



# Teleportation of atomic states via cavity quantum electrodynamics

E.S. Guerra \*

*Departamento de Física, Universidade Federal Rural do Rio de Janeiro, Cx. Postal 23851,  
Antiga Estrada Rio So Paulo 23890-000 Seropédica, RJ, Brazil*

Received 13 July 2004; received in revised form 17 August 2004; accepted 19 August 2004

## Abstract

In this paper, we discuss a scheme of teleportation of atomic states. The experimental realization proposed makes use of cavity quantum electrodynamics involving the interaction of Rydberg atoms with a micromaser cavity prepared in a coherent state. We start presenting a scheme to prepare atomic Bell states via the interaction of atoms with a cavity. In our scheme the cavity and some atoms play the role of auxiliary systems used to achieve the teleportation.

© 2004 Elsevier B.V. All rights reserved.

*PACS:* 03.65.Ud; 03.67.Mn; 32.80.-t; 42.50.-p

*Keywords:* Teleportation; Entanglement; Non-locality; Bell states; Cavity QED

## 1. Introduction

Entanglement and non-locality can have revolutionizing impact on our thinking about information processing and quantum computing [1,2]. Probably, one of the most notable and dramatic among various concepts developed through the application of quantum mechanics to the information science is teleportation, put forward by Bennett et al. [3] given rise to a new field of research. The essen-

tials of the teleportation scheme is that, given an unknown quantum state to the sender, making use of quantum entanglement and non-locality, it is possible to reproduce this state far apart in the quantum system of the receiver where, in the process, both the sender and the receiver follow a certain prescription and communicate with each other through a classical channel. In the end of the process the receiver has a quantum state similar to the quantum state of the sender and the quantum state of the sender is destroyed since, according to the no-cloning theorem [1,4] it is not possible to clone a quantum state. It is interesting to note that quantum teleportation does not allow one to transmit

\* Tel./fax: +55 21 256 74271.

E-mail address: [emerson@ufrj.br](mailto:emerson@ufrj.br).

information faster than light in accordance with the theory of relativity because to complete the teleportation process the sender must communicate the result of some sort of measurement she performs to the receiver through a classical channel. Quantum teleportation is an experimental reality and it holds tremendous potential for applications in the fields of quantum communication and computing [1,2]. For instance, it can be used to build quantum gates which are resistant to noise and is intimately connected with quantum error-correcting codes [1]. The most significant difficulty for quantum teleportation to become a useful tool in quantum communication and computing arises in maintaining the entanglement for the time required to transfer the classical message, that is, how to avoid decoherence effects [5,6]. For several proposals of realization schemes of teleportation, see [2]. A scheme of teleportation of atomic states, using cavity QED, has been proposed in [7].

In this paper, we present a scheme of teleportation close to the original scheme presented by Bennett et al. [3]. We will assume that Alice and Bob meet and then create a Bell atomic state involving atoms A2 and A4. Then Alice and Bob separate. Alice takes with her half of the Bell pair, that is, atom A2 and Bob keeps with him the other half of the Bell pair, that is, atom A4. Later on Alice is going to be able to teleport to Bob's atom A4 an unknown state of an atom A1 making use of her half of the Bell pair, that is, atom A2. As it will be clear, teleportation is possible due to a fascinating and at the same time intriguing feature of quantum mechanics: entanglement and its consequence, non-locality.

In the discussion which follows we are going to consider Rydberg atoms of relatively long radiative lifetimes [8]. We also assume a perfect microwave cavity and we neglect effects due to decoherence. Concerning this point, it is worth to mention that nowadays it is possible to build up niobium superconducting cavities with high quality factors  $Q$ . It is possible to construct cavities with quality factors  $Q \sim 10^8$  [9]. Even cavities with quality factors as high as  $Q \sim 10^{12}$  have been reported [10], which, for frequencies  $\nu \sim 50$  GHz gives us a cavity field lifetime of the order of a few seconds.

## 2. Bell states

First let us present a scheme to prepare Bell states and how to detect them. We start assuming that we have a cavity C prepared in coherent state  $|\alpha\rangle$ . Consider a three-level cascade atom  $A_k$  with  $|e_k\rangle$ ,  $|f_k\rangle$  and  $|g_k\rangle$  being the upper, intermediate and lower atomic states (see Fig. 1). We assume that the transition  $|f_k\rangle \rightleftharpoons |e_k\rangle$  is far enough from resonance with the cavity central frequency such that only virtual transitions occur between these states (only these states interact with field in cavity C). In addition we assume that the transition  $|e_k\rangle \rightleftharpoons |g_k\rangle$  is highly detuned from the cavity frequency so that there will be no coupling with the cavity field. Here, we are going to consider the effect of the atom-field interaction taking into account only levels  $|f_k\rangle$  and  $|g_k\rangle$ . We do not consider level  $|e_k\rangle$  since it will not play any role in our scheme. Therefore, we have effectively a two-level system involving states  $|f_k\rangle$  and  $|g_k\rangle$ . Considering levels  $|f_k\rangle$  and  $|g_k\rangle$ , we can write an effective time evolution operator [11]

$$U_k(t) = e^{i\varphi a^\dagger a} |f_k\rangle\langle f_k| + |g_k\rangle\langle g_k|, \quad (2.1)$$

where the second term above was put by hand just in order to take into account the effect of level  $|g_k\rangle$  and where  $\varphi = g^2\tau/\Delta$ ,  $g$  is the coupling constant,  $\Delta = \omega_e - \omega_f - \omega$  is the detuning where

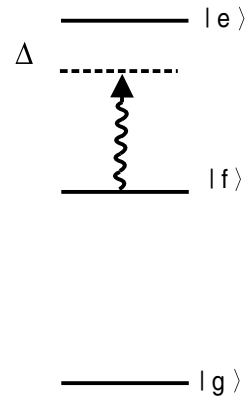


Fig. 1. Energy states scheme of a three-level atom where  $|e\rangle$  is the upper state with atomic frequency  $\omega_e$ ,  $|f\rangle$  is the intermediate state with atomic frequency  $\omega_f$ ,  $|g\rangle$  is the lower state with atomic frequency  $\omega_g$  and  $\omega$  is the cavity field frequency and  $\Delta = (\omega_e - \omega_f) - \omega$  is the detuning.

