Renewable and Sustainable Energy Reviews 14 (2010) 1490–1495

Contents lists available at ScienceDirect

Renewable and Sustainable Energy Reviews

journal homepage: www.elsevier.com/locate/rser

Wind power distributions: A review of their applications

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ARTICLE INFO

Article history:
Received 26 October 2009
Accepted 8 January 2010

Keywords:
Wind speed
Wind power
Mean density power
Weibull distributions
Rayleigh distributions

ABSTRACT

This paper presents the features of wind power distributions that have been analytically obtained from wind distribution functions. Simple equations establishing a relationship between mean power density and wind speed have been obtained for a given location and wind turbine (WT). Four different concepts relating wind power distribution functions are shown: the power transported by the wind; the theoretical maximum convertible power from it according to the Betz' law; the maximum convertible power from the wind considering more realistic limits that will be explained; finally an even more approximate limit to the maximum power obtained from a wind turbine, considering its parameters. Similarly, four different equations are obtained establishing relationships between the mean power density and the mean wind speed. These equations are very simple and very useful when discarding locations for wind turbine installation.

Contents

1. Introduction

WORLD wind power has increased its capacity due to the installation of a large number of wind turbines in recent years. As the rated speed of new WTs has continuously risen, it has become more and more important to pick the most suitable and worthwhile locations to install them [1]. In fact, one of the most important problems to be solved is whether a given location should be chosen or discarded for the installation of a generator system. For instance, a location can be valid from the point of view of wind energy potential, but not valid due to the lack of electrical infrastructure, whilst a not-so-good location according to wind speed potential can be in a much suitable place regarding electrical infrastructure. Due to these considerations, locations close to adequate electrical lines usually have to be analyzed in order to investigate their energy potential, whilst other can be dispensable if there are signs that they will not cover costs of installation, taxes, etc.

Wind power distributions have been widely investigated, employed and explained [2]. In many wind power studies the features of such distributions are used for design purposes. Both analytical and Monte Carlo simulation methods can be carried out, although they are generally used with features of wind power and not output power in mind. However, things can be planned from a different point of view, as similar distribution functions can be described for power, if wind distribution functions are taken into account, together with WT features, on the basis of data provided by the manufacturers.

On the basis that the distribution function of the wind speed in a certain location depends just on the mean wind speed, a distribution function of the wind power can be obtained for a given WT by using its power curve. Once the wind power

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1364-0321/$ – see front matter
doi:10.1016/j.rser.2010.01.005

distribution function is obtained, the mean power available is deduced. So as not to depend on the type of WT, this will be shown per unit of surface (mean power density). This process is performed in four different ways:

(1) obtaining of the power from the wind [3].
(2) consideration of Betz' law [4].
(3) consideration of realistic values [5], remembering that Betz' law is an upper limit.
(4) consideration of WT parameters such as Cut-In and Cut-Out wind speed, rated speed, and rated power [6].

The goal of this work is to obtain expressions that allow us to give response to questions about the mean value of the statistical distribution of the maximum power obtainable from the wind, regardless of the WT chosen, and also taking into account its features, when the only input value is the mean wind speed.

2. The proposed relationships

2.1. Relationship between wind speed and usable power from the wind

The power, P', transported by an airstream flowing with a given speed, U, can be calculated according to (1) as has been established in [7–9].

$$P' = \frac{1}{2} A \rho U^3 \tag{1}$$

where ρ is the air density and A the area of the airstream, measured in a perpendicular plane to the direction of the wind speed.

The calculation of the mechanical power that can be extracted by the rotor of a horizontal axis WT (HAWT), requires Betz' law to be taken into account. It states that a maximum portion of 16/27 of the power transported by the wind can be converted into mechanical power by means of such a converter.

