



# A new guidance law for trajectory tracking of an underactuated unmanned surface vehicle with parameter perturbations

Haibin Huang<sup>a</sup>, Mian Gong<sup>a,b</sup>, Yufei Zhuang<sup>a,\*</sup>, Sanjay Sharma<sup>c</sup>, Dianguo Xu<sup>b</sup>

<sup>a</sup> Harbin Institute of Technology, Weihai, PR China

<sup>b</sup> Harbin Institute of Technology, Harbin, PR China

<sup>c</sup> University of Plymouth, Plymouth, United Kingdom

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## ABSTRACT

This paper proposes a novel trajectory tracking controller for an underactuated unmanned surface vessel (USV) with multiple uncertainties and input constraints. Unlike previous dynamic surface control (DSC) methods that utilize a first-order filter to obtain derivatives and avoid “explosion of complexity”, we introduce nonlinear tracking differentiators (NTDs) that achieve satisfactory differential performance and fast tracking response and control the vessel to converge to the pseudo-yaw angle and surge. First, a new guidance law for yaw angle and surge is constructed. Second, inner and outer disturbances caused by uncertainties can be observed by reduced-order extended state observers. With the new guidance law, the design process of the controller is simplified and easy to implement. The simulation results show the trajectory tracking error can be stabilized to a certain extent under the parameter perturbations and other uncertainties, which verified the effectiveness of the proposed algorithm.

## 1. Introduction

A great amount of research in the field of unmanned surface vessel motion control has been witnessed in recent decades. Unlike the path-following problem, which only involves a spatial constraint, the aim of trajectory tracking is forcing the vehicle to follow a time-varying trajectory. Moreover, these vessels are usually underactuated, which means that the direct force for sway control is unavailable. All of these factors increase the difficulty of controller design for USV. The plane motion of unmanned underactuated underwater vehicles is similar to that of USVs. In previous trajectory tracking research, exponential asymptotical stability could be achieved with a backstepping design in Godhavn et al. (1998) and Pettersen and Nijmeijer (1999). In Lefeber (2000) and Lefeber et al. (2003), the tracking control strategies proposed were based on a cascading approach. In Pettersen and Nijmeijer (2001), the ship mode was transformed into a chained form system, and related experiments were carried out. In Zhong-Ping Jiang (2002), the inherent cascade structure of ship dynamics was analysed, and the tracking controller was designed using Lyapunov's direct method. Based on Barbalat's lemma and backstepping techniques, the controller can achieve global asymptotical and exponential tracking in Dong and Guo (2005). The early studies of trajectory tracking based on the cascade and back-stepping approaches require a persistent excitation (PE)

condition, i.e., a straight-line reference trajectory cannot be tracked. To resolve the PE problem, Do et al. (2015) and Do et al. (2002) relaxed the restricted conditions using Lyapunov's direct method.

Generally, there are two types of trajectory tracking methods: direct and indirect. Direct methods consider the problem as a stabilization problem for tracking error equations and apply adaptive, back-stepping or sliding mode control directly to determine the control law, while indirect methods divide the problem into a two-step process. First, the guidance laws, i.e., the desired surge, sway or yaw angle, are obtained. Then, the actual inputs are designed so that the surge, sway or yaw angle follow the desired pseudo-variables. Most of the literature on this topic adopts direct methods. The control laws obtained using this approach often result in complex algorithms and a large amount of calculations. In Ashrafiuon et al. (2008), the desired surge and sway velocities are designed as guidance laws. Then, sliding mode controllers are applied to guarantee asymptotic convergence to the desired values. However, the desired values proposed in Ashrafiuon et al. (2008) are special cases. For this, Yu et al. (2012) suggests that the desired surge and sway should relate to the position errors and be modified. Motivated by Yu et al. (2012), the desired surge and sway velocities are improved using hyperbolic tangent functions in Elmokadem et al. (2016a,b). Cui et al. (2017) proposed an integral sliding mode controller based on multiple-input and multiple-output extended state

\* Corresponding author.

E-mail address: [yufeizhuang9@gmail.com](mailto:yufeizhuang9@gmail.com) (Y. Zhuang).

observer to estimate the linear and angular velocities and unknown external disturbances. Terminal sliding mode control is adopted to increase the robustness of the system. Since the desired sway velocity is not forced by the control input directly, the correct heading of the USV is difficult to guarantee. In Chwa, D. (2011), a guidance law is proposed to solve the trajectory tracking problem. However, the reference trajectory is generated by a virtual ship that depends on exact model parameters. For fluctuating parameters, the lateral velocity and reference will deviate because control in the sway direction is not available.

To obtain a simpler control structure and ensure the correct heading, we must develop a guidance law for the surge and yaw angle. Furthermore, the law should track various reference trajectories without restrictions on the initial conditions. Finally, since the actual USV has to meet control input constraints, the influence of input saturation must be considered.

This paper is organized as follows. Section 2 states the underactuated USV mode and trajectory tracking control objective. Section 3 proposes a trajectory tracking guidance law based on surge and yaw angle. In section 4, controllers are proposed based on dynamic surface control (DSC) and active disturbance rejection control (ADRC). Section 5 provides a simulation to illustrate the proposed methods. Finally, conclusions are summarized in section 6.

