



Polarization properties of ‘slow’ light

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ABSTRACT

The propagation of the arbitrarily polarized pulse of the weak probe field through the resonant medium of Λ -type three-level atoms with degenerate levels adiabatically driven by the coherent coupling field is considered. It is shown that such pulse is decomposed in the medium into two orthogonally polarized dark-state polaritons propagating with different group velocities. The expressions for the polarizations and group velocities of these two polaritons are obtained. The dependence of these polarizations and group velocities on the values of the angular momenta of resonant levels, on the polarization of the coupling field and on the initial atomic state is studied.

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1. Introduction

The remarkable reduction of the group velocity of light pulses [1,2] based on the phenomenon of the electromagnetically induced transparency (EIT) [3,4] provided a number of applications, the most promising among them being the implementation of quantum memory [5–7]. The recent experiments [8–10] on EIT-based quantum memory demonstrate the continuously enhancing memory efficiency and fidelity, bringing it close to practical applications. Such memory is based on the concept of dark-state polaritons in the three-level Λ -type systems, proposed in [11,12] and soon realized in rubidium vapor in the experiment [13]. The group velocity of such polaritons may be controlled by the adiabatically varying intensity of the driving field to store single-photon pulses in the resonant media or to retrieve them. The most natural way for qubit encoding is provided by the photon two polarization degrees of freedom, as it was implemented in the experiments [9,10]. However the group velocity of the dark-state polariton may depend essentially on its polarization due to the optical anisotropy induced by the polarization of the driving field, while for the effective storage of the photon polarization qubit its both polarization components must be stopped in the medium simultaneously. The objective of the present paper is to study the polarization properties of the dark-state polaritons formed in the three-level Λ -type systems with degenerate levels, which are in many experiments the hyperfine structure components of alkali atoms degenerate in the projections of the atomic total angular momentum on the quantization axis.

2. Basic equations and relations

We consider the pulse of the weak probe field propagating along the sample axis Z with the carrier frequency ω , which is in resonance with the frequency ω_0 of an optically allowed transition $J_a \rightarrow J_c$ between the ground state J_a and the excited state J_c , while the strong coherent coupling field propagates in the same direction with the carrier frequency ω_c , which is in resonance with the frequency ω_{c0} of an optically allowed transition $J_b \rightarrow J_c$ between the long-lived state J_b and the same excited state J_c (Fig. 1). Here J_a, J_b and J_c are the values of the angular momenta of the levels. The electric field strength of the coupling field and that of the probe field may be put down as follows:

$$\mathbf{E}_c = e_c(t - z/c) \mathbf{l}_c e^{-i\omega_c(t - z/c)} + c.c., \quad (1)$$

$$\mathbf{E} = \mathbf{e}(t, z) e^{-i\omega(t - z/c)} + c.c., \quad (2)$$

where e_c is the slowly varying amplitude of the coupling field and \mathbf{l}_c is its constant unit polarization vector, while \mathbf{e} is the slowly varying vector amplitude of the probe pulse, which satisfies the equation:

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) \mathbf{e} = \frac{i\omega n_0 |d|}{2\epsilon_0} \text{tr} \{ \hat{\mathbf{g}} \hat{\rho} \}, \quad (3)$$

as it follows from Maxwell equations, while the evolution of the atomic slowly varying density matrix $\hat{\rho}$ in the rotating-wave approximation is described by the equation:

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{i}{2} [\hat{V}, \hat{\rho}] + \left(\frac{d\hat{\rho}}{dt} \right)_{\text{rel}}, \quad (4)$$

$$\hat{V} = 2(\Delta \hat{P}_c + \delta \hat{P}_b) + \hat{D} + \hat{D}^\dagger, \quad \hat{D} = \Omega_c \hat{g}_c + \hat{G}, \quad (5)$$

$$\hat{g}_c = \hat{\mathbf{g}}_c \mathbf{l}_c^*, \quad \hat{G} = (2|d|/\hbar) \hat{\mathbf{g}} \mathbf{e}^*. \quad (6)$$

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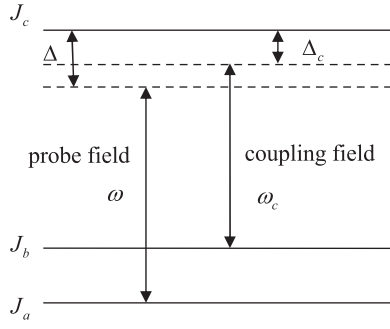


Fig. 1. The level diagram.

