



## Mixedness and entanglement for two-mode Gaussian states

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### ABSTRACT

We analytically exploit the two-mode Gaussian states nonunitary dynamics. We show that in the zero temperature limit, entanglement sudden death (ESD) will always occur for symmetric states (where initial single-mode compression is  $z_0$ ) provided that the two mode squeezing  $r_0$  satisfies  $0 < r_0 < \frac{1}{2} \log(\cosh(2z_0))$ . We also give the analytical expressions for the time of ESD. Finally, we show the relation between the single modes initial impurities and the initial entanglement, where we exhibit that the latter is suppressed by the former.

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### 1. Introduction

The concept of entanglement, a typically quantum mechanical property, is a natural consequence of the superposition principle for composite systems. It was discussed by Schrödinger who immediately realized its (at the time) seemingly “unphysical” consequences [1]. Nowadays the growing interest in the subject is related to quantum information theory and potential technological applications [2,3]. Thus a lot of effort has been put both in the quantification of entanglement and in studying the consequences of deleterious environmental effects on this property [4]. It is well known today, both theoretically and experimentally, that in general such effects tend to destroy quantum properties; in the case of systems with one degree of freedom this happens only asymptotically [5–7]. The studies concerning the degradation of quantum effects are vast in the literature, specially those related to continuous variables systems subjected to noisy channels, see [8,9]. Some recent works devote their attention to the robustness of Gaussian and non-Gaussian states under dissipative channels [10].

It was recently realized that in bipartite systems that entanglement may disappear suddenly, phenomenon called therefore “entanglement sudden death” (ESD) or finite-time disentanglement [11,12]. The phenomenon is strongly related to geometrical properties of the set of mixed quantum states [13], but from a physical point of view, the issue is still far from being closed.

In the present work we study more specifically the entanglement properties of two mode Gaussian states under a nonunitary evolution. In Ref. [5–7] the entropy growth of single-mode Gaussian and non-Gaussian states coupled to a reservoir has been presented in analytical form. We show that in the two-mode case, single-mode squeezing plays an important role in entanglement dynamics, even for fully symmetric channels (phenomenon that appears also in the case of qu-bits [14]). For the specific case of a zero temperature reservoir and symmetric states (where initial single mode squeezing of both modes is  $z_0$ ) we are able to show that ESD will always occur if  $0 < r_0 < \frac{1}{2} \log(\cosh(2z_0))$ , where  $r_0$  is the two-mode squeezing. We also show that there exists an upper limit for the degree of mixedness of a state so that it can exhibit entanglement. This result may turn out useful for experimental realizations of two-mode entangled Gaussian states. Finally, for symmetric and pure states evolving in a reservoir at zero temperature, we analytically present the time of ESD.

The paper is organized as follows: in Section 2 we present some well known properties concerning Gaussian states; in the next section we briefly review the analytical dynamics of two-mode Gaussian states; in Section 4 we present our results. Finally, some conclusions are drawn.

### 2. General properties of two-mode Gaussian states

A two-mode Gaussian state with vanishing averages  $\langle a_i \rangle = \text{tr}(a_i \rho) = 0$ ,  $i = 1, 2$ , can always be written as

$$\rho_G = S_1(z_1, z_2) S_2(r) \sigma(v_1, v_2) S_2^\dagger(r) S_1^\dagger(z_1, z_2), \quad (1)$$

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where  $S_1(z_1, z_2)$  is the single-mode squeezing operator,  $S_2(r)$  is the two-mode squeezing operator and  $\sigma(v_1, v_2)$  is the two-mode thermal state. More explicitly we have:

$$S_1(z_1, z_2) = \exp\left[\frac{z_1}{2}(a_1^{\dagger 2} - a_1^2)\right] \exp\left[\frac{z_2}{2}(a_2^{\dagger 2} - a_2^2)\right], \tag{2}$$

