B - h_A) + \frac{1}{2}k_L u_B^2 = 0 \tag{6}$$

where p is air pressure, u is flow velocity, g is acceleration due to gravity and h is the water height from the common reference level. All losses in the opening are represented by the non-dimensional pressure-loss coefficient k_L. Consequently, by assuming that $u_A = 0$, the flow through an opening with area dS is:

$$dQ = C_d \cdot \sqrt{2g(h_A - h_B)} \cdot dS \tag{7}$$

where the semi-empirical discharge coefficient is:

$$C_d = \frac{1}{\sqrt{1 + k_L}} \tag{8}$$

For cross-flooding ducts and pipes, the pressure-loss coefficient k_L is equal to the k-sum used in regulations, IMO (2013). For openings, such as open/collapsed doors, the industry standard discharge coefficient $C_d = 0.6$ is used. This corresponds to $k_L \approx 1.78$.

In this study, the air compression effects are ignored and the air pressure p in all rooms are assumed to be equal to the atmospheric pressure.

The calculation within a single time step is iterative and the algorithm changes the pressures in the flooded rooms until both Bernoulli's equation for each opening and the conservation of mass for each flooded room are satisfied with sufficient accuracy. The floating position of the ship is then calculated based on the volumes of floodwater in the compartments.

The leakage and collapse characteristics of closed non-watertight doors are accounted for by using the full-scale test results of the FLOODSTAND project, Jalonen et al. (2017), whereas in the conventional approach different intermediate stages are calculated by assuming the internal structures to be either practically watertight or to collapse immediately. Both approaches are described in detail by Ruponen and Lindroth (2016).

For progressive flooding through the openings, the floodwater volumes at each time step are kept constant in the calculation of the righting lever curve, thus assuming rapid heeling, also in the final stage of flooding.

Table 1
Summary of the applied calculation methods.

Method	Damage	Progressive flooding	Treatment of floodwater
Conv.	Damaged rooms open to sea (lost buoyancy)	Stages with intermediate filling phases	Final phase of each stage calculated with lost buoyancy; intermediate filling phases with a constant amount of floodwater
Sim. OTS	Damaged rooms open to sea (lost buoyancy)	Through openings based on Bernoulli's equation using time-domain simulation	Progressive flooding treated as a liquid moving mass, with a constant volume in the GZ calculation. Damaged rooms calculated with lost buoyancy
Sim. breach	Breach modelled as opening(s) based on zone extents	Through openings based on Bernoulli's equation using time-domain simulation	Floodwater in all compartments treated as a liquid moving mass, with a constant volume in the GZ calculation

4.4. Modelling of the damage in flooding simulation (Sim. OTS & Sim. Breach)

There are two alternative approaches for modelling the damage in time-domain flooding simulation, as illustrated in Fig. 9. The current SOLAS Chapter II-1 regulation, IMO (2017), assumes that the breached compartments are instantly flooded, i.e. open to sea (OTS). The same approach can also be used in simulation. The approach to flood compartments instantly before simulation commences is denoted by "Sim. OTS". Alternatively, the breach in the hull can be considered as openings, connecting the damaged compartments to the sea. Simulation with openings representing the initial breach is denoted by "Sim. Breach". This method has been widely used in flooding simulations, e.g. Dafermos and Papanikolaou (2016). In the present study, the breach extents are taken from the subdivision used for SOLAS, so that the results are comparable to the conventional calculations. However, it is noteworthy that with large breaches the dynamic motions in the transient flooding phase can be significant, Manderbacka and Ruponen (2016). Consequently, in the SOLAS Chapter II-1 calculations, the "Sim. Breach" method can result in unrealistically large breaches that will overestimate the effects of the transient flooding stage. In the present study, both approaches are used, and the results are compared and discussed.

4.5. Summary of applied calculation methods

In this study, three alternative methods are applied. The conventional method (Conv.), normally used in statutory calculations, is used as a reference. In addition, time-domain flooding simulation is used with two different approaches for modelling the damage (Sim OTS and Sim.

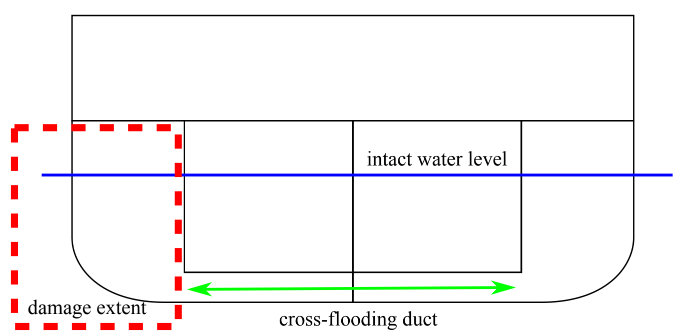

Fig. 10. Case A - cross-flooding in a large U-void.

