



## Wave-mixing-induced transparency with zero phase shift in atomic vapors

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## ABSTRACT

We present a wave-mixing induced transparency that can lead to a hyper-Raman gain-clamping effect. This new type of transparency is originated from a dynamic gain cancellation effect in a multiphoton process where a highly efficient light field of new frequency is generated and amplified. We further show that this novel dynamic gain cancellation effect not only makes the medium transparent to a probe light field at appropriate frequency but also eliminates the probe field propagation phase shift. This gain-cancellation-based induced transparency holds for many potential applications on optical communication and may lead to effective suppression of parasitic Raman/hyper-Raman noise field generated in high intensity optical fiber transmissions.

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## 1. Introduction

The terminology of *interference* describes a class of phenomena in which the amplitudes of various physical processes influence and affect each other, resulting pronounced coherent build-up of certain fields if some amplitudes are in-phase, or complete cancellation of certain features when the amplitudes of the physical processes are 180° out-of-phase. Quantum interference effect is the foundation of quantum mechanics [1]. In fact, the effect of interference between waves was the primary reason that quantum mechanics was also referred to as wave mechanics [2]. In the field of nonlinear optics it is known that quantum interference effects can profoundly modify the over-all system response and also lead to new physics [3]. For instance, the electromagnetically induced transparency (EIT) effect [4] can be considered as a destructive interference between the amplitudes of two excitation pathway involving a weak on-resonance probe field and a strong but also on-resonance coupling field. The consequence of such interference effect is the total cancellation of the absorption of the probe field as well as the elimination of the probe propagation phase shift when both one- and two-photon detuning are zero [5].

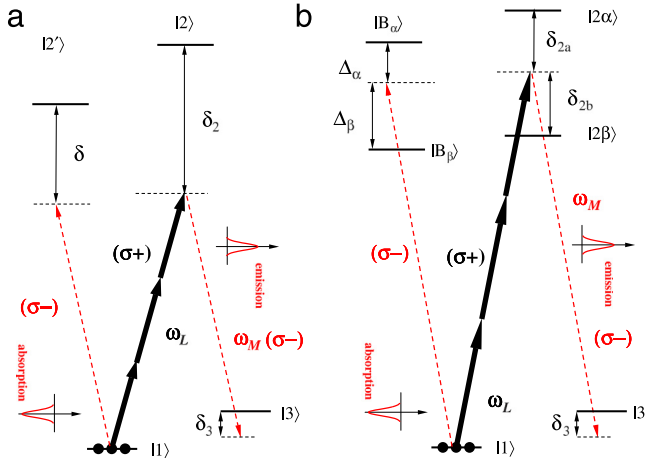
In the field of nonlinear optics of resonant media quantum interference effect between different excitation pathways have been shown to

lead to many novel and intriguing processes [3], including efficient odd-[6–14] and even-photon [15] destructive interference effects, enhanced and suppressed multi-photon ionization processes [11], forward-hyper-Raman suppression [13], suppression of various optical shifts [16] and nonclassical field generation [17–19]. It has been known, however, that such multi-wave mixing interference effects [20–25] usually do not occur if the hyper-Raman terminal state is not allowed to couple radiatively back to the ground state. Indeed, to date there has been no study, theoretical or experimental, on the suppression of forward hyper-Raman gain process by such multi-wave mixing processes.

In this work we show that it is possible to achieve a complete transparency in a hyper-Raman process where the back-coupling to the ground state is not allowed. We show that by carefully choosing operation parameters a nonlinear wave-mixing-induced transparency regime can be established via a dynamic gain cancellation effect. As a result, if a light field with the same frequency as the internally-generated field is injected into the medium it will experience no change in intensity and propagation phase. We emphasize that while this wave-mixing-induced transparency leads to propagation-independent properties similar to that resulted from the well-known odd-photon destructive interference effects [3,4,6–16] it is not an interference effect that leads to electromagnetically induced transparency. This gain-cancellation-based induced transparency may lead to effective suppression of parasitic

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**Fig. 1.** (Color online) (a) A right-circularly-polarized strong pump field  $E_L$  with angular frequency  $\omega_L$  couples the transition between states  $|1\rangle$  and  $|2\rangle$  via three-photon process. A left-circularly-polarized weak SFG field  $E_M$  with angular frequency  $\omega_M$  drives  $|1\rangle \leftrightarrow |2'\rangle$  and  $|2\rangle \leftrightarrow |3\rangle$  transitions simultaneously. (b) Energy levels and laser-field configurations for the experiment.