$\omega_e$  and  $\omega_f$  are the frequencies of the upper and intermediate levels, respectively, and  $\omega$  is the cavity field frequency and  $\tau$  is the atom-field interaction time. Let us take  $\varphi = \pi$ . We assume that we have a two-level atom A1 initially in the state  $|g_1\rangle$ , which is prepared in a coherent superposition according to the rotation matrix

$$R_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad (2.2)$$

and we have

$$|\psi\rangle_{A1} = \frac{1}{\sqrt{2}}(|f_1\rangle + |g_1\rangle). \quad (2.3)$$

Now, let us assume that we have a cavity C prepared in coherent state  $|\alpha\rangle$ . A coherent state  $|\beta\rangle$  is obtained applying the displacement operator  $D(\beta) = e^{(\beta a^\dagger - \beta^* a)}$  to the vacuum, that is  $|\beta\rangle = D(\beta)|0\rangle$ , and is given by [5,12]

$$|\beta\rangle = e^{-\frac{1}{2}|\beta|^2} \sum_{n=0}^{\infty} \frac{(\beta)^n}{\sqrt{n!}} |n\rangle. \quad (2.4)$$

Experimentally, it is obtained with a classical oscillating current in an antenna coupled to the cavity. Then, the system A1–C evolves to

$$|\psi\rangle_{A1-C} = \frac{1}{\sqrt{2}}(|f_1\rangle|\alpha\rangle + |g_1\rangle|-\alpha\rangle), \quad (2.5)$$

where we have used  $e^{za^\dagger}|\alpha\rangle = |e^z\alpha\rangle$  [12]. Now, if atom A1 enters a second Ramsey cavity R2 where the atomic states are rotated according to the rotation matrix

$$R_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad (2.6)$$

we have

$$\begin{aligned} |f_1\rangle &\rightarrow \frac{1}{\sqrt{2}}(|f_1\rangle - |g_1\rangle), \\ |g_1\rangle &\rightarrow \frac{1}{\sqrt{2}}(|f_1\rangle + |g_1\rangle), \end{aligned} \quad (2.7)$$

and therefore,

$$|\psi\rangle_{A1-C} = \frac{1}{2}\{|f_1\rangle(|\alpha\rangle + |-\alpha\rangle) - |g_1\rangle(|\alpha\rangle - |-\alpha\rangle)\}. \quad (2.8)$$

It is worth to mention at this point that if we define the non-normalized even and odd coherent states

$$\begin{aligned} |+\rangle &= |\alpha\rangle + |-\alpha\rangle, \\ |-\rangle &= |\alpha\rangle - |-\alpha\rangle, \end{aligned} \quad (2.9)$$

with  $N^\pm = \langle\pm|\pm\rangle = 2(1 \pm e^{-2|\alpha|^2})$  and  $\langle+|-\rangle = 0$  [13], we have already a Bell state involving the atomic states of A1 and the cavity field state, that is we have

$$|\psi\rangle_{A1-C} = \frac{1}{2}(|f_1\rangle|+\rangle - |g_1\rangle|-\rangle). \quad (2.10)$$

Now, let us prepare a two-level atom A2 in the Ramsey cavity R3. If atom A2 is initially in the state  $|g_2\rangle$ , according to the rotation matrix

$$R_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad (2.11)$$

we have

$$|\psi\rangle_{A2} = \frac{1}{\sqrt{2}}(|f_2\rangle + |g_2\rangle), \quad (2.12)$$

and let us send this atom through cavity C, assuming that for atom A2, as above for atom A1, the transition  $|f_2\rangle \rightleftharpoons |e_2\rangle$  is far of resonance with the cavity central frequency. Taking into account (2.1) with  $\varphi = \pi$ , after the atom has passed through the cavity we get

$$\begin{aligned} |\psi\rangle_{A1-A2-C} &= \frac{1}{2\sqrt{2}}\{|f_1\rangle(|f_2\rangle + |g_2\rangle)(|\alpha\rangle + |-\alpha\rangle) \\ &\quad + |g_1\rangle(|f_2\rangle - |g_2\rangle)(|\alpha\rangle - |-\alpha\rangle)\}. \end{aligned} \quad (2.13)$$

Then, atom A2 enters a Ramsey cavity R4 where the atomic states are rotated according to the rotation matrix

$$R_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad (2.14)$$

that is

$$\begin{aligned} \frac{1}{\sqrt{2}}(|f_2\rangle + |g_2\rangle) &\rightarrow |f_2\rangle, \\ \frac{1}{\sqrt{2}}(|f_2\rangle - |g_2\rangle) &\rightarrow -|g_2\rangle, \end{aligned} \quad (2.15)$$

and we get

$$|\psi\rangle_{A1-A2-C} = \frac{1}{2} \{ |f_1\rangle|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) - |g_1\rangle|g_2\rangle(|\alpha\rangle - |-\alpha\rangle) \}. \quad (2.16)$$

