



Periodically modulated single-photon transport in one-dimensional waveguide



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ABSTRACT

Single-photon transport along a one-dimension waveguide interacting with a quantum system (e.g., two-level atom) is a very useful and meaningful simplified model of the waveguide-based optical quantum devices. Thus, how to modulate the transport of the photons in the waveguide structures by adjusting certain external parameters should be particularly important. In this paper, we discuss how such a modulation could be implemented by periodically driving the energy splitting of the interacting atom and the atom–photon coupling strength. By generalizing the well developed time-independent full quantum mechanical theory in real space to the time-dependent one, we show that various sideband-transmission phenomena could be observed. This means that, with these modulations the photon has certain probabilities to transmit through the scattering atom in the other energy sidebands. Inversely, by controlling the sideband transmission the periodic modulations of the single photon waveguide devices could be designed for the future optical quantum information processing applications.

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1. Introduction

Single-photon propagation is one of the basic and important subjects in quantum optics. It is related to the designs and fabrications of various optical quantum devices for optical quantum information processings [1–3]. Recently, many theoretical and experimental works have been demonstrated to investigate the single-photon transport along a one-dimension waveguide with aside one- and multi- atoms as the scatters [4–9]. These investigations are directly related to various single-photon quantum device applications to implement, e.g., the single-photon routers, switches, and detectors, etc., [10–16], as well as quantum communications and quantum information applications [17–21]. Note that, almost all these works are based on a time-independent quantum theory, i.e., the Hamiltonians of the considered systems are time independent, and thus can only describe the elastic scatterings of the photons in the waveguide by the aside atom(s).

However, manipulatable single-photon devices are usually necessary for many practical applications, such as the quantum Zeno switches [22]. Therefore, the investigation of how to modulate the transport of the photons along the waveguide-atom structures by controlling certain external parameters should be meaningful. Physically, these modulations can be applied to either the energy splitting(s) of the

scattering atom(s) or the photon–atom interaction, or both of them. For example, in a recent experiment [23] the famous dynamical Casimir effect was verified by probing the sideband photons, generated by the microwave propagating along a coplanar waveguide terminated by a superconducting quantum interference device with fast changing magnetic flux.

It is noted that the time-dependent transport problem is usually encountered for the electron transport along the electronic waveguide in mesoscopic physics, and the relevant theory [24–28], including the so-called Floquet theory for periodic modulation [26–30], has been developed well by directly solving the time-dependent Schrödinger equation. A typical deduction for this theory is, due to the inelastic scatterings the electrons could be transmitted/reflected into the various energy sidebands (with the zero-sideband describing the elastic scattering of the electrons). As a consequence, the electronic transport could be modulated, in principle, from one sideband to the others.

Similar to the time-dependent electronic waveguide transport theory, in this paper we will develop a time-dependent single-photon transport theory to describe the photons propagating in the optical waveguides with certain time-modulations. Certainly, due to the present inelastic scatterings, the photons can also be propagated in various

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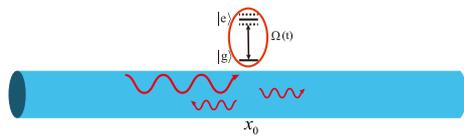


Fig. 1. A single photon transporting along a one-dimension waveguide is scattered by a two-level atom (at x_0) with the energy splitting between $|g\rangle$ and $|e\rangle$ being periodically modulated.

energy sidebands, and thus the total transmitted/reflected probability of the photon should be the sum of the ones in all the possible sidebands. Physically, the desired modulations could be achieved by adjusting the atomic level-splittings or the atom–photon coupling strength. As a consequence, the transport of the photons along the designed waveguides can be controlled at a single-photon level.

The paper is organized as follows. In Section 2, we present our model by considering a single waveguide photon scattered by a two-level atom with the periodic modulating energy splitting. With such a modulation we show that the photons could transmit through the atom in certain energy sidebands. In Section 3, keeping the transition frequency of the atom unchanged, we investigate how to control the transport of the photon by using the periodically modulated photon–atom interaction. In this case, we find that the transmission of the photon is mainly along the $n \neq 0$ sideband (due to the inelastic scattering), and the transmission in the zero-sideband (related to the elastic scattering) is negligible. Finally, in Section 4 we summarize our work and discuss the potential applications of the time-dependent single photon transport theory developed here.