Thus the maximum power that can be extracted from the airstream is given by (2).

$$P'' = \frac{8}{27} A \rho U^3 \tag{2}$$

Eqs. (1) and (2) express the instantaneous power, P, as a function of the instantaneous wind speed, U. However, the wind speed may vary during a period of time. To consider this effect we are going to work with the wind speed probability distribution function (PDF).

According to [10] the wind speed at a certain location may be represented by a Weibull distribution function with two parameters called scale parameter, C, and shape parameter, k. Occasionally, and under certain conditions, $k = 2$ is assumed, which constitutes the particular case of the Rayleigh distributions.

Weibull PDF [11] of the wind speed at a certain location can be expressed as in (3).

$$f_U(U) = \left\{ \begin{array}{ll} \frac{k}{C}\left(\frac{U}{C}\right)^{k-1} e^{-(U/C)^k} & U \geq 0 \\ 0 & U < 0 \end{array} \right\} \tag{3}$$

In order to obtain the wind power PDF, a change of variables may be operated. As can be read in [12] and in order to obtain the PDF of a variable $P = g(U)$, when g is monotonic, and where U has PDF $f_U(U)$, the operation given in (4) can be applied.

$$f_P(P) = \left| \frac{1}{g'(g^{-1}(U))} \right| f_U(g^{-1}(P)) \tag{4}$$

where g^{-1} denotes inverse and g' derivative function.

If (4) is applied to the case that is being analyzed in this paper, the PDF of the power may be expressed as a function of the variable P, i.e. P' of (1) or P'' of (2), considering its relationship with the wind speed U as in (5).

$$f_P(P) = \begin{cases} (k/3c_tC^3)\left(\frac{P}{c_tC^3}\right)^{(k/3)-1} e^{-(P/c_tC^3)^{k/3}} & P \geq 0 \\ 0 & P < 0 \end{cases} \tag{5}$$

where c_t can be $A\rho/2$ or $8A\rho/27$ depending on the case (again, P' or P'').

Therefore the power transported by the wind can be represented by a Weibull distribution with the parameters C' and k' given in (6).

$$C' = \frac{1}{2} A \rho C^3 \qquad k' = \frac{k}{3} \tag{6}$$

And the maximum power that can be extracted by a WT can be represented by a Weibull distribution with the parameters C'' and k'' given in (7).

$$C'' = \frac{8}{27} A \rho C^3 \qquad k'' = \frac{k}{3} \tag{7}$$

Moreover, as has been presented in [13–20], considering the features of Weibull distributions, the mean value, μ_U, can be expressed as a function of the parameters C, k, and the Gamma function, such as in (8).

$$U_{\text{mean}} = \mu_U = C\Gamma\left(1 + \frac{1}{k}\right) \tag{8}$$

Then, the mean value of an airstream wind power PDF can be expressed as in (9).

$$P'_{\text{mean}} = \mu_{P'} = C'\Gamma\left(1 + \frac{1}{k'}\right) \tag{9}$$

And the mean value of the PDF of the maximum power that can be extracted by a WT can be expressed as in (10).

$$P''_{\text{mean}} = \mu_{P''} = C''\Gamma\left(1 + \frac{1}{k''}\right) \tag{10}$$

As has been explained, the shape parameter, k, of a Weibull distribution of wind speeds in a certain location is sometimes considered to be 2, giving the particular case of the Rayleigh distributions. Under this assumption, (8)–(10) can be expressed as in (11)–(13) respectively.

$$U_{\text{mean}} = \frac{\sqrt{\pi}}{2} C \tag{11}$$

$$P'_{\text{mean}} = \frac{3}{8} \sqrt{\pi} A \rho C^3 \tag{12}$$

$$P''_{\text{mean}} = \frac{2}{9} \sqrt{\pi} A \rho C^3 \tag{13}$$

and it is also possible to write the mean power as a function of the mean wind speed as in (14) and (15).

