



# Simulation of blended nonlinear hydrodynamics forces using radial basis function in uniform moving frame

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## ABSTRACT

This study focuses on the development of a blended technique in moving frame which encompasses nonlinearities and real time simulation of the vital early design parameters using combined exact nonlinear and quasi-nonlinear forcing terms. Generally, a full three-dimensional problem needs to be solved for the precise forward speed correction. However, in this paper the forward speed end corrections are calculated by converting the two dimensional velocity potential into a three dimensional mathematical function using radial basis function then partial differentiation is performed with respect to the longitudinal direction. The difference between the forward speed correction used for time simulation in the blended method and the strip-theory in the frequency domain has been explained. The use of radial basis functions for the estimation of quasi-nonlinear combined radiation and diffraction pressures in moving frame and their conversion between two and three dimensions has been demonstrated and validated experimentally.

## 1. Introduction

Rapid and repetitive assessment of seakeeping parameters is needed during an early design phase of sea-going vessel. Along with the accuracy of modelling, time to complete the assessment cycle also becomes a dominant factor. Therefore, a fast and robust seakeeping code that must be capable of performing rigorous hydrodynamics calculations is always desirable.

Over the past several decades, numerous investigations have been conducted on hydrodynamic analysis of hull behaviour. They are classified into three major categories: analytical, experimental and numerical techniques. The numerical techniques are further divided into methods for boundary value problems and domain-dependent problems. Each technique is applicable only for a limited range of cases (Yousefi et al., 2013). Plethora of publications exists on the subject of BEMs applied for single hull hydrodynamics. Doctor et al. (Doctors, 1974) and Wang and Day (2007) utilized a distribution of pressure elements over

planning surfaces while Lai and Troesch (1996) and Benedict et al. (2002) applied vortex lattice methods. Several CFD-type analyses of specific high-speed multihulls can be also found in the literature (e.g., Zhou, 2003 (Zhou, 2003); (Kandasamy et al., 2011); Yousefi et al., 2014 (Yousefi et al., 2014)). Jiang et al. (2016) employed CFD code based on finite volume method to simulate and analyse the forward motion of the hull body with two degrees of freedom (heave and pitch). The main objective of the study was to analyse the flow mechanism and characteristics of the tunnel at different forward speeds. Predicted results were validated with experimental data and showed good agreement. Ghasabzadeh and Ghassemi (2014) used commercial software FLUENT to simulate a multi-hull tunnel vessel in calm water. The degrees of freedom in the simulation were pitch and heave motions and the hull position changed with a moving mesh. The simulated results of drag and trim were in good agreement with experimental data.

The evaluation of structural responses is key element in the design of ships and off shore structures. The total pressure to examine hull behaviour comprises of incident wave Froude-Kriloff pressure,

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Nomenclature	
$U$	Mean forward speed/velocity
$X$	Ship absolute motions vector
$F_n$	Froude number
$\widetilde{F}_R(t)$	Complex radiation force time series
$\widetilde{F}_D(t)$	Complex wave load force time series
$x, y, z$	Inertial coordinate system
$\eta_1$	Surge
$\eta_2$	Sway
$\eta_3$	Heave
$\eta_4$	Roll
$\eta_5$	Pitch
$\eta_6$	Yaw
$x_0$	X –axis in earth fixed coordinate
$t_0$	Time in earth fixed coordinate
$n$	Normal vector
$t$	Surface tangent
$\varphi_R$	Radiation velocity potential
$\varphi_D$	Diffraction velocity potential
$\varphi_I$	Incident wave velocity potential
$F$	Force
$S$	Surface/Segment
$\Phi$	Total Velocity potential
$\nabla$	Laplace operator
$D$	Total/cumulative derivative
$\varphi$	Perturbation velocity potential
$\varphi_s$	Steady perturbation velocity potential
$\varphi_R$	Unsteady perturbation velocity potential
$W$	Velocity flow vector
$\Omega$	Rotational velocity vector
$V$	Ship unsteady oscillatory velocity vector
$u, v, w$	Subscripts used for velocity in x, y and z directions $\Psi(r)$
	Subscripts used for velocity in x, y and z directions $\Psi(r)$
	Single basis function
$\xi$	Instantaneous wave profile
$p$	Dynamic pressure
$\omega_e$	Frequency of encounter
$\kappa$	Wave number
$\beta$	Heading angle
$z$	Complex points on the body surface
$z_0$	Complex points in the fluid water domain
$L$	Linear
$N$	Quasi-Nonlinear
$E$	Experimental
$r$	Complex data (gain)
$i$	Complex data (imaginary)

hydrostatic pressure and radiation and diffraction pressures. Hirdaris et al. (Hirdaris et al., 2014) presented a detailed review on the recent advances in the assessment of loads for ships and off shore structures for further understanding, validation and implementation by the academic and industrial communities.

Many researchers have worked to solve the instantaneous boundary value problem using desingularized Euler-Lagrange methods. Finn (2003) solved the boundary value problem which satisfies body boundary condition in the near field. The free surface condition is only satisfied within the reasonably wide range of free space of water surface, in the vicinity of the piercing body. The matching of far field at infinity and near field solutions has not been considered by assuming that the impact of the far field condition on the near field solutions would not be significant.

Kent (2005) used a pseudo-spectral method, where asymptotic technique is employed to seek the solution that satisfies the far field boundary conditions. This approach is close to the one used by Faltinsen (1971) for diffraction velocity potential for a piercing body in head seas where far field and near field conditions are matched asymptotically.

Salvesen et al. (1970) used conventional strip theory for seakeeping analysis of normal displacement ships. However, its validity can be questioned when it is used for ships with higher maximum operating speeds, such as for semi-displacement vessels (Faltinsen, 2005).

Savander et al. (2002) applied the boundary value problem to a planing plate and obtained relationships between potential perturbation and vortex distribution. They calculated the hydrodynamic pressure, lift and drag forces for the planing plate at different speeds. Ghassemi et al. (Ghassemi and Ghiasi, 2008; Ghassemi and Yu-min, 2008; Reza Kohansal and Ghassemi, 2010) have developed a computer code, based on BEM in conjunction with boundary layer, for hydrodynamic analysis of planing and non-planing hulls. This code was also utilized to study the wave pattern and pressure coefficients. However, the code was unable to take into account a two-phase flow model and cannot be applied for complex geometries and high Froude number cases.

Kihara et al. (2005) presented computations of hydrodynamic forces when the ship oscillates in head seas, although their focus is on the added resistance in waves and they simplified the description of sprays.

They simulated non-viscous flow separation from the knuckle of a ship section using 2D + t method together with a nonlinear BEM solver.

Giorgi and Ringwood (2018) discussed the importance and relevance of nonlinear Froude Kriloff force representation for axisymmetric wave energy converters, for vessels moving in surge, heave, and pitch. It is found that Froude-Kriloff integrals can be solved numerically by assuming small pitch angles which simplify the problem and achieve a considerably faster algebraic solution.

Yuan et al. (2014) investigated the hydrodynamic interactions between two ships arranged side by side with forward speed using boundary element program based on 3-D Rankine source method. The radiation condition is satisfied by using a modified Sommerfeld radiation condition which takes into account the Doppler shift of the scattered waves. This new radiation condition is applicable to a wide range of forward speeds. Yuan et al. (2015) also investigated the hydrodynamic interaction between two vessels with forward speed arranged side by side in shallow water. The motion responses of both ships were calculated and compared to these obtained from commercial software and experimental results.

Rajendran et al. (2015) investigated the effects of surge hydrodynamic forces and surge mode of motion on the vertical responses of a container ship and a chemical tanker in waves. It was found that the longitudinal forces have insignificant effect on the vertical moment about the lateral axis at the mean water level and on the flexural normal stress at the deck. Rajendran et al. (2016) also developed a code based on strip theory to calculate vertical ship responses of a containership in extreme sea conditions. The numerical method calculates the radiation forces based on Cummins formulation. Radiation, diffraction, Froude–Krylov and hydro static forces for instantaneous wetted surface area of the hull were calculated. It is observed that the body nonlinear radiation/diffraction forces significantly improve prediction of the ship responses in extreme waves.

Khalid et al. (2016) investigated the role of blended method to solve two-dimensional boundary value problem for multi-bodies radiation and diffraction velocity potentials. It was found that blended technique is an efficient and accurate alternate method to provide time simulations of ship motion and other essential parameters for design optimization.

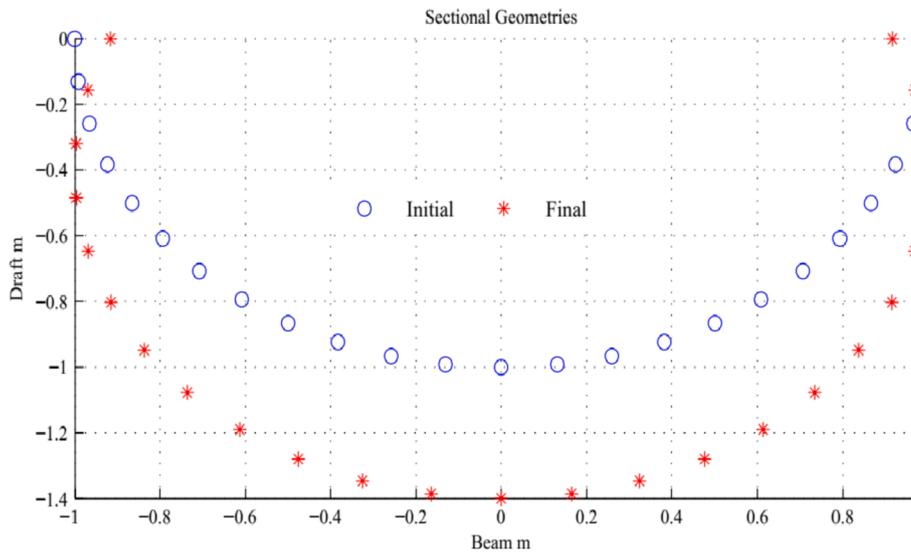


Fig. 1. Two circular-sections,  $\lambda_i$  based on the original body-section to estimate the function corresponding to changed or final body-section, submergence = .4R.