Here n_0 is the concentration of resonant atoms, $\hat{\mathbf{g}}$ and $\hat{\mathbf{g}}_c$ are the dimensionless electric dipole moment operators for the transitions $J_c \rightarrow J_a$ and $J_c \rightarrow J_b$, $d = d(J_a J_c)$ and $d_c = d(J_b J_c)$ being the reduced matrix elements of the electric dipole moment operators for these transitions, $\Delta = \omega - \omega_0$ and $\Delta_c = \omega_c - \omega_{c0}$ are the frequency detunings from resonance of the probe and of the coupling fields, while $\delta = \Delta - \Delta_c$, \hat{P}_α is the projector on the subspace of the atomic level J_α ($\alpha = a, b, c$), $\Omega_c = 2|d_c|e_c/\hbar$ is the reduced Rabi frequency for the coupling field. The matrix elements of the circular components \hat{g}_q and \hat{g}_{cq} ($q = 0, \pm 1$) of vector operators $\hat{\mathbf{g}}$ and $\hat{\mathbf{g}}_c$ are expressed through Wigner 3J-symbols [14]:

$$(\hat{g}_q)_{m_a m_c}^{ac} = (-1)^{J_a - m_a} \begin{pmatrix} J_a & 1 & J_c \\ -m_a & q & m_c \end{pmatrix}, \quad (7)$$

$$(\hat{g}_{cq})_{m_b m_c}^{bc} = (-1)^{J_b - m_b} \begin{pmatrix} J_b & 1 & J_c \\ -m_b & q & m_c \end{pmatrix}. \quad (8)$$

Finally, the term $(d\hat{\rho}/dt)_{rel}$ in Eq. (4) describes the irreversible relaxation. Initially the atoms are at the ground state a the atomic density matrix being

$$\hat{\rho}(0) = \hat{\rho}_a.$$

In the linear approximation for the probe field we obtain from Eqs. (4)–(5) for the elements of the atomic density matrix the following equations:

$$\left(\frac{\partial}{\partial t} + \gamma - i\Delta\right)\hat{\rho}^{ca} = \frac{i}{2} \left(\Omega_c \hat{g}_c^\dagger \hat{\rho}^{ba} + \hat{G}^\dagger \hat{\rho}_a\right), \quad (9)$$

$$\left(\frac{\partial}{\partial t} + \Gamma - i\delta\right)\hat{\rho}^{ba} = \frac{i}{2} \Omega_c \hat{g}_c \hat{\rho}^{ca}, \quad (10)$$

where

$$\hat{\rho}^{\alpha\beta} = \hat{P}_\alpha \hat{\rho} \hat{P}_\beta, \quad \alpha, \beta = a, b, c,$$

while the irreversible relaxation is simply characterized by the two real relaxation rates $-\gamma$ for the optically allowed transition $J_a \rightarrow J_c$ and Γ for the optically forbidden transition $J_a \rightarrow J_b$:

$$\left(\frac{d\hat{\rho}}{dt}\right)_{rel}^{ca} = -\gamma \hat{\rho}^{ca}, \quad \left(\frac{d\hat{\rho}}{dt}\right)_{rel}^{ba} = -\Gamma \hat{\rho}^{ba}.$$