$$P'_{\text{mean}} = \frac{3}{\pi} A \rho U^3_{\text{mean}} \tag{14}$$

$$P''_{\text{mean}} = \frac{16}{9\pi} A \rho U^3_{\text{mean}} \tag{15}$$

The power can be calculated per surface unit, in order to establish a criterion to assess the mean power. If the value of $\rho = 1.225\ \text{kg/m}^3$ for the air density (at 15 °C) is introduced in the previous Eqs. (16) and (17) are obtained and can be considered as

theoretical limits.

$$\frac{P'_{\text{mean}}}{A} = 1.17U_{\text{mean}}^3 \tag{16}$$

$$\frac{P''_{\text{mean}}}{A} = 0.69U_{\text{mean}}^3 \tag{17}$$

The use of (16) and (17) requires using m/s as wind speed units, in order to obtain W/m^2 as power units. Eq. (16) gives the mean value of the PDF of the power per surface unit in a location with a given mean wind speed. Eq. (17) gives the maximum power that can be extracted, per surface unit, according to Betz' limit.

2.2. Relationship between wind speed and power produced by a HAWT

The relationship between wind and power in a WT has been widely studied and depends on several items such as the type of WT, the air density and the temperature.

The type of WT influences because the area swept by the rotor depends on the blade size. It also influences on the power coefficient, C_p, which is the ratio between the power produced by a WT and the power carried by the free airstream.

The power that a HAWT can finally extract from the wind can be expressed as in (18).

$$P''' = \frac{1}{2}A^2\rho U^3 C_p(\lambda, \beta) \tag{18}$$

The power coefficient is always between 0 and the Betz' limit, 16/27.

In order to obtain similar results to those in the above section, the term $C_{P_{\max}}$ will be used. This term is a constant that will represent a maximum value for the power coefficient. Then, the power P''' is just a function of the third power of the wind speed and, therefore, it has a Weibull PDF with the parameters expressed in (19).

$$C''' = \frac{1}{2}A\rho C_{P_{\max}} C^3 \qquad k''' = \frac{k}{3} \tag{19}$$

And the mean power, considering more realistic limits than Betz' law, may be shown as a function of the parameter C (20).

$$P'''_{\text{mean}} = \frac{3}{8}\sqrt{\pi}A\rho C_{P_{\max}} C^3 \tag{20}$$

It may also be expressed as a function of the mean wind speed (21).

$$P'''_{\text{mean}} = \frac{3}{\pi}A\rho C_{P_{\max}} U_{\text{mean}}^3 \tag{21}$$

When establishing the criteria to evaluate the mean power per surface unit, the more realistic limit taken is shown in (22). If a value of 0.45 is assumed for $C_{P_{\max}}$, then:

$$\frac{P'''_{\text{mean}}}{A} = 0.53U_{\text{mean}}^3 \tag{22}$$

2.3. Relationship between wind speed and power produced by a WT operating at maximum power capacity considering Cut-In and Cut-Out limits

An even more realistic analysis of the wind power curves shows that these curves can be split in four different parts.

- The first part corresponds to low wind speeds, and it is not possible, by means of any control system, to obtain power from the wind. The upper limit of this part is the Cut-In speed. We will denote this wind speed as $U_{\text{Cut-In}}$.
- The second part corresponds to an interval of wind speeds, where maximum coefficient power can be reached (for a given variety of WTs). Generally, no pitch control is operated within this interval.
- The third part of the curve corresponds to the maximum power that the WT will generate, and pitch control generally operates in this part. The upper limit of this part is given by a Cut-Out point and the corresponding wind speed will be denoted as $U_{\text{Cut-Out}}$.
- The fourth part corresponds to wind speeds of values $U > U_{\text{Cut-Out}}$, and no power is generated for the sake of security.

Table 1
Output power in a WT depending on wind speed.