## 2. Problem formulation

This section presents the kinematic and dynamic models of an underactuated USV with three degrees of freedom.

To study an underactuated surface vessel moving in the horizontal plane, the kinematic and dynamic models can be described with 3 degrees of freedom. Detailed derivation procedures can be found in Fossen (2002). The kinematic equations for an USV can be described as:

$$\begin{cases} \dot{x} = u \cos \psi - v \sin \psi \\ \dot{y} = u \sin \psi + v \cos \psi \\ \dot{\psi} = r \end{cases} \quad (1)$$

where  $x$  and  $y$  represent the inertial coordinates of the USV.  $\psi$  is the yaw angle.  $u$  and  $v$  denote the surge and sway velocities, respectively, and  $r$  is the yaw velocity.

Considering the internal parameters perturbations, unmodeled dynamics and external environmental disturbances, the dynamic equations can be modified as follows:

$$\begin{aligned} \dot{u} &= \frac{m_{22} + \Delta m_{22}}{m_{11} + \Delta m_{11}} vr - \frac{d_{11} + \Delta d_{11}}{m_{11} + \Delta m_{11}} u + \frac{\tau_u + \Delta \tau_u}{m_{11} + \Delta m_{11}} + \Delta_u = \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{\tau_u}{m_{11}} \\ &\quad + F_u \\ \dot{v} &= -\frac{m_{11} + \Delta m_{11}}{m_{22} + \Delta m_{22}} ur - \frac{d_{22} + \Delta d_{22}}{m_{22} + \Delta m_{22}} v + \frac{\Delta \tau_v}{m_{22} + \Delta m_{22}} \Delta_v = -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v + F_v \\ \dot{r} &= \frac{(m_{11} + \Delta m_{11}) - (m_{22} + \Delta m_{22})}{m_{33} + \Delta m_{33}} uv - \frac{d_{33} + \Delta d_{33}}{m_{33} + \Delta m_{33}} r + \frac{\tau_r + \Delta \tau_r}{m_{33} + \Delta m_{33}} + \Delta_r \\ &= \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{\tau_r}{m_{33}} + F_r \end{aligned} \quad (2)$$

Where  $\tau_u$  and  $\tau_r$  denote the control inputs. Obviously, there is not a control force in the sway direction, therefore, the USV model is an underactuated system. Positive constant parameters  $m_{ii}$  and  $d_{ii}$  are the ship inertia including the added mass and hydrodynamic damping terms, respectively.  $\Delta m_{ii}$  and  $\Delta d_{ii}$  represent the parameter perturbations in the vessel model.  $\Delta \tau_u$ ,  $\Delta \tau_v$  and  $\Delta \tau_r$  are the unknown external environmental disturbances, and  $\Delta_u$ ,  $\Delta_v$  and  $\Delta_r$  denote other unmodeled dynamics.  $F_u$ ,  $F_v$ , and  $F_r$  are the lumped uncertainties.

**Assumption 1.** For the lumped uncertainties  $F = [F_u \ F_v \ F_r]^T$  in (2), there are positive constants  $\bar{F}_u$ ,  $\bar{F}_v$ ,  $\bar{F}_r$ , such that  $F_u$ ,  $F_v$ ,  $F_r$  satisfy  $\left| \frac{d^k F_i}{dt^k} \right| \leq \bar{F}_i$ ,  $i = u, v, r$ ,  $k = 0, 1$ .

The control objective of trajectory tracking is to design control laws

for the surge force  $\tau_u$  and yaw torque  $\tau_r$ , and ensure the USV tracks a desired, time-varying, smooth trajectory.

## 3. Trajectory tracking guidance law

The along-track error  $x_e$  and the cross-track error  $y_e$  can be defined as follows:

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} = \begin{bmatrix} \cos \psi_d & -\sin \psi_d \\ \sin \psi_d & \cos \psi_d \end{bmatrix}^T \begin{bmatrix} x - x_d(t) \\ y - y_d(t) \end{bmatrix} \quad (3)$$

where  $\psi_d(\omega) = \text{atan2}(y'_d(t), x'_d(t)) \in [-\pi, \pi]$  denotes the tangent angle of the trajectory.

Taking the time derivative of  $x_e$ , we have

$$\begin{aligned} \dot{x}_e &= (\dot{x} - \dot{x}_d) \cos \psi_d - \dot{\psi}_d (x - x_d) \sin \psi_d + (\dot{y} - \dot{y}_d) \sin \psi_d \\ &\quad + \dot{\psi}_d (y - y_d) \cos \psi_d = \dot{x} \cos \psi_d + \dot{y} \sin \psi_d - \underbrace{\left( \dot{x}_d \cos \psi_d + \dot{y}_d \sin \psi_d \right)}_{U_d} \\ &\quad + \dot{\psi}_d \left[ \underbrace{-(x - x_d) \sin \psi_d + (y - y_d) \cos \psi_d}_{y_e} \right] \end{aligned} \quad (4)$$