2. A single waveguide photon scattered by a two-level atom with periodic modulated transition frequency

At the first, let us consider a simplest model, i.e., a single-photon with the fixed frequency transporting along a one-dimension waveguide and being scattered by an ideal two-level atom (i.e., without any atomic decay), whose eigenfrequency is periodically modulated. The system is sketched in Fig. 1, wherein the energy splitting between the ground state $|g\rangle$ and the excited state $|e\rangle$ of the atom, locating at x_0 , is periodically modulated. The Hamiltonian of the system can be written as ($\hbar = 1$) :

$$H = \int dx \left[c_R^\dagger(x) (-iv_g \frac{\partial}{\partial x}) c_R(x) + c_L^\dagger(x) (iv_g \frac{\partial}{\partial x}) c_L(x) \right] + \int dx V \delta(x - x_0) [c_R(x) \sigma^+ + c_L(x) \sigma^+ + H.c.] + \Omega(t) \sigma^+ \sigma^- \tag{1}$$

Here, $c_R^\dagger(x)$ ($c_R(x)$) and $c_L^\dagger(x)$ ($c_L(x)$) are the bosonic creation (annihilation) operators of the single-photon propagating right and left directions, respectively. v_g is the group velocity of the photon, V is the coupling strength between the waveguide photon and the atom, and σ^+ (σ^-) the atomic raising (lowering) ladder operator. The atomic transition frequency Ω between the ground and excited states is now periodically modulated, i.e., $\Omega(t) = \Omega[1 + f \cos(\omega t)]$ with $f \ll 1$ being the modulated amplitude and ω the modulated frequency. The atom–photon coupling strength V is kept unchanged and the dissipations of the system are neglected also for simplicity.

The generic solution to the time-dependent Schrödinger equation with the Hamiltonian (1) can be expressed as

$$|\Psi\rangle = \int dx \left[\phi_R(x, t) c_R^\dagger(x) + \phi_L(x, t) c_L^\dagger(x) \right] |\emptyset\rangle + e(t) \sigma^+ |\emptyset\rangle, \tag{2}$$

with $|\emptyset\rangle$ being the vacuum state, i.e., without any photon in the waveguide and the atom stays at its ground state $|g\rangle$, and $\phi_{R/L}(x, t)$ and $e(t)$ standing for the time-dependent probabilistic amplitudes of the

photon propagating along the R/L direction and the atomic excitation, respectively. The time-dependent coefficients in the above wave function are determined by the following equations:

$$i \frac{\partial}{\partial t} \phi_R(x, t) = -iv_g \frac{\partial}{\partial x} \phi_R(x, t) + V \delta(x) e(t), \tag{3}$$

$$i \frac{\partial}{\partial t} \phi_L(x, t) = iv_g \frac{\partial}{\partial x} \phi_L(x, t) + V \delta(x) e(t) \tag{4}$$

$$i \frac{\partial}{\partial t} e(t) = \Omega [1 + f \cos(\omega t)] e(t) + V [\phi_R(0, t) + \phi_L(0, t)]. \tag{5}$$

As the incident single-photon is now scattered by a time-dependent atom and thus its energy should be no longer conservation. This implies that the photon could be transmitted/reflected into the different energy states, i.e., energy sidebands. The above probabilistic amplitudes of the photon propagating along the R/L direction could be taken generically as

$$\phi_R(x, t) = \theta(-x + x_0) e^{i(q_0 x - \omega_0 t)} + \theta(x - x_0) \psi_R(x, t), \tag{6}$$

$$\phi_L(x, t) = \theta(-x + x_0) \psi_L(x, t), \tag{7}$$

where $\psi_R(x, t)$ and $\psi_L(x, t)$ stand for the transmitted and reflected parts of the scattered photon, respectively. Also, ω_0 is the frequency of the incident photon with the wave vector $q_0 = \omega_0/v_g$.