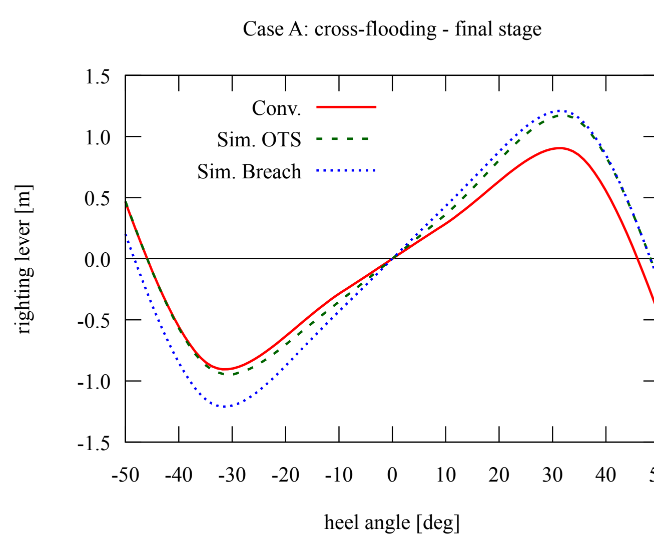

Fig. 11. Comparison of GZ curve for the final equilibrium in the cross-flooding case.

breach). A summary of the applied methods is presented in Table 1. In all methods, the righting lever is calculated using constant (intact) displacement. The applied range of heeling angles covers both sides. All calculations were performed with the NAPA software.

5. Case studies

5.1. Case A - cross-flooding

The first case study concentrates on a simple cross-flooding scenario. A large U-shaped void is damaged on the port side, and there is a cross-flooding duct in the double bottom that enables equalizing flooding to the undamaged side of the void, Fig. 10. The damage is limited both vertically and transversally, so that the upper decks remain intact, and consequently, only the U-void is flooded.

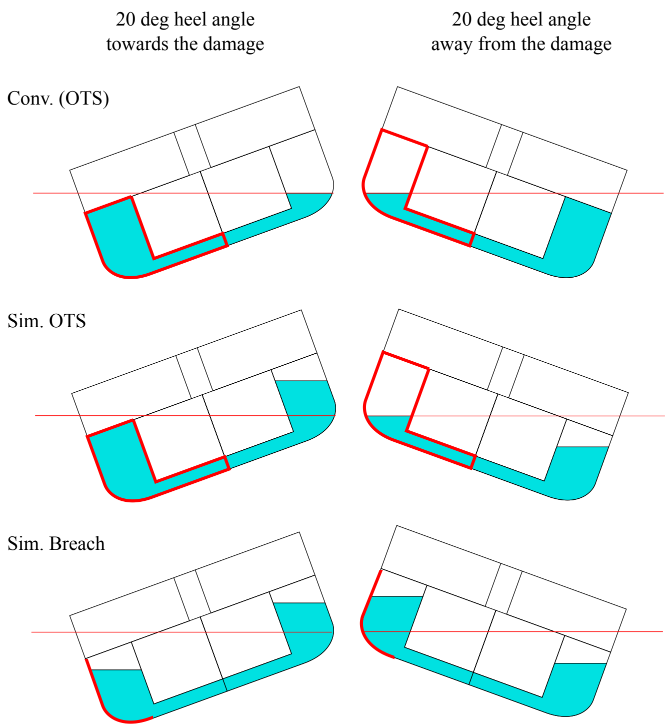

Fig. 12. Comparison of floodwater levels at 20° heel angle with different calculation methods for the final condition after cross-flooding in Case A.

The GZ curves for the final stage of flooding with different calculation methods are presented in Fig. 11. As expected, all methods give exactly the same final floating position. However, simulation with a constant amount of floodwater results in larger righting lever values. The reasons for this are illustrated in Fig. 12. When the ship is heeled towards the damage (positive angles), the additional floodwater above the sea level on the intact side of the ship increases the righting moment. On the other hand, when the ship is heeled away from the damage (negative angles), the righting lever is increased since the amount of water on the intact side is not increased. These effects are more notable when the initial breach is modelled as an opening (Sim. Breach).

5.2. Case B – longitudinal bulkheads

The second studied scenario is a large collision damage to a single zone. There is cross-flooding in the double bottom and the two upper decks are divided by longitudinal A-class bulkheads, Fig. 13. There are four hinged A-class fire doors, denoted by D1 … D4, that connect the rooms, as illustrated in Fig. 13. All these doors are considered to be closed. The leakage and collapse parameters from the FLOODSTAND project, Jalonen et al. (2017), are used.