Wind speed interval	Output power
$U < U_{\text{Cut-In}}$	$P = 0$
$U_{\text{Cut-In}} < U < U_{P_{\max}}$	$P = \frac{1}{2}A\rho U^3 C_p(\lambda, \beta)$
$U_{P_{\max}} < U < U_{\text{Cut-Out}}$	$P = P_{\max}$
$U < U_{\text{Cut-Out=}}$	$P = 0$

According to this, and in order to obtain the PDF of the power generated by a WT, the following considerations must be assumed: when $U < U_{\text{Cut-In}}$ no power can be generated. When $U_{P_{\max}} < U < U_{\text{Cut-Out}}$ the power generated by the WT may be considered constant and equal to this maximum power, $P_{\max}$.

Thus, the PDF of the wind speed may be divided into four main intervals which can be seen in Table 1.

The probability of the wind speed being lower than the Cut-In point or greater than the Cut-Out one can be calculated as in (23) and (24), where probability has been denoted as Pr(·).

$$Pr(U \le U_{\text{Cut-In}}) = F_U(U_{\text{Cut-In}}) = 1 - e^{-(U_{\text{Cut-In}}/C)^k} \tag{23}$$

$$Pr(U_{\text{Cut-Out}} \le U) = 1 - F_U(U_{\text{Cut-Out}}) = e^{-(U_{\text{Cut-Out}}/C)^k} \tag{24}$$

The probability of the power being equal to zero coincides with the probability of the wind speed being lower than $U_{\text{Cut-In}}$ and higher than $U_{\text{Cut-Out}}$, expressed in (25).

$$Pr(P = 0) = 1 + e^{-(U_{\text{Cut-Out}}/C)^k} - e^{-(U_{\text{Cut-In}}/C)^k} \tag{25}$$

The probability of the wind speed, U, being higher than $U_{P_{\max}}$ but lower than $U_{\text{Cut-Out}}$ is calculated in (26) according to the Weibull cumulative distribution function (CDF).

$$\begin{aligned} Pr(U_{P_{\max}} \le U \le U_{\text{Cut-Out}}) &= F_U(U_{\text{Cut-Out}}) - F_U(U_{P_{\max}}) = \ldots \\ &= e^{-(U_{P_{\max}}/C)^k} - e^{-(U_{\text{Cut-Out}}/C)^k} \end{aligned} \tag{26}$$

Therefore, the probability of the output power being equal to $P_{\max}$ coincides with the probability of the wind speed being higher than $U_{P_{\max}}$ but lower than $U_{\text{Cut-Out}}$, that has been expressed in (26).

The probability of the output power being between the limits $P_{\min}$ and $P_{\max}$ will be taken as in Section 2.2. $P_{\min}$ is the output power obtainable from $U_{\text{Cut-In}}$, $P_{\min} = (1/2)\rho ACP\max U_{\text{Cut-In}}{}^3$.

The PDF is completed with two cases, i.e. when the output power is between 0 and $P_{\min}$, and when it is greater than $P_{\max}$.

Summarizing, the PDF of the output power is defined in (27).

$$f_P(P) = \begin{cases} (1 + e^{-(U_{\text{Cut-Out}}/C)^k} - e^{-(U_{\text{Cut-In}}/C)^k})\delta(P) & P = 0 \\ 0 & 0 < P \le P_{\min} \\ (k/3c_tC^3)(P/c_tC^3)^{(k/3)-1}e^{-(P/c_tC^3)^{k/3}} & P_{\min} < P < P_{\max} \\ (e^{-(U_{P_{\max}}/C)^k} - e^{-(U_{\text{Cut-Out}}/C)^k})\delta(P - P_{\max}) & P = P_{\max} \\ 0 & P_{\max} < P \end{cases} \tag{27}$$