Without loss of the generality, we take $x_0 = 0$ for simplicity. By substituting Eqs. (6) and (7) into Eqs. (3) and (4), we have

$$\psi_R(0, t) = \psi_L(0, t) + e^{-i\omega_0 t}, \tag{8}$$

$$V e(t) = iv_g \psi_L(0, t). \tag{9}$$

Furthermore, with Eq. (5) we get

$$\frac{\partial}{\partial t} e(t) = -i\Omega [1 + f \cos(\omega t)] e(t) - \frac{V^2}{v_g} e(t) - iV e^{-i\omega_0 t}. \tag{10}$$

A particular solution to the homogeneous differential equation on $e(t)$ reads

$$e(t) = e^{-i\Omega t - \frac{V^2}{v_g} t} e^{-i \frac{f\Omega}{\omega} \sin(\omega t)}. \tag{11}$$

By using the Jacobi–Anger expansion [31]

$$e^{iu \sin x} = \sum_n J_n(u) e^{inx}, \tag{12}$$

with $J_n(u)$ being the first kind Bessel function of the n -order, the generic solution to the Eq. (10) reads

$$e(t) = \sum_{n,l} \frac{V J_l(\frac{f\Omega}{\omega}) J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma} e^{-i\omega_n t}, \tag{13}$$

with $\Delta = \omega_0 - \Omega$, $\gamma = V^2/v_g$ and $\omega_n = \omega_0 + n\omega$. Here, Δ and γ are the detuning and the effective coupling strength between the photon and the periodically-modulated atom, respectively. As a consequence,

$$\psi_L(x, t) = \sum_n e^{-i(q_n x + \omega_n t)} \left[\sum_l \frac{-i\gamma J_l(\frac{f\Omega}{\omega}) J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma} \right], \tag{14}$$

$$\psi_R(x, t) = \sum_n e^{i(q_n x - \omega_n t)} \left[\sum_l \frac{-i\gamma J_l(\frac{f\Omega}{\omega}) J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma} + \delta_{n,0} \right], \tag{15}$$

with $q_n = \omega_n/v_g$. It is seen that many energy sidebands appear in the reflected and transmitted coefficients of the scattered photon. With Eqs. (6) and (7) one can easily see that the quantities defined in the square brackets in Eqs. (14) and (15) are just the reflected and transmitted amplitudes in the n th sideband, i.e.,

$$r_n = \sum_l \frac{-i\gamma J_l(\frac{f\Omega}{\omega}) J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma}, \tag{16}$$

$$t_n = \sum_l \frac{-i\gamma J_l(\frac{f\Omega}{\omega}) J_{n+l}(\frac{f\Omega}{\omega})}{\Delta - l\omega + i\gamma} + \delta_{n,0}. \tag{17}$$

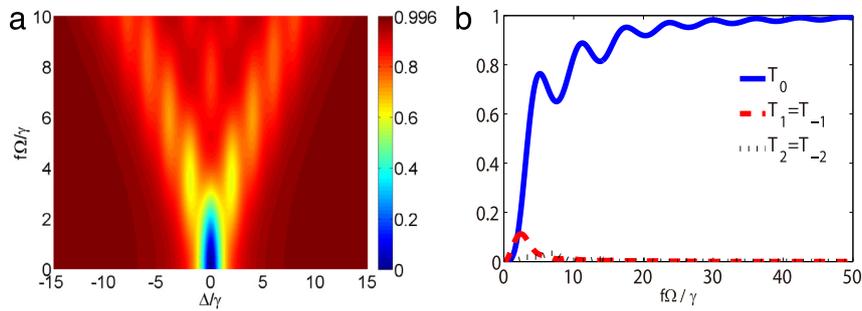


Fig. 2. The transmission probabilities of the incident waveguide photon scattered by an amplitude modulated atom $f\Omega/\gamma$ with a fixed modulated frequency $\omega/\gamma = 2$. (a) The total transmitted spectrum of the photon. (b) The transmitted probabilities for various possible photonic sidebands of the incident resonant photon with a fixed frequency: $\omega_0 = \Omega$.