In the adiabatic approximation, when the coupling field varies slowly:

$$\gamma T \gg 1, \quad \Omega_c^2 T \gg \gamma,$$

$T \simeq |\Omega_c / \dot{\Omega}_c|$ being the characteristic time scale for the variation (switching on or off) of the coupling field, while the relaxation at the forbidden transition remains negligible

$$\Gamma T \ll 1,$$

in the case of single-photon and two-photon resonances

$$\Delta \leq \gamma, \quad \delta T \ll 1,$$

from (9)–(10) it follows:

$$\Omega_c \hat{g}_c^\dagger \hat{\rho}^{ba} + \hat{G}^\dagger \hat{\rho}_a = 0, \quad (11)$$

$$\frac{\partial \hat{\rho}^{ba}}{\partial t} = \frac{i}{2} \Omega_c \hat{g}_c \hat{\rho}^{ca}. \quad (12)$$

Eq. (11), which is the approximation of the relation

$$\hat{D}^\dagger \hat{\rho} = 0,$$

linear in the probe field, means that only the dark states contribute to the solution of Eq. (12). In order to express $\hat{\rho}^{ca}$ through $\hat{\rho}^{ba}$ we multiply both parts of Eq. (12) by the matrix \hat{g}_c^\dagger from the left and consider the orthonormal set of eigenvectors $|c_n\rangle$ of the hermitian operator $\hat{g}_c^\dagger \hat{g}_c$ acting at the subspace of the excited level c :

$$\hat{g}_c^\dagger \hat{g}_c |c_n\rangle = \sigma_n^2 |c_n\rangle, \quad n = 1, \dots, 2J_c + 1, \quad (13)$$

the corresponding eigenvalues being non-negative $\sigma_n^2 \geq 0$. Then, after multiplying both parts of Eq. (12) from the left by the matrix

$$\hat{D}_c = \sum_n \frac{1}{\sigma_n^2} |c_n\rangle \langle c_n|, \quad (14)$$

we obtain

$$\hat{P}_c^b \hat{\rho}^{ca} = -\frac{2i}{\Omega_c} \frac{\partial}{\partial t} (\hat{D}_c \hat{g}_c^\dagger \hat{\rho}^{ba}), \quad (15)$$

where

$$\hat{P}_c^b = \sum_n |c_n\rangle \langle c_n|, \quad (16)$$

while the summation in Eqs. (14) and (16) is carried out only over eigenvectors $|c_n\rangle$ with non-zero eigenvalues $\sigma_n^2 > 0$, \hat{P}_c^b being the projector on the subspace formed by such eigenvectors. The eigenvectors $|c_n\rangle$ with zero eigenvalues $\sigma_n^2 = 0$ are not affected by the coupling field and may be neglected under the assumed approximation. Now let us introduce the vector field

$$\mathbf{p} = \text{tr}\{\hat{g}_c \hat{D}_c \hat{g}_c^\dagger \hat{\rho}^{ba}\}, \quad (17)$$

describing the induced coherence at the forbidden transition $J_a \rightarrow J_b$. With the two orthonormal vectors \mathbf{l}_i in the polarization plane XY ($\mathbf{l}_j \mathbf{l}_k^* = \delta_{jk}$, $j, k = 1, 2$) we obtain from (3), (15) and (11) for the components $e_k = \mathbf{e} \mathbf{l}_k^*$ and $p_k = \mathbf{p} \mathbf{l}_k^*$ the following equations:

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z}\right) e_k = \frac{\omega n_0 |d|}{\varepsilon_0 \Omega_c} \frac{\partial p_k}{\partial t}, \quad (18)$$

$$p_k = -\frac{2|d|}{\hbar \Omega_c} \sum_j a_{kj} e_j, \quad (19)$$

$$a_{kj} = \text{tr}\{\hat{\rho}_a \hat{g}_k \hat{D}_c \hat{g}_j^\dagger\}, \quad \hat{g}_k = \hat{\mathbf{g}} \mathbf{l}_k^*. \quad (20)$$