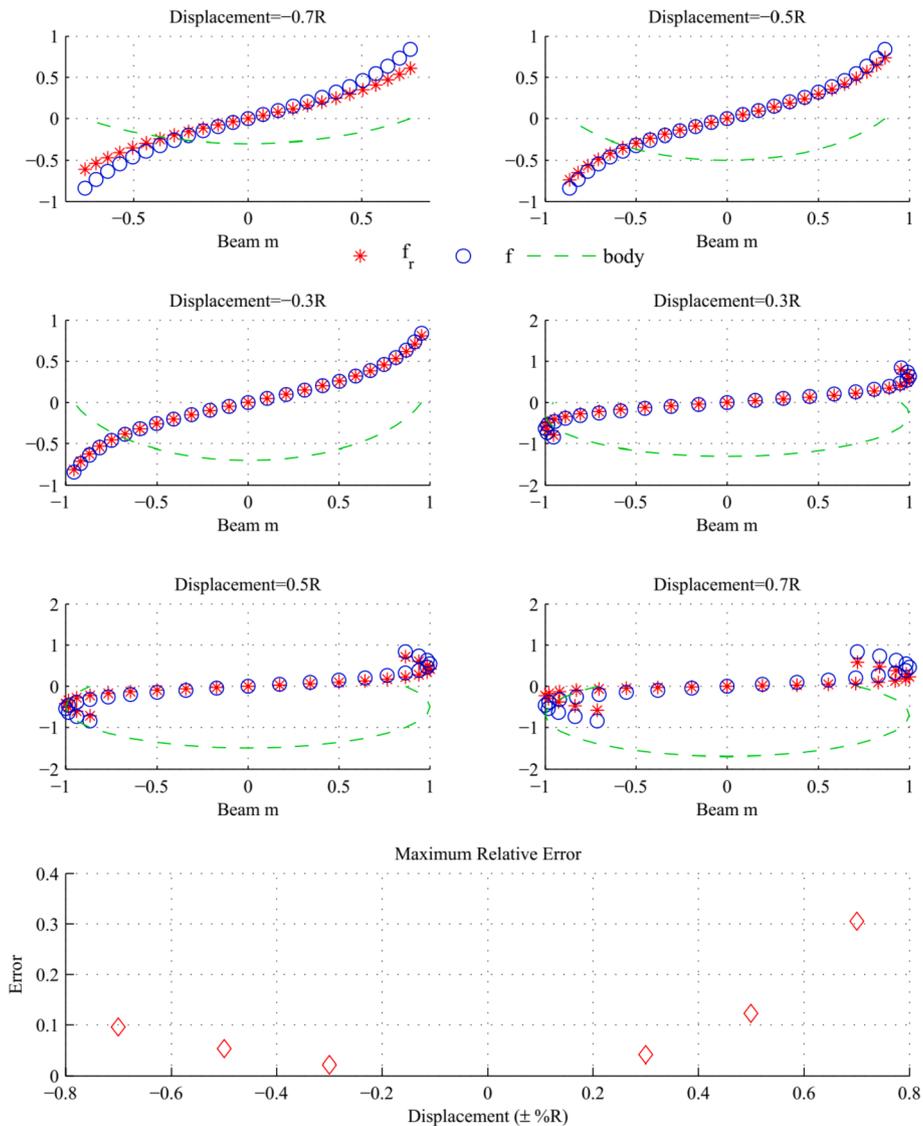


Fig. 2. Comparing nodal values of an exact function  $f$  vs estimated function  $f_r$  on a varying cylindrical section.

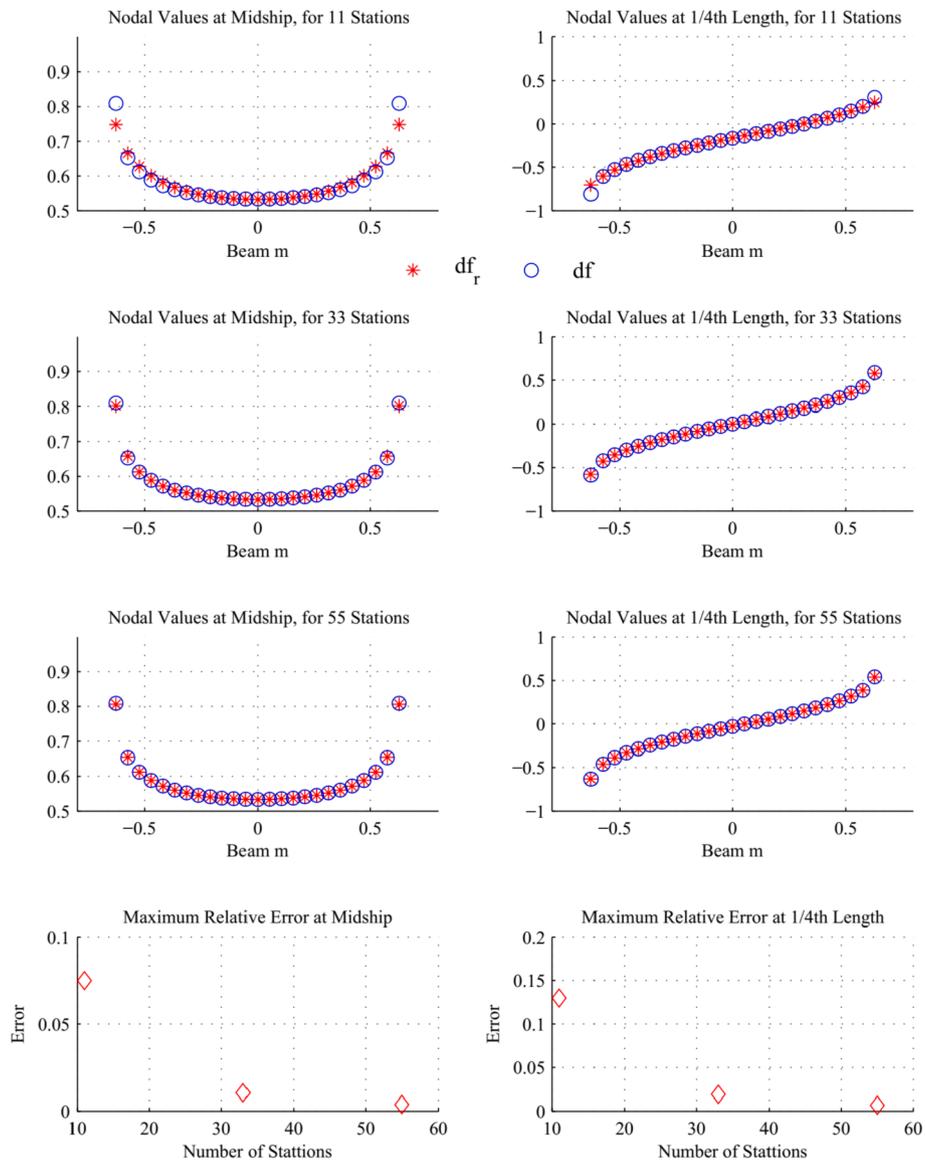


Fig. 3. Comparing nodal values of an exact function  $df = \frac{\partial f(x,y,z)}{\partial x}$  vs. estimated function  $df_r = \frac{\partial f_r(x,y,z)}{\partial x}$  on a cylinder varying the number of station along length.

In this paper, blended technique using radial basis function has been developed and employed to calculate hydrodynamic forces in moving frame. This technique encompasses nonlinearities and real time simulation of the vital early design parameters using combined exact nonlinear and quasi-nonlinear forcing terms. The difference between the forward speed correction used for time simulation in the blended method and the strip-theory in the frequency domain has been explained. The use of radial basis functions for the estimation of quasi-nonlinear combined radiation and diffraction pressures in moving frame and their conversion between two and three dimensions has been demonstrated and validated experimentally.

## 2. Radial basis function and convergence studies

### 2.1. Radial basis function

The functions expressed in the distance variable are usually termed as radial basis functions (RBFs) in the literature. The idea of radial-basis function derives from the theory of functions. A radial-basis function approach is formed by taking a linear combination of a single basis function  $\Psi(r)$ , which is radially symmetric about its center. The only geometric property used in the radial basis function method is the

pairwise distances between points. Wright and Fornberg (2006) have shown that the interpolation can be accurate for approximating derivatives. By imposing some mild restrictions on  $\Psi(r)$ , the radial-basis function interpolation converges to polynomials. The radial basis function is a weighted sum of the radially symmetric basic functions located at the centers/nodes and a low degree polynomial. The radial basis function evaluates the nodal value pressure by keeping track of the new relative position of all the other nodes in the domain of interest. It also saves an enormous amount of time by not solving the boundary value problem at each full and intermediate time step. Wide range of radial basis functions can be found in the literature with different kinds of constraints and utilities such as bi-harmonic, tri-harmonic, quadratic, multi-quadratic, inverse multi-quadratic, wend-land and generalized Duchon spline radially symmetric functions (Wright and Fornberg, 2006; Carr et al., 2003; Carret al., 2001; Tolstykh and Shirobokov, 2003). Khalid (2007) in his thesis evaluated dynamic and static pressures acting on a floating body while the body is experiencing large motions in six degrees of freedom and extreme random seas. A blended scheme is introduced to calculate total hydrodynamic pressure acting on an arbitrary body. The method employs quasi-nonlinear radiation and diffraction models while hydrostatic and Froude-Kirloff pressures are fully nonlinear. The quasi-nonlinear radiation and diffraction pressures

are estimated using radial-basis function without solving the boundary-value problems.

## 2.2. Convergence studies

In this study, radial-basis function is used for the estimation of body exact instantaneous radiation and diffraction pressures. There are two distinct applications, the first is to capture the changing sectional geometries and the second is to evaluate partial derivative  $\frac{\partial}{\partial x}$  of the radiation and diffraction velocity-potentials needed for the evaluation of forward-speed corrections.

Typical convergence investigations are illustrated using real part of a three dimensional incident wave velocity-potential  $f(x, y, z) = e^z \sin(-x - y)$ , where the wave amplitude and wave number are assumed to be unity. The initial radial-basis coefficients  $\lambda_i$  are evaluated for the mean position of a half-submerged cylinder of radius  $R = 1$ . The cylinder is then further submerged by a factor of  $0.4R$ . The previously computed  $\lambda$ 's corresponding to mean position are used to estimate the function  $f_r(x, y, z)$  for the changed body-section; the variation of body geometry is shown in Fig. 1.

The extent of change in the body geometry relative to its mean position is also investigated. For the same function and cylindrical body, various emersions and immersions are shown in Fig. 2 using 25 nodal points on a section. The estimates compare reasonably well for extreme geometry changes. The estimate for extreme geometry reduction due to emersion of  $0.7R$  of the cylinder is 10% of the exact value; for an immersion of  $-0.7R$ , the error is 30% of exact value. In general for moderate changes of geometry the error in estimates remains below 3% of exact values.

