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# Seepage effects on bedload sediment transport rate by random waves

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ABSTRACT

The mean net bedload sediment transport rate beneath random waves is predicted taking into account the effect of seepage flow. This is achieved by using wave half-cycle bedload sediment transport formulas valid for regular waves together with a modified Shields parameter including the effect of seepage flow. The Madsen and Grant (1976) bedload sediment transport formula is used to demonstrate the method. An example using data typical to field conditions is included to illustrate the approach. The analytical results can be used to make an assessment of seepage effects on the mean net bedload sediment transport based on available wave statistics. Generally, it is recommended that a stochastic approach should be used rather than using the *rms* values in an otherwise deterministic approach.



## 1. Introduction

At intermediate and shallow water depths the bottom wave boundary layer is a thin flow region dominated by friction arising from the combined action of the wave-induced near-bottom flow and the bottom roughness. The wave boundary layer flow determines the bottom shear stress, which affects many phenomena in coastal engineering, e.g. sediment transport and the assessment of the stability of scour protection in the wave environment. A review is e.g. given in Holmedal et al. (2003).

In coastal areas the seabed is often sandy and permeable, and seepage flow in the seabed may occur naturally due to the horizontal pressure gradient caused by the difference between the pressure at the seabed under the wave crest and the wave trough, respectively. This will vary in time and space following the wave motion, inducing flow into the seabed as the wave crest passes and out of the seabed as the wave trough passes. This seepage flow has two opposing effects: first, the seepage flow into and out of the bed modifies the wave boundary layer, causing the bed shear stress to increase and decrease, respectively; second, the seepage flow exerts a vertical force on the sediments as the seepage flow into and out of the bed stabilizes and destabilizes the sediment, respectively.

Effects of seepage flow due to waves have been discussed by Sleath (1984), Soulsby (1997) and Nielsen (1992, 2009), for example. Moreover, Conley and Inman (1992) observed a wave crest–wave trough asymmetry in the fluid–sediment boundary layer development due to seepage flow in field measurements. These observations were supported by Conley and Inman (1994) in laboratory experiments. Lohmann et al. (2006) performed Large Eddy Simulation of a fully developed turbulent wave boundary layer subject to seepage flow, and obtained results in accordance with the experimental results of Conley and Inman (1994). Nielsen (1997) was the first to quantify the two opposing effects of seepage flow by defining a modified Shields parameter. He used the shear stress experiments of Conley (1993) and the slope stability experiments of Martin and Aral (1971) to derive the coefficients to use in this modified Shields parameter. Nielsen et al. (2001) used this modified Shields parameter together with their own experiments to investigate the seepage effects on the mobility of sediments on a flat bed under waves. Obhrai et al. (2002) extended this work to investigate the seepage effects on suspended sediments over a flat and a rippled bed.

For the prediction of the inception of motion or transport of seabed material under random waves, a commonly used procedure is to use the root-mean-square (*rms*) value of the wave height ($H_{rms}$) or the near-bed orbital velocity amplitude ($U_{rms}$) in an otherwise deterministic approach. However, this approach does not account for the stochastic feature of the processes included.

The purpose of the present paper is to provide a practical approach by which the stochastic properties of the net bedload sediment transport rate due to seepage flow can be derived from the irregular wave motion outside the seabed wave boundary layer. For regular waves there is a variety of sediment transport formulas available (see e.g. Soulsby, 1997). However, the purpose here is not to examine the details of them, but to demonstrate how such formulas can be used to find the mean net bedload sediment transport due to seepage flow under random waves. The approach

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is based on assuming the waves to be a stationary Gaussian narrow-band random process, using a wave half-cycle sediment transport rate formula for regular waves including the effect of seepage flow by adopting the Nielsen (1997) modified Shields parameter. The formulation is general, while the Madsen and Grant (1976) wave half-cycle bedload transport formula has been chosen to illustrate the method. An example is also included to demonstrate the applicability of the results for practical purposes using data typical for field conditions.