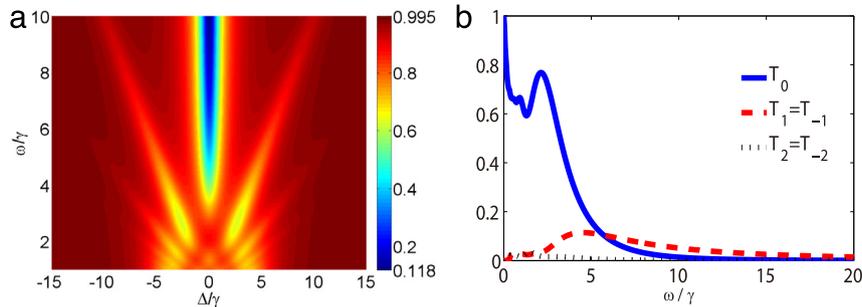


Fig. 3. The transmitted probability of the photon scattered by the frequency modulated atom with the fixed modulated amplitude $f\Omega/\gamma = 5$. (a) The total transmitted spectrum. (b) The transmitted probabilities of the resonant incident photon (with $\Delta = 0$) in different photonic sidebands: the zero-sideband and the sidebands with $n = 1, -1; 2, -2$.

Physically, the total transmitted and reflected probabilities of the waveguide photon scattered by the atom reads

$$R = \int dx \psi_L^*(x, t) \psi_L(x, t) = \sum_n R_n, \quad R_n = |r_n|^2, \quad (18)$$

$$T = \int dx \psi_R^*(x, t) \psi_R(x, t) = \sum_n T_n, \quad T_n = |t_n|^2. \quad (19)$$

With Eqs. (18) and (19), we can check that the usual completeness condition: $T + R = 1$, is still satisfied for the present time-dependent quantum scattering system. Certainly, when $f\omega = 0$, the above results reduce to those for the usual time-independent scattering problem [5,32], in which the resonant incident photon (i.e., $\omega_0 = \Omega$) is reflected completely by the atom and any photonic sideband is excited. In what follows we specifically investigate how the periodic modulation of the atomic energy splitting excites the energy sidebands of the photon, and then discuss numerically how the sideband transport of the photon can be controlled.

i) For a fixed modulated periodic/frequency of atomic energy splitting, e.g., $\omega/\gamma = 2$ with a relatively-low modulated frequency, Fig. 2(a) shows how the transmission spectrum changes with the modulated amplitude f . It is seen that the transmission spectrum of the photon scattered by the atom with the energy splitting modulation is obviously different from that scattered by a time-independent atom; the transmitted probability of the incident photon with the frequency $\omega_0 = \Omega$ is no longer zero but a finite value, which can be approached to the 1 (i.e., complete transmission) for the sufficiently-strong modulated amplitude. This is because that the energy splitting of the scattering atom is adjusted from the resonance to non-resonance with the incident photon. Interestingly, Fig. 2(b) shows that, even for the weak amplitude modulation, the incident photon with the fixed frequency (i.e., $\omega_0 = \Omega$) could still be transmitted in different photonic sidebands with the frequency $\omega_n = \omega_0 \pm n\omega, n = 1, 2, 3, \dots$. Physically, this phenomenon is originated from inelastic scattering of the modulated atom. Typically, the transmitted probability T_0 (blue solid line) contributed from the zero-sideband (i.e., $n = 0$) is dominant for the sufficiently large amplitude modulations (e.g., $f\Omega/\gamma > 4$ in Fig. 2(b)). Inversely, the

transmission in the sideband (e.g., $n = 1/-1$ marked as the red dashed line in Fig. 2(b)) is significantly strong for the relatively-low amplitude modulations (e.g., $f\Omega/\gamma < 2$ in the figure). For example, the transmission in the sidebands with $n = 1, -1$ can be larger than that of the zero-sideband (i.e., $n = 0$ corresponding to the elastic scattering of the photon) for certain modulated parameters, although the transmissions in the higher sidebands T_n, T_{-n} (with $|n| \geq 2$, black dotted line in the figure) are still negligible.