Convergence studies for the second application of radial-basis function to evaluate  $\frac{\partial}{\partial x}$ , is demonstrated using the partial derivative  $\frac{\partial f}{\partial x}$  of the previously used function  $f(x, y, z)$  for a cylindrical ship-shape. The ship's length equals the length of the wave, i.e  $L = \lambda = 2\pi$  and the radius of the each section is  $R = \frac{\lambda}{10}$ . The exact  $\frac{\partial f(x,y,z)}{\partial x}$  is known at different nodes on various sections along the ship's length.

The radial-basis coefficients  $\lambda_i$  are evaluated for the function  $f(x, y, z)$ . Then the estimated function  $f_r(x, y, z)$  is constructed using these coefficients. The partial derivative  $\frac{\partial f_r}{\partial x} = \frac{\partial f_r(x,y,z)}{\partial x}$ , is performed on the mathematical or algebraic form of the function  $f_r(x, y, z)$ . The estimates are increasingly good for an increase in number of stations and the convergence is fast as shown in Fig. 3. The relative error is reasonably small for moderate numbers of stations along the ship's length.

## 3. Mathematical formulation

### 3.1. Fully nonlinear steady perturbation problem

To account for the forward speed corrections, a steady perturbation velocity potential  $\varphi_s$  is found through solving a steady flow problem in moving frame. The velocity vector  $W$  of steady flow relative to the moving frame is defined in Equation (1). Since the dynamic pressure given by Bernoulli's Equation holds for all time, setting its substantial derivative equal to zero gives an alternative fully nonlinear free surface boundary condition (2). Substituting  $U(\varphi_s - x)$  for velocity potential  $\Phi$  in Eq. (2), one may write the free surface,  $z_0 = \xi(x, y, t)$  boundary and body boundary conditions at mean body position in terms of steady velocity vector  $W$  as in Eqs. (3) and (4), where  $n$  is unit normal vector. In this text  $n$  is vector unless written subscripted as  $n_j$ .

$$W = U\nabla(\varphi_s(x, y, z) - x) \quad (1)$$

$$\frac{\partial^2 \Phi}{\partial t^2} + \nabla\Phi \cdot \nabla\Phi_t + \frac{1}{2} \nabla\Phi \cdot \nabla(\nabla\Phi \cdot \nabla\Phi) + g\Phi_{z_0} = 0 \quad \text{on } z_0 = \xi \quad (2)$$

$$\frac{1}{2} W \cdot \nabla(W^2) + g \frac{\partial \varphi_s}{\partial z_0} = 0 \quad \text{on } z_0 = \xi \quad (3)$$

$$W \cdot n = 0 \quad \text{on } S_0 \quad (4)$$

To include all the nonlinearities, one should solve for the combined steady and unsteady  $\varphi_s R = U\varphi_s + \varphi_R$  velocity potential subject to combined free surface and instantaneous body boundary conditions (6 & 7) in terms of the ship absolute velocity and steady flow vector  $W$ . However, the assumption of small motions leads to decomposition of unsteady velocity potential  $\varphi_R$  into its components for each of six degrees of rigid body motions. Under such conditions, one may solve the steady flow problem separately and the unsteady velocity potential separately with the free surface boundary condition (5) and the body boundary condition (7). This is accomplished by replacing velocity flow vector  $W$  with  $\tilde{U}i$  and neglecting the steady perturbation velocity potential  $\varphi_s$ ,

$$\left[ \left( \frac{\partial}{\partial t} - U \frac{\partial}{\partial x} \right)^2 + g \frac{\partial}{\partial z} \right] \varphi_R = 0 \quad (5)$$

$$\frac{\partial^2 \varphi_R}{\partial t^2} + 2W \cdot \nabla \frac{\partial \varphi_R}{\partial t} + \frac{1}{2} (W + \nabla \varphi_R) \cdot \nabla W^2 + W \cdot \nabla (W \cdot \nabla \varphi_R) + g \frac{\partial (\varphi_R + \varphi_s)}{\partial z} = 0 \quad \text{on } z_0 = \xi \quad (6)$$

$$\frac{\partial \varphi_R}{\partial n} = \nabla \varphi_R \cdot n = (V - W) \cdot n \quad \text{on } S(t) \quad (7)$$

Newman (1979) mentioned two first order contributions of  $W \cdot n$  in the instantaneous body boundary condition (7) as (a) rotation of the ship fixed coordinate system and (b) gradient of the steady flow. Therefore, one may express the last term  $W \cdot n$  used in Eq. (7) as vectorial representation of coordinate rotation and gradient of the steady flow. Using Eq. (8), where  $\Omega$  is the rotational velocity vector representing roll, pitch and yaw velocities and  $X$  represents the ship absolute motions vector. After substitution of Eq. (8) into Eq. (7), the body boundary condition (7) can be applied to steady state body position and to the instantaneous body position. Eq. (8) is valid on both steady state and instantaneous body positions. By invoking Eq. (4) and using vector equalities  $\Omega \times W = (W \cdot \nabla) X$ , the alternative of Eq. (8) may be written as Eq. (9)

$$W \cdot n \cong (W - \Omega \times W + (X \cdot \nabla)W) \cdot n \quad (8)$$

$$W \cdot n \cong (- (W \cdot \nabla)X + (X \cdot \nabla)W) \cdot n \quad (9)$$

Bernoulli's equation for dynamic pressure in the coordinate system moving with velocity vector  $W$  will be written as in Eq. (10). The radiation velocity potential  $\varphi_R$  in Eq. (10) should be obtained by solving a boundary value problem subject to fully nonlinear boundary conditions (6 & 7) after substitution of Eq. (9) into Eq. (7)

$$p = -\rho \left( \frac{\partial \varphi_R}{\partial t} + W \cdot \nabla \varphi_R + \frac{1}{2} \nabla \varphi_R \cdot \nabla \varphi_R + g z_0 \right) + p_a \quad (10)$$

### 3.2. Linear approximation and decomposition

To avoid the complexities of the three-dimensional problem, one may tend to neglect the steady flow field due to the ship presence. Assuming  $\varphi_s = 0$  the velocity flow vector will be  $W = -\tilde{U}i$ , the body boundary condition (7) will reduce to Eq. (11). It is observed here that with the assumption of  $\varphi_s = 0$ , one of the two first order contributions of  $W \cdot n$  is lost completely. The only first order contribution retained in the body boundary condition is due to the rotation of the coordinate system. The flow velocity vector is constant  $W = -\tilde{U}i$ ; its gradient will vanish.

$$\frac{\partial \varphi_R}{\partial n} = \nabla \varphi_R \cdot n = \left( V - U \frac{\partial X}{\partial x} \right) \cdot n \quad \text{on } S(t) \quad (11)$$

The assumption of small unsteady motions leads to decomposition of  $\varphi R$  into separate six degrees of freedom and the body boundary conditions for six cases would be as follows:

$$\frac{\partial \varphi R}{\partial n} = \nabla \varphi R \cdot n = \dot{\eta}_1 n_1 \quad \text{on } S_o \quad (12)$$

$$\frac{\partial \varphi R}{\partial n} = \nabla \varphi R \cdot n = \dot{\eta}_2 n_2 \quad \text{on } S_o \quad (13)$$

$$\frac{\partial \varphi R}{\partial n} = \nabla \varphi R \cdot n = \dot{\eta}_3 n_3 \quad \text{on } S_o \quad (14)$$

$$\frac{\partial \varphi R}{\partial n} = \nabla \varphi R \cdot n = \dot{\eta}_4 n_4 \quad \text{on } S_o \quad (15)$$

$$\frac{\partial \varphi R}{\partial n} = \nabla \varphi R \cdot n = \dot{\eta}_5 n_5 + U \eta_3 n_3 \quad \text{on } S_o \quad (16)$$

$$\frac{\partial \varphi R}{\partial n} = \nabla \varphi R \cdot n = \dot{\eta}_5 n_5 - U \eta_2 n_2 \quad \text{on } S_o \quad (17)$$

The term proportional to  $U$  may be interpreted as the product of the ship forward speed and angle of attack due to pitch and yaw motions. With the additional restriction that unsteady motions are sinusoidal oscillations with  $\omega_e$  as the frequency of encounter, the velocities and displacements are related as  $V = i\omega_e X$ , Eq (16, 17) and Bernoulli's Eq. (10) may be further linearized as follows:

$$\frac{\partial \varphi R}{\partial n} = \nabla \varphi R \cdot n = \dot{\eta}_5 n_5 + \frac{U}{i\omega_e} \dot{\eta}_3 n_3 \quad \text{on } S_o \quad (18)$$

$$\frac{\partial \varphi R}{\partial n} = \nabla \varphi R \cdot n = \dot{\eta}_5 n_5 - \frac{U}{i\omega_e} \dot{\eta}_2 n_2 \quad \text{on } S_o \quad (19)$$

$$p = -\rho \left( \frac{\partial \varphi R}{\partial t} - U \frac{\partial \varphi R}{\partial x} + g z_o \right) + p_a \quad (20)$$

$$\frac{\partial \varphi R_j}{\partial n} = \nabla \varphi R_j \cdot n = i\omega_e n_j \quad (21)$$

$$\left( \frac{\partial^2}{\partial t_o^2} + g \frac{\partial}{\partial z_o} \right) \varphi R_j = 0 \quad \text{at } Z = \zeta \quad (22)$$

The last terms in each of Eqs. (18) and (19) represent the body boundary conditions (13 & 14) with a constant multiplier  $\pm \frac{U}{i\omega_e}$  respectively. One may intend to solve the six boundary value problems, without explicit involvement of forward speed, using a simple two-dimensional Green's function given by (Frank, 1967) and subjected to simple free surface and body boundary conditions (21 & 22). However doing so will result in an inherent anomaly in free surface pressure. The free surface pressure would be slightly different than absolute zero value because of not satisfying the consistent linearized free surface boundary condition (5) involving the forward speed  $U$  after neglecting the steady perturbation velocity  $\varphi_s$  in the boundary value problem. After solving the boundary value problem for  $\varphi_R^o$  using body boundary conditions (12 through 17) with  $U = 0$  and consistent linearized free surface boundary condition (5), the pitch and yaw radiation potentials then may be corrected for the constant multiplier as shown in Eqs. (23) and (24). There is additional forward speed correction present in linearized dynamic pressure Eq. (20) due to moving frame of reference.