where  $U_d = \dot{x}_d \cos \psi_d + \dot{y}_d \sin \psi_d = \sqrt{\dot{x}_d^2 + \dot{y}_d^2} \cos(\psi_d + \alpha) = \sqrt{\dot{x}_d^2 + \dot{y}_d^2}$ . Note that  $\alpha = -\psi_d$ . Substituting kinematic equation (1) into (4), we have

$$\begin{aligned} \dot{x}_e &= u \cos \psi \cos \psi_d - v \sin \psi \cos \psi_d + u \sin \psi \sin \psi_d + v \cos \psi \sin \psi_d - U_d \\ &\quad + y_e \dot{\psi}_d = u \cos(\psi - \psi_d) - v \sin(\psi - \psi_d) - U_d + y_e \dot{\psi}_d \\ &= U \cos(\psi - \psi_d + \beta) - U_d + y_e \dot{\psi}_d \end{aligned} \quad (5)$$

where  $U = \sqrt{u^2 + v^2}$  represents total speed of the USV.  $\beta = \text{atan2}(v, u)$  represents the sideslip angle between the vessel heading and the orientation of the velocity vector.

Similarly, taking the time derivative of  $y_e$ , and substituting kinematic equation (1) into it, we have

$$\begin{aligned} \dot{y}_e &= U \sin(\psi - \psi_d + \beta) - x_e \dot{\psi}_d + (\dot{x}_d \sin \psi_d - \dot{y}_d \cos \psi_d) \\ &= U \sin(\psi - \psi_d + \beta) - x_e \dot{\psi}_d \end{aligned} \quad (6)$$

Finally, the time derivative of (3) becomes

$$\begin{cases} \dot{x}_e = U \cos(\psi - \psi_d + \beta) - U_d + y_e \dot{\psi}_d \\ \dot{y}_e = U \sin(\psi - \psi_d + \beta) - x_e \dot{\psi}_d \end{cases} \quad (7)$$

Up to this point, pseudo-variables  $\psi_{\text{pseudo}}$  and  $u_{\text{pseudo}}$  are designed to make the tracking errors  $x_e$  and  $y_e$  converge to zero.

**Proposition 1.** Let the desired yaw angle and surge be such that

$$\begin{cases} \psi_{\text{pseudo}} = \psi_d + \arctan\left(-\frac{y_e}{\Delta}\right) - \beta \\ u_{\text{pseudo}} = \sqrt{U_{\text{pseudo}}^2 - v^2} \end{cases} \quad (8)$$

where  $U_{\text{pseudo}} = \frac{(U_d - kx_e)\sqrt{y_e^2 + \Delta^2}}{\Delta}$ ,  $\Delta$  is the look-ahead distance.  $k > 0$  is the controller gain.

If the following error signals

$$E_\psi = \psi - \psi_{\text{pseudo}} \quad (9)$$

$$E_u = u - u_{\text{pseudo}} \quad (10)$$

converge to zero, the convergence of the position error is guaranteed.

**Proof** Consider the following Lyapunov function candidate:

$$V_1 = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2 \quad (11)$$

Differentiating (11), and substituting (7) into it, yields

$$\begin{aligned}\dot{V}_1 &= x_e \dot{x}_e + y_e \dot{y}_e = x_e [U \cos(\psi - \psi_d + \beta) - U_d + y_e \dot{\psi}_d] \\ &+ y_e [U \sin(\psi - \psi_d + \beta) - x_e \dot{\psi}_d] = -kx_e^2 - \frac{Uy_e^2}{\sqrt{y_e^2 + \Delta^2}}\end{aligned}\quad (12)$$

Since  $k > 0$ , it is obvious that  $\dot{V}_1 \leq 0$ . Therefore, it can be concluded that  $x_e$  and  $y_e$  converge to zero.

**Remark.** Different from the existing trajectory tracking guidance laws in Ashrafiun et al. (2008), Yu et al. (2012) and Elmokadem et al. (2016), the proposed guidance law has the following advantages.  $\psi$  and  $u$  can be controlled directly by the control inputs  $\tau_u$  and  $\tau_r$  as shown in dynamic equation (2). Therefore, the complexity of the control law can be reduced. Additionally, the correct yaw angle can be guaranteed. In other words, if  $u$  and  $v$  are selected as the desired values, the heading angle could be in the wrong direction. In Chwa, D. (2011), a guidance law is proposed for  $\psi$  and  $u$ . The main difference between our paper and Chwa, D. (2011) is the reference trajectory generation, with Chwa, D. (2011) relying on the model parameters of a virtual ship, as discussed in the introduction section. However, considering the limited performance of USV, the reference trajectory in this paper cannot be generated randomly.