The conventional approach to A-class structures does not recognize the doors as the routes for progressive flooding. Instead, all possible combinations of either fully watertight or collapsed bulkheads are calculated. For this damage case, it means five alternative intermediate stages, as well as the final stage with all A-class bulkheads collapsed. All the flooding stages are illustrated in Fig. 14. The so-called A-class stages are denoted with #n.

A constant time step of 5.0 s is used in the simulations. The closed fire doors on the lower deck (D1 and D2 in Fig. 13) collapse within 90 s, but the doors on the upper deck (D3 and D4) only leak since the pressure head does not reach the collapse threshold. Consequently, the time to flood is quite long (about 35 min). The development of the heeling angle from the simulation results is shown in Fig. 15, and for different stages and phases of flooding with the conventional approach in Fig. 16. The modelling of the damage, either as damaged rooms open to sea (Sim. OTS) or with the breach as an opening (Sim. Breach), affects only the initial phases of flooding, and after the first minute, the simulation results with both approaches for damage modelling are practically identical.

This case demonstrates the problem of simultaneous cross-flooding and leakage/collapse of non-watertight structures. Time-domain

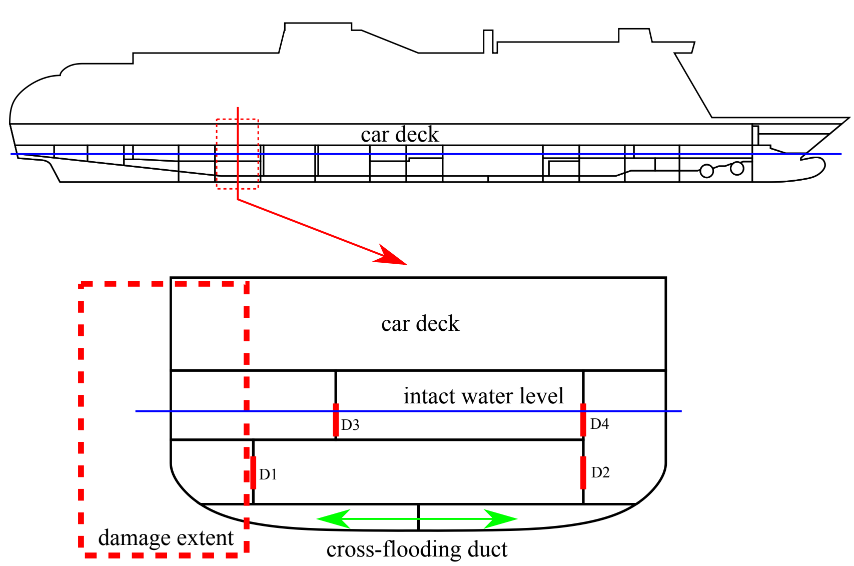

Fig. 13. Damage and openings for Case B.

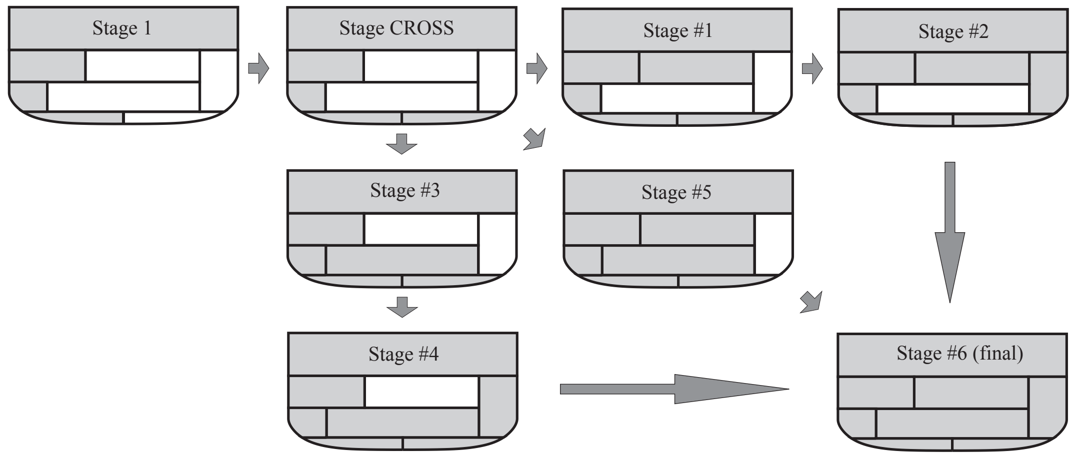

Fig. 14. Intermediate stages of flooding in Case B with the conventional approach (the shaded rooms are open to sea at the end of the stage).