$$\varphi R_5 \cong \varphi_{R_5}^o + \frac{U}{i\omega_e} \varphi_{R_5}^o \quad (23)$$

$$\varphi R_6 \cong \varphi_{R_6}^o - \frac{U}{i\omega_e} \varphi_{R_6}^o \quad (24)$$

#### 4. Blended method for radiation forces in moving frame

In blended method, fully nonlinear Euler equations of motion are solved with nonlinear hydrodynamic forces acting on multi-hulled vessels. Lid is employed over the body segment to suppress the eigenvalue mode, thus eliminating singularities in source strength being used on multi-hull bodies presenting geometrical discontinuities (Khalid et al., 2016). In the nonlinear time simulation, added-mass and damping coefficients are not used. Instead, total nodal radiation pressure is estimated using an appropriate radial-basis function for exact instantaneous body position. Six boundary value problems are solved separately without involving forward speed. Then after correcting the  $\varphi R_j$  for pitch and yaw modes by invoking Eqs. (23) and (24), the total radiation velocity potential in rotating frame may be expressed as shown in Eq. (25). The substitution of this  $\varphi R$  in Eq. (20) gives the dynamic pressure in moving-coordinates.

$$\varphi R = \sum_{j=1}^6 \varphi R_j \eta_j \quad (25)$$

The diffracted velocity potential  $\varphi D$  is also solved for simple free surface and body boundary conditions (21 & 22). For the diffraction boundary value problem, the body is held in steady state position against the incident wave, the forward speed correction  $-U \frac{\partial \varphi_i}{\partial x}$  in the body boundary condition (11) for a rotating frame will vanish, as there is no rotation of the coordinate system. However the forward speed correction  $U \frac{\partial}{\partial x}$  of linearized dynamic pressure Eq. (20) due to steady moving-coordinate is absolutely applicable for the diffracted velocity potential as well. Similarly, due to the moving frame of reference for the incident velocity potential, the only forward speed correction needed is  $U \frac{\partial}{\partial x}$  in the linearized dynamic pressure Eq. (20). In the blended method, an appropriate radial-basis function is used to convert two-dimensional radiation and diffraction velocity potentials into their three-dimensional equivalents and the partial derivative  $U \frac{\partial}{\partial x}$  in Eq. (20) is performed explicitly for the combined radiation and diffracted velocity potentials.

##### 4.1. Validations

For the purpose of validation a comparison is presented between linear strip-theory of (Salvesen et al., 1970) and the blended method adopted in this research. In strip theory (Salvesen et al., 1970), the added-mass and damping coefficients may be written as in Eq. (26), where  $\varphi R_j$  has been corrected for rotation of coordinate system as in Eqs. (23) and (24).

$$\omega_e^2 A_{ij} - i\omega_e B_{ij} = -\rho \iint \left( i\omega_e \varphi R_j - U \frac{\partial \varphi R_j}{\partial x} \right) n_i dS \quad (26)$$

For ease of understanding, one may split the  $\varphi R_j$  into two parts as  $R_j = \varphi_{R_j}^o + \varphi_j^u$ . Where  $\varphi_{R_j}^o$  is the solution of the boundary value problem obtained from six body boundary conditions (12 through 17) with  $U = 0$  and free surface boundary condition (5) and  $\varphi_j^u$  is forward speed correction occurring in the body boundary condition due to rotation of the coordinate system. Considering Eqs. (23) and (24), the velocity potential  $\varphi_j^u = 0$  for  $j = 1, 2, 3, 4$  and is nonzero for pitch  $\varphi_5^u = \frac{\varphi_{R_5}^o}{i\omega_e}$  and yaw  $\varphi_6^u = -\frac{\varphi_{R_6}^o}{i\omega_e}$  modes.

The forward speed correction for the radiation velocity potential arises because of axes rotation, this correction is applied to the mean position of the body in the strip theory (Salvesen et al., 1970) while in the blended method the same correction is estimated for the instantaneous position of the body by using an appropriate radial-basis scheme with similar inconsistencies with regard to free surface boundary condition. The unsteady velocity potentials  $\varphi_{R_j}^o$  are calculated using zero speed Green's function given in (Frank, 1967). The actual difference

between the linear strip-theory (Salvesen et al., 1970) and the blended method is because of the partial derivative term,  $U \frac{\partial}{\partial x}$  present in the linearized dynamic pressure Eq. (20). In the blended method the partial derivative  $U \frac{\partial}{\partial x}$  is performed explicitly, while in the linear strip theory of (Salvesen et al., 1970), this derivative is circumvented using an alternate form of Stoke's theorem as proposed by Ogilvie (Ogilvie and Tuck, 1969). Ogilvie (1977) suggested to evaluate the surface integral term

$U \iint \left( \frac{\partial \varphi_{R_i}}{\partial x} \right) n_i dS$  of Eq. (26) as follows:

$$U \iint \left( \frac{\partial \varphi_{R_i}}{\partial x} \right) n_i dS = -U \iint \varphi_{R_j} m_j dS - U \int \varphi_{R_j} [(t \times n) \cdot W] n_i dl \quad (27)$$

The last term in Eq. (27) is a line integral over the boundary of mean position of the body at the intersection of the ship hull and the calm water line; where  $t$  and  $n$  are surface tangent and normal vectors respectively.

Newman (1979) showed that for a wall-sided ship  $[(t \times n) \cdot W] = \frac{\partial \varphi_s}{\partial z} \cong 0$  at  $z = 0$ , the vertical velocity due to the steady perturbation potential is of higher-order at the intersection of body and calm water line. This line integral is of higher-order and therefore may be neglected. Salvesen et al. (1970) have also suggested to neglect the line integral along the water line and retaining the line integral of the end section only. There is experimental evidence also, which strongly suggest completely ignoring the line integral present in Eq. (27). If the steady perturbation velocity potential  $\varphi_s = 0$  then the line integral in Eq. (27) should be identically zero. In the linear part of the code developed for this study, the line integral shown in Eq. (27) has been completely neglected.

It may be recognized that the term  $U \frac{\partial}{\partial x}$  has reduced from  $W \cdot \nabla$  due to setting  $\varphi_s = 0$  and linear approximations and decomposition of the full three-dimensional problem. Using the theorem given in appendix A of (Ogilvie and Tuck, 1969), it can be shown that  $\iint W \cdot (\nabla \varphi_R) n dS \cong \iint \varphi_R (n \cdot \nabla) W dS$ . The term  $m_i$  of Eq. (27) may be written in vector form as  $(m_1, m_2, m_3) = -n \cdot \nabla W$  and  $(m_4, m_5, m_6) = -n \cdot \nabla (r \times W) = r \times (m_1, m_2, m_3) + W \times n$ , where  $r$  is the three-dimensional position vector. If the steady-perturbation velocity potential is neglected by setting  $\varphi_s = 0$  and let  $U = 1$ , then from the above mentioned vector operations it can be shown that  $m_i = 0$  for  $i = 1, 2, 3, 4$  and  $m_5 = n_3, m_6 = -n_2$ . Eq. (26) can be rewritten in terms of  $\varphi^o R_j$ ,  $n_i$ ,  $\varphi_j^u$  and  $m_i$  for  $i = 1 \dots 6$  and  $j = 1 \dots 6$  as following:

$$\omega_e^2 A_{ij} - i \omega_e B_{ij} = -\rho i \omega_e \iint \left[ \varphi^o R_j n_i + U \left( \varphi_j^u n_i - \frac{\varphi^o R_j}{i \omega_e} m_i \right) - \frac{U^2}{i \omega_e} \varphi_j^u m_i \right] dS \quad (28)$$

As already defined the  $\varphi^u = 0$  for  $j = 1 \dots 4$ ,  $\varphi_5^u = \frac{\varphi_3}{i \omega_e}$  and  $\varphi_6^u = -\frac{\varphi_2}{i \omega_e}$ . Similarly  $m = 0$  for  $j = 1 \dots 4$ ,  $m_5 = n_3$  and  $m_6 = -n_2$ . The velocity potential  $\varphi_j^u$  has resulted from body boundary condition in a rotating frame of reference. The term  $m_i$  is the consequence of replacing the integrand term  $\widehat{U} i \cdot \nabla (\varphi^o R_j + \varphi_j^u) n$  of Eq. (26) inside the vectorial surface integral of the linearized dynamic pressure given in Eq. (20) due to moving frame, with the term  $(\varphi^o R_j + \varphi_j^u) (n \cdot \nabla) \widehat{U} i$  for surge, and heave modes. For roll, pitch and yaw modes, the integrand term  $\widehat{U} i \cdot \nabla (\varphi^o R_j + \varphi_j^u) (r \times n)$  is replaced with the term  $(\varphi^o R_j + \varphi_j^u) [(r \times n \cdot \nabla) \widehat{U} i + \widehat{U} i \times n]$ . Ogilvie et al. (Ogilvie and Tuck, 1969) showed mathematical validation for the interchangeability of the two terms inside a surface integral using the properties of velocity flow vector  $\widehat{U} i$ . For the explicit evaluation of pressure one cannot take the advantage of interchangeability of these two terms. In the case of pressure, the partial derivative  $U \frac{\partial}{\partial x}$  needs to be performed explicitly on the velocity potentials  $\varphi^o R_j + \varphi_j^u$ . The comparison between the radiation forces due to blended method and strip theory of (Salvesen et al., 1970) depends on how closely the

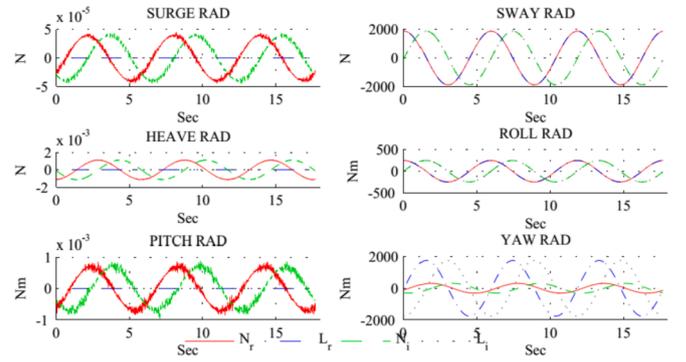


Fig. 4. 0.05 m Forced sway radiation, linear  $L$  vs quasi-nonlinear  $N$ ,  $\omega_e = 1.066$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