## 2. Bedload sediment transport rate by regular waves

The wave half-cycle bedload sediment transport rate for regular waves is given as

$$\Phi = \alpha(\theta_w - \theta_{cr})^\gamma \tag{1}$$

where $\alpha$ and $\gamma$ depend on the formula considered, and

$$\Phi = \frac{q_b}{[g(s-1)d_{50}^3]^{1/2}} \tag{2}$$

$$\theta_w = \frac{u_{*0}^2(1-\kappa(w/u_{*0}))}{g(s-1-\beta(w/K))d_{50}} \tag{3}$$

Here $\Phi$ is the dimensionless bedload transport rate, $\theta_w$ is the Shields parameter including the effect of seepage, $q_b$ is the volumetric bedload transport rate per unit width [m$^2$/s], $g$ is the acceleration due to gravity, $s$ is the sediment density to fluid density ratio, and $d_{50}$ is the median grain size diameter. The effect of seepage is taken into account by adopting a modified Shields parameter originally suggested by Nielsen (1997) (and re-presented by Nielsen et al. (2001)) where $u_{*0}=(\tau_{w0}/\rho)^{1/2}$ is the friction velocity with no seepage, $\tau_{w0}$ is the maximum bottom shear stress with no seepage, $\rho$ is the density of the fluid, $w$ is the vertical seepage velocity taken as positive upwards, $\kappa$ and $\beta$ are dimensionless coefficients recommended as $(\kappa,\beta)=(16,0.4)$, and $K$ is the hydraulic conductivity of the sand. Eq. (3) is based on obtaining $u_*^2/u_{*0}^2 = 1-16w/u_{*0}$ as the best fit to the Conley (1993) data for $-0.05 < w/u_{*0} < 0.025$, where $u_*=(\tau_w/\rho)^{1/2}$ is the friction velocity with seepage, and $\tau_w$ is the maximum bottom shear stress with seepage. Moreover, $\beta=0.4$ was determined using the slope stability experiments of Martin and Aral (1971). This modified Shields parameter includes two opposing effects. First, the flow into and out of the bed will make the boundary layer thinner and thicker and thereby the bed shear stress increases and decreases, respectively. Second, the flow into and out of the bed stabilizes and destabilizes the sediments, respectively. The numerator in Eq. (3) includes the change in the bed shear stress, i.e. to increase the shear stress for downward seepage ($w<0$) and to reduce it for upward seepage ($w>0$). The denominator includes the change in the effective weight due to the seepage, i.e. to stabilize the particles for downward seepage and to destabilize the particles for upward seepage. It should also be noticed that Eq. (3) is valid for non-breaking waves over a horizontal bed; see Nielsen et al. (2001) for more details.

Eq. (1) was originally given with Eq. (3) for $w=0$, and is applicable to bedload transport and applies if $\theta_w$ is larger than the threshold value $\theta_{cr} \approx 0.05$. A review of wave half-cycle bedload sediment transport rate formulas is given in e.g. Soulsby (1997); Soulsby proposed $\alpha=5.1$ and $\gamma=3/2$; Madsen and Grant (1976) proposed $\alpha = 12.5 w_s d_{50}/[g(s-1)d_{50}^3]^{1/2}$, $\gamma=3$ and $\theta_{cr}=0$, where $w_s$ is the grain settling velocity. More discussion will be given in Section 3.2.

The maximum bottom shear stress within a wave-cycle without seepage is taken as

$$\frac{\tau_{w0}}{\rho} = \frac{1}{2} f_w U^2 \tag{4}$$

where $U$ is the orbital velocity amplitude at the seabed, and $f_w$ is the wave friction coefficient taken as (Myrhaug et al., 2001)

$$f_w = c\left(\frac{A}{z_0}\right)^{-d} \tag{5}$$

$$(c,d) = (18,1) \text{ for } 20 \lesssim A/z_0 \lesssim 200 \tag{6}$$

$$(c,d) = (1.39,0.52) \text{ for } 200 < A/z_0 \lesssim 11{,}000 \tag{7}$$

$$(c,d) = (0.112,0.25) \text{ for } 11{,}000 < A/z_0 \tag{8}$$

where $A=U/\omega$ is the orbital displacement amplitude at the seabed, and $z_0=2.5d_{50}/30$ is the bed roughness based on the median grain size diameter $d_{50}$. Note that Eq. (7) corresponds to the coefficients given by Soulsby (1997) obtained as best fit to data for $10 \lesssim A/z_0 \lesssim 10^5$. The advantage of using this friction factor for rough turbulent flow is that it is possible to derive the stochastic approach analytically. One should note that all the wave-related quantities in Eqs. (1)–(5), i.e., $\tau_{w0}$, $U$ and $A$ are the quantities associated with the harmonic motion. Thus a stochastic approach based on the harmonic wave motion is feasible, as will be outlined in the forthcoming.