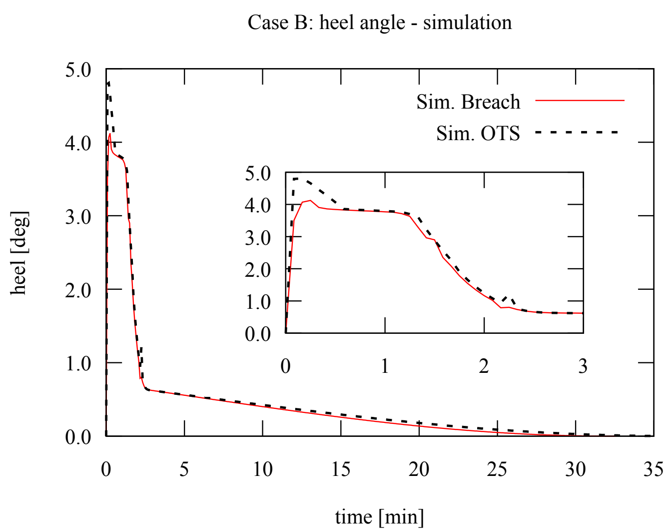

Fig. 15. Time history of heel angle from the simulation results of Case B.

simulation is evidently the only realistic approach to calculate the intermediate stages of flooding for this kind of damage scenarios. The revised Explanatory Notes to SOLAS, IMO (2017), recommends that if both passive cross-flooding devices and collapsing structures are faced at the same time, cross-flooding should be calculated first. This is reasonable since these devices are designed to allow for a fast equalizing flooding, whereas the leakage/collapse of non-watertight structures can take a much longer time, Ruponen (2017).

Righting lever curves for the different calculation methods are shown in Fig. 17 for the final equilibrium condition. Modelling the damage as a breach results in much larger righting moment levers, especially at larger heel angles. This is caused by the large damaged room (car deck) above the sea level at the final equilibrium, Fig. 18. The breach to this room is large, and consequently, free communication with the sea is a realistic approach, since the car deck would be rapidly flooded if the breach was temporary submerged due to an external heeling moment.

5.3. Case C – extensive progressive flooding

The third studied scenario is an extensive three-compartment damage to the aft part of a large passenger ship, Fig. 19. The studied ship design is a slightly modified version of the FLOODSTAND sample ship A, Kujanpää and Routi (2009). The damaged compartments contain various store areas, and there are several A-class fire rated steel bulkheads providing

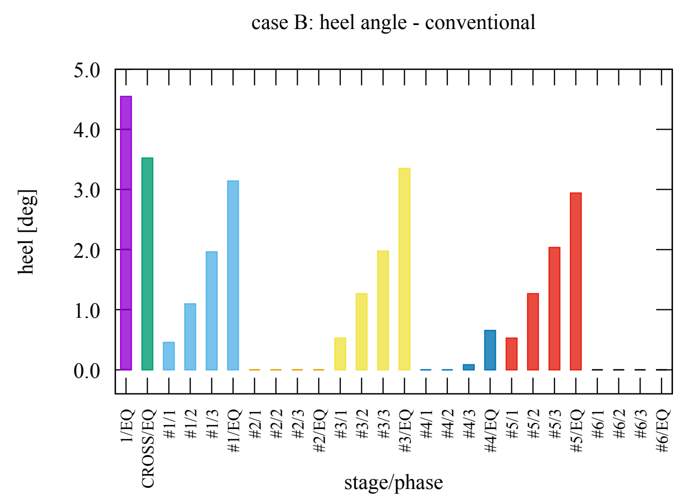

Fig. 16. Heel angle for different flooding stages and phases with the conventional method for Case B.

non-watertight subdivision inside the watertight compartments. The damage considered is one similar to the ones defined for the probabilistic damage stability calculations in SOLAS, based on the WT subdivision

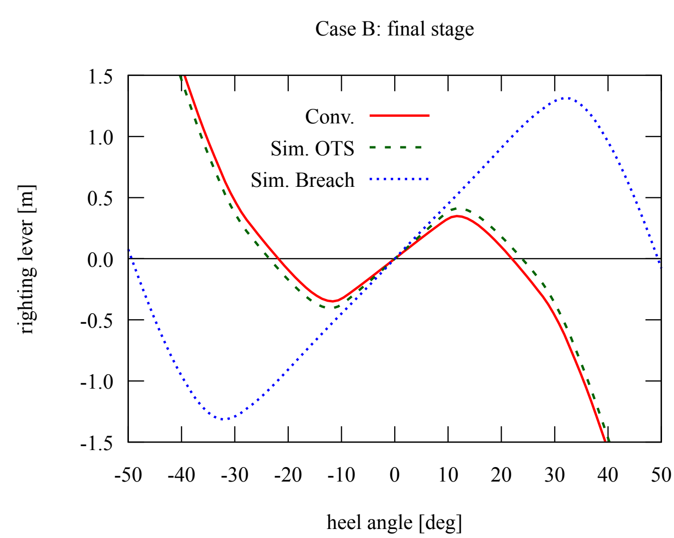

Fig. 17. GZ curves for the final stage of Case B.