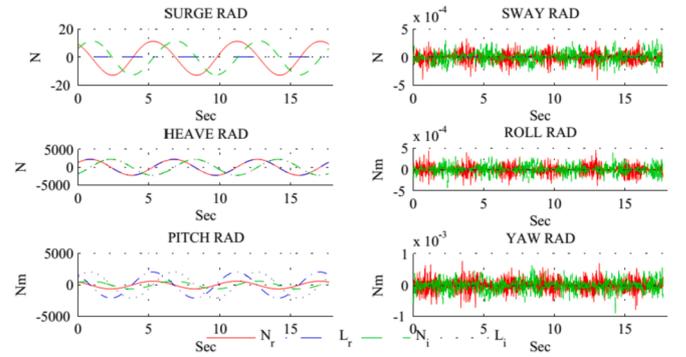


Fig. 5. 0.05 m Forced heave radiation, linear  $L$  vs quasi-nonlinear  $N$ ,  $\omega_e = 1.066$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

numerical equivalence  $\iint U \frac{\partial (\varphi^o R_j + \varphi_j^u)}{\partial x} n_i dS \cong \iint U (\varphi_j^o + \varphi_j^u) m_i dS$  holds between the interchangeable two terms discussed above in the light of theorem given in (Ogilvie and Tuck, 1969).

$$F_j = \int_{-\frac{L}{2}}^{\frac{L}{2}} \left[ \int_{S(t)} P n_j d\delta \right] dx \quad (29)$$

The blended scheme presented in this study focuses on explicit evaluation of dynamic pressures. The forces are then calculated by using a surface integral on the product of pressures and the body normal as shown in Equation (29). For the purpose of a comparison between the blended method and (Salvesen et al., 1970) strip-theory, Wigley-hull offsets were generated using Equation (30), where  $B = 2m$ ,  $L = 20m$ ,  $T = -1$  m are the beam, length and the draft of the ship respectively.

$$Y = \frac{-B}{2} \left[ 1 - 0.95 \left( \frac{X}{0.5L} \right)^2 \right] \left[ 1 - \left( \frac{Z}{T} \right)^2 \right] \quad (30)$$

For the forced oscillation in sway mode, the comparison between linear and blended method is shown in Fig. 4. It can be seen that sway into sway and sway into roll compare well, while sway into yaw does not. In sway into yaw mode, the linear complex force is significantly bigger than the radiation force predicted by blended method. In the (Salvesen et al., 1970) strip theory model, the cross coupling radiation force is coming from the term  $i \omega_e U \left( \varphi_j^u n_i - \frac{\varphi^o R_j}{i \omega_e} m_i \right)$  of Equation (28), for  $i = 6, j = 2$  this term reduces to  $-U \varphi_2 m_6$ . This forward speed correction is only due to dynamic pressure in moving frame. Body boundary forward speed correction would be zero, because rotation of axis-system is zero. For the reduced term  $-U \varphi_2 m_6 = U \varphi_2 n_2$ , effectively the partial derivative has been performed only on  $n_6$ . While in the blended method, the partial derivative  $U \frac{\partial}{\partial x}$  is performed on  $\varphi_{R_2}$  and  $n_6$  remains unchanged. Numerically,  $\iint U \frac{\partial \varphi_{R_2}}{\partial x} dS \neq \iint U \varphi_{R_2} n_2 dS$ . This explains the

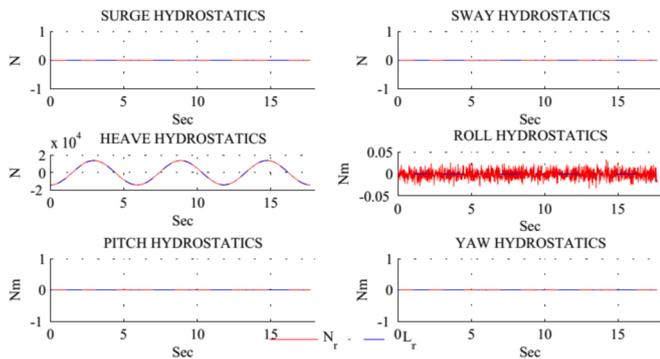


Fig. 6. 0.05 m Forced heave hydrostatic, linear  $L$  vs quasi-nonlinear  $N$ ,  $\omega_e = 1.066$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

difference in sway into yaw mode shown in Fig. 1 between linear and blended method.

For the forced heave oscillation, the comparison between linear and blended method is shown in Figs. 5 and 6, respectively for resulting radiation and hydrostatic heave forces. The radiation and hydrostatic forces in all the modes compare well except for heave into pitch as in the linear model the partial derivative  $U_{\partial x}$  is performed only on  $n_5$  instead of taking the partial derivative  $U_{\partial x}$  of  $\phi R_3$ .

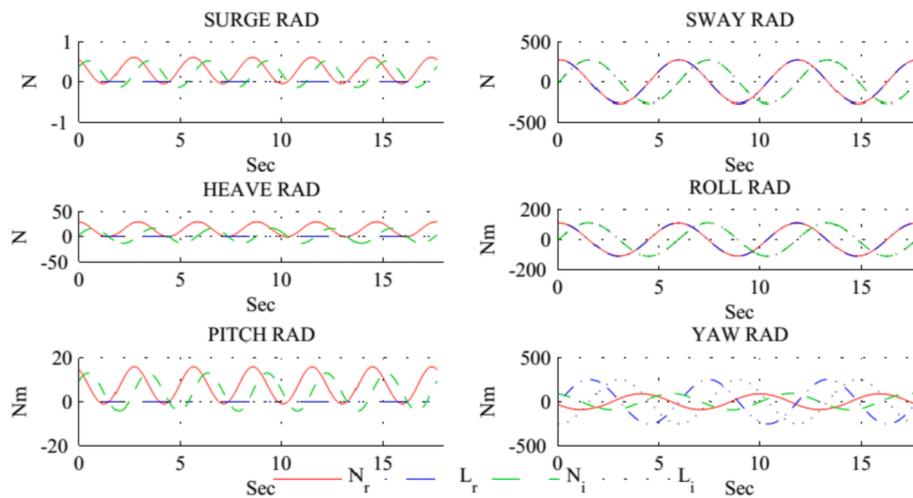


Fig. 7. 0.05 rad Forced roll radiation, linear  $L$  vs quasi-nonlinear  $N$ ,  $\omega_e = 1.066$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

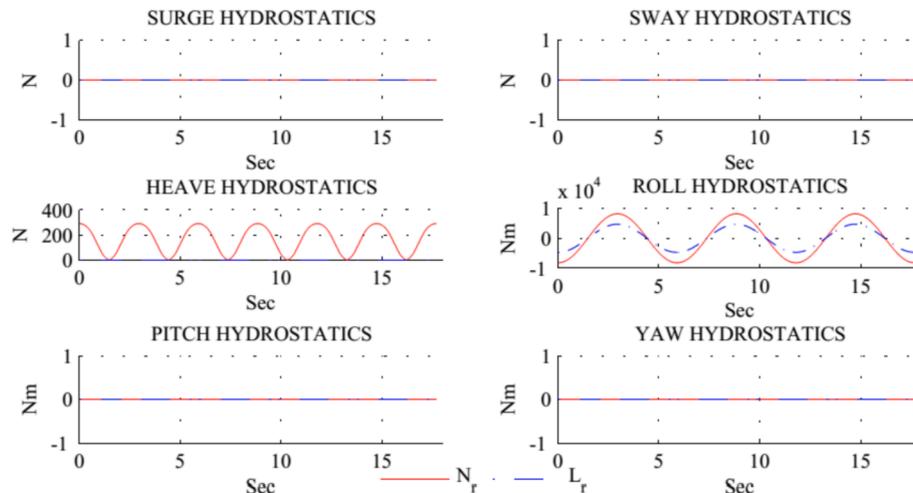


Fig. 8. 0.05 rad Forced roll hydrostatic, linear  $L$  vs quasi-nonlinear  $N$ ,  $\omega_e = 1.066$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

Similarly the forced roll oscillation, comparison between linear and blended method is shown in Figs. 7 and 8 respectively for resulting radiation and hydrostatic forces. In Fig. 8, the roll and heave hydrostatic forces are not in good agreement. The Wigley-hull being non wall-sided is the reason for this small difference. When forced roll motion is given to a non-wall-sided body about a fixed axis, the vertical hydrostatic would not be in balance, unless the body is allowed to heave. In Fig. 7, the radiation forces of roll into roll and roll into sway are in good agreement. However, in case of roll into yaw, the blended method has predicted lower radiation force. The forward speed correction due to the body boundary condition in rotating-frame of reference is zero because of  $\phi_s = 0$ . In (Salvesen et al., 1970) strip theory, the only forward speed correction captured is due to linearized dynamic pressure in the moving coordinate. It can be shown again using similar reasoning already discussed for the case of sway into yaw radiation mode, that the partial derivative  $U_{\partial x}$  is performed only on  $n_6$  instead of taking the partial derivative  $U_{\partial x}$  of  $\phi_4$  in (Salvesen et al., 1970) strip theory.

In Figs. 9 and 10, the comparison between linear and blended method is shown for resulting radiation and hydrostatic forces respectively for the forced pitch oscillation. In Fig. 10, the pitch into heave force is not in good agreement. The reason for this vertical unbalanced hydrostatic is the non-wall-sided hull form is forced to pitch about a fixed axis without allowing to heave. This vertical unbalanced hydrostatic will vanish if the ship is allowed to heave freely. In Fig. 9 the

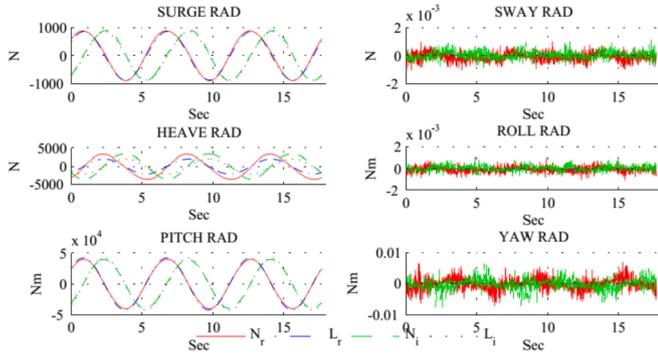


Fig. 9. 0.05 rad Forced pitch radiation, linear L vs quasi-nonlinear N,  $\omega_e = 1.066$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

radiation force pitch into pitch and pitch into surge are in good agreement. But the blended method has predicted higher force in pitch into heave mode. In the case of the linear model, the cross coupling radiation force is a result of the term  $i\omega_e U \left( \varphi_j^H n_i - \frac{\varphi_i^R}{i\omega_e} m_i \right)$  of Equation (28). For  $i = 3, j = 5$  this term reduces to  $i\omega_e U \varphi_5^H n_3 = U \varphi R_3 n_3$ . This forward speed correction is coming from the body boundary condition in the rotating axis-system. It is observed that the forward speed correction arising from dynamic-pressure due to moving frame of reference could not be captured appropriately in (Salvesen et al., 1970) linear strip-theory model. However, the blended method is adequately taking care of both the forward speed corrections arising from the rotating body boundary condition and dynamic pressure in moving frame of reference. This is the reason that the blended method is predicting higher pitch into heave radiation force than (Salvesen et al., 1970) linear strip theory.