ii) For a fixed modulated amplitude, e.g., $f\Omega/\gamma = 5$, we numerically show the transmitted spectrum of the incident photon in Fig. 3(a). One can see that, under the relatively-high frequency modulations (e.g., $\omega/\gamma > 6$) the incident resonant photon (with the frequency $\omega_0 = \Omega$) approaches to the complete reflection, except certain relatively-weak sideband transmissions. However, for the relatively-low frequency modulation the total transmission of the photon is significantly enhanced. Again, this is because the detuning between the atom and the incident photon has been effectively modulated, and the completely-reflected effect of the resonant photon is suppressed robustly. Specifically, Fig. 3(b) shows again that, due to the scattering of the periodic modulated atom, the resonant incident photon could transmit along various photonic sidebands. Compared with the transmitted probability of the zero-sideband T_0 (blue solid line), the transmitted probabilities of the sidebands $n = 1, -1$, i.e., $T_{1,-1}$ (red dashed line), cannot be neglected for certain frequency modulations. Again, the transmissions along the higher frequency sidebands (black dotted line) are still unimportant for the present parameter conditions.

3. Single-photon transport with the periodic modulated atom–photon interaction

We now consider another mechanism to modulate the photon transport by the periodically-driven atom–photon interaction generated by, e.g., the photonic wavepacket across the atom, and the oscillating distance between the waveguide and the aside atom. The configuration

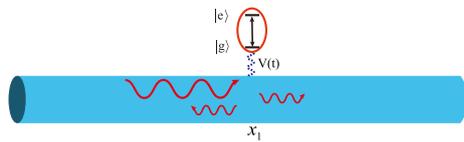


Fig. 4. Single-photon transport with the periodic modulated atom–photon interaction $V(t)$ at the location $x_1 = 0$.

considered here can be schematized in Fig. 4, with the Hamiltonian

$$\begin{aligned}
 H = & \int dx \left[c_R^\dagger(x) \left(-i v_g \frac{\partial}{\partial x} \right) c_R(x) + c_L^\dagger(x) \left(i v_g \frac{\partial}{\partial x} \right) c_L(x) \right] \\
 & + \int dx V(t) \delta(x - x_1) \left[c_R(x) \sigma^+ + c_L(x) \sigma^+ + H.c. \right] \\
 & + \Omega \sigma^+ \sigma^-, \tag{20}
 \end{aligned}$$

where $V(t) = V [1 + \lambda \cos(\omega_s t)]$ is the periodic modulated atom–photon interaction (with λ being the modulated amplitude and ω_s the frequency). The generic wave function and also the probabilistic amplitudes for the photon propagating along the R/L direction take the same forms in Eqs. (2), (6) and (7), respectively. But, the relevant bound conditions are now replaced as

$$\psi_R(0, t) = \psi_L(0, t) + e^{-i\omega_0 t}, \tag{21}$$

$$\psi_L(0, t) = \frac{-iV}{v_g} [1 + \lambda \cos(\omega_s t)] e(t), \tag{22}$$

$$i \frac{\partial}{\partial t} e(t) = \Omega e(t) + V [1 + \lambda \cos(\omega_s t)] [\psi_L(0, t) + e^{-i\omega_0 t}]. \tag{23}$$

Again, by using the expansion formula [31]:

$$e^{x \sin \theta} = \sum_n i^{-n} I_n(x) e^{in\theta}, \tag{24}$$

with $I_n(x)$ being the modified Bessel function of order n . Specially, under the practical condition: $\gamma \lambda^2 \ll 4\omega_s$ for the fast convergence of the expanded Bessel functions, we have $I_l(\frac{\gamma \lambda^2}{4\omega_s}) \approx \delta_{l,0}$ and $I_j(\frac{\gamma \lambda^2}{4\omega_s}) \approx \delta_{j,0}$. Furthermore, we let $I_k(\frac{2\gamma \lambda}{\omega_s}) = I_k$ for simplicity. The reflected and transmitted amplitudes can be reduced to