Raman/hyper-Raman noise field generated in high intensity optical fiber transmissions.

## 2. Model

We begin by first considering a life-time broadened (such as laser-cooled) three-state atomic medium in a sum-frequency-generation (SFG) configuration (Fig. 1a) where three photons of frequency  $\omega_L$  from a strong pump field  $E_L$  are absorbed simultaneously during the transition  $|1\rangle \rightarrow |2\rangle$ . As the consequence of this three-photon excitation and with appropriate choice of light polarization the dipole-allowed transition  $|2\rangle \rightarrow |3\rangle$  leads to a SFG field that terminates at the lower excited state  $|3\rangle$ . Since the SFG field terminal state  $|3\rangle$  is not allowed to radiatively couple back to the ground state  $|1\rangle$  the SFG field is also referred to as a hyper-Raman field. For this reason we interchangeably use SFG and hyper-Raman field in this work in describing this generated field. Indeed, this “inelastically-generated” [26] sum-frequency wave-mixing field  $E_M$  with frequency  $\omega_M$  has the feature of a Raman-like process. It propagates co-linearly and grows coherently with the pump field. The main objective of the present work is to show that with appropriate selected states for the hyper-Raman field an additional nearby state can open an effective absorption channel which results in novel features not known before. We show that under suitable operation conditions, a nonlinear third-order effect arising from non-adiabatic correction to the polarization at frequency  $\omega_M$  can provide an effective gain cancellation mechanism, leading to a mixing-wave-induced transparency where the generated field becomes both concentration and propagation independent.

The energy-level diagram with laser couplings of the system studied is given in Fig. 1a. While the transitions require circularly polarized pump and SFG seed field to enable an additional background absorption channel a mathematical treatment assuming linearly polarized pump and SFG fields is more transparent in demonstrating the underlying physics [27]. Indeed, there is no loss of key physics using this simplified treatment which can also be verified using a density matrix approach. Based on these considerations we write the equations of motion for the state amplitudes under rotating-wave approximation as

$$\dot{A}_1 = i\Omega_{21}^{(3)*} A_2, \quad (1)$$

$$\dot{A}_2 = id_2 A_2 + i\Omega_{21}^{(3)} A_1 + i\Omega_{23} A_3, \quad (2)$$

$$\dot{A}_3 = id_3 A_3 + i\Omega_{23}^* A_2, \quad (3)$$

where  $A_j$  is the amplitude of the state  $|j\rangle$ ,  $d_i = \delta_j + i\gamma_i$  with  $\delta_2$  and  $\delta_3$  being the one- and two-photon detunings, respectively. Here,  $\gamma_i$  is the decay rate of state  $|i\rangle$ .  $\Omega_{21}^{(3)}$  is the effective three-photon Rabi frequency by the pump field and  $\Omega_{23} = \Omega_M = D_{23} E_M / \hbar$  is the Rabi frequencies of the hyper-Raman/SFG fields with electric dipole moments of  $D_{23}$ . Typically, in a hyper-Raman process  $\delta_3 \approx 0$ . The additional absorption channel can be introduced if the SFG field is weakly affected by a nearby state  $|2'\rangle$  (see Fig. 1a) offered by a background buffer gas. Mathematically, such a perturbative effect by the background gas can be satisfactorily treated by adding a corresponding term in the source term for the Maxwell equation of the hyper-Raman/SFG field. One primary example of such a treatment is the index compensation in sum frequency generation in pure alkali metal vapor with inert background gas for index matching. We note that Eqs. (1)–(3) derived using rotating wave approximations have been successfully used to explain various nonlinear optical multi-photon destructive interference effects, validating the use of such a treatment for the problem presented here.