#### 4. Trajectory tracking control law

The reduced-order extended state observer (ESO) proposed in Huang and Xue (2014) has been studied extensively in Shao and Wang (2015) and Liu et al. (2017). Compared with the traditional ESO, the reduced-order ESO yields faster response with the same bandwidth as that in Shao and Wang (2015). More importantly, the peaking phenomenon during the initial phase may result in performance deterioration and even lead to system instability. This issue can be totally eliminated using first-order reduced-order ESO. Therefore, in this paper, we use the reduced-order ESO to estimate the lumped uncertainties in dynamic equation (2).

$$\begin{cases} \dot{\hat{p}}_1 = -\omega_1 \hat{p}_1 - \omega_1^2 u - \omega_1 \left( \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{\tau_u}{m_{11}} \right) \\ \hat{F}_u = \omega_1 u + \hat{p}_1 \end{cases}\quad (13)$$

$$\begin{cases} \dot{\hat{p}}_2 = -\omega_2 \hat{p}_2 - \omega_2^2 r - \omega_2 \left( \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{\tau_r}{m_{33}} \right) \\ \hat{F}_r = \omega_2 r + \hat{p}_2 \end{cases}\quad (14)$$

where  $\omega_1 > 0$  and  $\omega_2 > 0$  are the observer bandwidths to be determined.

**Lemma 1.** Consider the proposed reduced-order ESO in (13) and (14), if Assumption 1 is satisfied, then

$|\tilde{F}_x(t)| \leq \zeta_k \frac{1}{\omega_k}$ , where  $t > t_0 + \max\left\{0, \frac{\ln \omega_k}{\omega_k}\right\}$ ,  $\tilde{F}_x = \hat{F}_x - F_x$ ,  $\zeta_k > 0$  is a positive constant, ( $x = u, r$ ;  $k = 0, 1$ ).

Lemma 1 implies that properly increasing the observer gains  $\omega_1$  and  $\omega_2$  can reduce the estimation error  $\tilde{F}_x$ . However, the observer gains cannot be too large. The increase in gains is subject to noise effects. Ultimately, this approach is a reasonable compromise based on estimation quality and the noise effect, see Liu et al. (2017).

To facilitate controller design, the trajectory tracking guidance law was established in the previous section. In this section, our motivation is very clear: design  $\tau_u$  and  $\tau_r$ , and make the USV yaw angle  $\psi$  and surge velocity  $u$  converge to pseudo-variables  $\psi_{\text{pseudo}}$  and  $u_{\text{pseudo}}$ , respectively. The guidance law is designed according to control inputs  $\tau_u$  and  $\tau_r$ , so many control approaches such as adaptive control, sliding mode control, and PID, can be employed easily. In this paper, to avoid direct derivation of intermediate variable values and ease implementation, the DSC technique is applied to the dynamic level.

It is well known that discrete systems are widely applied in actual engineering practice. To obtain high-quality tracking and differential signals from virtual control commands, nonlinear tracking differentiators (NTDs) were chosen to implement the functions of a first-order filter for DSC. NTD is a time-optimal solution in a discrete form proposed in Han (2009) and is given in the following form:

where  $h$  is the sampling period,  $R$  is an acceleration factor,  $h_0$  can be adjusted individually according to the noise signal.  $\text{fhan}(x_1(k) - v(k), \dot{x}_1(k), R, h_0)$  is

$$\begin{cases} \text{fhan} = \text{fhan}(x_1(k) - v(k), \dot{x}_1(k), R, h_0) \\ x_1(k+1) = x_1(k) + h \cdot \dot{x}_1(k) \\ \dot{x}_1(k+1) = \dot{x}_1(k) + h \cdot \text{fhan} \end{cases}\quad (15)$$

where  $h$  is the sampling period,  $R$  is an acceleration factor,  $h_0$  can be adjusted individually according to the noise signal.  $\text{fhan}(x_1(k) - v(k), \dot{x}_1(k), R, h_0)$  is

$$\begin{cases} d = Rh \\ d_0 = hd \\ y = x_1 + h \dot{x}_1 \\ a_0 = \sqrt{d^2 + 8R|y|} \\ a = \begin{cases} \dot{x}_1 + \frac{a_0 - d}{2} \text{sign}(y), & |y| > d_0 \\ \dot{x}_1 + \frac{y}{h}, & |y| \leq d_0 \end{cases} \\ \text{fhan} = \begin{cases} R \text{sign}(a), & |y| \leq d_0 \\ R \frac{a}{d}, & |y| > d_0 \end{cases} \end{cases}\quad (16)$$

More detailed discussion and study of NTD can be found in Guo and Zhao. (2011).

Choose equation (10) as the error surface and let  $u_{\text{pseudo}}$  pass through NTD to obtain the command signal  $u_{\text{cmd}}$  and its derivative  $\dot{u}_{\text{cmd}}$ :

$$\begin{cases} \text{fhan} = \text{fhan}(u_{\text{cmd}}(k) - u_{\text{pseudo}}(k), \dot{u}_{\text{cmd}}(k), R, h_0) \\ u_{\text{cmd}}(k+1) = u_{\text{cmd}}(k) + h \cdot \dot{u}_{\text{cmd}}(k) \\ \dot{u}_{\text{cmd}}(k+1) = \dot{u}_{\text{cmd}}(k) + h \cdot \text{fhan} \end{cases}\quad (17)$$

Take the derivative of

$$\tilde{E}_u = u - u_{\text{cmd}}\quad (18)$$

We have

$$\dot{\tilde{E}}_u = \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{\tau_u}{m_{11}} + \tilde{F}_u - \dot{u}_{\text{cmd}}\quad (19)$$

Therefore,  $\tau_u$  can be designed as

$$\tau_u = m_{11} \left( -\frac{m_{22}}{m_{11}} vr + \frac{d_{11}}{m_{11}} u - \tilde{F}_u + \dot{\tilde{E}}_u \right)\quad (20)$$

where  $\tilde{\tau}_u = -k_u \tilde{E}_u + \dot{u}_{\text{cmd}}$ ,  $k_u > 0$ .