Now, we inject  $|-\alpha\rangle$  in cavity C which mathematically is represented by the operation  $D(\beta)|\alpha\rangle = |\alpha + \beta\rangle$  [12] and this gives us

$$|\psi\rangle_{A1-A2-C} = \frac{1}{2} \{ |f_1\rangle|f_2\rangle(|0\rangle + |-2\alpha\rangle) - |g_1\rangle|g_2\rangle(|0\rangle - |-2\alpha\rangle) \}. \quad (2.17)$$

In order to disentangle the atomic states of the cavity field state, we now send a two-level atom A3, resonant with the cavity, with  $|f_3\rangle$  and  $|e_3\rangle$  being the lower and upper levels respectively, through C. If A3 is sent in the lower state  $|f_3\rangle$ , under the Jaynes–Cummings dynamics [5] we know that the state  $|f_3\rangle|0\rangle$  does not evolve, however, the state  $|f_3\rangle|-2\alpha\rangle$  evolves to  $|e_3\rangle|\chi_e\rangle + |f_3\rangle|\chi_f\rangle$ , where  $|\chi_f\rangle = \sum_n C_n \cos(gt\sqrt{n})|n\rangle$  and  $|\chi_e\rangle = -i\sum_n C_n \sin(gt\sqrt{n+1})|n\rangle$  and  $C_n = e^{-\frac{1}{2}|2\alpha|^2} (-2\alpha)^n / \sqrt{n!}$ . Then we get

$$|\psi\rangle_{A1-A2-C} = \frac{1}{2} \{ |f_1\rangle|f_2\rangle(|f_3\rangle|0\rangle + |e_3\rangle|\chi_e\rangle + |f_3\rangle|\chi_f\rangle) - |g_1\rangle|g_2\rangle(|f_3\rangle|0\rangle - |e_3\rangle|\chi_e\rangle + |f_3\rangle|\chi_f\rangle) \}, \quad (2.18)$$

and if we detect atom A3 in state  $|e_3\rangle$  finally we get the Bell state

$$|\Psi^+\rangle_{A1-A2} = \frac{1}{\sqrt{2}} (|f_1\rangle|f_2\rangle + |g_1\rangle|g_2\rangle), \quad (2.19)$$

which is an entangled state of atoms A1 and A2, which in principle may be far apart from each other.

In the above disentanglement process we can choose a coherent field with a photon-number distribution with a sharp peak at average photon number  $\langle n \rangle = |\alpha|^2$  so that, to a good approximation,  $|\chi_f\rangle \cong C_{\bar{n}} \cos(\sqrt{\bar{n}}g\tau)|\bar{n}\rangle$  and  $|\chi_e\rangle \cong C_{\bar{n}} \sin(\sqrt{\bar{n}}g\tau)|\bar{n}\rangle$ , where  $\bar{n}$  is the integer nearest  $\langle n \rangle$ , and we could choose, for instance  $\sqrt{\bar{n}}g\tau = \pi/2$ , so that we would have  $|\chi_e\rangle \cong C_{\bar{n}}|\bar{n}\rangle$  and  $|\chi_f\rangle \cong 0$ . In this case atom A3 would be detected in state  $|e_3\rangle$  with almost 100% of probability. Therefore, proceeding this way, we can guarantee that the atomic and field states will be disentangled successfully as we would like.

Notice that starting from (2.16) if we had injected  $|\alpha\rangle$  in the cavity and detected  $|e_3\rangle$  we would get the Bell state

$$|\Psi^-\rangle_{A1-A2} = \frac{1}{\sqrt{2}} (|f_1\rangle|f_2\rangle - |g_1\rangle|g_2\rangle). \quad (2.20)$$

Now, if we apply an extra rotation on the states of atom A2 in (2.19) in a Ramsey cavity R5, according to the rotation matrix

$$R_5 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad (2.21)$$

that is

$$R_5 = |f_2\rangle\langle g_2| - |g_2\rangle\langle f_2|, \quad (2.22)$$

we get

$$|\Phi^-\rangle_{A1-A2} = \frac{1}{\sqrt{2}} (|f_1\rangle|g_2\rangle - |g_1\rangle|f_2\rangle), \quad (2.23)$$

and applying (2.22) on (2.20) we get

$$|\Phi^+\rangle_{A1-A2} = \frac{1}{\sqrt{2}} (|f_1\rangle|g_2\rangle + |g_1\rangle|f_2\rangle). \quad (2.24)$$

The states (2.19), (2.20), (2.23) and (2.24) form a Bell basis [1,14] which are a complete orthonormal basis for atoms A1 and A2.

These states show that quantum entanglement implies non-locality. The manifestation of non-locality shows up when we perform a measurement on one of the atoms. For instance, from (2.19) it is clear that if we detect atom A1 in state  $|f_1\rangle$  then atom A2 collapses instantaneously to the state  $|f_2\rangle$  and if we detect atom A1 in state  $|g_1\rangle$  then atom A2 collapses instantaneously to the state  $|g_2\rangle$ , no matter how distant they are from each other. The same applies to the other states (2.20), (2.23) and (2.24). The Bell basis play a central role in the original teleportation scheme proposed in [3].