Now let us choose the two orthonormal vectors \mathbf{l}_i as the two eigenvectors of the hermitian 2×2 matrix a_{jk} , defined by (20). Then

$$a_{kj} = a_k \delta_{kj}, \quad (21)$$

where a_k ($k=1,2$) are the two real eigenvalues of this matrix. By introducing via canonical transformation the field of the dark-state polariton [12]:

$$\Psi_k = \cos \theta_k e_k - \sin \theta_k \lambda_k p_k, \quad (22)$$

where

$$\lambda_k = \sqrt{\frac{\hbar \omega n_0}{2 \varepsilon_0 a_k}}, \quad (23)$$

and the angle θ_k is determined by the equation

$$\tan \theta_k = \frac{2|d| a_k \lambda_k}{\hbar \Omega_c}, \quad (24)$$

neglecting the retardation of the coupling field $\Omega_c(t-z/c) \approx \Omega_c(t)$, we obtain from (18)–(24) the following equation:

$$\frac{\partial \Psi_k}{\partial t} + c \cos^2 \theta_k \frac{\partial \Psi_k}{\partial z} = 0, \quad (25)$$

which describes the propagation of the dark-state polariton with the group velocity

$$V_k^{gr} = c \cos^2 \theta_k = \frac{c}{1 + n_k^{gr}}, \quad (26)$$

$$n_k^{gr} = \tan^2 \theta_k = \frac{2|d|^2 n_0 \omega a_k}{\hbar \Omega_c^2 \varepsilon_0}. \quad (27)$$

In the case when the probe field (2) represents itself a single-photon pulse the classical field $\mathbf{e}(t, z)$ must be replaced by the operator $\hat{\mathbf{e}}(t, z)$ [12]. All the results of the present section remain true for such single-photon pulses with the substitution of the photon-field function

$$\mathbf{e}(t, z) = \langle 0 | \hat{\mathbf{e}}(t, z) | f \rangle$$

instead of the classical field $\mathbf{e}(t, z)$, where $|0\rangle$ and $|f\rangle$ stand for the vacuum and single-photon field states respectively.

3. Discussion

As it follows from (22)–(27), the arbitrarily polarized pulse of the weak probe field is decomposed under the action of the strong coupling field into two components polarized along the two eigenvectors of tensor (20) propagating with different group velocities, which difference is determined by the difference in the two eigenvalues of tensor (20). So the polarization properties of the dark-state polaritons are totally determined by the hermitian 2×2 tensor a_{jk} defined by Eq. (20). This tensor in its turn is determined by the values of the angular momenta of resonant levels, by the polarization of the coupling field and by the initial atomic state.

In the experiments on the EIT-based quantum memory the coupling field is adiabatically switched off as the photon pulse enters the resonant medium so that the mixing angle θ_k , defined by Eq. (24), varies from 0 to $\pi/2$. Then, according to Eq. (22), the field component e_k is stopped in the medium and mapped to the atomic state p_k determined by Eq. (17). To retrieve this photon-field component from the medium the coupling field is adiabatically switched on decreasing the mixing angle θ_k from $\pi/2$ to 0. In order to store the qubit encoded in the polarization state of the photon both its polarization components must be stopped in the medium simultaneously, which implies the equality $a_1 = a_2$ of the two eigenvalues of the tensor (20).

Let us now calculate the relative difference $1 - a_2/a_1$ in the eigenvalues a_1 and a_2 ($a_1 > a_2$) and the corresponding eigenvectors \mathbf{l}_1 and \mathbf{l}_2 of the tensor a_{jk} for some transitions $J_a \rightarrow J_c \rightarrow J_b$ with the angular momenta characteristic for the experiments on the hyperfine structure components of atomic levels. In the case of equilibrium initial atomic state

$$\hat{\rho}_a = \frac{\hat{P}_a}{2J_a + 1}$$

we obtain:

$J_a \rightarrow J_c \rightarrow J_b$	\mathbf{l}_c	$1 - a_2/a_1$	\mathbf{l}_1	\mathbf{l}_2
$0 \rightarrow 1 \rightarrow 1$	\mathbf{l}_x	1	\mathbf{l}_y	\mathbf{l}_x
$1 \rightarrow 1 \rightarrow 2$	\mathbf{l}_x	0.125	\mathbf{l}_x	\mathbf{l}_y
$1 \rightarrow 2 \rightarrow 2$	\mathbf{l}_x	0.25	\mathbf{l}_x	\mathbf{l}_y
$2 \rightarrow 2 \rightarrow 3$	\mathbf{l}_x	0.3	\mathbf{l}_x	\mathbf{l}_y
$2 \rightarrow 3 \rightarrow 3$	\mathbf{l}_x	0.396	\mathbf{l}_x	\mathbf{l}_y
$3 \rightarrow 3 \rightarrow 4$	\mathbf{l}_x	0.347	\mathbf{l}_x	\mathbf{l}_y

$3 \rightarrow 4 \rightarrow 4$	\mathbf{l}_x	0.439	\mathbf{l}_x	\mathbf{l}_y
$0 \rightarrow 1 \rightarrow 1$	\mathbf{l}_{+1}	1	\mathbf{l}_{+1}	\mathbf{l}_{-1}
$1 \rightarrow 1 \rightarrow 2$	\mathbf{l}_{+1}	0.625	\mathbf{l}_{+1}	\mathbf{l}_{-1}
$1 \rightarrow 2 \rightarrow 2$	\mathbf{l}_{+1}	0.577	\mathbf{l}_{+1}	\mathbf{l}_{-1}
$2 \rightarrow 2 \rightarrow 3$	\mathbf{l}_{+1}	0.568	\mathbf{l}_{+1}	\mathbf{l}_{-1}
$2 \rightarrow 3 \rightarrow 3$	\mathbf{l}_{+1}	0.402	\mathbf{l}_{+1}	\mathbf{l}_{-1}
$3 \rightarrow 3 \rightarrow 4$	\mathbf{l}_{+1}	0.529	\mathbf{l}_{+1}	\mathbf{l}_{-1}
$3 \rightarrow 4 \rightarrow 4$	\mathbf{l}_{+1}	0.307	\mathbf{l}_{+1}	\mathbf{l}_{-1}

Here \mathbf{l}_x and \mathbf{l}_y denote the unit vectors of the Cartesian axes, while

$$\mathbf{l}_{\pm 1} = \mp \frac{\mathbf{l}_x \pm i \mathbf{l}_y}{\sqrt{2}}$$

are the two circular vectors.

As it follows from this table, the photon polarization qubit cannot be recorded in a single medium sample in all the considered cases ($a_1 \neq a_2$). However the two different polarization components of the photon may be recorded separately in the two different samples providing thus the entanglement between the two spatially separated atomic ensembles. For such a purpose the greatest difference in the group velocities of the two polarization components is preferable. The greatest difference is achieved for the most simple transition $J_a = 0 \rightarrow J_c = 1 \rightarrow J_b = 1$. On such transitions with the coupling field linearly polarized along the axis X ($\mathbf{l}_c = \mathbf{l}_x$) the three states $|c_n\rangle$ ($n = 1, 2, 3$) at the excited level c , defined by Eq. (13), are as follows:

$$|c_1\rangle = \frac{1}{\sqrt{2}}(|m_c = -1\rangle + |m_c = 1\rangle),$$

$$|c_2\rangle = |m_c = 0\rangle,$$

$$|c_3\rangle = \frac{1}{\sqrt{2}}(|m_c = -1\rangle - |m_c = 1\rangle),$$

the corresponding eigenvalues being

$$\sigma_1^2 = \sigma_2^2 = \frac{1}{6}, \quad \sigma_3^2 = 0.$$

The X-component of the probe field couples the ground state $|m_a = 0\rangle$ only to the state $|c_3\rangle$ of the excited level, which remains unaffected by the coupling field:

$$\hat{g}_x |c_1\rangle = \hat{g}_x |c_2\rangle = 0, \quad \hat{g}_x |c_3\rangle \neq 0, \quad \hat{g}_c |c_3\rangle = 0,$$

so this component passes through the medium with the group velocity close to the vacuum light speed c ($a_2 = a_x = 0$). The Y-component of the probe field couples the ground state $|m_a = 0\rangle$ only to the state $|c_1\rangle$ of the excited level, which is affected by the coupling field:

$$\hat{g}_y |c_2\rangle = \hat{g}_y |c_3\rangle = 0, \quad \hat{g}_y |c_1\rangle \neq 0, \quad \hat{g}_c |c_1\rangle \neq 0,$$

so the group velocity of this component is reduced by the coupling field ($a_1 = a_y = 2$). In the case of more complex transitions both polarization components of the probe field couple the states of the ground level a to the states of the excited level c , which are all affected by the coupling field, providing the reduction of the group velocity for both components, though differently.

For the purposes of quantum memory, however, the group velocity of the single-photon pulse must not depend on its polarization, which cannot be achieved with the probe and coupling fields both polarized in the same plane, as it may be seen from the table. In the experiment [9] the coupling field was linearly polarized in the direction of propagation of the probe field: $\mathbf{l}_c = \mathbf{l}_z$, while it propagated in the perpendicular direction. With such a π -polarized coupling field ($\mathbf{l}_c = \mathbf{l}_z$) and with the diagonal initial atomic state

$$\hat{\rho}_a = \sum_{m=-J_a}^{J_a} n_m^a |J_a, m\rangle \langle J_a, m|$$

the tensor a_{kj} (20) is transformed to the expression

$$a_{kj} = \sum_{m,q} n_m^a G_q^m l_{k,q} l_{j,q}^*, \quad (28)$$

where

$$G_q^m = \frac{\begin{pmatrix} J_a & 1 & J_c \\ -m & q & m-q \end{pmatrix}^2}{\begin{pmatrix} J_b & 1 & J_c \\ q-m & 0 & m-q \end{pmatrix}^2}. \quad (29)$$

The two eigenvectors of the tensor a_{kj} (28) are the two circular vectors \mathbf{l}_q ($q = \pm 1$) with the corresponding eigenvalues

$$a_q = \sum_m n_m^a G_{-q}^m. \quad (30)$$

Since $G_q^m = G_{-q}^{-m}$, then

$$a_{-q} = \sum_m n_{-m}^a G_{-q}^m,$$

and both polarization components of the probe pulse will propagate with the same group velocity $a_{+1} = a_{-1}$ in the case when the Zeeman components of the ground state are initially symmetrically populated:

$$n_m^a = n_{-m}^a. \quad (31)$$

In the experiment [9] the atoms were prepared at the pure Zeeman state with the zero projection on the quantization axis: $n_m^a = 0$, which is in compliance with the condition (31). However the condition (31) is also fulfilled for the unprepared atoms, which are initially in the equilibrium state with equally populated Zeeman sublevels: $n_m^a = 1/(2J_a + 1)$. So, rather a complicated stage of preparation of the atom at the initial pure state $n_m^a = 0$ may be avoided.

4. Conclusions

In the present paper we consider the propagation of the arbitrarily polarized pulse of the weak probe field through the resonant medium of Λ -type three-level atoms with degenerate levels adiabatically driven by the coherent coupling field. It is shown that such pulse is decomposed in the medium into two orthogonally polarized dark-state polaritons propagating with

different group velocities. The polarizations of these two polaritons and the difference in their group velocities are determined by the values of the angular momenta of resonant levels, by the polarization of the coupling field and by the initial atomic state.

For the purposes of the EIT-based quantum memory the group velocity of the single-photon pulse must not depend on its polarization. It is shown that in the case of the π -polarized coupling field this condition will be fulfilled for the unprepared atoms, which are initially in the equilibrium state with equally populated Zeeman sublevels, enabling thus to omit rather a complicated stage of preparation of the atoms at the initial pure state.

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