$$t_n = r_n + \delta_{n0}, \tag{25}$$

$$\begin{aligned}
 r_n = & \sum_k \frac{-i^{-n-2k+1} I_k \left[I_{n+k} + \frac{i\lambda}{2} I_{n+k-1} + \frac{i^{-1}\lambda}{2} I_{n+k+1} \right]}{\frac{\Delta}{\gamma} - k \frac{\omega_s}{\gamma} + i(1 + \frac{\lambda^2}{2})} \\
 & + \sum_k \frac{-i^{-n-2k+1} \frac{\lambda}{2} I_k \left[i^{-1} I_{n+k+1} + \frac{\lambda}{2} I_{n+k} + \frac{i^{-2}\lambda}{2} I_{n+k+2} \right]}{\frac{\Delta}{\gamma} - (k+1) \frac{\omega_s}{\gamma} + i(1 + \frac{\lambda^2}{2})} \\
 & + \sum_k \frac{-i^{-n-2k+1} \frac{\lambda}{2} I_k \left[i I_{n+k-1} + \frac{i^2\lambda}{2} I_{n+k-2} + \frac{\lambda}{2} I_{n+k} \right]}{\frac{\Delta}{\gamma} - (k-1) \frac{\omega_s}{\gamma} + i(1 + \frac{\lambda^2}{2})}. \tag{26}
 \end{aligned}$$

Substituting these coefficients into the Eqs. (18) and (19), the total reflected and transmitted probabilities of the waveguide photon in the present configuration are obtained.

In order to show how the periodically-modulated photon–atom interaction influences the photon transport, we investigate the transmitted spectra of the photon for two typical cases. (i) $\lambda \omega_s = 0$, i.e., the atom–photon coupling strength is not time modulation for the below comparisons, the transmitted spectra schematized in Fig. 5 shows that the resonant incident photon is always reflected completely and the stronger coupling strength corresponds the wider dip of the transmitted spectrum. (ii) $\lambda, \omega_s \neq 0$, i.e., the atom–photon coupling strength is periodically modulated. Without loss of the generality, we set $\gamma \lambda^2 \ll 4\omega_s$ for simplicity.

Firstly, we investigate the influence from the frequency modulation for the fixed modulation amplitude $\lambda = 0.1$. Fig. 6(a) shows clearly that

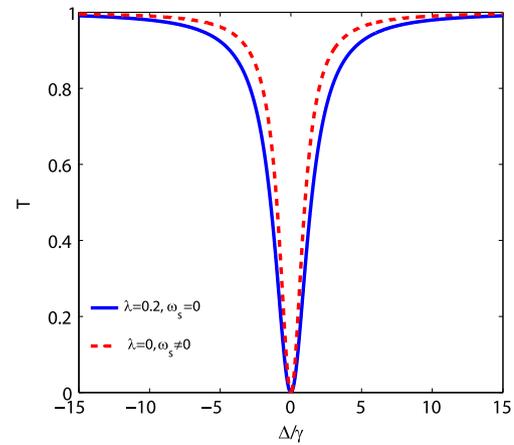


Fig. 5. The transmitted spectra of the photon for different atom–photon interaction strengths with $\lambda \omega_s = 0$.

the resonant incident photon can be transmitted with certain probabilities (i.e., the total transmitted probability marked as the blue solid line), which increase with the modulation frequency ω_s . Specifically, one can see that such a transmission is mainly originated from the transmissions in the $n = 1$ or $n = -1$ sideband (the red dashed line), and the contributions from the $n = 0$ sideband (the green dot-dashed line) and $|n| \geq 2$ sidebands (the black dotted line) are negligible. Furthermore, we shows how the transmissions in these sidebands depend on the detuning parameter $\Delta = \omega_0 - \Omega$ in Fig. 6(b), which indicates that the resonant incident photon is maximally transmitted into the sidebands and the transmission in the $n = 0$ sideband is negligible. Also, the transmitted probabilities are symmetrically distributed around the detuning $\Delta = 0$ and the maximized transmission reaches 0.005 for the modulated frequency $\omega_s/\gamma \geq 10$.

Secondly, we discuss how the modulated amplitude λ influence on the sideband transmissions for the modulated frequency $\omega_s/\gamma = 10$. The transmissions of the resonant incident photon varies with the λ -parameter is shown in Fig. 7(a), wherein the total transmitted probability is marked as the blue solid line, and the transmissions in $n = 0, n = 1, -1$ and $n = 2, -2$ sidebands are represented by the green dot-dashed, red dashed, and black dotted lines, respectively. Note that the sideband transmissions (mainly in the $n = -1, 1$ sidebands) increase monotonously with the modulation amplitude λ . One can see from Fig. 7(b) that the total transmitted probability in the $n = 1, -1$ sidebands is $T_1 + T_{-1} = 0.005$ for the typical experimental parameters; $\Delta = 0, \lambda = 0.1$.