Choose equation (9) as the error surface and let  $\psi_{\text{pseudo}}$  pass through NTD.

Based on Chwa, D. (2011), let us choose

$$r_{\text{pseudo}} = k_\psi \sin(\tilde{E}_\psi/2) + \psi_{\text{cmd}}\quad (21)$$

where  $k_\psi > 0$ ; then, let  $r_{\text{pseudo}}$  pass through NTD.

$$\begin{cases} \text{fhan} = \text{fhan}(r_{\text{cmd}}(k) - r_{\text{pseudo}}(k), \dot{r}_{\text{cmd}}(k), R, h_0) \\ r_{\text{cmd}}(k+1) = r_{\text{cmd}}(k) + h \cdot \dot{r}_{\text{cmd}}(k) \\ \dot{r}_{\text{cmd}}(k+1) = \dot{r}_{\text{cmd}}(k) + h \cdot \text{fhan} \end{cases}\quad (22)$$

Take the derivative of

$$\tilde{E}_r = r - r_{\text{cmd}}\quad (23)$$

Then, we have

$$\dot{\tilde{E}}_r = \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{\tau_r}{m_{33}} + \tilde{F}_r - \dot{r}_{\text{cmd}}\quad (24)$$

Therefore,  $\tau_r$  can be designed as

$$\tau_r = m_{33} \left( -\frac{m_{11} - m_{22}}{m_{33}} uv + \frac{d_{33}}{m_{33}} r - \tilde{F}_r + \dot{\tilde{E}}_r \right)\quad (25)$$

where  $\tilde{\tau}_r = -2\xi k_r \tilde{E}_r - k_r^2 \sin(\tilde{E}_\psi/2) + \dot{r}_{\text{cmd}}$ ,  $\xi > 0$ ,  $k_r > 0$ .