Now let us see how we can perform measurements in order to distinguish the four Bell states (2.19), (2.20), (2.23) and (2.24). First notice that, defining

$$\Sigma_x = \sigma_x^1 \sigma_x^2, \quad (2.25)$$

where

$$\sigma_x^k = |f_k\rangle\langle g_k| + |g_k\rangle\langle f_k|, \quad (2.26)$$

we have

$$\begin{aligned}\Sigma_x |\Psi^\pm\rangle_{A1-A2} &= \pm |\Psi^\pm\rangle_{A1-A2}, \\ \Sigma_x |\Phi^\pm\rangle_{A1-A2} &= \pm |\Phi^\pm\rangle_{A1-A2}.\end{aligned}\quad (2.27)$$

Therefore, we can distinguish between  $(|\Psi^+\rangle_{A1-A2}, |\Phi^+\rangle_{A1-A2})$  and  $(|\Psi^-\rangle_{A1-A2}, |\Phi^-\rangle_{A1-A2})$  performing measurements of  $\Sigma_x = \sigma_x^1 \sigma_x^2$ . In order to do so we proceed as follows. We make use of

$$K_k = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad (2.28)$$

or

$$K_k = \frac{1}{\sqrt{2}} (|f_k\rangle\langle f_k| - |f_k\rangle\langle g_k| + |g_k\rangle\langle f_k| + |g_k\rangle\langle g_k|), \quad (2.29)$$

to gradually unravel the Bell states.

The eigenvectors of the operators  $\sigma_x^k$  are

$$|\psi_x^k, \pm\rangle = \frac{1}{\sqrt{2}} (|f_k\rangle \pm |g_k\rangle), \quad (2.30)$$

and we can rewrite the Bell states as

$$\begin{aligned}|\Psi^\pm\rangle_{A1-A2} &= \frac{1}{2} [|\psi_x^1, +\rangle(|f_2\rangle \pm |g_2\rangle) \\ &\quad + |\psi_x^1, -\rangle(|f_2\rangle \mp |g_2\rangle)], \\ |\Phi^\pm\rangle_{A1-A2} &= \frac{1}{2} [|\psi_x^1, +\rangle(|g_2\rangle \pm |f_2\rangle) \\ &\quad + |\psi_x^1, -\rangle(|g_2\rangle \mp |f_2\rangle)].\end{aligned}\quad (2.31)$$

Let us take for instance (2.19)

$$|\Psi^+\rangle_{A1-A2} = \frac{1}{\sqrt{2}} (|f_1\rangle|f_2\rangle + |g_1\rangle|g_2\rangle). \quad (2.32)$$

Applying  $K_1$  to this state we have

$$\begin{aligned}K_1 |\Psi^+\rangle_{A1-A2} &= \frac{1}{2} \{ |f_1\rangle(|f_2\rangle - |g_2\rangle) + |g_1\rangle \\ &\quad \times (|f_2\rangle + |g_2\rangle) \}. \end{aligned}\quad (2.33)$$

Now, we compare (2.33) and (2.31). We see that the rotation by  $K_1$  followed by the detection of  $|g_1\rangle$  corresponds to the detection of the state  $|\psi_x^1, +\rangle$  whose eigenvalue of  $\sigma_x^1$  is +1. After we detect  $|g_1\rangle$ , we get

$$|\psi\rangle_{A2} = \frac{1}{\sqrt{2}} (|f_2\rangle + |g_2\rangle), \quad (2.34)$$

that is, we have got

$$|\psi\rangle_{A2} = |\psi_x^2, +\rangle. \quad (2.35)$$

If we apply (2.29) for  $k = 2$  to the state (2.35), we get

$$K_2 |\psi\rangle_{A2} = |g_2\rangle. \quad (2.36)$$

We see that the rotation by  $K_2$  followed by the detection of  $|g_2\rangle$  corresponds to the detection of the state  $|\psi_x^2, +\rangle$  whose eigenvalue of  $\sigma_x^2$  is +1. The same applies to (2.24).

We can repeat the above procedure and see that we have only two possibilities which are presented schematically below, where on the left, we present the possible sequences of atomic state rotations through  $K_k$  and detections of  $|f_k\rangle$  or  $|g_k\rangle$  and on the right, we present the sequences of the corresponding states  $|\psi_x^k, \pm\rangle$  where  $k = 1$  and 2 which corresponds to the measurement of the eigenvalue of the operator  $\Sigma_x$ , +1, given by (2.27), which corresponds to the detection of (2.19) or (2.24)

$$\begin{aligned}(K_1, |g_1\rangle)(K_2, |g_2\rangle) &\leftrightarrow |\psi_x^1, +\rangle|\psi_x^2, +\rangle, \\ (K_1, |f_1\rangle)(K_2, |f_2\rangle) &\leftrightarrow |\psi_x^1, -\rangle|\psi_x^2, -\rangle.\end{aligned}\quad (2.37)$$

Considering (2.20) and (2.23), we have

$$\begin{aligned}(K_1, |g_1\rangle)(K_2, |f_2\rangle) &\leftrightarrow |\psi_x^1, +\rangle|\psi_x^2, -\rangle, \\ (K_1, |f_1\rangle)(K_2, |g_2\rangle) &\leftrightarrow |\psi_x^1, -\rangle|\psi_x^2, +\rangle,\end{aligned}\quad (2.38)$$

which corresponds to the measurement of the eigenvalue of the operator  $\Sigma_x$ , -1, given by (2.27).