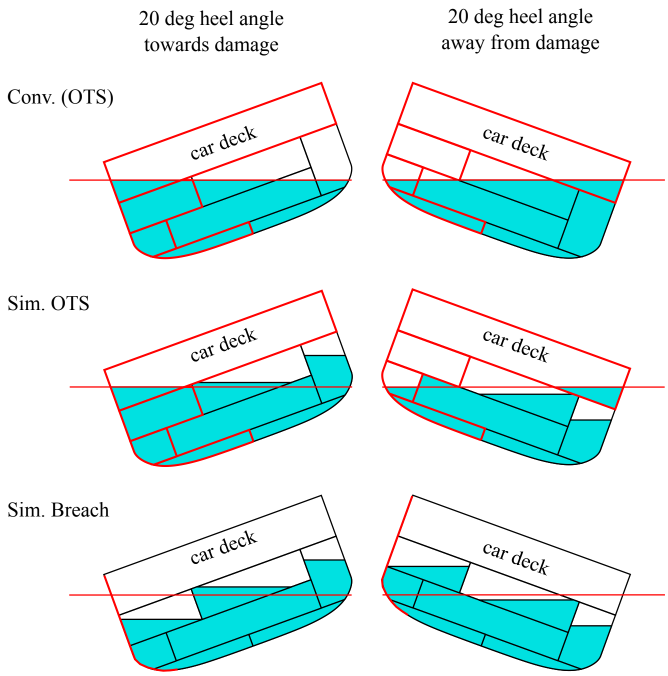

Fig. 18. Comparison of floodwater levels at 20° heel angle with different calculation methods in the final equilibrium in Case B.

limits. For this particular case, the damage length is 44 m and the penetration is 8.7 m. The damage extends vertically from the base line up to the upper limit of the buoyant hull.

The time-domain simulations were done with a constant time step of 5.0 s. Based on Ruponen and Lindroth (2016), a shorter time step would not significantly improve the accuracy in similar flooding cases. The applied leakage and collapse characteristics are based on the FLOODSTAND results, Jalonen et al. (2017). Furthermore, all doors are assumed closed.

The final stable equilibrium floating position is reached in about 2 h 40 min. The development of the heel angle from the simulation results is shown in Fig. 20 for both approaches of modelling the damage. The results are nearly identical. In addition, in both simulations the same eight non-watertight doors collapse and the remaining 13 doors are only leaking. Most of the collapsed doors (seven) are located on Deck 2, whereas most of the doors that do not collapse are on Deck 3. The doors collapse within 2 min, but due to the slow leakage, the overall time to flood is very long. A similar observation was made in a previous study, Ruponen (2017), where the status (open/closed) of the non-watertight doors was found to be a key parameter affecting the flooding time.

The conventional approach with all possible combinations of collapsing A-class structures on the other hand becomes significantly more complex compared to the simulation. The total number of stages generated exceeds 1000, even with minor simplifications in the arrangement where some adjacent rooms are considered to flood simultaneously. The heeling angles for all calculated phases of all stages are presented in Fig. 21. The initial heel quickly drops during the cross-flooding stage, similar to the simulations. This is followed by the stages representing the alternative combinations of flooding through the collapsing A-class boundaries. A wider spread of heeling angles is obtained as the asymmetry of flooding varies between different stages. All the heel angles in Fig. 21 are between the maximum and minimum heel angles in the simulation results, Fig. 20. At the final equilibrium stage, all internal structures have collapsed, and the rooms damaged are open to sea (lost buoyancy).