Solving Eqs. (1)–(3) using the first order non-adiabatic theory in non-depleted ground state approximation we obtained polarization source term for the Maxwell equation for the inelastic SFG field. To the leading order of nonlinear contribution and the first order background absorption, this can be expressed as [28]

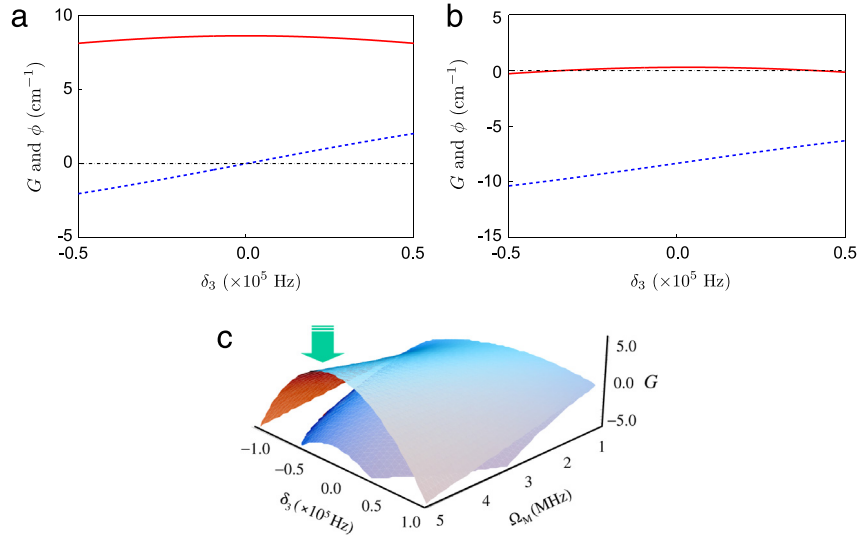
$$\begin{aligned} \frac{\partial \Omega_M}{\partial z} + \frac{i}{2k_z} \nabla_{\perp}^2 \Omega_M &= i\kappa_{32} A_2 A_3^* - i \frac{\kappa_b}{\delta + i\gamma} \Omega_M \\ &= -i\kappa_{32} \left( 1 + \frac{|\Omega_M|^2}{d_2 d_3} \right) \frac{|\Omega_{21}^{(3)}|^2}{|d_2|^2} \frac{\Omega_M}{d_3^*} - i \frac{\kappa_b}{\delta + i\gamma} \Omega_M \\ &= (G + i\phi) \Omega_M, \end{aligned} \quad (4)$$

where  $\delta$  and  $\gamma$  are the detuning and the dephasing rate of the state  $|2'\rangle$  respectively, and  $\kappa_b \approx \kappa_{32} = 2\pi\omega_{23} N |D_{23}|^2 / (\hbar c)$  with  $N$  being the atom density and the transverse contribution to the wave equation has been neglected. In Eq. (4) we have defined two real quantities

$$\begin{aligned} G &= \kappa_{32} \left[ 1 - \frac{|\Omega_M|^2 \gamma_2}{(\delta_2^2 + \gamma_2^2) \gamma_3} \right] \frac{|\Omega_{21}^{(3)}|^2}{|d_2|^2 \gamma_3} - \frac{\kappa_b \gamma}{\delta^2 + \gamma^2} \\ &= (G_0 - \alpha_0) - \alpha_1 |\Omega_M|^2, \\ \phi &= -\kappa_{32} \left[ \frac{|\Omega_M|^2 \delta_2}{(\delta_2^2 + \gamma_2^2) \gamma_3} \right] \frac{|\Omega_{21}^{(3)}|^2}{|d_2|^2 \gamma_3} - \frac{\kappa_b \delta}{\delta^2 + \gamma^2} = \phi_0 + \phi_1 |\Omega_M|^2. \end{aligned}$$

with  $G_0 = \kappa_{32} |\Omega_{21}^{(3)}|^2 / (|d_2|^2 \gamma_3)$ ,  $\alpha_0 = \kappa_b \gamma / (\delta^2 + \gamma^2)$ ,  $\alpha_1 = \kappa_{32} \gamma_2 |\Omega_{21}^{(3)}|^2 / [\gamma_3^2 |d_2|^2 (\delta_2^2 + \gamma_2^2)]$ ,  $\phi_0 = -\kappa_b \delta / (\delta^2 + \gamma^2)$  and  $\phi_1 = -\kappa_{32} \delta_2 |\Omega_{21}^{(3)}|^2 / [\gamma_3^2 |d_2|^2 (\delta_2^2 + \gamma_2^2)]$ .