## 3. Bedload sediment transport rate by random waves

### 3.1. Outline of stochastic method

The present approach is based on the following assumptions: (1) the free surface elevation $\zeta(t)$ associated with the harmonic motion is a stationary Gaussian narrow-band random process with zero expectation described by the single-sided spectral density $S_{\zeta\zeta}(\omega)$, and (2) the formulas for bedload sediment transport rate for regular waves given in the previous section, are valid for irregular waves as well.

Based on the present assumptions, the (instantaneous) time-dependent bed orbital displacement and velocity, $a(t)$ and $u(t)$, respectively, associated with the harmonic motion, are both stationary Gaussian narrow-band processes with zero expectations and with single-sided spectral densities

$$S_{aa}(\omega) = \frac{S_{\zeta\zeta}(\omega)}{\sinh^2 kh} \tag{9}$$

$$S_{uu}(\omega) = \omega^2 S_{aa}(\omega) = \frac{\omega^2 S_{\zeta\zeta}(\omega)}{\sinh^2 kh} \tag{10}$$

Now the orbital displacement amplitude at the seabed, $A$, the orbital velocity amplitude at the seabed, $U$, and the wave height, $H$, are Rayleigh-distributed with the cumulative distribution function ($cdf$) given by

$$P(\hat{x}) = 1-\exp(-\hat{x}^2), \quad \hat{x} = x/x_{rms} \geq 0 \tag{11}$$

where $x$ represents, $A$, $U$ or $H$, and $x_{rms}$ is the $rms$ value of $x$ representing $A_{rms}$, $U_{rms}$ or $H_{rms}$. Now $A_{rms}$, $U_{rms}$ and $H_{rms}$ are related to the zeroth moments $m_{0aa}$, $m_{0uu}$ and $m_{0\zeta\zeta}$ of the amplitude, velocity and free surface elevation spectral densities, respectively (corresponding to the variances of the amplitudes ($\sigma_{aa}^2$), the velocity ($\sigma_{uu}^2$) and the free surface elevation ($\sigma_{\zeta\zeta}^2$)), given by

$$A_{rms}^2 = 2m_{0aa} = 2\sigma_{aa}^2 = 2\int_0^\infty S_{aa}(\omega)d\omega \tag{12}$$

$$U_{rms}^2 = 2m_{0uu} = 2\sigma_{uu}^2 = 2\int_0^\infty S_{uu}(\omega)d\omega \tag{13}$$

$$H_{rms}^2 = 8m_{0\zeta\zeta} = 8\sigma_{\zeta\zeta}^2 = 8\int_0^\infty S_{\zeta\zeta}(\omega)d\omega \tag{14}$$

From Eqs. (13) and (10) it also appears that $m_{0uu}=m_{2aa}$, where $m_{2aa}$ is the second moment of the amplitude spectral density. Thus, the mean zero-crossing frequency for the bed orbital displacement, $\omega_z$, is obtained from the spectral moments of $a(t)$ as

$$\omega_z = \left(\frac{m_{2aa}}{m_{0aa}}\right)^{1/2} = \left(\frac{m_{0uu}}{m_{0aa}}\right)^{1/2} = \frac{U_{rms}}{A_{rms}} \tag{15}$$

where Eqs. (12) and (13) have been used. This result is valid for a stationary Gaussian random process. Note that this zero-crossing frequency will generally be smaller than for the surface elevation due to greater attenuation of high frequencies. However, for a narrow-band process these two zero-crossing frequencies will be equal.

It should be noted that $U_{rms}$ used by Soulsby (1997) corresponds the standard deviation $\sigma_{uu}$ used here.