$$S_2(r) = \exp[r(a_1^{\dagger} a_2^{\dagger} - a_1 a_2)], \tag{3}$$

$$\sigma(v_1, v_2) = \frac{1}{v_1 + 1} \frac{1}{v_2 + 1} \sum_n \sum_m \left(\frac{v_1}{v_1 + 1}\right)^n \left(\frac{v_2}{v_2 + 1}\right)^m |n\rangle \langle n| \otimes |m\rangle \langle m|,$$

where  $z_i$  is the single-mode squeezing parameter of the mode  $i$ ,  $a_i^{(\dagger)}$  is the annihilation (creation) operator of the  $i$ th mode,  $r$  is the two-mode squeezing and  $v_i$  is the “mixedness” of the  $i$ th mode (by “mixedness” we mean the initial number of thermal photons of the state), related to the thermal two-mode state  $\sigma$ . We assume  $\langle a_i \rangle = 0$  in this work, since the entanglement properties are independent of them, and in our equations of motion the second momenta evolution decouple from the first momenta. We also choose, without loss of generality in this case, the squeezing parameters  $z_i$  and  $r$  to be real.

This state is entirely described by the parameters given above. However, in order to handle entanglement properties, it is most convenient to write these parameters in terms of the corresponding covariance matrix (CM):

$$V_\rho = \begin{pmatrix} n_1 + \frac{1}{2} & m_1 & m_s & m_c \\ m_1^* & n_1 + \frac{1}{2} & m_c^* & m_s^* \\ m_s^* & m_c & n_2 + \frac{1}{2} & m_2 \\ m_c^* & m_s & m_2^* & n_2 + \frac{1}{2} \end{pmatrix}, \tag{4}$$

where  $n_i = \langle a_i^\dagger a_i \rangle$ ,  $m_i = -\langle a_i^2 \rangle$ ,  $m_s = -\langle a_1 a_2^\dagger \rangle$ ,  $m_c = \langle a_1 a_2 \rangle$ , and  $\langle \xi \rangle$  denotes the quantum expectation value  $\text{tr}(\xi \rho)$  of an observable  $\xi$ . The CM can also be written as

$$V_\rho = \begin{pmatrix} V_1 & C \\ C^\dagger & V_2 \end{pmatrix}, \tag{5}$$

where  $V_i$  is a  $2 \times 2$  matrix related to the mode  $i$ , and  $C$  is a  $2 \times 2$  matrix that gives the correlations (both quantum and classical) between the modes. For later use, we define some invariants of the covariance matrix as [7,15]:

$$I_V = \det V_\rho,$$

$$I_{1,2} = \det V_{1,2},$$

$$I_3 = \det C,$$

$$I_4 = \text{tr}[V_1 Z C V_2 Z C^\dagger Z]. \tag{6}$$

These quantities are invariants under local unitary operations and  $Z = \text{diag}(1, -1)$ . Next we give the explicit connection between the parameters of the Gaussian state and the matrix elements of the covariance matrix:

$$z_i = \frac{1}{2} \text{arctanh} \left[ \frac{m_i}{n_i + \frac{1}{2}} \right] \tag{7}$$

$$v_1 = \frac{1}{2} (\det V_1 - \det V_2) + \frac{1}{2} \sqrt{1 - x^2} (\det V_1 + \det V_2) - \frac{1}{2}, \tag{8}$$

$$v_2 = \frac{1}{2} (\det V_2 - \det V_1) + \frac{1}{2} \sqrt{1 - x^2} (\det V_1 + \det V_2) - \frac{1}{2}, \tag{9}$$

$$r = \frac{1}{2} \text{arctanh}[x], \tag{10}$$

where we have defined

$$x = \frac{2m_s}{(\sqrt{\det V_1} + \sqrt{\det V_2}) \sinh(z_1 + z_2)}. \tag{11}$$

Once the evolution of the covariance matrix is obtained, the evolution of the parameters of the state can be inferred.

Since we are working with Gaussian states, there are *necessary and sufficient* criterion to determine whether the state is entangled [16–18]. Simon [16] has shown that for any two-mode Gaussian state, if the following inequality is observed:

$$S(V_\rho) = I_1 I_2 + (1/4 - |I_3|)^2 - I_4 - 1/4(I_1 + I_2) \geq 0, \tag{12}$$

the state is separable. For the purposes of this work, Simon’s criterion is enough to study the entanglement dynamics.