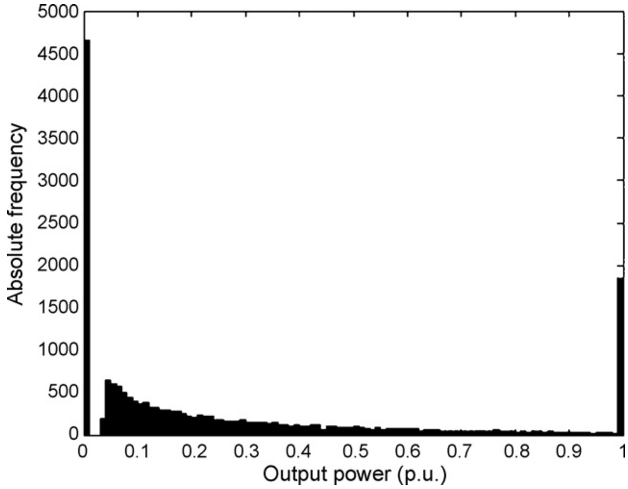

Fig. 1. PDF of the output power in a WT according to (27).

where $c_t = 1/2AC_{P_{\max}}$, C and k are the parameters defined in (3), P is the Output power, and δ is Dirac's delta, a generalized function, defined in Appendix A.

A part of the result given in (27) has also been obtained in [16,19].

For a more detailed illustration of how the power PDF behaves, the result of a 20,000-sample Monte Carlo simulation is shown in Fig. 1.

In order to obtain the mean output power such as in previous cases, it is necessary to solve (28).

$$P_{\mathrm{mean}} = \sum_{i=0}^{\infty} P_i Pr(P_i) = P_1 + P_2 \tag{28}$$

where the terms P_1 and P_2 are expressed, respectively, in (29) and (30).

$$\int_{P_{\min}}^{P_{\max}} P(k/3c_tC^3)\left(\frac{P}{c_tC^3}\right)^{(k/3)-1} e^{-\left(\frac{P}{c_tC^3}\right)^{k/3}} dP \tag{29}$$

$$(e^{-(U_{P\max}/C)^k} - e^{-(U_{\mathrm{Cut\text{-}Out}}/C)^k})P_{\max} \tag{30}$$

In order to solve (29), the Maclaurin power series are applied [21], so utilizing (31), the integral shown in (29) is converted into (32).

$$e^{-(P/c_tC^3)^{k/3}} = \sum_{n=0}^{\infty} (-1)^n P^{\frac{nk}{3}} \Big/ n!(c_tC^3)^{\frac{nk}{3}} \tag{31}$$

$$P_1 = \int_{P_{\min}}^{P_{\max}} k/3 \sum_{n=0}^{\infty} (-1)^n P^{\frac{(n+1)k}{3}} \Big/ n!(c_tC^3)^{\frac{(n+1)k}{3}} dP \tag{32}$$

which can also be converted into (33).

$$P_1 = \frac{k}{3}\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(c_tC^3)^{(n+1)k/3}} \int_{P_{\min}}^{P_{\max}} P^{(n+1)k/3} dP \tag{33}$$

that, once solved, can be written as in (34).

$$P_1 = \sum_{n=0}^{\infty} \frac{k(-1)^n \left[P^{(nk+k+3)/3}\right]_{P_{\min}}^{P_{\max}}}{n!(c_tC^3)^{(n+1)k/3}(nk+k+3)} \tag{34}$$

In (34) the limits can be taken into account to give (35).

$$P_1 = \sum_{n=0}^{\infty} \frac{k(-1)^n (P_{\max}^{(nk+k+3)/3} - P_{\min}^{(nk+k+3)/3})}{n!(c_tC^3)^{(n+1)k/3}(nk+k+3)} \tag{35}$$

In the case of a Rayleigh distribution ($k = 2$), (35) results in (36).

$$P_1 = \sum_{n=0}^{\infty} \frac{2(-1)^n (P_{\max}^{(2n+5)/3} - P_{\min}^{(2n+5)/3})}{n!(c_tC^3)^{2(n+1)/3}(2n+5)} \tag{36}$$

Equation (36) can be written depending on wind speed, resulting in (37).