For a narrow-band process, $A=U/\omega$ where $\omega$ is replaced with $\omega_z$ from Eq. (15). Then, by substitution in Eqs. (3)–(5), Eq. (3) can be re-arranged to give the Shields parameter for individual narrow-band random waves as

$$\theta_w = \frac{u_{*rms}^2 \hat{U}^{2-d}}{g(s-1-\beta(w/K))d_{50}}\left(1-\kappa\frac{w}{u_{*rms}}\hat{U}^{-(2-d)/2}\right) \tag{16}$$

where $\hat{U}=U/U_{rms}$ and

$$u_{*rms}^2 = \frac{1}{2}c\left(\frac{A_{rms}}{z_0}\right)^{-d} U_{rms}^2 \tag{17}$$

Moreover, a Shields parameter can be defined where the wave-related quantities are replaced by their *rms*-values, i.e.,

$$\theta_{wrms} = \frac{u_{*rms}^2}{g(s-1)d_{50}} \tag{18}$$

Furthermore, by taking $\omega=\omega_z$ and $k=\bar{k}$, where $\bar{k}$ is the mean wave number determined from $\omega_z^2 = g\bar{k}\tanh\bar{k}h$, then $U_{rms} = \omega_z H_{rms}/(2\sinh\bar{k}h)$.

The net bedload sediment transport rate due to the effect of seepage is then obtained as the difference between the wave half-cycle transport beneath the wave crest ($w<0$) and the wave half-cycle transport beneath the wave trough ($w>0$), which for individual narrow-band random waves is obtained by substituting Eq. (16) in Eq. (1) as (notice that $w$ is taken as positive in the following)

$$\Phi_{net} = \alpha\left[\frac{u_{*rms}^2}{C_+}\hat{U}^{2-d}(1+b\hat{U}^{-(2-d)/2})-\theta_{cr}\right]^\gamma - \alpha\left[\frac{u_{*rms}^2}{C_-}\hat{U}^{2-d}(1-b\hat{U}^{-(2-d)/2})-\theta_{cr}\right]^\gamma \tag{19}$$

where

$$b = \kappa\frac{w}{u_{*rms}} \tag{20}$$

$$C_\pm = g\left(s-1\pm\beta\frac{w}{K}\right)d_{50}\ [\mathrm{m}^2/\mathrm{s}^2] \tag{21}$$

Now Eq. (1) is valid for $\theta_w \geq \theta_{cr} \approx 0.05$. By defining $\theta_{w0}$ as the Shields parameter without seepage, i.e. given by Eq. (3) with $w=0$, it follows that Eq. (19) is valid for $\theta_{w0} = \theta_{wrms}\hat{U}^{2-d} \geq \theta_{cr} \approx 0.05$, i.e. for

$$\hat{U} \geq \hat{U}_1 = \left(\frac{0.05}{\theta_{wrms}}\right)^{1/(2-d)} \tag{22}$$

Consequently, $\hat{U}$ follows the truncated Rayleigh distribution given by the *cdf*

$$P(\hat{U}) = \frac{\exp(-\hat{U}_1^2)-\exp(-\hat{U}^2)}{\exp(-\hat{U}_1^2)}, \quad \hat{U}\geq\hat{U}_1 \tag{23}$$

A statistical quantity of interest is the expected (mean) value of the net bedload sediment transport rate given as

$$E[\Phi_{net}(\hat{U})] = \int_{\hat{U}_1}^\infty \Phi_{net}(\hat{U})p(\hat{U})\,d\hat{U} \tag{24}$$

where $p(\hat{U})$ is the probability density function (*pdf*) of $\hat{U}$ given by $p(\hat{U})=dP(\hat{U})/d\hat{U}$ where $P(\hat{U})$ is given in Eq. (23) and $\hat{U}_1$ is given in Eq. (22). The integral in Eq. (24) can be evaluated numerically. However, in the following section the Madsen and Grant (1976) model referred to in Section 2 will be used to facilitate an analytical approach.

### 3.2. Application of the Madsen and Grant (1976) formula

The Madsen and Grant (1976) formula is adopted in order to facilitate an analytical approach and to serve the purpose of demonstrating the effect of seepage on bedload sediment transport. Now Eq. (1) yields

$$\Phi = \alpha\theta_w^3, \quad \alpha = \frac{12.5w_s d_{50}}{[g(s-1)d_{50}^3]^{1/2}} \tag{25}$$

and is only applied if $\theta_w \gg \theta_{cr}$. It should be noted that Horikawa et al. (1982) found good agreement between experiments at very high sediment transport rates and Eq. (25).