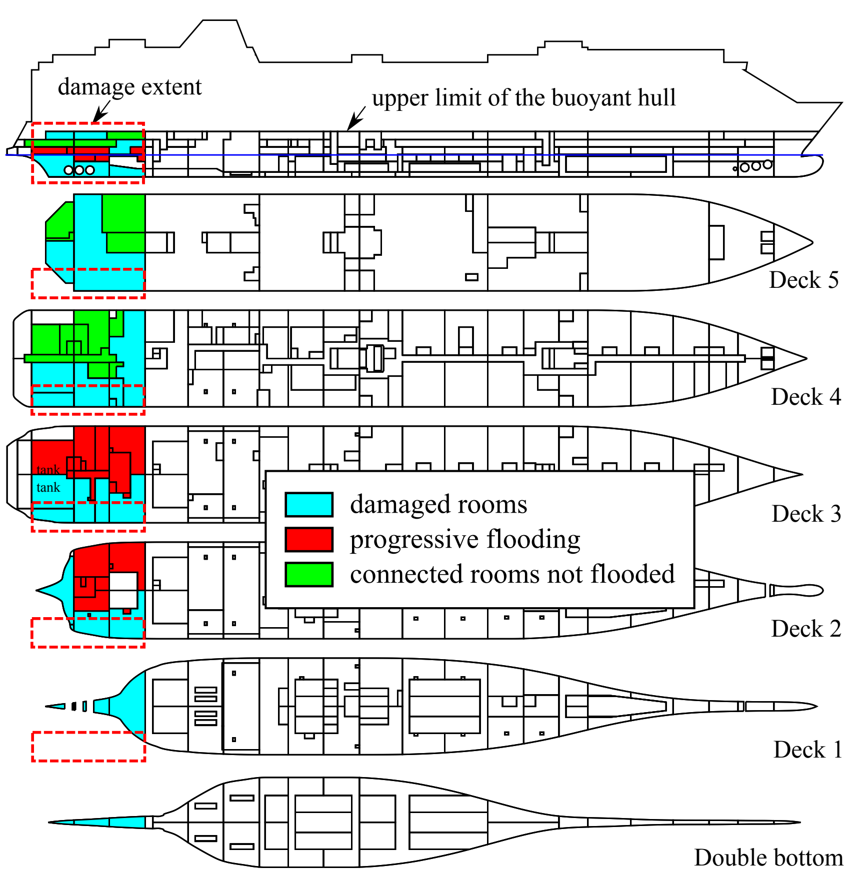

Fig. 19. Extensive flooding in a large passenger ship, Case C.

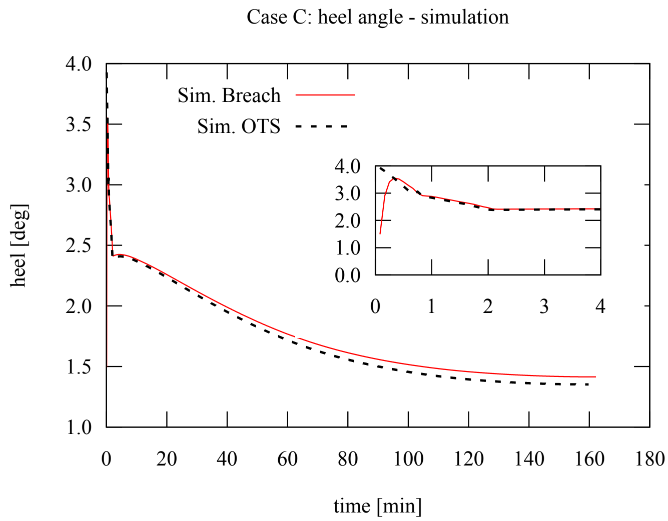

Fig. 20. Development of heel angle in time-domain simulation for Case C.

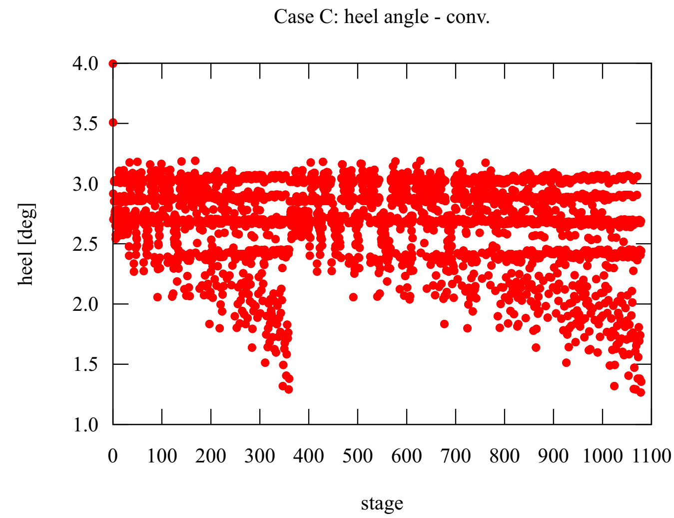

Fig. 21. Heel angle for different stages with the conventional method for Case C.

The number of phases calculated in the simulation (about 2000 with a time step of 5 s) are far less than the number of phases in the conventional calculation (1000+ stages with 3 phases each). From a performance point of view, it is noteworthy that the generation of a damage case with 1000+ alternative stages also takes a considerable time.