### 3. Nonunitary dynamics and its analytical solution

The degradation of the quantum information content of Gaussian states is a subject of interest, both for the technological and/or experimental applications as well as for fundamental quantum mechanics in what concerns the classical limit.

The usual approach to non-unitary dynamics is by means of master equations, which has found several successful applications in other areas of physics. Our master equation reads

$$\dot{\rho} = \mathcal{L}\rho, \tag{13}$$

where  $\mathcal{L}$  is a superoperator given by:

$$\mathcal{L} = \sum_{i=1,2} \left( \gamma_i (n_i^\beta + 1) (2a_i \bullet a_i^\dagger - a_i^\dagger a_i \bullet - \bullet a_i^\dagger a_i) + \gamma_i n_i^\beta (2a_i^\dagger \bullet a_i - a_i a_i^\dagger \bullet - \bullet a_i a_i^\dagger) \right). \tag{14}$$

In the equation above,  $\gamma_i$  is the dissipation constant of the reservoir related to the mode  $i$ ,  $n_i^\beta$  is related to the temperature of the thermal bath of the mode  $i$ , and  $a_i^\dagger(a_i)$  is the creation (annihilation) operator of the respective mode.

Eq. (14) models a linear coupling between the state (the two-mode Gaussian state in our case) with a thermal bath of quantum harmonic oscillators. The evolution of each term of the covariance matrix, evolving under the dynamics described above, is given by

$$n_i = e^{-2\gamma_i t} ((-1 + e^{2\gamma_i t}) n_i^\beta + \cosh(2z_i^0) (v_i^0 \cosh^2 r_0 + (1 + v_k^0) \sinh^2 r_0) + \sinh^2 z_i^0), \tag{15}$$

$$m_i = -e^{-2\gamma_i t} (v_i^0 - v_k^0 + (1 + v_i^0 + v_k^0) \cosh(2r_0)) \cosh z_i^0 \sinh z_i^0, \tag{16}$$

$$m_c = \frac{1}{2} e^{-t(\gamma_1 + \gamma_2)} (1 + v_1^0 + v_2^0) \cosh(z_1^0 + z_2^0) \sinh(2r_0), \tag{17}$$

$$m_s = -\frac{1}{2} e^{-t(\gamma_1 + \gamma_2)} (1 + v_1^0 + v_2^0) \sinh(2r_0) \sinh(z_1^0 + z_2^0). \tag{18}$$

In the equations above,  $i=1$  or  $2$  (mode 1 or mode 2) and  $k \neq i$ . The parameters are such that  $z_i^0$  is the initial *single mode squeezing*,  $\gamma_i$  is the dissipation constant,  $v_i^0$  is related to the *initial mixedness* and  $n_i^\beta$  is the reservoir temperature, where all the quantities refer to the  $i$ th mode, as denoted by the subscript  $i$ . Also, we have that  $r_0$  is the initial *two-mode squeezing*.

### 4. Results

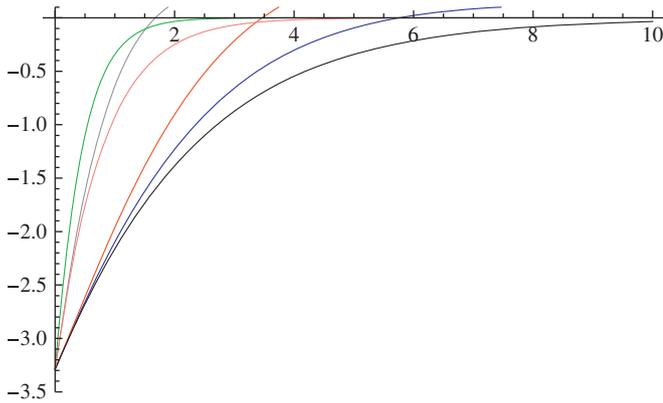
#### 4.1. Entanglement dynamics and single-mode squeezing

In order to get a clear picture and to gain physical insight into the rich and multifaceted aspects of the non-unitary dynamics of general two-mode Gaussian states, we consider firstly the simplest case, *i.e.* the two-mode squeezed vacuum in dissipative channel.