By taking $\gamma=3$ and $\theta_{cr}=0$, Eq. (19) can be re-arranged to

$$\phi_{net} \equiv \frac{\Phi_{net}}{\alpha u_{*rms}^6} = P_-\hat{U}^{3(2-d)} + 3bP_+\hat{U}^{5(2-d)/2} + 3b^2P_-\hat{U}^{2(2-d)} + b^3P_+\hat{U}^{3(2-d)/2} \tag{26}$$

where

$$P_\pm = \frac{1}{C_+^3} \pm \frac{1}{C_-^3}\ [\mathrm{s}^6/\mathrm{m}^6] \tag{27}$$

Since Eq. (25) is valid for $\theta_w \gg \theta_{cr} \approx 0.05$, i.e. an order of magnitude larger than $\theta_{cr}$, it follows that Eq. (26) is valid for $\theta_w \gg \theta_c \approx 0.5$, or similarly to Eq. (22), corresponding to

$$\hat{U} > \hat{U}_c = \left(\frac{0.5}{\theta_{wrms}}\right)^{1/(2-d)} \tag{28}$$

Thus the expected (mean) value of the net bedload sediment transport rate is given from Eq. (26) as

$$E[\Phi_{net}] = \frac{12.5w_s d_{50} u_{*rms}^6}{[g(s-1)d_{50}^3]^{1/2}} E[\phi_{net}] \tag{29}$$

where

$$E[\phi_{net}] = \int_{\hat{U}_c}^\infty \phi_{net}(\hat{U})p(\hat{U})\,d\hat{U} = \exp(\hat{U}_1^2)\Big[P_-\Gamma(4-\tfrac{3}{2}d,\hat{U}_c^2) + 3bP_+\Gamma\left(3.5-\tfrac{5}{4}d,\hat{U}_c^2\right) + 3b^2P_-\Gamma(3-d,\hat{U}_c^2) + b^3P_+\Gamma\left(2.5-\tfrac{3}{4}d,\hat{U}_c^2\right)\Big] \tag{30}$$

and $\Gamma(\bullet,\bullet)$ is the incomplete gamma function. Moreover, $\hat{U}_c$ is given in Eq. (28); $\theta_{wrms}$ in Eq. (18); $u_{*rms}$ in Eq. (17); $b$ in Eq. (20) where $w$ needs to be specified according to values given in the range $-0.05 < w/u_{*rms} < 0.025$. Here the integral is evaluated analytically by using the results in Appendix A; i.e. by using Eq. (A4), the result of Eqs. (24), (26) and (23) is obtained as in Eq. (30).

For the prediction of the net bedload sediment transport beneath irregular waves, a commonly used procedure is to use the *rms* values of the wave-related quantities in an otherwise deterministic approach. In the present case this corresponds to taking $U = U_{rms}$, i.e. $\hat{U} = U/U_{rms} = 1$, which substituted in Eq. (26) gives

$$\phi_{net,\text{det}} = P_- + 3bP_+ + 3b^2P_- + b^3P_+ \quad (31)$$

Consequently

$$\Phi_{net,det} = \frac{12.5 w_s d_{50} u_{*rms}^6}{[g(s-1)d_{50}^3]^{1/2}} \phi_{net,det} \quad (32)$$

## 4. Example of results

To the authors' knowledge no data exist in the open literature for random wave-induced bedload transport due to the effect of seepage flow. Hence an example of calculating the net bedload sediment transport rate due to the effect of seepage based on the results in Section 3.2 is provided by using the following given flow conditions:

- horizontal bed with water depth, $h = 15$ m
- significant wave height, $H_s = 5$ m
- mean wave period, $T_z = 8.9$ s
- median grain diameter (fine sand according to Soulsby (1997, Fig. 4)) $d_{50} = 0.20$ mm
- $s = 2.65$ (as for quartz sand)
- median grain settling velocity (according to Soulsby (1997, Eq. SC(102)), $w_s = 0.02$ m/s