The GZ curves for the final equilibrium stage after flooding with the different calculation methods are shown in Fig. 22. Similarly to the previous cases, the modelling of the breach as openings in flooding simulation results in larger righting lever values, especially at larger heeling angles. The differences between the two methods for modelling the damage in flooding simulation are much smaller than in Case B. The reason for this is that the damaged rooms above the bulkhead deck are also smaller. Considering the fact that only one of the 13 closed non-watertight doors on Deck 3 collapses in this case, the assumption of free communication with the sea seems to be quite unrealistic, at least for the rooms above Deck 2.

6. Discussion

Stability reserves in the form of a range and area in the GZ curve are needed to provide a restoring force against time-dependent external loads, mainly waves and wind. However, the response of the ship, e.g. roll motion, under such loads is dependent on the inertia of the ship, which in a damaged case is generally increased by the floodwater. The time to roll the ship to the maximum heel angle is thus related to the roll period of the damaged ship, Manderbacka et al. (2015b), which in turn relates to the restoring force, i.e. the GZ curve. As has been pointed out earlier, the definition of the GZ curve depends, among other things, on the assumptions made on the height of the internal floodwater surfaces and volumes. Either the surfaces are assumed to be equalized, or alternatively not allowing any exchange of floodwater at all. The time to equalize the floodwater surfaces is dependent on the proportion of the opening size with respect to the room volume. This matter was pointed out by Manderbacka and Ruponen (2016), and the consequences were studied with time-domain flooding simulations accounting for the dynamics.

Moreover, due to the complicated matter of dynamic interactions, the floodwater motions, namely sloshing, may be important for wide rooms, where the time needed to equalize the internal surface is dependent on the natural period of sloshing in the room. It is nearly impossible to define this period for a damaged room with a varying amount of floodwater. Such effects should be separately studied with an aid of designated numerical simulation tools, Fonfach et al. (2016). The dynamic effects are, however, beyond the scope of the presented research, addressing and providing recommendations of practical means of estimating the residual stability of a damaged ship.

Flooding simulation has several advantages when compared to the conventional calculation methods. For example, previous research on survivability of damaged passenger ships, Vassalos et al. (2005), has brought up multiple free surfaces as a dangerous failure mode for ships with several decks below the bulkhead deck. This phenomenon cannot be captured by the conventional approach with the lost buoyancy method applied to all flooded rooms. However, time-domain simulation with the breach modelled as openings can model this condition realistically.

In reality, the breach shapes are arbitrary with petalling of the hull plating, Li et al. (2014). The extreme damage extent is used in statistics, and regulatory calculations are normally based on the assumption that the whole WT zone is damaged, resulting in a much larger breach size compared to reality, as illustrated in Fig. 23. This should be considered when selecting how the damage in flooding simulation is modelled.

Based on the presented examples, the use of a constant amount of floodwater for the final stage results in larger righting levers than the conventional approach with the lost buoyancy method, especially at large heel angles. However, the effects on the SOLAS calculations might be smaller since the immersion of unprotected openings limits the applied range of the GZ curve. Further studies on these effects are still needed.

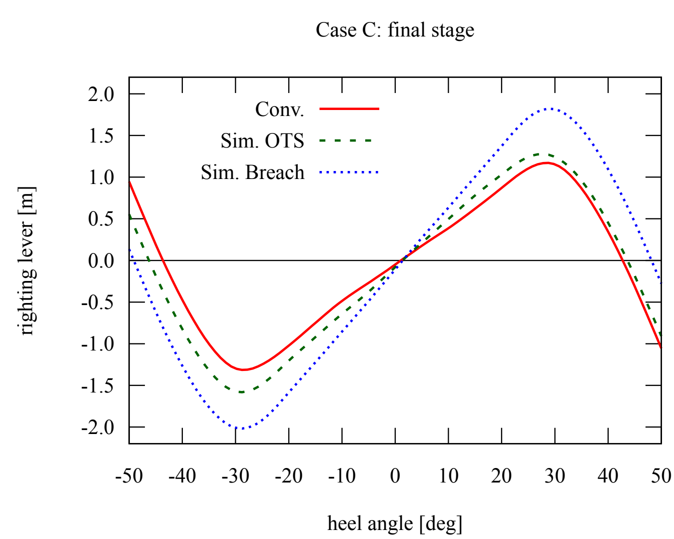

Fig. 22. GZ curves for the final stage of Case C.