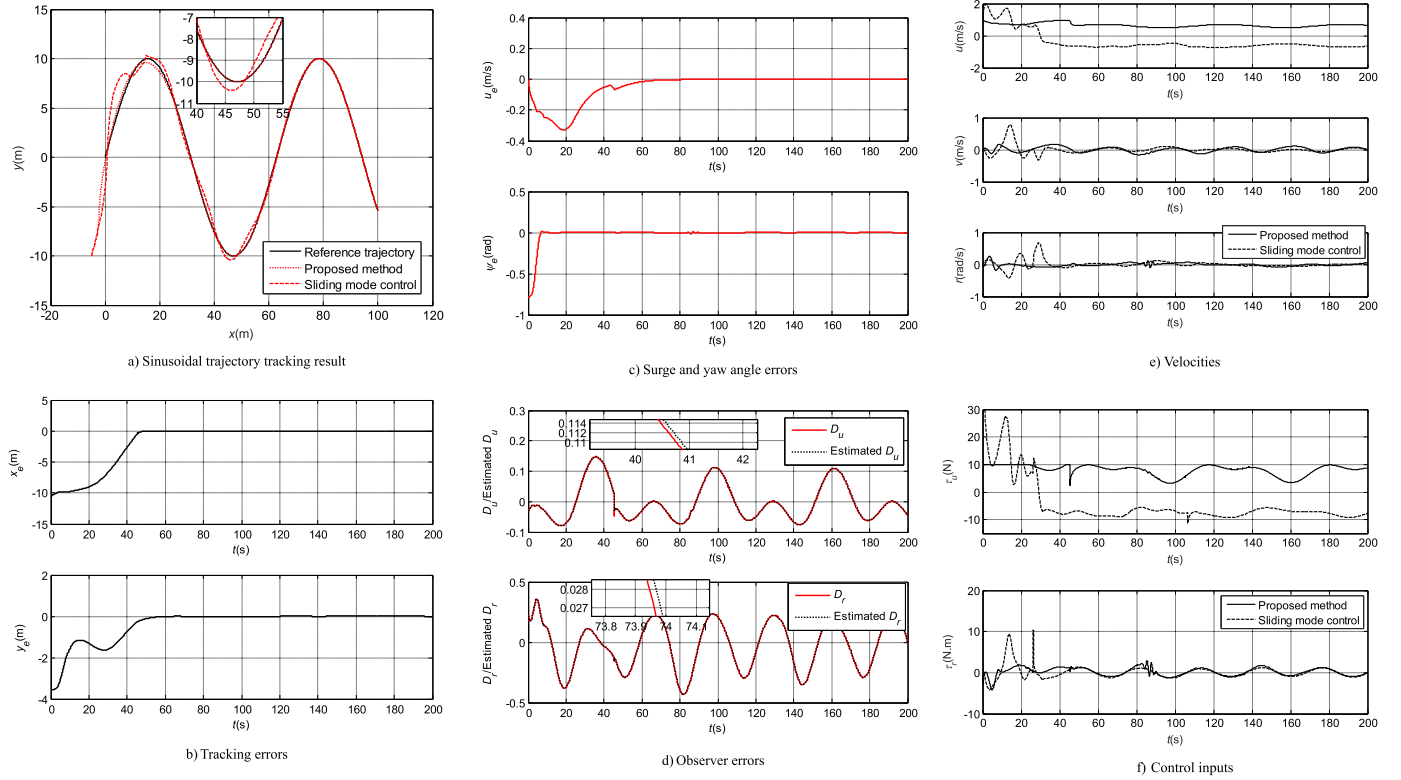


Fig. 1. Sinusoidal trajectory tracking performance.

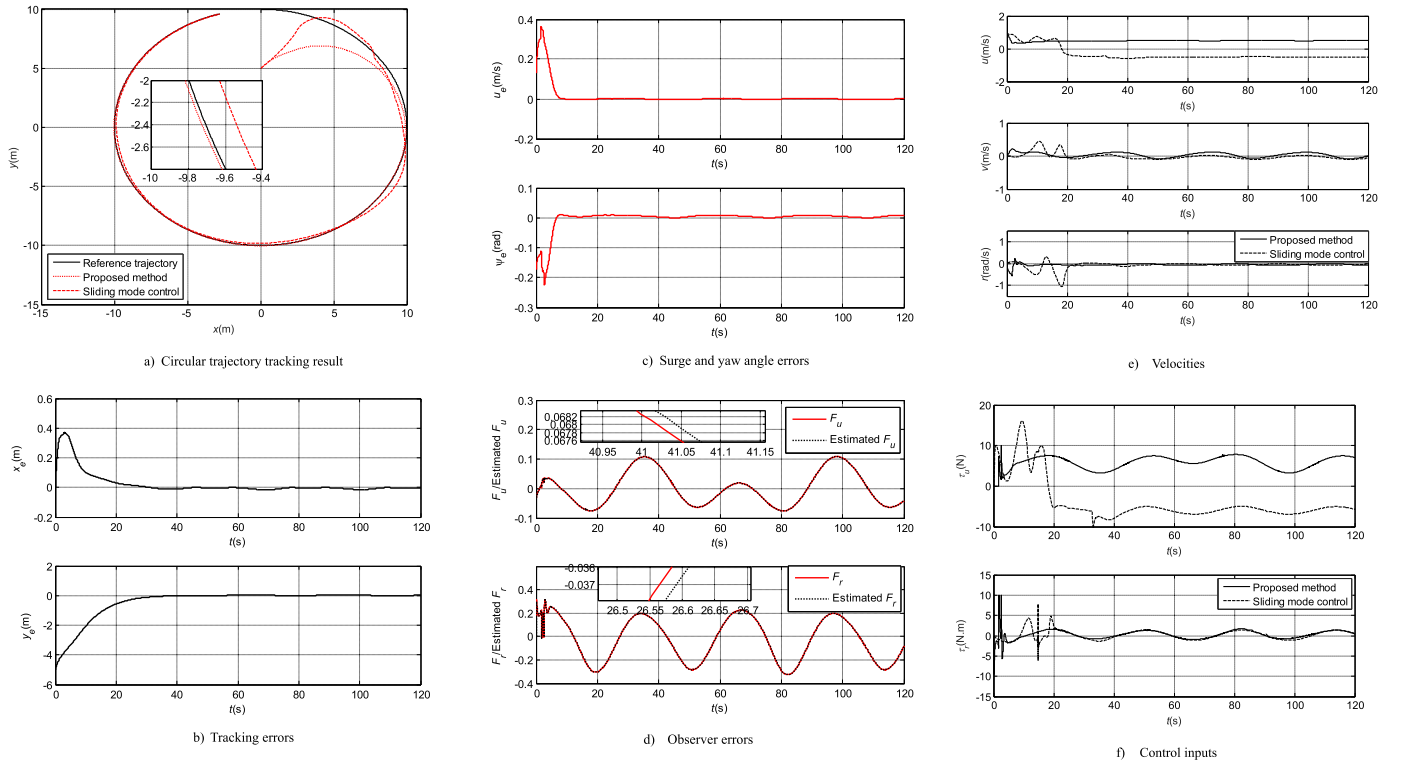


Fig. 2. Circular trajectory tracking performance.