$$P_1 = \sum_{n=0}^{\infty} \frac{2(-1)^n c_t (U_{P\max}^{(2n+5)} - U_{\mathrm{Cut\text{-}In}}^{(2n+5)})}{n! C^{2(n+1)}(2n+5)} \tag{37}$$

Moreover, by substituting parameter C as a function of U_{mean}, (38) is now obtained.

$$P_1 = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{n+1} c_t (U_{P_{\max}}^{2n+5} - U_{\mathrm{Cut\text{-}In}}^{2n+5})}{2^{2n+1} n! U_{\mathrm{mean}}^{2(n+1)}(2n+5)} \tag{38}$$

Finally, replacing c_t for its value, (39) may be treated as a final result.

$$P_1 = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{n+1} A\rho C_{P_{\max}} (U_{P_{\max}}^{2n+5} - U_{\mathrm{Cut\text{-}In}}^{2n+5})}{2^{2(n+1)} n! U_{\mathrm{mean}}^{2(n+1)}(2n+5)} \tag{39}$$

As in former sections, the mean power density is obtained in (40), by taking into account both terms, P_1, given in (39), derived from (29), and P_2, given in (30), and finally, writing $P_{\max}$ as a function of ρ, A and $C_{P_{\max}}$. In this case it depends on $U_{\mathrm{Cut\text{-}In}}$, $U_{P_{\max}}$, $U_{\mathrm{Cut\text{-}Out}}$ and U_{mean}.

$$\begin{aligned}\frac{P_{\mathrm{mean}}}{A} &= \frac{P_1 + P_2}{A} \\ &= \rho C_{P_{\max}}\left(\frac{e^{-(U_{P\max}/C)^k} - e^{-(U_{\mathrm{Cut\text{-}Out}}/C)^k}}{2} U_{P\max}^3 \right. \\ &\quad \left. + \cdots + \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{n+1} (U_{P_{\max}}^{2n+5} - U_{\mathrm{Cut\text{-}In}}^{2n+5})}{2^{2(n+1)} n! U_{\mathrm{mean}}^{2(n+1)}(2n+5)}\right)\end{aligned} \tag{40}$$

It can be seen that there is a dependency on parameters that have to be given by manufacturers in WT data sheets. In section III a real WT is analyzed by using these equations.

3. Application to a real WT

Eqs. (16), (17) and (22) show very useful relationships between the mean power density and the cube of the mean wind speed.

The mean power obtained in each case has a different meaning. It should be remembered that the mean power that the wind carries is obtained in (16); the maximum mean power that may be obtained theoretically through a real WT is calculated in (17); and the maximum value of the mean power that may be obtained with a real WT, considering its current limits, is deduced from Eq. (22).

A more realistic approach considers the maximum output power of the WT and the wind speed Cut-In and Cut-Out limits of the WT. In that case, and, in order to obtain an equation similar to those described above, the specific data of a WT have to be considered.

The WT used in the simulation tests is a 2 MW Vestas V80, with $U_{\mathrm{Cut\text{-}In}}$ = 4 m/s, $U_{\mathrm{Cut\text{-}Out}}$ = 25 m/s. These data are given by Vestas in the machine data sheet.

As the values of $U_{\mathrm{Cut\text{-}In}}$, $U_{\mathrm{Cut\text{-}Out}}$, and $P_{\max}$ are specified, the results obtained are shown as a function of U_{mean}. The values obtained for that WT are given in Table 2, and they are also displayed in Fig. 2. The results are similar to those obtained in [22].

Table 2
Values Obtained Using Eqs. (16), (17), (22) and (40).