4. Summary and conclusions

A time-dependent counterpart of the usual quantum theory of the time-independent photon transport in real-space is developed to describe the photon propagations modulated periodically by the atomic level splitting and the atom–photon interaction. We showed that, due to the existence of the inelastic scatterings, the photon can be transmitted/reflected into its sidebands. This leads certain novel transport phenomena of the photon in the waveguides. For example, in the framework of the time-independent quantum theory, the resonant incident photon (i.e., its frequency is equivalent to the transition frequency of the scattering atom) is reflected completely. However, if the atomic transition frequency or the atom–photon interaction is time-dependent modulation, the resonant incident photon could transmit across the atom in various energy sidebands. Thus, the complete reflection does not appear. Specifically, we numerically found that, under certain conditions, i.e., requiring the expanded Bessel functions to be fast convergence, the contributions of transmission in lower energy sidebands, e.g., the $n = 0, \pm 1, \pm 2$ are relatively-important.

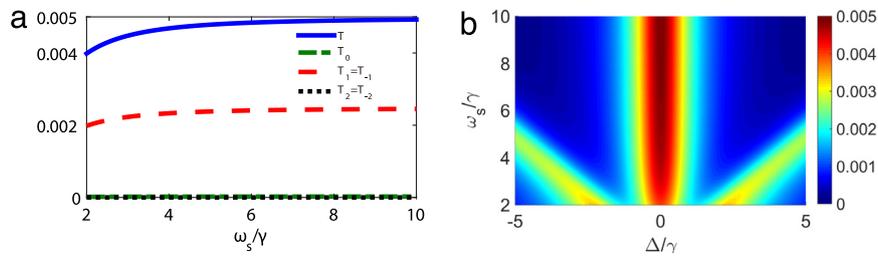


Fig. 6. The transmitted probabilities of the photon versus the modulated frequency ω_s/γ of the atom–photon interaction with the fixed modulated amplitude $\lambda = 0.1$. (a) The total transmitted probabilities of the resonant incident photon. (b) The spectrum $T_1 + T_{-1}$ of the photon transmitted into $n = 1$ and $n = -1$ sidebands.

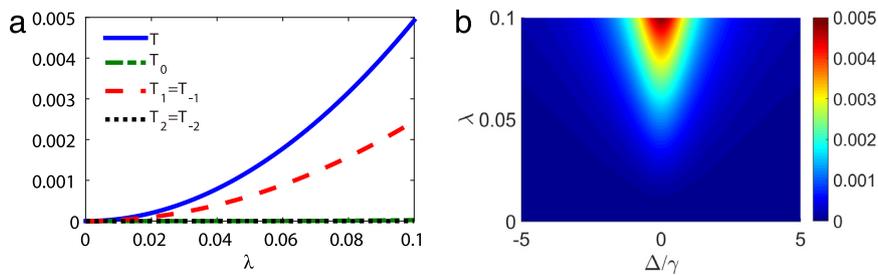


Fig. 7. The transmitted probabilities of the photon changes with the modulated atom–photon interaction amplitude λ with the fixed modulated frequency $\omega_s/\gamma = 10$. (a) The transmitted probabilities of the resonant incident photon in different sidebands. (b) The total transmitted probability $T_1 + T_{-1}$ of the photon in the $n = 1, -1$ sidebands controlled by the modulated amplitude and detuning between the photon and atom.

In principle, the time-modulation of photon transport could be applied to various realistic physical problems, typically, e.g., the controllable single-photon quantum devices. For example, with the time-dependent Jaynes–Cummings interaction [33] photon transport through a cavity could be controlled by manipulating the time-dependent interaction between the cavity and the containing atom. As a consequence, single-photon transport devices such as photonic switch could be realized at a single atom level. More interestingly, the recent experiment [23] demonstrated successfully the dynamical Casimir effect by observing the microwave photon propagating along a coplanar waveguide terminated by a flux-modulating superconducting quantum interference device, which might be regarded as an artificial atom with time-modulating energy splittings. The generated and observed photon, i.e., the sideband photon, is identical to the one calculated in the dynamical Casimir effect for a single oscillating mirrors [34].

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