## 5. Simulation results

This section presents the simulation results to validate the effectiveness and robustness of the proposed control strategies. CyberShip II, developed by Norwegian University of Science and Technology, is

selected as the simulation USV, with the following parameters:  $m_{11} = 25.8$ ,  $m_{22} = 33.8$ ,  $m_{33} = 6.2$ ,  $d_{11} = 12$ ,  $d_{22} = 17$ , and  $d_{33} = 0.5$ . The internal parameter uncertainties are generated as follows:

$$\begin{aligned} \Delta m_{11} &= 0.2m_{11} \sin(0.1t), & \Delta m_{22} &= 0.2m_{22} \sin(0.2t + \pi/4), & \Delta \\ m_{33} &= 0.2m_{33} \sin(0.1t + \pi/6), & \Delta d_{11} &= 0.2d_{11} \sin(0.1t + \pi/3), & \Delta \end{aligned}$$

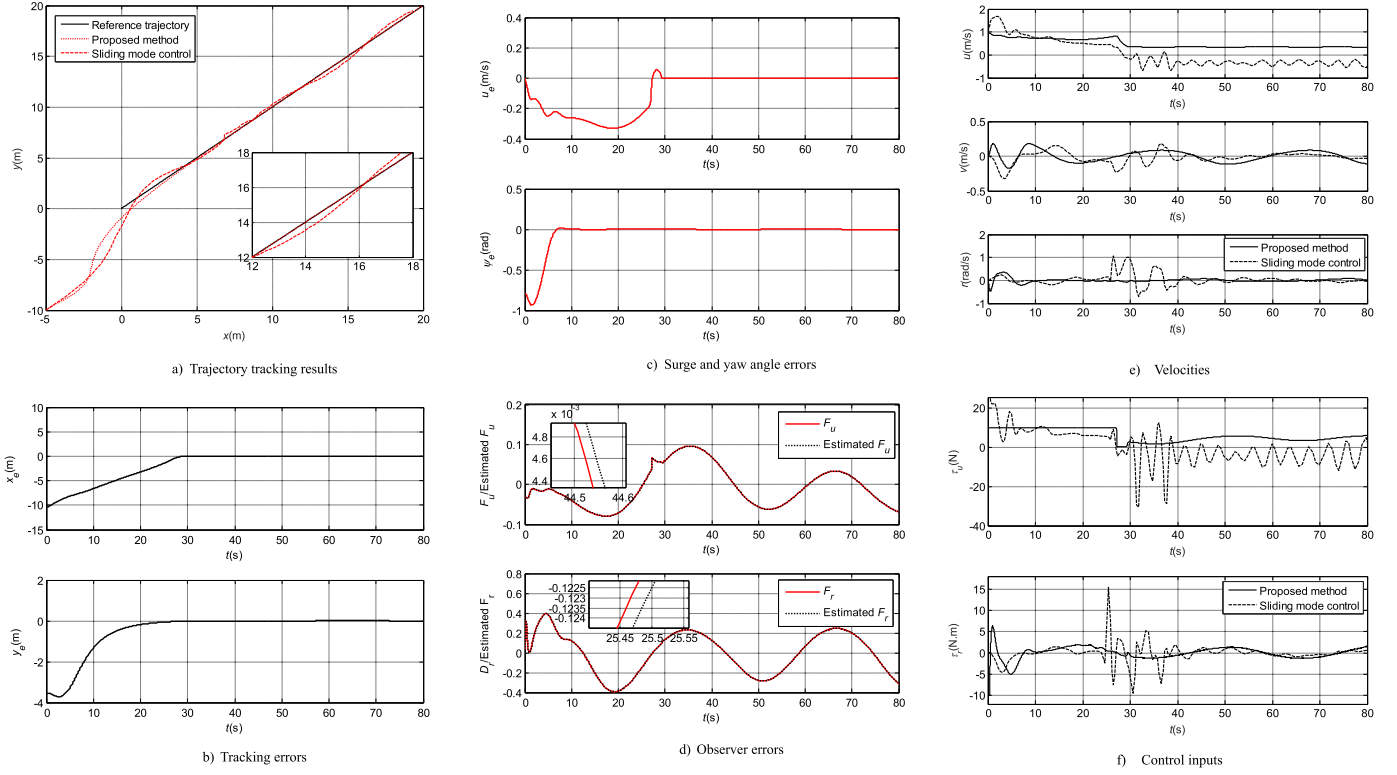


Fig. 3. Straight trajectory tracking performance.

$d_{22} = 0.2d_{22} \sin(0.2t + \pi/2)$ , and  $\Delta d_{33} = 0.2d_{33} \sin(0.1t + \pi/5)$ . The external disturbances are assumed to be

$$\begin{cases} \Delta \tau_u = \sin(0.2t) + \cos(0.2t + \pi/4) + \sin(0.2t + \pi/6) \\ \Delta \tau_v = \sin(0.2t) + \cos(0.2t + \pi/4) + \sin(0.2t + \pi/6) \\ \Delta \tau_r = \sin(0.2t) + \cos(0.2t + \pi/4) + \sin(0.2t + \pi/6) \end{cases}$$