Now, let us see how we can make distinction between (2.19), (2.20), (2.24) and (2.23). For this purpose we are going to consider (2.1) for  $\varphi = \pi$  and a cavity C prepared in the state  $|\alpha\rangle$ . Let us first apply  $K_1$  to (2.19), (2.20), (2.24) and (2.23), that is

$$\begin{aligned}K_1 |\Psi^+\rangle_{A1-A2} &= |\Psi^+\rangle_{A1-A2-K1} \\ &= \frac{1}{2} \{ |f_1\rangle(|f_2\rangle - |g_2\rangle) + |g_1\rangle(|f_2\rangle + |g_2\rangle) \}, \\ K_1 |\Psi^-\rangle_{A1-A2} &= |\Psi^-\rangle_{A1-A2-K1} \\ &= \frac{1}{2} \{ |f_1\rangle(|f_2\rangle + |g_2\rangle) + |g_1\rangle(|f_2\rangle - |g_2\rangle) \}, \\ K_1 |\Phi^+\rangle_{A1-A2} &= |\Phi^+\rangle_{A1-A2-K1} \\ &= \frac{1}{2} \{ -|f_1\rangle(|f_2\rangle - |g_2\rangle) + |g_1\rangle(|f_2\rangle + |g_2\rangle) \}, \\ K_1 |\Phi^-\rangle_{A1-A2} &= |\Phi^-\rangle_{A1-A2-K1} \\ &= \frac{1}{2} \{ |f_1\rangle(|f_2\rangle + |g_2\rangle) - |g_1\rangle(|f_2\rangle - |g_2\rangle) \}.\end{aligned}\quad (2.39)$$

Then, we pass atom A2 through C and, taking into account (2.1), we get

$$\begin{aligned}
 |\Psi^+\rangle_{A1-A2-K1-C} &= \frac{1}{2} \{ |f_1\rangle(|f_2\rangle|\alpha\rangle - |g_2\rangle|-\alpha\rangle) \\
 &\quad + |g_1\rangle(|f_2\rangle|\alpha\rangle + |g_2\rangle|-\alpha\rangle) \}, \\
 |\Psi^-\rangle_{A1-A2-K1-C} &= \frac{1}{2} \{ |f_1\rangle(|f_2\rangle|\alpha\rangle + |g_2\rangle|-\alpha\rangle) \\
 &\quad + |g_1\rangle(|f_2\rangle|\alpha\rangle - |g_2\rangle|-\alpha\rangle) \}, \\
 |\Phi^+\rangle_{A1-A2-K1-C} &= \frac{1}{2} \{ -|e_1\rangle(|f_2\rangle|\alpha\rangle - |g_2\rangle|-\alpha\rangle) \\
 &\quad + |g_1\rangle(|f_2\rangle|\alpha\rangle + |g_2\rangle|-\alpha\rangle) \}, \\
 |\Phi^-\rangle_{A1-A2-K1-C} &= \frac{1}{2} \{ |f_1\rangle(|f_2\rangle|\alpha\rangle + |g_2\rangle|-\alpha\rangle) \\
 &\quad - |g_1\rangle(|f_2\rangle|\alpha\rangle - |g_2\rangle|-\alpha\rangle) \}.
 \end{aligned} \quad (2.40)$$

Now, we apply a rotation on the states of A2, that is, we apply

$$R = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad (2.41)$$

which gives us

$$\begin{aligned}
 |f_2\rangle &\rightarrow \frac{1}{\sqrt{2}}(|f_2\rangle - |g_2\rangle), \\
 |g_2\rangle &\rightarrow \frac{1}{\sqrt{2}}(|f_2\rangle + |g_2\rangle),
 \end{aligned} \quad (2.42)$$

and we get

$$\begin{aligned}
 |\Psi^+\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|-\alpha\rangle + |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \}, \\
 |\Psi^-\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|\alpha\rangle + |-\alpha\rangle)] \}, \\
 |\Phi^+\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ -|f_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|-\alpha\rangle + |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \}, \\
 |\Phi^-\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \\
 &\quad - |g_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|\alpha\rangle + |-\alpha\rangle)] \}.
 \end{aligned} \quad (2.43)$$

$$\begin{aligned}
 |\Psi^+\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|\alpha\rangle + |-\alpha\rangle)] \}, \\
 |\Psi^-\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|\alpha\rangle + |-\alpha\rangle)] \}, \\
 |\Phi^+\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ -|f_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|-\alpha\rangle + |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \}, \\
 |\Phi^-\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \\
 &\quad - |g_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|\alpha\rangle + |-\alpha\rangle)] \}.
 \end{aligned} \quad (2.44)$$

$$\begin{aligned}
 |\Psi^+\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ -|f_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|-\alpha\rangle + |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \}, \\
 |\Psi^-\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|\alpha\rangle + |-\alpha\rangle)] \}, \\
 |\Phi^+\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ -|f_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|-\alpha\rangle + |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \}, \\
 |\Phi^-\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \\
 &\quad - |g_1\rangle[|f_2\rangle(|\alpha\rangle - |-\alpha\rangle) - |g_2\rangle(|\alpha\rangle + |-\alpha\rangle)] \}.
 \end{aligned} \quad (2.45)$$

$$\begin{aligned}
 |\Phi^-\rangle_{A1-A2-K1-C-R} &= \frac{1}{2\sqrt{2}} \{ |f_1\rangle[|f_2\rangle(|\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|-\alpha\rangle - |\alpha\rangle)] \\
 &\quad + |g_1\rangle[|f_2\rangle(|-\alpha\rangle + |-\alpha\rangle) + |g_2\rangle(|\alpha\rangle \\
 &\quad + |-\alpha\rangle)]|\alpha\rangle - |\alpha\rangle \}.
 \end{aligned} \quad (2.46)$$