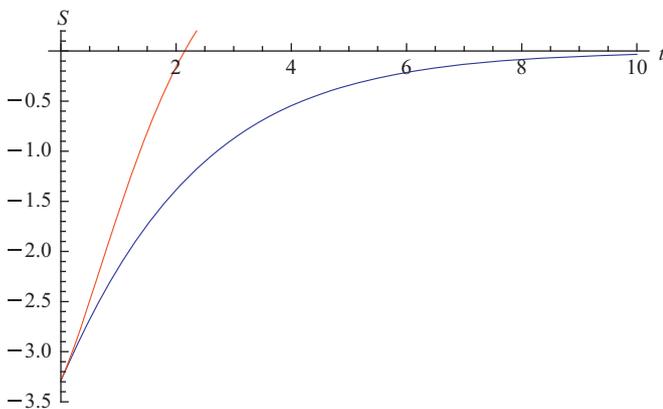
Let us consider the case of a two-mode vacuum state without single-mode squeezing (i.e.  $z_1 = z_2 = 0$ ). As shown in Fig. 1, for symmetric and asymmetric channels, with temperatures  $n_1^B$  and  $n_2^B$  different from zero, there will always be a finite time when entanglement vanishes. This can be understood using a geometrical picture of entanglement decay [13]. In fact, in this case, the long-time state is a separable mixed state, well within the set of separable states. Thus, if an initial state is entangled, it necessarily crosses the border of separable states in finite time. For zero-temperature baths, i.e. if  $n_1^B = n_2^B = 0$ , even if one uses different dissipation constants  $\gamma_1$  and  $\gamma_2$ , the entanglement only disappears asymptotically.

Next we introduce single-mode squeezing, i.e.  $z_1^0 \neq 0$  and/or  $z_2^0 \neq 0$ . We note in Fig. 2 that, even for the zero temperature case, one observes entanglement sudden death (ESD). We note that there is a dynamical entropy increase in the squeezed mode, caused by the reservoir, which acts in such a way that, for a relatively short time interval, this mode's entropy grows and then decays to the vacuum. We have observed that this dynamical entropy growth [5,6] causes the entanglement disappearance in finite time.

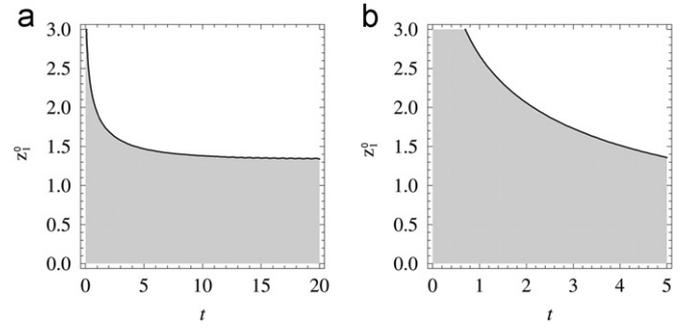
Since single-mode squeezing turns out to play a significant role on ESD. In Fig. 3 we show Simon's criterion evolution for



**Fig. 1.** Simon criterion for several cases of initial parameters. The common parameters are:  $z_1^0 = z_2^0 = v_1^0 = v_2^0 = 0, r_0 = 1$ . In each curve: gray:  $\gamma_1 = 0.1, \gamma_2 = 0.5, n_1 = 0.2, n_2 = 0.2$ ; blue:  $\gamma_1 = 0.1, \gamma_2 = 0.1, n_1 = 0.2, n_2 = 0.2$ ; green:  $\gamma_1 = 0.5, \gamma_2 = 0.5, n_1 = 0.0, n_2 = 0.0$ ; red:  $\gamma_1 = 0.1, \gamma_2 = 0.1, n_1 = 1.0, n_2 = 0.0$ ; black:  $\gamma_1 = 0.1, \gamma_2 = 0.1, n_1 = 0.0, n_2 = 0.0$ ; pink:  $\gamma_1 = 0.1, \gamma_2 = 0.5, n_1 = 0.0, n_2 = 0.0$ . (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)