For the forced yaw oscillation in Fig. 11, the comparison is shown between the linear strip theory and blended method. The resulting radiation forces for the modes yaw into sway and yaw into roll are not in good agreement, the blended method predicts higher forces. The reason for these higher cross coupling terms can be explained based on the argument presented in the previous paragraph for the pitch into heave mode. In linear strip theory model, the forward speed corrections resulting from dynamic-pressure in moving frame have been almost neglected in yaw into roll and yaw into sway modes. The blended method by comparison, partially accounts for the forward speed corrections in the moving frame after setting  $\varphi_s = 0$ .

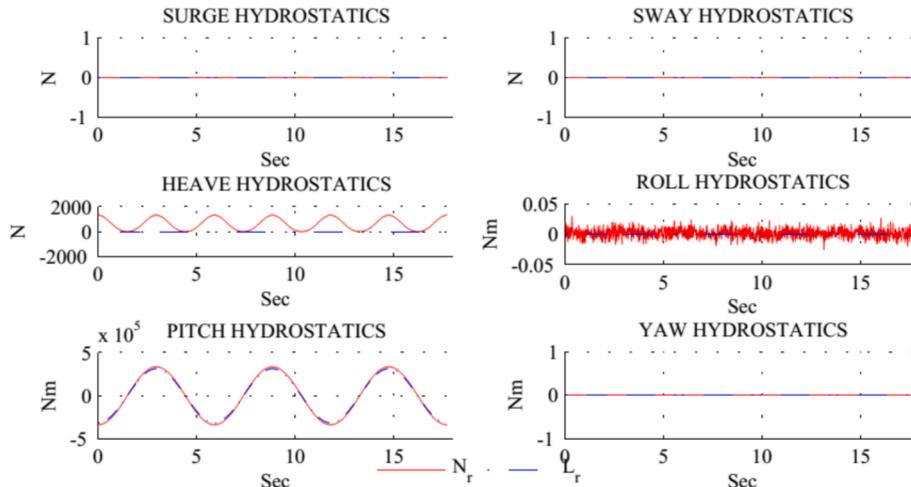


Fig. 10. 0.05 rad Forced pitch hydrostatic, linear L vs quasi-nonlinear N,  $\omega_e = 1.066$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

#### 4.2. Blended method for diffraction and Froude-Kriloff forces in moving frame

The scattered or diffracted velocity potential  $\varphi_D$  is obtained by solving the Laplace Equation (31) with the free surface and body boundary conditions 22 and 33. The scattering phenomenon may be described as the restrained body in the steady moving frame with the velocity  $U \neq 0$  or in the earth fixed coordinate with the velocity  $U \equiv 0$  exposed to incident waves. There is no rotation of the axis-system; the forward speed correction arising in the body boundary conditions should vanish. This can be seen from Equation (34), where the term  $[(W \cdot \nabla)X] \cdot n$  vanishes explicitly for  $X = 0$  or for the non-rotating body. The nonlinear body boundary condition for the scattered velocity potential including the steady perturbation potential would be as follows:

$$\nabla^2 \Phi(x, y, z, t) = 0 \tag{31}$$

$$\frac{D}{Dt} S(x, y, z, t) = 0 \quad \text{at } z = \zeta \tag{32}$$

where  $\nabla^2$  is Laplace operator and D is the total/cumulative derivative.

$$\frac{\partial \varphi_D}{\partial n} = \nabla \varphi_D \cdot n = -\nabla \varphi_I \cdot n \tag{33}$$

$$\frac{\partial \varphi_D}{\partial n} = \nabla \varphi_D \cdot n = -\nabla \varphi_I \cdot n + [(W \cdot \nabla)X - (X \cdot \nabla)W] \cdot n \tag{34}$$

In the steady moving frame, the terms involving W and X would be identically zero if  $\varphi_s$  is neglected. Effectively Equation (34) reduces to Equation (33). For the diffraction problem in a moving frame of

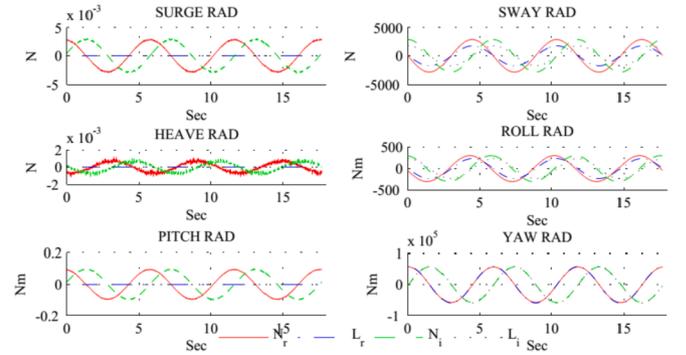


Fig. 11. 0.05 rad Forced yaw radiation, linear L vs quasi-nonlinear N,  $\omega_e = 1.066$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

reference, the only forward speed correction would be from dynamic-pressure Equation (10) or its linearized version in Equation (20) after substituting  $\varphi_D$  for  $\varphi_R$ . For the blended method, the diffraction force is as follows:

$$F_j^D = -\rho \iint \left[ \frac{\partial \varphi_D}{\partial t} + W \cdot \nabla \varphi_D + \frac{1}{2} \nabla \varphi_D \cdot \nabla \varphi_D \right] n_j dS \quad (35)$$

The surface integral in Equation (35) is for the instantaneous submerged body. If the steady perturbation velocity potential is neglected  $\varphi_S$ , the term  $W \cdot \nabla \varphi_D$  in Equation (35), may be simplified as  $-\hat{U}i \cdot \nabla \varphi_D = -U \frac{\partial \varphi_D}{\partial x}$ . In the blended method, the partial derivative  $-U \frac{\partial}{\partial x}$  is performed numerically using radial basis function.

$$F_{D_j} = \rho \int_{L/2}^{L/2} \left[ \int_{S_o} \nabla \varphi_I \cdot n_j \varphi_R d\delta \right] dx \quad (36)$$

In case of the linear strip-theory model, the scattering velocity potential is not known explicitly. The linear diffraction forces are calculated from the incident and radiation velocity potentials after applying Green's second identity on the surface integral involving all the bounding surfaces of the linear boundary value problem, as shown in Haskind-relation 36 for zero forward speed. For the problem involving forward speed, one may assume that the speed dependent terms in the linearized free surface boundary condition are of higher-order and therefore could be neglected. The linear approximation of Equation (35) may be rewritten as:

$$F_j^D = -\rho \iint [i\omega_e \varphi_D - \hat{U}i \cdot \nabla \varphi_D] n_j dS \quad (37)$$

$$\nabla \varphi(x, y, z, t) \cdot n = V \cdot n \quad \text{at } S_o \quad (38)$$

where  $V$  denotes ships unsteady oscillatory velocity in the moving frame.

Considering body boundary condition 38, the first term  $[i\omega_e \varphi_D] n_j$  of Equation (37) can be replaced by  $(-\nabla \varphi_{R_j} \cdot n) \varphi_D$ . The equality  $\nabla \varphi_{R_j} \cdot n \varphi_D = \nabla \varphi_D \cdot n \varphi_{R_j}$  for the integrand of surface integral holds because of the second identity of Green's theorem applied to all the bounding surface of the boundary value problem. From the body boundary condition 33 of the scattering problem  $\nabla \varphi_D \cdot n = -\nabla \varphi_I \cdot n$ , where  $\varphi_I$  is incident wave velocity potential defined in Equation (39). The first term  $[i\omega_e \varphi_D] n_j$  in Equation (37) represents exactly the Haskind-relation of Equation (36) for zero speed.

$$\varphi_I = \frac{ig\alpha}{\omega} e^{kz} e^{-ik(x\cos\beta + y\sin\beta)} e^{i\omega_e t} \quad \text{at } z = \zeta \quad (39)$$

where  $k = 2\pi/\lambda$  is the wave number and  $\beta$  is the heading angle.

The second term of Equation (37) may be written in vector notation as  $[-\hat{U}i \cdot \nabla \varphi_D] n$ . The equality  $\iint [-\hat{U}i \cdot \nabla \varphi_D] n dS = \iint [\varphi_D (n \cdot \nabla)] (-\hat{U}i) dS$  can be shown by using properties of flow vector  $W$  and theorem given in appendix A of (Ogilvie and Tuck, 1969). The term  $[\varphi_D (n \cdot \nabla)] (-\hat{U}i)$  is identically zero for surge sway and heave modes of motion. For roll, pitch and yaw, the second term of Equation (37) may be written in vector form as  $[-\hat{U}i \cdot \nabla \varphi_D] (r \times n)$ . Again, using (Salvesen et al., 1970) theorem, one can show the equivalence  $\iint [-\hat{U}i \cdot \nabla \varphi_D] (r \times n) dS = \iint \varphi_D [(r \times n) \cdot \nabla] (-\hat{U}i) + (n \times -\hat{U}i) dS$ , which is zero for roll, and in pitch and yaw modes are  $U\varphi_D n_3 = -U\varphi_D n_2$  respectively.