Now, for (2.43) we displace the cavity injecting  $|\alpha\rangle$  and send a two-level atom A3 resonant with the cavity through C. If A3 is sent in the lower state  $|f_3\rangle$  and after it crosses the cavity we detect the upper  $|e_3\rangle$  we get

$$|\Psi^+\rangle_{A1-A2} = \frac{1}{2} \{ (|f_1\rangle + |g_1\rangle)(|f_2\rangle - |g_2\rangle) \}, \quad (2.47)$$

and if we apply (2.29) to atoms A1 and A2 we get

$$|\Psi^+\rangle_{A1-A2-K1-K2} = |g_1\rangle|f_2\rangle. \quad (2.48)$$

For (2.44), we displace the cavity injecting  $|-\alpha\rangle$  and, as above, sending a two-level atom A3 through C in the lower state  $|f_3\rangle$  and after it crosses the cavity detecting the upper state  $|e_3\rangle$  we get

$$|\Psi^-\rangle_{A1-A2} = \frac{1}{2} \{ (|f_1\rangle - |g_1\rangle)(|f_2\rangle + |g_2\rangle) \}, \quad (2.49)$$

and if we apply (2.29) to atoms A1 and A2 we get

$$|\Psi^-\rangle_{A1-A2-K1-K2} = |f_1\rangle|g_2\rangle. \quad (2.50)$$

For (2.45), we displace the cavity injecting  $|\alpha\rangle$  and sending a two-level atom A3 through C in the lower state  $|f_3\rangle$  and after it crosses the cavity detecting the upper state  $|e_3\rangle$ , we get

$$|\Phi^+\rangle_{A1-A2} = \frac{1}{2} \{ (-|f_1\rangle + |g_1\rangle)(|f_2\rangle - |g_2\rangle) \}, \quad (2.51)$$

and if we apply (2.29) to atoms A1 and A2 we get

$$|\Phi^+\rangle_{A1-A2-K1-K2} = |f_1\rangle|f_2\rangle. \quad (2.52)$$

And finally, for (2.46) we displace the cavity injecting  $|-\alpha\rangle$  and sending a two-level atom A3 through C in the lower state  $|f_3\rangle$  and after it crosses the cavity detecting the upper state  $|e_3\rangle$ , we get

$$|\Phi^-\rangle_{A1-A2} = \frac{1}{2} \{ (|f_1\rangle + |g_1\rangle)(|f_2\rangle + |g_2\rangle) \}, \quad (2.53)$$

and if we apply (2.29) to atoms A1 and A2 we get finally

$$|\Phi^-\rangle_{A1-A2-K1-K2} = |g_1\rangle|g_2\rangle. \quad (2.54)$$

As we see, the discrimination between (2.19), (2.20), (2.24) and (2.23), is made detecting, after the process described above,  $(|g_1\rangle|f_2\rangle)$ ,  $(|f_1\rangle|g_2\rangle)$ ,  $(|f_1\rangle|f_2\rangle)$  or  $(|g_1\rangle|g_2\rangle)$ .

### 3. Teleportation

In this section we are going to discuss a teleportation scheme that is closely similar to the original scheme suggested by Bennett et al. [3]. Let us assume that Alice and Bob meet and then they build up a Bell state involving two-level atoms A2 and A4 as described in Section 2 (we use the notation A3 for the two-level atom used to disentangle the atomic states from the cavity state as in the previous section). That is, as in the previous section they make use of a cavity prepared initially in a coherent state and send A2 and A4 through this cavity where the atoms interact dispersively with the cavity, and following the recipe presented in that section, they get

$$|\Psi^+\rangle_{A2-A4} = \frac{1}{\sqrt{2}}(|f_2\rangle|f_4\rangle + |g_2\rangle|g_4\rangle). \quad (3.1)$$

Now, let us assume that Alice keeps with her the half of this Bell state consisting of atom A2 and Bob keeps with him the other half of this Bell state, that is, atom A4. Then they separate and let us assume that they are far apart from each other. Later on, Alice decides to teleport the state of an atom A1 prepared in an unknown state

$$|\psi\rangle_{A1} = \zeta|f_1\rangle + \xi|g_1\rangle \quad (3.2)$$

to Bob. For this purpose let us write the state formed by the direct product of the Bell state and this unknown state  $|\Psi^+\rangle_{A2-A4}|\psi\rangle_{A1}$ , that is,

$$\begin{aligned} |\psi\rangle_{A1-A2-A4} = \frac{1}{\sqrt{2}} \{ & \zeta(|f_1\rangle|f_2\rangle|f_4\rangle \\ & + |f_1\rangle|g_2\rangle|g_4\rangle) + \xi(|g_1\rangle|f_2\rangle|f_4\rangle \\ & + |g_1\rangle|g_2\rangle|g_4\rangle) \}. \end{aligned} \quad (3.3)$$

First Alice prepares a cavity C in a coherent state  $|\alpha\rangle$ . Taking into account (2.1) with  $\varphi = \pi$ , after atoms A1 and A2 fly through the cavity we have

$$\begin{aligned} |\psi\rangle_{A1-A2-A4C} = \frac{1}{\sqrt{2}} \{ & \zeta(|f_1\rangle|f_2\rangle|-\alpha\rangle|f_4\rangle \\ & + |f_1\rangle|g_2\rangle|\alpha\rangle|g_4\rangle) + \xi(|g_1\rangle|f_2\rangle|\alpha\rangle|f_4\rangle \\ & + |g_1\rangle|g_2\rangle|-\alpha\rangle|g_4\rangle) \}. \end{aligned} \quad (3.4)$$