**Fig. 2.** Simon criterion for two set of initial parameters, say:  $v_1^0 = v_2^0 = 0, r_0 = 1$ . Blue:  $\gamma_1 = \gamma_2 = 0.1, n_1^B = n_2^B = 0, z_1^0 = z_2^0 = 0$ ; Red:  $\gamma_1 = \gamma_2 = 0.1, n_1^B = n_2^B = 0, z_1^0 = 2, z_2^0 = 0$ . (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)



**Fig. 3.** Simon criterion for values of the initial parameter  $z_1^0$  and time  $t$ , where we have used  $z_1^0 = z_2^0$ . For the shaded area the Simon criterion  $S$  is negative (entangled state), in the white area  $S$  is positive. Note that there is a limit for  $z_1$  in which the state will have ESD. In this case:  $z_1 \approx 1.4$ . Parameters:  $r_0 = 1, v_1^0 = v_2^0 = 0, n_1^B = n_2^B = 0, \gamma_1 = \gamma_2 = 0.1$ . Graphs (a) and (b) are the same function, in a different time scale.

states with compression in only one of the modes, for reservoirs at null temperature. The vertical axis corresponds to the initial parameter  $z_1^0$  and the plot shows, for each instant of time, whether the evolved state is entangled (shaded area) or not (blank area). For instance, if the initial state has  $z_1^0 = 2$ , Simon's criterion will change from negative to positive in finite time; if one have  $z_1^0 = 1.0$ , the entanglement will vanish asymptotically. We will show an analytical relation between single-mode squeezing and ESD hereafter.

In the following we want to discuss the instant of time in which the state becomes separable, or when occurs ESD, in the case of zero temperature and symmetrical states ( $z_i^0 = 0, r_0 > 0, v_i^0 = v_0$  and  $\gamma_i = \gamma, n_i^B = 0$ , for  $i=1,2$ ). In this case the elements of the covariance matrix, which do not vanish, depend on

$$n = n(t) = \frac{1}{2}e^{-2\gamma t}((2v_0 + 1)\cosh(2r_0) - 1) = n_0 e^{-2\gamma t}, \quad (19)$$

$$m = m(t) = \frac{1}{2}e^{-2\gamma t}(2v_0 + 1)\sinh(2r_0) = m_0 e^{-2\gamma t}. \quad (20)$$

In this case Simon  $S$  can be factorized as  $S = (m+n)(m+n+1)(m-n)(m-n-1)$ . Since the first two factors are clearly positive, we can see that  $S$  is negative if  $n < m < n+1$ . If this inequality is satisfied at the initial time,  $n_0 < m_0 < n_0 + 1$ . Now, multiplying by  $e^{-2\gamma t}$  we get  $n < m < n + e^{-2\gamma t} < n + 1$ . Thus, if the state is initially entangled, the evolved state is also entangled for any finite time. Since Simon  $S$  vanishes asymptotically  $\lim_{t \rightarrow \infty} S(t) = 0$ , we see that either the entanglement decay is asymptotic, or the initial state is already separable. A two-mode squeezed vacuum,  $v_0 = 0$ , is always entangled; hence, it separates asymptotically.