Using body boundary condition 33, and Green's second identity for surface integral over all the linearized bounding surfaces, the forward speed correction can be written in terms of radiation and incident velocity potentials for pitch and yaw as  $\frac{U}{i\omega_e} (\nabla \varphi_I \cdot n) \varphi_{R_3}$  and  $-\frac{U}{i\omega_e} (\nabla \varphi_I \cdot n) \varphi_{R_2}$ , respectively. These are the same forward speed corrections as given by (Salvesen et al., 1970) derived from the body boundary condition due to rotation of axis-system. The forward speed corrections derived here are arising from dynamic pressure in moving

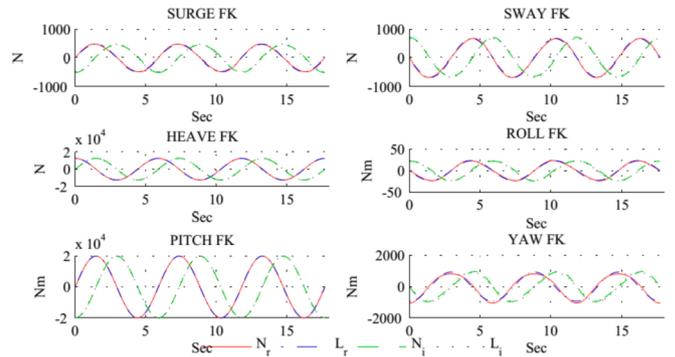


Fig. 12. Froude-Kriloff forces, Linear L vs quasi-nonlinear N,  $\alpha = 0.05m$ ,  $\beta = 130^\circ$ ,  $\omega = 1$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

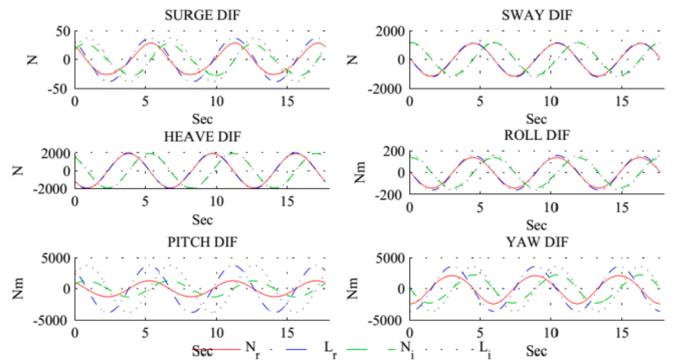


Fig. 13. Diffraction forces, Linear L vs quasi-nonlinear N,  $\alpha = 0.05m$ ,  $\beta = 130^\circ$ ,  $\omega = 1$  rad/s,  $V = 1$  m/s,  $Fn = 0.0714$ , Wigley-hull.

frame of reference.

Similarly, the Froude-Kriloff forward speed corrections are also from the dynamic-pressure due to the moving frame. It can be derived analytically from incident velocity potential as given in Equation (39). The comparison between blended and (Salvesen et al., 1970) linear strip-theories for Froude-Kriloff and diffraction forces are shown in Figs. 12 and 13 respectively. The Froude-Kriloff forces compare well in all six modes. However, the diffraction forces are significantly different for the pitch and yaw modes. The reason for the difference is the numerical inequality between the term  $-U \frac{\partial \varphi_D}{\partial x}$  and the terms used in the pitch and yaw modes  $\frac{U}{i\omega_e} (\varphi_I \cdot n) \varphi_{R_3}$  and  $-\frac{U}{i\omega_e} (\varphi_I \cdot n) \varphi_{R_2}$  respectively.

#### 4.3. Comparison between blended method and experimental data

As shown in the previous two sections, the cross coupling terms for radiation forces resulting from the blended method developed in this study, do not compare well with Ogilvie (Ogilvie and Tuck, 1969) and Salvesen (Salvesen et al., 1970) results. Similarly, diffraction forces in pitch and yaw modes also shows disagreement with Ogilvie (Ogilvie and Tuck, 1969) and Salvesen (Salvesen et al., 1970) findings. In this section the blended method for the forward speed correction is compared with the available experimental data.

Journee (I. J and Journee, 1992) conducted experiments on four different Wigley-hull forms for three different ship speeds corresponding to Froude numbers  $F_n = 0.2$ ,  $F_n = 0.3$  and  $F_n = 0.4$  in head seas. The author also gave his judgement on reliability of experimental data. A comparison between blended method and experimental data for Wigley-hull-III for the radiation and wave loads in head seas has been demonstrated in this section. For the forced oscillations, the experimental data is given in terms of added-mass and damping coefficients. For the purpose of comparison with the blended method, a complex radiation force time series is generated using the relationship  $\hat{F}_R(t) =$

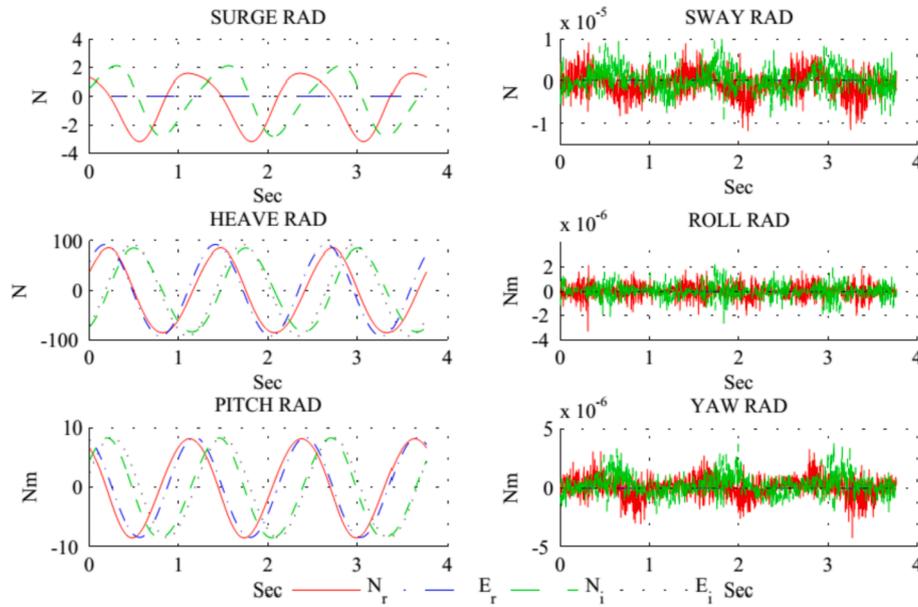


Fig. 14. 0.05m Forced heave radiation, experimental  $E$  vs blended  $N$ ,  $\omega_e = 5.0076$  rad/s,  $V = 1.0848$  m/s, Wigley-hull-III.

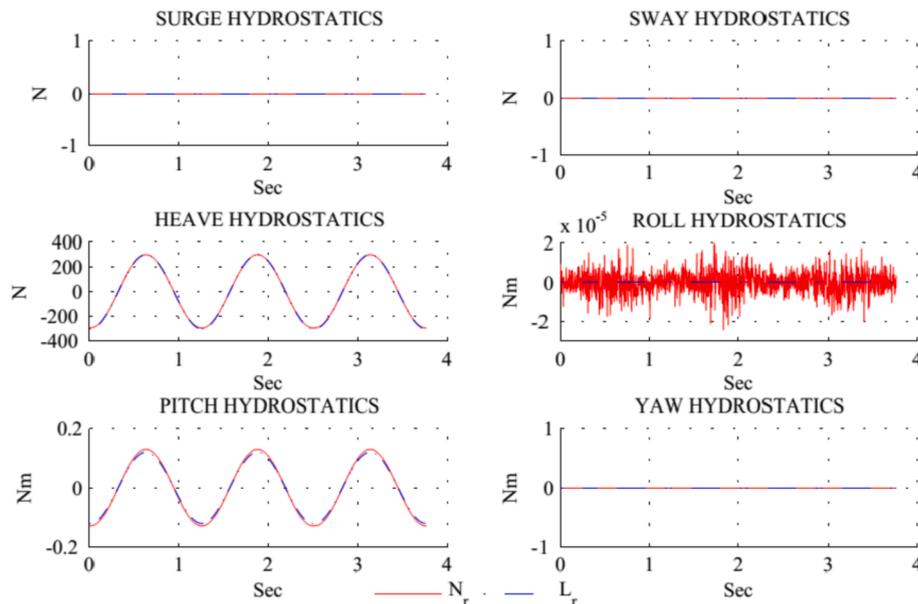


Fig. 15. 0.05m Forced heave hydrostatic, linear  $L$  vs blended  $N$ ,  $\omega_e = 5.0076$  rad/s,  $V = 1.0848$  m/s, Wigley-hull-III.

$(\omega_e^2 A - i\omega_e B)e^{i\omega_e t}$ . Similarly, the wave load data is given as normalized amplitude and phase, and a complex wave load force series is generated using the relationship  $\tilde{F}_D(t) = |F| e^{i(\omega_e t + \theta)}$ . The offsets for the Wigley-hull-III are obtained using Equation (40) as given in (I. J and Journee, 1992).

$$Y = -\frac{B}{2} \left[ 1 - \left( \frac{X}{0.5L} \right)^2 \right] \left[ 1 - \left( \frac{Z}{T} \right)^2 \right] \left[ 1 + 0.2 \left( \frac{Z}{T} \right)^2 \right] \quad (40)$$

Radiation and wave loads calculated by the blended method are in good agreement with the experimental data given in (I. J and Journee, 1992), for the various forward speeds. For the forced heave motion given to Wigley-hull-III moving with Froude number  $F_n = 0.2$ , the resulting radiation forces are comparing well as shown in Fig. 14. A similar comparison for the forced pitch motion and Froude number  $F_n = 0.4$  is shown in Fig. 16. The cross coupling terms obtained from the blended method developed in the present research and experiments by

Journee (I. J and Journee, 1992) are in good match as seen in Figs. 14 and 16. The comparison between linear and blended method, for the accompanying hydrostatic forces is shown in Figs. 15 and 17. A good agreement between the blended method and experiments by Journee (I. J and Journee, 1992) for the wave loads with forward speed corresponding to Froude number  $F_n = 0.3$  is found as shown in Fig. 18. The wave loads shown in Fig. 18 are the sum of diffraction, Froude-Kriloff and hydrostatic forces in the head seas. In the wave loads, the Froude-Kriloff is the dominating force, the diffraction and hydrostatic forces are approximately 20 and 1 percent of the total wave loads as found numerically using the blended method.