The control parameters are  $h = 0.001$ ,  $h_0 = 10h$ ,  $R = 150$ ,  $k_u = 0.1$ ,  $k_\psi = 5$ ,  $k_r = 5$ , and  $\xi = 0.8$ . In practicality, the saturation problem frequently occurs in engineering systems. In this paper, the bounds of control inputs  $\tau_u$  and  $\tau_r$  are set as  $0 < \tau_u < \tau_{u \max}$ ,  $|\tau_r| < \tau_{r \max}$ , where  $\tau_{u \max} = 10$ ,  $\tau_{r \max} = 10$ . Input saturation can be found in many industrial processes, which may destroy the stability of the closed-loop systems. To make this problem tractable, the desired pseudo-variable proposed in (8) can be limited. In this paper, we choose

$$u_{\text{pseudo}} = \begin{cases} u_{\text{pseudo}}, & u_{\text{pseudo}} \leq 1 \\ 1, & u_{\text{pseudo}} > 1 \end{cases} \quad (26)$$

To demonstrate the advantage of the proposed guidance law, the previous method using  $u$  and  $v$  as the desired values presented in Elmokadem et al. (2016a,b) is selected for comparison.

The desired reference trajectories of the three scenarios are as follows:

#### (1) Sinusoidal trajectory

$$\begin{cases} x_d(t) = 0.5t \\ y_d(t) = 10 \sin(0.05t) \end{cases} \quad (27)$$

The initial velocities are  $[u(0) \ v(0) \ r(0)] = [1 \ 0 \ 0]$ . The initial position and yaw angle are  $[x(0) \ y(0) \ \psi(0)] = [-5 \ -10 \ \pi/4]$ .

The sinusoidal trajectory can be tracked under the various disturbances and constraints by the proposed method, as shown in Fig. 1. Fig. 1a) and b) indicate the errors between the reference and actual trajectory asymptotically decrease to approximately zero. The performance of yaw angle and surge velocity controllers is shown in Fig. 1 c). The observer performance for the reduced-order ESOs is shown in Fig. 1

d), which demonstrates that the designed observer can quickly and accurately estimate the lumped disturbance. The surge sway and yaw velocities are shown in Fig. 1 e). Note that the sway velocity is bounded. Although there is no actuation in the sway direction, the sway velocity  $v$  will remain bounded. This effect is due to the inputs and disturbance being bounded. Fig. 1 f) indicates that the proposed control system performs well despite the existence of limit bounds on the control inputs. The desired pseudo-variables and control input gains are adjusted until the constraint requirements are met. Note that the previous method did not show good performance for long times even though input saturation did not exist. Fig. 1e) and f) show that the tracking control is not in the correct direction because the yaw angle of the ship cannot be controlled directly.

#### (2) Circular trajectory

$$\begin{cases} x_d(t) = 10 \sin(0.05t) \\ y_d(t) = 10 \cos(0.05t) \end{cases} \quad (28)$$

The initial velocities are  $[u(0) \ v(0) \ r(0)] = [1 \ 0 \ 0]$ . The initial position and yaw angle are  $[x(0) \ y(0) \ \psi(0)] = [0 \ 5 \ \pi/4]$ .

Fig. 2 shows the tracking results for a circular trajectory. From Fig. 2a) and b), using the proposed method, the tracking errors are sufficiently small and can be maintained even in the presence of parametric uncertainties and external disturbances. Fig. 2 a) shows that the previous method has higher convergence speed because the inputs are constrained. However, the previous method has low convergence accuracy. Fig. 2e) and f) show that the tracking control cannot maintain the correct direction. The circular trajectory is generated with a constant velocity  $u$ ,  $v$  and  $r$ . The actual control inputs shown in Fig. 2 f) are time-varying to compensate for the time-varying disturbance.

#### (3) Straight trajectory

$$\begin{cases} x_d(t) = 0.25t \\ y_d(t) = 0.25t \end{cases} \quad (29)$$



The initial velocities are  $[u(0) \ v(0) \ r(0)] = [1 \ 0 \ 0]$ . The initial position and yaw angle are  $[x(0) \ y(0) \ \psi(0)] = [-5 \ -10 \ \pi/4]$ .

Fig. 3 shows the tracking results for a straight line. As is shown in Fig. 3a) and b), good performance is achieved even in the case of large initial errors and the abovementioned constraints. As is shown in Fig. 3c) and f), when thrust surplus appears, the controller adjusts quickly. However, the previous method cannot maintain a straight-line motion under the various disturbances. Additionally, it is difficult to maintain the correct yaw angle under the various disturbances.

## 6. Conclusion

This paper addressed the problem of trajectory tracking control for an underactuated USV on the horizontal plane in the presence of parameter perturbation and environmental disturbances. Compared with the traditional trajectory tracking control strategy, the new approach proposed in this paper divides the problem into a guidance law and a control loop. This approach can effectively avoid complex manufacturing processes and difficulty in the actual application. We believe that the indirect method with a suitable guidance law provides a simpler solution to the trajectory tracking problem than that of the direct method. By introducing the ADRC technique, we can both reduce the dependence on USV mathematical models and achieve improvement of the DSC. According to the simulation results from the reference sinusoidal, circular, and straight trajectory, input constraints can be effectively handled by adjusting the desired pseudo-variables and control gains. Since the proposed control law does not require a persistent excitation condition and special initial states, more types of reference trajectories can be tracked. Future work will include sea trials to further validate the control algorithm.

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