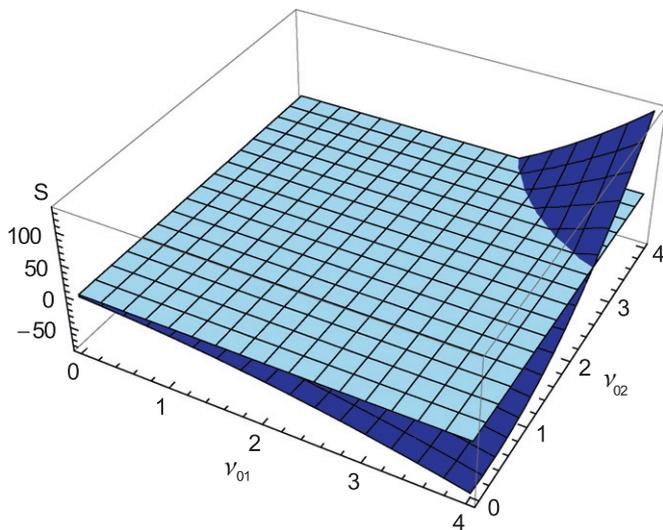
Now we study  $t_{ESD}$  for states containing single-mode squeezing, i.e.  $z_1 = z_2$ . Here one can find for  $t_{ESD}$ :

$$e^{-2\gamma t_{ESD}} = \frac{2 \exp(r_0) \cosh 2z_0 \sinh r_0 - 2 \sinh^2 z_0}{\exp(2r_0)(\cosh 2r_0 - \sinh 2z_0)}, \quad (21)$$

where we consider  $r_0 > 0$ , and  $z_1^0 = z_2^0 = z_0$ . In terms of the new variables  $\eta = \exp(2r_0)$  and  $\zeta = \exp(2z_0)$ , the disentanglement time

$$e^{-2\gamma t_{ESD}} = \frac{\eta(1 + \zeta^2 - 2\eta\zeta)}{\eta - \zeta - \eta^2\zeta + \zeta^2\eta},$$

is much easier to analyze. This equation has a valid solution when the right-hand side varies between 0 and 1, that is, when  $\eta$  satisfies the inequality  $1 \leq \eta \leq \frac{1}{2}(\zeta + 1/\zeta)$ . Going back to the original variables we conclude that the initial state separates at a finite time if  $0 < r_0 < \frac{1}{2} \log(\cosh(2z_0))$ . In this case, one can see clearly that the effect of single-mode squeezing is crucial to determine when the entangle will vanish.



**Fig. 4.** Simon criterion for the two-mode Gaussian state, Eq. (1). Note that impurities values sufficiently high can suppress the effect of entanglement given by the two-mode squeezing parameter  $r$ . Parameter:  $r_0 = 1$ .

#### 4.2. Effects of the mixedness in the initial state

Recently [5,6] it has been shown that in the case of single-mode Gaussian states, there is an upper limit for the degree of global purity (represented by  $v_i^0$ ) of the state above which no quantum properties are visible. We now show that an analogous result holds for two-mode Gaussian systems also. Following the steps in Ref. [5,6] we can show that the initial state is entangled if the following inequality is satisfied:

$$r_0 > \frac{1}{4} \cosh^{-1} \left( \frac{(1+v_2^0)^2 + 2v_1^0(1+v_2^0)(1+4v_2^0) + (v_1^0)^2(1+8v_2^0(1+v_2^0))}{(1+v_1^0+v_2^0)^2} \right). \quad (22)$$

In Fig. 4 we show the entanglement in the initial state as measured by  $S$ , with  $r_0 = 1$ . Notice that there is an upper limit on the initial state impurity above which the state becomes separable.

### 5. Conclusion

In this paper we review some aspects of two-mode Gaussian states, showing both the evolution of the state parameters (that entirely characterize the state) under nonunitary evolution and how entanglement of the state, characterized by the Simon criterion (12), depends on the initial state parameters. We show that, even in completely symmetrical reservoirs and zero temperature, the entanglement can vanish in finite time, depending on the single-mode squeezing of the state. We give a condition for ESD relating single-mode squeezing and two-mode squeezing, that is  $0 < r_0 < \frac{1}{2} \log(\cosh(2z_0))$ . We analytically present the time when occurs ESD for symmetrical states, evolving in a reservoir with zero temperature. We also show that entanglement can be “suppressed” by the initial mixedness of the modes,  $v_i^0$ : the initial two-mode squeezing has an upper limit as a function  $v_i^0$ , and for values above this limit the state is separable. This can be helpful for experimental procedures, since any state will have some minimal impurity.

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