It may be noticed that in case of cross coupling terms, the blended method is comparing well with the experimental data and not with the results obtained using slender body forward speed corrections suggested by Ogilvie et al. (Ogilvie and Tuck, 1969). To find the reason for this somewhat surprising outcome, one needs to explore the underlying

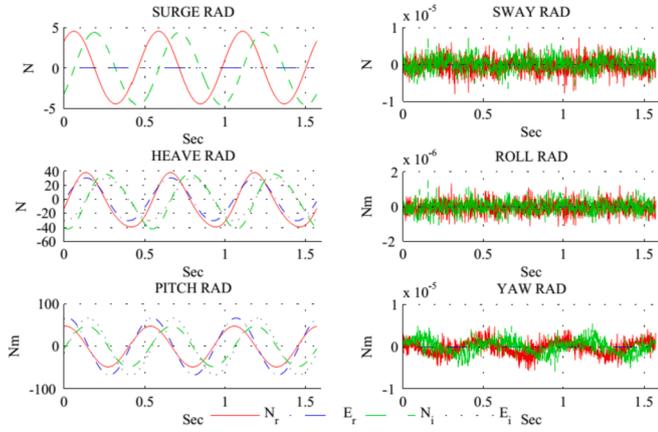


Fig. 16. 0.0262 rad Forced pitch radiation, experimental  $E$  vs blended  $N$ ,  $\omega_e = 12$  rad/s,  $V = 2.1696$  m/s, *Wigley-hull-III*.

assumptions, conditions and properties of different functions used in the derivation to prove the equality of the integrals  $\iint \hat{U}i \cdot \nabla(\varphi_{R_3}^o + \varphi_j^u)(r \times n) dS = \iint (\varphi_{R_3}^o + \varphi_j^u)[(r \times n \cdot \nabla)\hat{U}i + \hat{U}i \times n]dS$  of a surface integral by Ogilvie et al. (Ogilvie and Tuck, 1969). This equality does not hold in general for all surface integrals or their integrands.

### 5. Discussion

For the mode, heave into heave, the first component of the forward speed correction is evidently zero. After integrating along the ship length, the net effect of the second component of the forward speed correction  $U \frac{\partial \varphi_{R_3}}{\partial x}$  also becomes zero. The velocity potential  $\varphi_{R_3}$  is an even function with respect to  $x$ , its derivative  $\frac{\partial \varphi_{R_3}}{\partial x}$  would be an odd function. The  $n_3$ , a component of the normal  $n$  is an even function with respect to  $x$ . The product  $\frac{\partial \varphi_{R_3}}{\partial x} n_3$  is an odd function with respect to  $x$ . Therefore its integration along the longitudinal axis of the ship is zero. Effectively, there is no forward speed correction in the mode heave into heave. The slender-body theory, experiments and the blended method are all in good agreement. However in the blended method, the derivative  $\frac{\partial \varphi_{R_3}}{\partial x}$  is calculated to predict the nodal pressure inclusive of the forward speed

effects. After integration of the pressures along the ship length, the forward speed effects vanish.

In case of pitch into heave, the forward speed correction comprises both of the components. In the blended method, the integrand of this correction may be expressed as the product  $U \left( \frac{\partial \varphi_{R_5}}{\partial x} + \frac{\partial \varphi_{R_3}}{\partial x} \right) n_3$ .  $\varphi_{R_3}$  is the correction due to body boundary condition and the derivative with respect to  $x$  accounts for the evaluation of pressure in the moving frame. The second term of the integrand  $\frac{\partial \varphi_{R_3}}{\partial x} n_3$  is an odd function with respect to  $x$ , the net effect vanishes after integrating along the ship length. Thus in the blended method, the net effective forward speed correction is proportional to the surface integral of the term  $\frac{\partial \varphi_{R_5}}{\partial x} n_3$ . By assuming  $\varphi_{R_5} \approx -x\varphi_{R_3}(y, z)$ , one can show that this is identically equal to  $-\varphi_{R_3} n_3$ , which is the same as suggested by slender body theory. This approximation holds only if  $\varphi_{R_3}$  is assumed to be the function  $y, z$  only. There is no explicit or implicit dependence on  $x$ , while this is not the case in the blended method and in the real physics of the problem. The two-dimensional  $\varphi_{R_3}(y, z)$  is transformed into its corresponding three-dimensional quantity by using an appropriate radial basis function. Then the derivative  $\frac{\partial \varphi_{R_3}(x, y, z)}{\partial x} n_3$  is calculated on a three-dimensional quantity. This approach is closer to the real physics of the

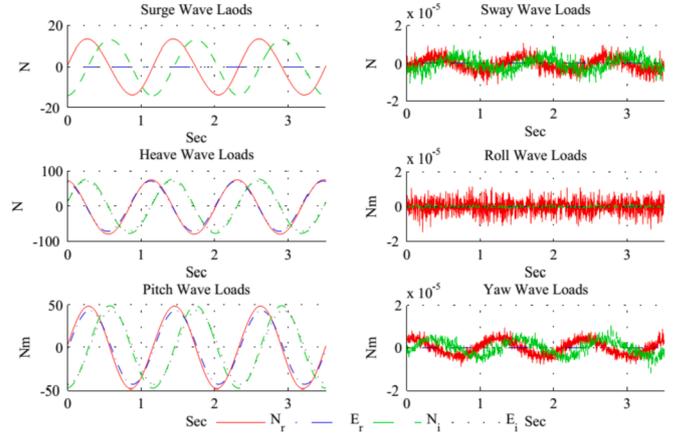


Fig. 18. Wave loads for  $\alpha = 0.023m$ , experimental  $E$  vs blended  $N$ ,  $\beta = 180^\circ$ ,  $\omega = 3.4259$  rad/s,  $\omega = 5.4246$  rad/s,  $V = 1.6272$  m/s, *Wigley-hull-III*.

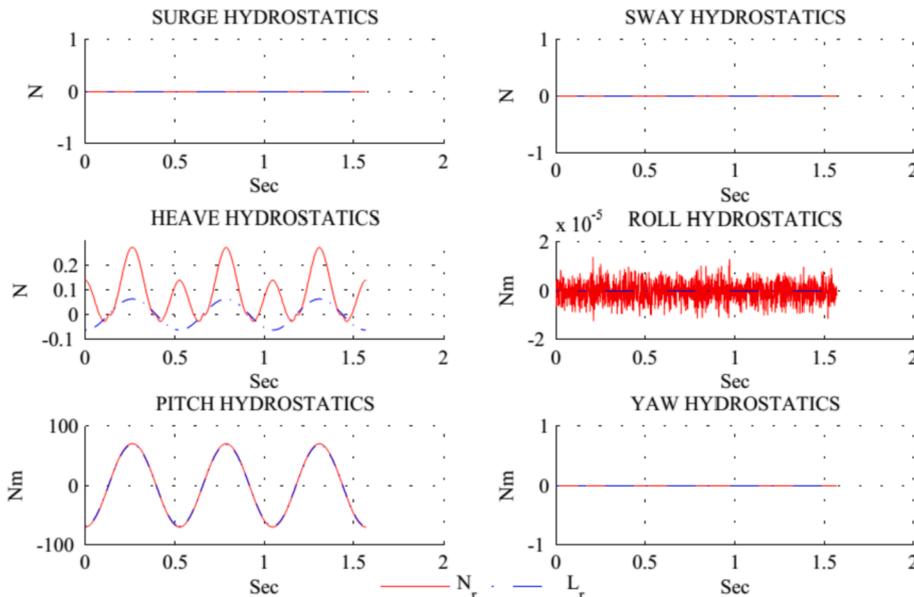


Fig. 17. 0.0262 rad Forced pitch hydrostatic, linear  $L$  vs blended  $N$ ,  $\omega_e = 12$  rad/s,  $V = 2.1696$  m/s, *Wigley-hull-III*.

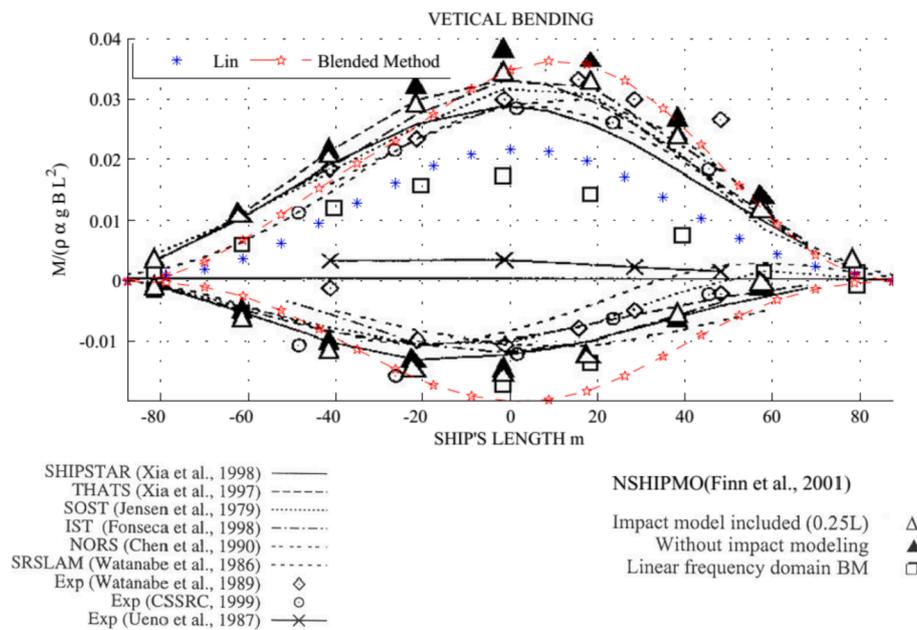


Fig. 19. Nonlinear sagging ( $M \geq 0$ ) and hogging ( $M \leq 0$ ) bending moments of SR175 container Ship for  $\lambda = 1.2L$ ,  $F_n = 0.25$ ,  $\alpha = L/60$ , old results extracted from (Finn et al., 2001).

problem. That is why the results from the blended method are in good agreement with the experimental results.

ISSC (ISSC, 2000) and Finn et al. (2001) have shown comparisons of vertical bending moment on SR175 container ship with various numerical calculations and experimental data. Their results are reproduced using blended method and are superimposed on them as shown in Fig. 19. It is observed that the maximum sagging moment is approximately twice the value of maximum hogging moment. These results are obtained by approximating the sectional weight distribution with the corresponding sectional buoyancy forces. The results from the blended method are comparing well with the results of other researchers.

## 6. Conclusions

In this paper the difference between the forward speed correction used for time simulation in the blended method and the strip-theory presented by Salvesen et al. (1970) in the frequency domain has been explained. Using the previous work of Newman (1979) and Ogilvie et al. (Ogilvie and Tuck, 1969), a basis is developed for the comparison between the blended method and the frequency domain linear model. The calculated results for the steady and unsteady problems are compared with published model test results. Good agreement is obtained, which validates the present numerical method. The comparison of the calculated results with the experimental data shows that the blended method is effective and feasible. The present method can be further extended to dynamic structural design and maneuvering problems.

## Author contribution

M. Saeed Khalid: Conceptualization, Methodology, Software and Validation Salman Nisar: Writing – Original Draft Preparation, Visualization Sohaib Zia Khan: Writing – Review and Editing Muhammad Ali Khan: Writing – Review and Editing Armin W. Troesch: Supervision, Resources.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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