Now, we make use of the Bell basis involving atom A1 and A2 and we can write

$$\begin{aligned} |f_1\rangle|f_2\rangle &= \frac{1}{\sqrt{2}}(|\Psi^+\rangle_{A1-A2} + |\Psi^-\rangle_{A1-A2}), \\ |g_1\rangle|g_2\rangle &= \frac{1}{\sqrt{2}}(|\Psi^+\rangle_{A1-A2} - |\Psi^-\rangle_{A1-A2}), \\ |f_1\rangle|g_2\rangle &= \frac{1}{\sqrt{2}}(|\Phi^+\rangle_{A1-A2} + |\Phi^-\rangle_{A1-A2}), \\ |g_1\rangle|f_2\rangle &= \frac{1}{\sqrt{2}}(|\Phi^+\rangle_{A1-A2} - |\Phi^-\rangle_{A1-A2}). \end{aligned} \quad (3.5)$$

Therefore, we can rewrite (3.4) as

$$\begin{aligned} |\psi\rangle_{A1-A2-A4C} = \frac{1}{\sqrt{2}} \{ & |\Psi^+\rangle_{A1-A2}[\zeta|f_4\rangle + \xi|g_4\rangle]|-\alpha\rangle \\ & + |\Psi^-\rangle_{A1-A2}[\zeta|f_4\rangle - \xi|g_4\rangle]|-\alpha\rangle \\ & + |\Phi^+\rangle_{A1-A2}[\zeta|g_4\rangle + \xi|f_4\rangle]|\alpha\rangle \\ & + |\Phi^-\rangle_{A1-A2}[\zeta|g_4\rangle - \xi|f_4\rangle]|\alpha\rangle \}. \end{aligned} \quad (3.6)$$

Notice that atoms A1 and A2 are with Alice and she wants to teleport the state of atom A1 (3.2) to Bob's atom A4. Inspecting (3.6) we see that all Alice has to do is to inject in the cavity  $|\alpha\rangle$  or  $|\alpha\rangle$  and to perform a measurement of one of the Bell states that form the Bell basis. We have already seen how to detect the Bell states in Section 2. Then proceeding according to the prescription detailed in Section 2 Alice, in the end, has four possibilities. First let us assume that Alice injects in the cavity  $|\alpha\rangle$ . Then, she sends a two-level atom A3 resonant with the cavity in the lower state  $|f_3\rangle$  and after it crosses the cavity she detects the upper state  $|e_3\rangle$  and she gets

$$\begin{aligned} |\psi\rangle_{A1-A2-A4} = \frac{1}{N} \{ & |\Psi^+\rangle_{A1-A2}[\zeta|f_4\rangle + \xi|g_4\rangle] \\ & + |\Psi^-\rangle_{A1-A2}[\zeta|f_4\rangle - \xi|g_4\rangle] \}, \end{aligned} \quad (3.7)$$

where  $N$  is a normalization constant. Then, she has four alternatives, that is, applying (2.29) to A1 and A2 respectively, according to (2.37) and (2.38) if she gets  $(K_1, |g_1\rangle)(K_2, |g_2\rangle)$  or  $(K_1, |f_1\rangle)(K_2, |f_2\rangle)$  this corresponds to the detection of



$|\Psi^+\rangle_{A1-A2}$  and if she gets  $(K_1, |f_1\rangle)(K_2, |g_2\rangle)$  or  $(K_1, |g_1\rangle)(K_2, |f_2\rangle)$  this corresponds to the detection of  $|\Psi^-\rangle_{A1-A2}$ . Therefore, after the detection of the states of the atoms A1 and A2 Alice calls Bob and informs him that she has injected  $|\alpha\rangle$  in the cavity and the result of her atomic detection so that Bob knows what to do to get the right state, that is, a state similar to (3.2). If she detects  $(|f_1\rangle|f_2\rangle)$  or  $(|g_1\rangle|g_2\rangle)$  Bob gets

$$|\psi\rangle_{A4} = \zeta|f_4\rangle + \xi|g_4\rangle, \quad (3.8)$$

and he has to do nothing else. If she detects  $(|f_1\rangle|g_2\rangle)$  or  $(|g_1\rangle|f_2\rangle)$  Bob gets

$$|\psi\rangle_{A4} = \zeta|f_4\rangle - \xi|g_4\rangle, \quad (3.9)$$

and he must apply a rotation in the Ramsey cavity R4

$$R_4 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad (3.10)$$

in order to get a state like (3.8).

Now let us assume that Alice injects in the cavity  $|\alpha\rangle$ . Then she sends a two-level atom A3 resonant with the cavity in the lower state  $|f_3\rangle$  and after it crosses the cavity she detects the upper state  $|e_3\rangle$  and she gets

$$|\psi\rangle_{A1-A2-A4} = \frac{1}{N} \{ |\Phi^+\rangle_{A1-A2} [\zeta|g_4\rangle + \xi|f_4\rangle] + |\Phi^-\rangle_{A1-A2} [\zeta|g_4\rangle - \xi|f_4\rangle] \}. \quad (3.11)$$