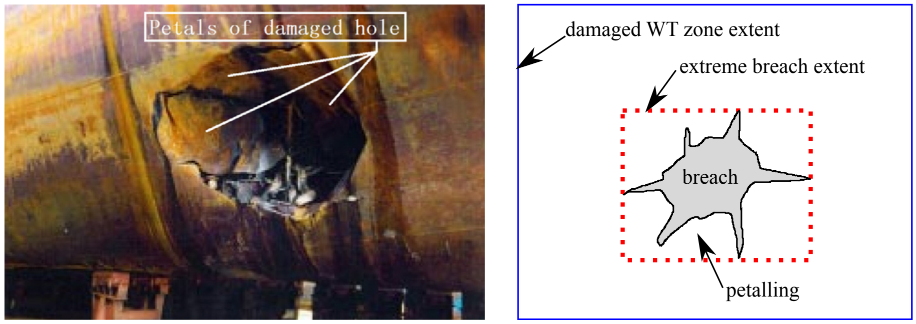

Fig. 23. Real breach extent compared to regulatory damage extent, photo on left has been adopted from Li et al. (2014).

The results from the FLOODSTAND project, Jalonen et al. (2017), demonstrate that non-watertight doors can withstand pressure heads up to 3.5 m before they collapse. The presented case studies and previous research, e.g. Ruponen (2017), show that several closed non-watertight doors may not even collapse during the flooding. However, leakage through doors results in a slow progressive flooding. This indicates that the assumption of free communication with the sea (method of lost buoyancy) is not always realistic.

Hybrid methods, where a heeling velocity (°/s) is associated with the GZ calculation in flooding simulation, could also be applied, as suggested by Dankowski (2013). However, this approach makes the calculations more complicated, and the suitable heeling velocity is difficult to define. In addition, the result should be independent of the set of heeling angles, where the righting levers are calculated. Therefore, the lost buoyancy and constant volume methods are considered more suitable, especially for regulatory calculations.

7. Conclusions

The righting lever curve and its properties are essential for damage stability assessments according to the current SOLAS regulations. With the increasing computational capacity, the amount of analyses has expanded, allowing for more realistic investigations of the intermediate stages of flooding. Especially, in passenger ships the conventional approach to calculate all possible intermediate stages due to the non-watertight internal structures can be computationally challenging.

Time-domain flooding simulation has been recognized as the best option to study the residual stability of a damaged ship during the intermediate stages of flooding. Several simulation tools have been developed and successfully validated. The next step in this development is to extend the use of time-domain analyses in damage stability assessment within the regulatory framework. Also for such studies, the righting lever curve can be used to evaluate the stability characteristics. In addition, further studies on the effects of the calculation method on the s-factors and the attained subdivision index are needed. After all, the thresholds for the current s-factors were derived based on calculations for sample ships with the conventional approach, Tagg and Tuzcu (2003).

In general, it can be concluded that the method of lost buoyancy is suitable for the final stage of flooding in damage cases, without internal structures that significantly restrict the free flow of water in the flooded compartments. This applies to tanks, voids, cargo holds, and possibly even to large engine rooms. However, for the accommodation and store areas this assumption is often not realistic since the non-watertight structures can significantly restrict the free flow of floodwater when the ship is heeled. Instead, the use of constant volumes of floodwater for evaluation of the righting lever curve is considered to be more realistic in these cases.

Based on the presented results, it is recommended to model the damaged compartments as lost buoyancy in regulatory analyses. This approach ensures that also the damaged rooms on the upper decks are flooded at large heel angles. If the actual breach is modelled as openings, the damaged rooms that are not flooded at the equilibrium floating position are considered empty also in the calculation of the GZ curve. This assumption can provide overly optimistic results in the damage cases that involve large rooms above the sea level, such as a car deck. However, if the real breach extents are known, as in accident analyses, Krüger (2016), or Monte Carlo methods, Dankowski and Krüger (2013), the breach should be modelled as openings, since this approach can accurately capture also the initial stages of the flooding process.

Regulatory calculations are based on simplified breach sizes that are derived from damage statistics. On the other hand, in operative damage stability assessment the breaches can be estimated based on measured inflow of floodwater and visual observations. Here the exact damage extent is unknown, but the breach size can still be estimated as presented in Ruponen et al. (2017). This approach is considered to provide more realistic prediction of progressive flooding than the assumption of lost buoyancy for the damaged rooms.

Considering the increased computational capacity, also damage stability calculations are developing towards more realistic approaches that properly account for the internal structures and progressive flooding in time-domain. Therefore, an explicit requirement for using the lost buoyancy method in the regulations should be avoided. A more relevant issue is to specify the treatment of floodwater in the evaluation of the righting lever curve. The most realistic approach depends on the studied case, and the regulations should have enough room for interpretations, in order to allow for feasible, yet safe, design and operation of ships.

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