The first term in the parenthesis on the right side of Eq. (4) represents the gain characteristics known to hyper-Raman and inelastic-wave-mixing processes [26] whereas the second term is the leading contribution by the third-order correction to the lowest adiabatic approximation of  $A_2$  and  $A_3$ . The last term containing  $\kappa_b$  arises from the background absorption arises from the near-by state  $|2'\rangle$  state (see the left red-dashed arrow in Fig. 1a [29]). Usually, such a background state can be engineered using two species of atomic gases with similar properties. However, an example of single element serves the purpose of illustrating the underlying physics. Consider, for instance, an atomic vapor system with  $F = 1, 2$  ground state manifold and assume that the population is initially in the  $F = 1, m_F = -1$  state. If we take the three-photon detuning  $\delta_2$  (a  $\sigma^{(+)}$  pump) to be about 1 GHz on the high energy side of the  $F' = 2$  manifold in a  $D_1$ -line-like transition and assume that the hyper-Raman field (with  $\sigma^{(-)}$  polarization) terminates on the  $F = 2, m_F = +1$  state that is 1.3 GHz above the  $F = 1$  manifold, then the hyper-Raman field can also couple the ground state  $F = 1, m_F = -1$  to  $F' = 2, m_{F'} = -2$  state with a smaller red detuning  $\delta = -300$  MHz. The idea is to use this background state to introduce an additional absorption that can slightly under-compensate the efficient hyper-Raman gain, i.e.,  $G_0 - \alpha_0 \sim 0^+$ , making the nonlinear third-order correction term an effective trigger for total propagation gain cancellation even with a weak generated field. This may lead to efficient suppression of parasitic Raman/hyper-Raman generation in fiber optical data links.



**Fig. 2.** Dispersion properties of (a) the non-background-state-compensated scheme and (b) back-ground-compensated scheme. Here, the red solid curve denotes the gain coefficient  $G$  per unit length and the blue dashed curve denotes the phase shift  $\phi$  per unit length. The system parameters are chosen as  $\gamma_3/2\pi = 200$  kHz,  $\kappa_{32} = \kappa_b = 5 \times 10^8$  cm<sup>-1</sup>s<sup>-1</sup>,  $\gamma_2/2\pi = 6$  MHz,  $\delta_2/2\pi = 60$  MHz. (c) Nonlinear switching effect with background-compensated scheme. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

### 3. Wave mixing induced transparency with zero phase shift

For simplicity, let us first neglect transverse contributions and take uniform density distribution. Defining  $G = G_0 - \alpha_0 > 0$  and writing the SFG field as  $\Omega_M = \Lambda_M e^{i\zeta}$  where  $\Lambda_M = |\Omega_M|$  and  $\phi$  are real quantities, Eq. (4) becomes

$$\frac{\partial}{\partial z} \begin{pmatrix} \Lambda_M \\ \zeta \end{pmatrix} = \begin{pmatrix} G\Lambda_M - \alpha_1\Lambda_M^3 \\ \phi_0 + \phi_1\Lambda_M^2 \end{pmatrix}. \quad (5)$$

The first equation in Eq. (5) yields an analytical solution for the SFG field

$$\Lambda_M^2 = |\Omega_M|^2 = \frac{|\Lambda_M^{(0)}|^2 e^{2Gz}}{1 + Q|\Lambda_M^{(0)}|^2 (e^{2Gz} - 1)} \approx \frac{|\Lambda_M^{(0)}|^2 e^{2Gz}}{1 + Q|\Lambda_M^{(0)}|^2 e^{2Gz}}, \quad (6)$$

where  $|\Lambda_M^{(0)}|^2 = |\Omega_M^{(0)}|^2$  is proportional to the intensity of the initial hyper-Raman field propagating co-linear with the pump field and  $Q = \alpha_1/G$ . In the last step of Eq. (6) we have assumed a significant coherent propagation gain for the hyper-Raman field so that  $e^{2Gz} \gg 1$ . In the initial growth regime with a small propagation distance we have  $1 > Q|\Lambda_M^{(0)}|^2 e^{2Gz}$  and Eq. (6) reduces to the well-known Raman-like/inelastic wave-mixing growth characteristics  $\Lambda_M^2 = |\Lambda_M^{(0)}|^2 e^{2Gz}$  [3].