The calculated quantities are given in Table 1, where $H_{rms} = H_s/\sqrt{2}$ when $H$ is Rayleigh-distributed. It appears that the flow corresponds to sheet flow conditions, i.e. $\theta_{wrms} = 1.121 > 0.8$. Thus, in this example it is justified to apply the Madsen and Grant (1976) formula in Eq. (25) due to the corresponding high sediment transport rate. The vertical seepage velocity (taken as positive upwards) is taken as $w = \pm 0.025 u_{*rms}$, which is in the validity range of Eq. (3). Moreover, the hydraulic conductivity of the sand is taken as $K = w/0.15$ based on Horn et al. (1998), who found that $w < 0.15$ K corresponds to modest seepage rates generated by uprush of swash during a rising tide (see more discussion in Nielsen et al. (2001)). It should be noted that the values of $w$ and $K$ are taken as characteristic values for all the individual random waves, which is consistent with the narrow-band assumption.

From Table 1 it appears that the expected value of the dimensionless net bedload transport rate based on using $\theta_c = 0.5$ (i.e. corresponding to $\hat{U}_c$ in Eq. (28)) is $E[\Phi_{net}] = 31.29$. This corresponds to a mean net bedload transport rate $E[q_{bnet}] = [g(s-1)d_{50}^3]^{1/2}\ E[\Phi_{net}] = 3.56 \times 10^{-4}$ m$^2$/s. It should be noted that the mean dimensionless net bedload transport rate based on using $\theta_{cr} = 0.05$ (i.e. corresponding to $\hat{U}_1$ in Eq. (22)) is $E[\Phi_{net}] = 31.46$. That is, the difference between using a threshold value of 0.05 (corresponding to when Eq. (25) shall be applied) is insignificant for these conditions due to the high transport rates. This is a consequence of that for almost all the waves in this "storm", $\theta_c = 0.05$ is exceeded. It should be noted that the net bedload transport is zero beneath sinusoidal waves when the effect of seepage is not taken into account.

In this example it is the effect of bed shear stress which dominates, i.e. the numerator in Eq. (3) contains the parenthesis $(1 \pm 0.4)$, while the denominator contains the parenthesis $(1.65 \pm 0.06)$. Thus the effect of increased and reduced bed shear stress for flow into and out of the bed, respectively, dominates the stabilizing and destabilizing effect of the sediments, respectively. Consequently there is a net bedload transport in the wave propagation direction.

The net bedload transport rate based on the deterministic method is $q_{bnet,det} = 1.57 \times 10^{-4}$ m$^2$/s. Thus the stochastic to deterministic method ratio is 2.27, suggesting that a stochastic method should be used to assess the effect of seepage on the net bedload sediment transport rates beneath random waves.

**Table 1**
Example of results for seepage effects on the mean net bedload sediment transport rate by random waves based on the Madsen and Grant (1976) formula in Eq. (25).

| | |
|---|---|
| $H_{rms}$ (m) | 3.54 |
| $k_p$ (rad/m) | 0.0667 |
| $U_{rms}$ (m/s) | 1.06 |
| $A_{rms}$ (m) | 1.50 |
| $A_{rms}/z_0$ | 90,000 |
| $c,d$ | 0.112, 0.25 |
| $u_{*rms}$ (m/s) (Eq. (17)) | 0.0603 |
| $\theta_{wrms}$ (Eq. (18)) | 1.121 |
| $w = \pm 0.025 u_{*rms}$ (m/s) | $\pm 0.00151$ |
| $K = w/0.15$ (m/s) | 0.0101 |
| $b$ (Eq. (20)) | 0.4 |
| $P_+$ ((m/s)$^{-6}$) (Eq. (27)) | $-6.459 \times 10^6$ |
| $P_-$ ((m/s)$^{-6}$) (Eq. (27)) | $59.419 \times 10^6$ |
| $\hat{U}_1$ (Eq. (22)) | 0.169 |
| $\hat{U}_c$ (Eq. (28)) | 0.630 |
| $E[\Phi_{net}]$ (Eqs. (29) and (30)) | 31.29 |
| $E[q_{bnet}]$ (m$^2$/s) | $3.56 \times 10^{-4}$ |
| $\Phi_{net,det}$ (Eqs. (31) and (32)) | 13.80 |
| $q_{bnet,det}$ (m$^2$/s) | $1.57 \times 10^{-4}$ |

## 5. Comments

It should be noted that the present theory is valid for linear waves only, suggesting that it should be applied outside the surf zone where the non-linearities become less important. The implications of including non-linearities (e.g. Stokes wave asymmetry with higher wave crests and shallower wave troughs) are expected to enhance the seepage effect leading to larger bedload transport than for linear waves.