MWS	MPD (16)	MPD (17)	MPD (22)	MPD (40)
4	75	44	34	30
5	146	86	66	60
6	253	149	114	97
7	401	237	182	134
8	599	353	271	168
9	853	503	386	198
10	1170	690	530	222
11	1557	918	705	240
12	2022	1192	916	253
13	2571	1516	1164	260
14	3211	1893	1454	262
15	3949	2329	1789	261
16	4792	2826	2171	257
17	5748	3390	2604	251
18	6823	4024	3091	243
19	8025	4733	3635	235
20	9360	5520	4240	226
21	10,835	6390	4908	217
22	12,458	7347	5643	207
23	14,235	8395	6449	198
24	16,174	9539	7327	189
25	18,281	10,781	8281	180

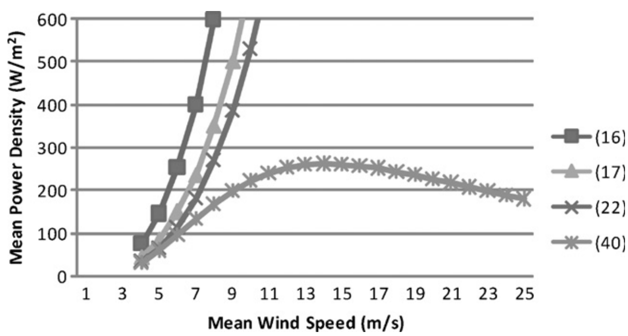

Fig. 2. Mean power density as a function of the mean wind speed according to Eqs. (16), (17), (22) and (40).

In Table 2, MWS means "mean wind speed", given in m/s, and MPD means "mean power density", given in W/m^2.

Although there are not so many locations with mean wind speeds as high as 15 m/s or more, the results are interesting in order to show how a WT responds to these wind speed values, having a maximum around 14 m/s (for that WT) and decreasing smoothly beyond this value.

Table 2 shows that, for example, for a location with a mean wind speed of 7 m/s, there are 401 W/m^2 available in the wind stream; no more than 237 W/m^2 may be obtained through a WT theoretically; no more than 182 W/m^2 would be available considering real technical limits of a WT; and less than 134 W/m^2 could be obtained through a Vestas V80-2.0, which has been chosen for the example.

4. Conclusions

The dependency between wind speed and maximum power obtained from a WT is described in this paper.

In a first step an analysis of the wind power PDF has been carried out. It has been expressed as a function of the wind speed PDF, assumed to be a Weibull function. The mean power density has been calculated by means of an expression, which has been given in (16).

The maximum power that a WT can extract from the wind was treated in a similar way. Betz' law was obviously taken into account. The result was given in (17).

The next step consisted of establishing a coefficient, $C_{P_{\max}}$, in an attempt to fix a more realistic limit for the power that can be extracted from a WT. This limit was fixed at 0.45 for this work, but, technological advances will see it reaching values closer to the Betz' limit in future years. The results by using this coefficient were given in (22).

Furthermore, considering an even more realistic case, where parameters of a real WT are $U_{\text{Cut-In}}$, $U_{\text{Cut-Out}}$, $U_{P_{\max}}$ and $P_{\max}$ a PDF was obtained for the power extracted by a WT, and the mean density power was obtained as a function of those parameters and the mean wind speed.

As a result of this work, the following questions can be answered by using the result obtained:

(1) Given the mean wind speed as a unique feature of a location, what is the maximum mean power that may be extracted from the wind?
(2) Given the mean wind speed as a unique feature of a location, what is the maximum mean power that can be extracted from the wind, considering a given WT?
(3) As long as the WT's are being developed and power coefficients are increasing, what is the theoretical limit for the mean power extracted from the wind in that situation?

Acknowledgement

The financial support given by the autonomous government *Xunta de Galicia*, under the contract *INCITE 08REM009303PR* is gratefully acknowledged by the authors.

Appendix A. Appendix

A.1. Dirac's delta

The Dirac delta [21] is a generalized function or density distribution function that can be defined as follows:

$$\begin{aligned} &\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \\ &\int_{-\infty}^{\infty} \delta(t)dt = \int_{-\varepsilon}^{\varepsilon} \delta(t)dt = 1 \qquad \varepsilon > 0 \\ &\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a) \end{aligned} \tag{A.1}$$

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