For sufficient hyper-Raman gain (yet still away from pump depletion), however, we have  $QR_0^2 e^{2Gz} \gg 1$ . In this regime, Eq. (6) yields

$$\Lambda_M^2 = |\Omega_M|^2 \rightarrow \frac{1}{Q} = \frac{G}{\alpha_1} \rightarrow \frac{\gamma_3}{\gamma_2} |\delta_2|^2. \quad (7)$$

This relation shows that when the gain cancellation effect occurs the generated field stops growing and exhibits density and propagation distance independent characteristics. If a light field of the same frequency as this saturated field was injected into the medium such prepared it would experience no change in intensity as if the medium is not present. This is a multi-photon wave mixing induced transparency [30]. It is, however, important to note that while the density/propagation-independent property resulted from the gain cancellation effect is similar to the density/propagation-independent property resulted from the electromagnetically induced transparency effect [4] and multi-photon destructive interference effects in multi-wave-mixing processes [3] it is fundamentally different from the latter effects. Indeed, EIT and multi-photon destructive interference effects are the interference effect between amplitudes of different excitation pathways and channels.

Clearly, the gain cancellation effect described here is not an interference effect [31].

By integrating the second equation in Eq. (5) we obtain the phase of the generated field in the gain cancellation regime

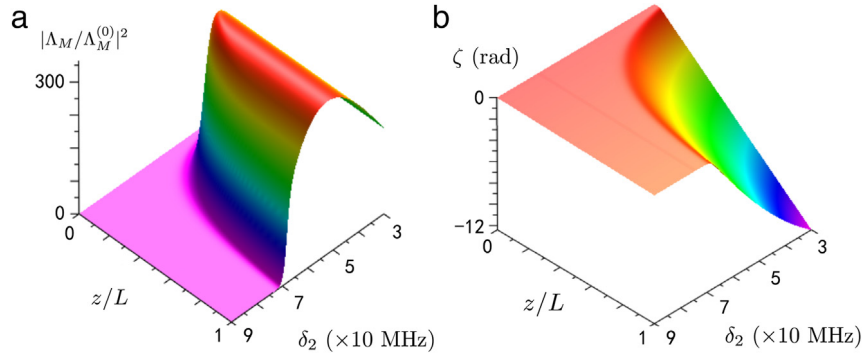
$$\zeta(z) = \phi_0 \left[ z + \frac{2R_S}{GQ\gamma_0^2\gamma_2} \ln \left( 1 + Q|\Lambda_M^{(0)}|^2 e^{2Gz} \right) \right], \quad (8)$$

where the pump rate  $R_S = |\Omega_{21}^{(3)}|^2 \gamma_2/4|d_2|^2$ . It can be shown from Eq. (8) that in the limit of wave-mixing-induced-transparency or gain-clamping regime the phase of the generated field is inversely proportional to the pump detuning. This suggests that by choosing two upper excited states with an appropriate separation and by tuning the pump light in between these two states it is possible to achieve complete cancellation of the phase (see Fig. 1(b)). As a result the amplitude-clamped wave-mixing field with no propagation phase is achieved, an outcome very similar to the EIT process except this is not an interference effect.