Due to the seepage effect there will be a net bedload transport beneath sinusoidal waves in coastal areas with sandy and permeable seabeds. The importance of the seepage effect on the sediment transport depends on the flow conditions in a non-trivial way. Here it is inherent in the modified Shields parameter in Eq. (3), i.e. how the seepage effect changes the bed shear stress in the numerator and the effective weight in the denominator. More discussion of the seepage effect on sediment transport is given in e.g. Nielsen (1997).

Generally, it is recommended to use a stochastic approach rather than using the *rms*-values in an otherwise deterministic approach since this will provide global stochastic features associated with the random waves. In the example it is shown that the stochastic method results in higher bedload transport than the deterministic approach. One reason for this is that the contribution from the higher waves is accounted for more correctly by using the stochastic than the deterministic method. The deterministic method will agree better with the stochastic method if a characteristic statistical value larger than the rms-value is used. However, this statistical value will depend on the flow conditions and can only be determined by performing an analysis spanning out a wide parameter range. All this information is contained in a stochastic method, and hence this should be used to make assessment of seepage effects on bedload sediment transport under random waves based on available wave statistics.

## 6. Summary

The mean net bedload sediment transport rate beneath random waves is predicted taking into account the effect of seepage flow. The method is based on assuming the waves to be a stationary Gaussian narrow-band random process, using wave half-cycle bedload sediment transport rate formulas for regular waves including seepage flow effects by adopting the Nielsen (1997) modified Shields parameter. The formulation is general, but the Madsen and Grant (1976) wave half-cycle bedload sediment transport formula is used to serve the purpose of illustrating the method. An example is also included to demonstrate the applicability of the results for practical purposes using data typical to field conditions. In this example the effect of increased and reduced bed shear stress for flow into (under the wave crest) and out of (under the wave trough) the bed, respectively, dominates the stabilizing and destabilizing effects of the sediments, respectively. In this case the net bedload transport is in the wave propagation direction. The present analytical results can be used to make assessment of seepage effects on the mean net bedload sediment transport under random waves based on available wave statistics. Generally, it is recommended to use a stochastic approach rather than using the *rms* values in an otherwise deterministic approach.

Although simple, the present method should be useful as a first approximation to represent the stochastic properties of the mean net bedload sediment transport due to seepage flow under random waves. However, comparisons with data are required before a conclusion regarding the validity of this approach can be given. In the meantime, the method should be useful as an engineering tool for the assessment of seepage effects on the mean net bedload sediment transport by random waves.

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## Appendix A

Let $x$ be Weibull distributed with the *pdf*

$$p(x) = \beta x^{\beta-1}\exp(-x^{\beta}), \quad x \geq 0,\ \beta > 0 \tag{A1}$$

The expected value of the $(1/n)$th largest values of $x$ is given as

$$E[x_{1/n}] = n\int_{x_{1/n}}^{\infty} xp(x)dx \tag{A2}$$

where $x_{1/n}$ is the value of $x$ which is exceeded by the probability $1/n$.

Moreover, from Abramowitz and Stegun (1972, Chapter 6.5, Eq. (6.5.3)) it is given that

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t}t^{a-1}\,dt \tag{A3}$$

By utilizing this, the following result is obtained

$$\int_{x_1}^{\infty} x^m p(x)dx = \int_{x_1}^{\infty} x^m \beta x^{\beta-1}\exp(-x^{\beta})dx = \Gamma\left(1+\frac{m}{\beta}, x_1^{\beta}\right) \tag{A4}$$

by using Eq. (A1), and where $\Gamma(.,.)$ is the incomplete gamma function: $\Gamma(x,0)=\Gamma(x)$ where $\Gamma$ is the gamma function. The result in Eq. (A4) is obtained by substituting $t=x^{\beta}$ in the second integral in Eq. (A4) and using Eq. (A3).

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