Again she has four alternatives, that is, applying (2.29) to A1 and A2 respectively, according to (2.37) and (2.38) if she gets  $(K_1, |g_1\rangle)(K_2, |g_2\rangle)$  or  $(K_1, |f_1\rangle)(K_2, |f_2\rangle)$  this corresponds to the detection of  $|\Phi^+\rangle_{A1-A2}$  and if she gets  $(K_1, |f_1\rangle)(K_2, |g_2\rangle)$  or  $(K_1, |g_1\rangle)(K_2, |f_2\rangle)$  this corresponds to the detection of  $|\Phi^-\rangle_{A1-A2}$ . Therefore, after the detection of the states of the atoms A1 and A2, Alice calls Bob and informs him that she has injected  $|\alpha\rangle$  in the cavity and the result of her atomic detection to Bob. If she detects  $(|f_1\rangle|f_2\rangle)$  or  $(|g_1\rangle|g_2\rangle)$  Bob gets

$$|\psi\rangle_{A4} = \zeta|f_4\rangle + \xi|g_4\rangle, \quad (3.12)$$

and he has to apply a rotation in the Ramsey cavity R4

$$R_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad (3.13)$$

to get (3.8). Finally, if she detects  $(|f_1\rangle|g_2\rangle)$  or  $(|g_1\rangle|f_2\rangle)$  Bob gets

$$|\psi\rangle_{A4} = -\xi|f_4\rangle + \zeta|g_4\rangle, \quad (3.14)$$

and he has to apply a rotation in the Ramsey cavity R4

$$R_4 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad (3.15)$$

to get (3.8). Notice that the original state (3.2) is destroyed in the end of the teleportation process (it evolves to  $|f_1\rangle$  or  $|g_1\rangle$ ) in accordance with the no-cloning theorem [1,4].

In Fig. 2, we present the scheme of the teleportation process we have discussed above.

Concluding, we have presented a scheme of realization of atomic state teleportation making use of cavity QED. A nice alternative scheme also making

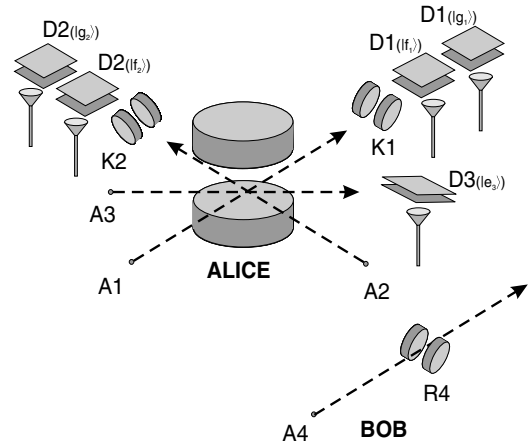


Fig. 2. Set-up for teleportation process. Alice and Bob meet and generate a Bell state involving atoms A2 and A4. Alice sends atoms A1 and A2 through a cavity C prepared initially in a coherent state  $|\alpha\rangle$ . After atoms A1 and A2 have flown through C Alice must inject  $|\alpha\rangle$  or  $|\alpha\rangle$  in the cavity, send a two-level atom A3 resonant with the cavity through C in the lower state  $|f_3\rangle$  and detect the upper state  $|e_3\rangle$ . Then she must perform a measurement of the remaining Bell states of the Bell basis. For this purpose she sends atom A1 through the Ramsey cavity K1 and A2 through Ramsey cavity K2. Then, she calls Bob and informs him which coherent field she has injected in C1 and the result of her atomic detections in detectors D1 and D2. Depending on the results of the Alice's atomic detections and which coherent state she injected in the cavity, Bob has or not to perform an extra rotation in the Ramsey cavity R4 on the states of his atom A4.



use of atoms interacting with electromagnetic cavities has also been proposed in [7]. In our scheme, we use atoms interacting with superconducting cavities prepared in a coherent state which are states relatively easy to be prepared and handled. In [7] it is used atoms interacting with cavities prepared in Fock states which are state of the electromagnetic field which are sensitive to decoherence. We think that both this schemes could be realized experimentally in the future.

## References

- [1] M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge, 2000.
- [2] G. Chen, R.K. Brylinski (Eds.), *Mathematics of Quantum Computation*, Chapman & Hall/CRC Press, London/Boca Raton, FL, 2002.
- [3] C.H. Bennett, C. Brassard, C. Crépeau, R. Jozsa, A. Peres, W.K. Wootters, *Phys. Rev. Lett.* 70 (1993) 1895.
- [4] W.K. Wootters, W.H. Zurek, *Nature* 299 (1982) 802.
- [5] M. Orszag, *Quantum Optics*, Springer, Berlin, 2000.
- [6] W.H. Zurek, *Phys. Today* 44 (October) (1991) 36; *Phys. Rev. D* 24 (1981) 1516; *Phys. Rev. D* 26 (1982) 1862.
- [7] L. Davidovich, N. Zagury, M. Brune, J.M. Raimond, S. Haroche, *Phys. Rev. A* 50 (1994) R895.
- [8] A.A. Radzig, B.M. Smirnov, *Reference Data on Atoms Molecules and Ions*, Springer, Berlin, 1985; T.F. Gallagher, *Rydberg Atoms*, Cambridge University Press, Cambridge, 1984.
- [9] M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J.M. Raimond, S. Haroche, *Phys. Rev. Lett.* 76 (1996) 1800.
- [10] G. Rempe, F. Schmidt-Kaler, H. Walther, *Phys. Rev. Lett.* 64 (1990) 2783.
- [11] C.C. Gerry, *Phys. Rev. A* 53 (1996) 2857; *Phys. Rev. A* 54 (1996) R2529.
- [12] W.H. Louisell, *Quantum Statistical Properties of Radiation*, Wiley, New York, 1973.
- [13] E.S. Guerra, B.M. Garraway, P.L. Knight, *Phys. Rev. A* 55 (1997) 3482.
- [14] S.L. Braustein, A. Mann, M. Revzen, *Phys. Rev. Lett.* 68 (1992) 3259.