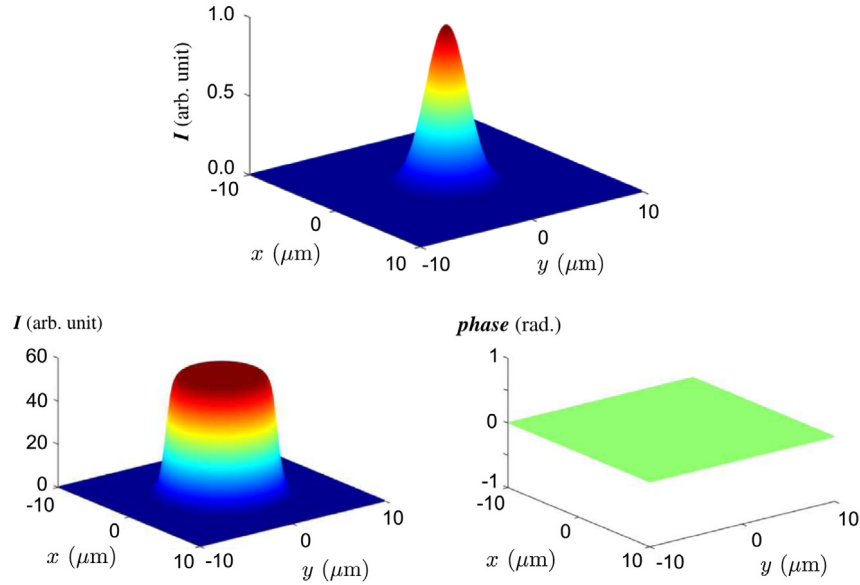
As a numerical example we consider a life-time broadened atomic gas column with a length of 5 cm and a uniform density distribution of  $N = 10^{10}$ /cm<sup>3</sup>. We take  $\gamma_3/2\pi = 200$  kHz,  $\kappa_{32} = \kappa_b = 5 \times 10^8$  cm<sup>-1</sup>s<sup>-1</sup>,  $\gamma_2/2\pi = 6$  MHz,  $\delta_2/2\pi = 60$  MHz (so that  $\gamma_2/\delta_2 = 0.1$ ). Assuming the intensity of the generated field propagating co-linearly with the pump ( $\Omega_{21}^{(3)}/2\pi = 5.6$  MHz) and nearby background state detuning  $\delta/2\pi = 10$  MHz (we take  $\gamma = \gamma_2$ ) we plot in Fig. 2 the dispersion properties of a background-state-compensated scheme (Fig. 2(a)) and a non-background-state-compensated scheme (Fig. 2(b)). It is clear to see that with the background-state-compensation, the gain of the SFG field decreases significantly, so that the nonlinear third-order correction can compensate the linear gain to realize a total propagation gain cancellation. In Fig. 2(c) the nonlinear switching effect (see the green arrow) is shown for the scheme depicted in Fig. 1(a). We show that the gain of the SFG field changes to an absorption as the SFG field increases. It is worthy to notice that this switching effect attributes to the third-order nonlinearity, which is different from the quantum interference effect.

Fig. 3 shows the behavior of the generated hyper-Raman/SFG field and the corresponding propagation phase as functions of normalized propagation distance and one-photon detuning for a fixed pump intensity. As expected from Eqs. (4) and (6), the hyper-Raman/SFG field saturates (see the rim of the plot in Fig. 3(a)) and the phase changes linearly once the cancellation effect occurs [Fig. 3(b)].

In Fig. 4 we plot the result Eq. (6) as a function of the propagation distance  $z$  and pump rate  $R_S$  using the propagation phase cancellation



**Fig. 3.** (Color online) (a). Plot of mixing-wave intensity distribution as a function of three-photon detuning and propagation distance. The rim on this 3D plot represents the condition of induced transparency where the growth of the hyper-Raman field stops. (b). Plot of the phase of the mixing-wave as a function of three-photon detuning and propagation distance. Once the induced transparency is achieved the phase changes linearly as  $z$ .



**Fig. 4.** (Color online) Wave-mixing-induced-transparency effect (see Fig. 1b). Top: Intensity distribution at the end of the medium when the wave-mixing-induced-transparency is not effect due to the lack of balanced gain/loss mechanism. Bottom Left: wave-mixing-induced-transparency develops from the center of the medium where the hyper-Raman field is the strongest and the nonlinear third-order term triggers the wave-mixing-induced-transparency which grows radially outward. Bottom right: The doublet scheme in Fig. 1b cancels the propagation gain, leading to zero propagation phase shift.

scheme given in Fig. 1(b). Experimentally, Fig. 1(b) can be realized using Zeeman magnetic sub-levels. For instance,  $|1\rangle = |3, m_F = 0\rangle$ ,  $|2\alpha\rangle = |4, m_F = +3\rangle$ ,  $|2\beta\rangle = |3, m_F = +3\rangle$  (i.e.,  $\sigma^{(+)}$  pump), and  $|3\rangle = |3, m_F = +2\rangle$  (i.e.,  $\sigma^{(+)}$  SFG). The background contribution states are  $|B_\alpha\rangle = |4, m_F = -1\rangle$  and  $|B_\beta\rangle = |3, m_F = -1\rangle$ , respectively (which may be in some cases be provided and also adjusted by changing the axial magnetic field). The main advantage of doublet upper state is to cancel out propagation dependent phase [see Eq. (8)]. The top panel shows the hyper-Raman field intensity as a function of  $x$  and  $y$  under the condition that the gain clamping is not effective. In lower left panel of Fig. 4 the hyper-Raman nonlinear gain has reached a level where the gain clamping effect develops from the center of the field intensity distribution outward. As expected, contributions to the total propagation phase from the doublet cancel out, resulting in a wave-mixing-induced-transparency that has nearly constant propagation phase (see the lower right panel in Fig. 4). We emphasize that while these characteristics are very similar to that from an EIT-based process they are achieved without interference-based cancellation effect as required by any EIT-based process.

The principle described in this work may have applications in suppression of parasitic Raman/hyper-Raman background. Assuming a signal field propagating in a fiber which has a core of a few  $\mu\text{m}$ , resulting in significantly high local light intensity. This can lead to

strong generation of parasitic field via Raman, FWM, and hyper-Raman processes. Suppose this signal carried is the  $\Omega_{21}^{(3)}$  and the noise generation is described by  $\Omega_M$ , then it is possible to apply the principle described here to cap the growth of the parasitic noise field  $\Omega_M$  even at fairly weak level.

#### 4. Conclusions

In conclusion, we have shown a wave-mixing induced transparency that can lead to a suppression of forward hyper-Raman processes by the nonlinear optical effect. The propagation-critical gain cancellation effect demonstrated is fundamentally different from the interference effect such as EIT and the “accidental” linear absorption/phase cancellation in the particular region in between  $D_1$  and  $D_2$  transitions, both latter effects do not depend on propagation. As a result of this gain cancellation effect, if a light field with the same frequency  $\omega_M$  is injected into the medium it will experience no absorption with no medium-dependent propagation phase as if the medium is not present. We have also shown that by carefully choosing the system parameters and using the doublet excitation states, the medium is not only transparent to the probe field but also immune to the transient propagation phase shift. This new transparency have many applications in suppression of the parasitic noise generation.

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- [27] While mathematically the effective three-photon Rabi frequency resembles the one-photon Raman scheme the three-photon SFG scheme with no intermediate one and two-photon enhancements avoids the strong absorption of the pump field in the usual one-photon Raman scheme. This is the key reason why such an induced transparency cannot occur in a usual one-photon pumping scheme.
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- [29] For instance if the pump laser has  $\sigma^{(+)}$  polarization and the SFG field has  $\sigma^{(-)}$  polarization then an additional absorption channel for the SFG field from the ground state is enabled (see the red-dashed arrow on the left near the state  $|2'\rangle$  in Fig. 1a).
- [30] The propagation-dependent multi-photon induced transparency is fundamentally different from the usual EIT which is not a propagation-based effect. The Multi-photon induced transparency does not occur in the entire medium. It is established only after some propagation distance when the internally generated field becomes sufficiently strong. When a second field with the same frequency of the probe is injected the probe is effectively stronger and the transparency is established earlier in the medium. This leads to the less loss of the first probe which compensates the absorption of the second field at the same frequency. This is why effectively the second field experiences no presence of the medium.
- [31] The transparency described is also fundamentally different from the trivial linear transparency pint in between  $D_1$  and  $D_2$  levels which does not rely on nonlinear propagation of the generated field.