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Perturbative approach for diffraction due to a periodically corrugated boundary between vacuum and a negative phase-velocity material

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Abstract

Using a perturbative approach, we study the changes that the transformation of a refracting medium from positive/negative phase velocity to negative/positive phase velocity has on the diffraction of a plane wave due to a periodically corrugated boundary. In contrast to specular reflection, we find that nonspecular reflection, albeit weak for weakly corrugated boundaries, is highly affected by the type of refracting medium, due to the involvement of evanescent waves in the troughs of the corrugated boundary.

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1. Introduction

The phase velocity vector is opposed in direction to the time-averaged Poynting vector in certain isotropic dielectric–magnetic materials [1]. Though several names have been proposed for this class of materials, we think that the most descriptive is: negative phase-velocity (NPV) materials [2]. In contrast, the phase velocity and the time-averaged Poynting vectors are co-parallel in positive phase-velocity (PPV) materials. PPV materials are, of course, commonplace and require no introduction.

During the last four years, a spate of research publications have appeared on electromagnetic fields in NPV materials [3,4]. Many interesting effects have been predicted, with some experimental backing as well [5–7]. The intrinsic difference

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between NPV and PPV materials is easily gauged from the reflection and refraction of a plane wave due to a homogeneously filled half-space. Let vacuum be the medium of incidence, while ϵ_2 and μ_2 denote the relative permittivity and relative permeability of the medium of refraction. Let a linearly polarized plane wave be incident on the planar interface of the two mediums at an angle θ_0, ($|\theta_0| < \pi/2$), from the normal to the interface, and $\rho(\theta_0)$ be the reflection coefficient. If the transformation $\{\epsilon_2 \to -\epsilon_2^*, \mu_2 \to -\mu_2^*\}$ is implemented, then $\rho(\theta_0) \to \rho^*(\theta_0)$, where the asterisk denotes the complex conjugate [8]. Thus, the replacement of a NPV/PPV refracting half-space by an analogous PPV/NPV half-space changes the phase of the reflection coefficient but not its magnitude.

To see what would happen if the surface were corrugated, we study here the case of surface-relief gratings [9]. Restricting ourselves to slightly rough surfaces, we use a perturbative approach based on the Rayleigh hypothesis [10]. This perturbative approach has been used for periodically corrugated hard surfaces [11], as well as for periodically corrugated boundaries between either two dielectric non-magnetic materials [12,13] or two index-matched dielectric–magnetic materials [14]. Our intention here is to elucidate the mechanism for the behavioral differences between periodically corrugated NPV and PPV half-spaces, rather than to put forward the perturbative approach as a design paradigm. An $\exp(-i\omega t)$ time–dependence is implicit, with ω as the angular frequency.

2. Analysis

2.1. Boundary value problem

In a rectangular coordinate system (x, y, z), we consider the periodically corrugated boundary $y = g(x) = g(x + d)$ between vacuum and a homogeneous, isotropic, linear material, with d being the corrugation period. The region $y > g(x)$ is vacuous, whereas the medium occupying the region $y < g(x)$ is characterized by complex-valued scalars $\epsilon_2 = \epsilon_{2R} + i\epsilon_{2I}$ and $\mu_2 = \mu_{2R} + i\mu_{2I}$. If this medium is of the NPV type, then [2,15]

$$\epsilon_{2R}|\mu_2| + \mu_{2R}|\epsilon_2| < 0 \tag{1}$$

otherwise

$$\epsilon_{2R}|\mu_2| + \mu_{2R}|\epsilon_2| > 0. \tag{2}$$

As the refracting medium is being modeled here as a continuum, its microstructural features are assumed to be considerably smaller in size than the undulations of the boundary. A linearly polarized electromagnetic plane wave is incident on this boundary from the region $y > g(x)$ at an angle θ_0, ($|\theta_0| < \pi/2$), with respect to the y axis.

Let the function $f(x, y)$ represent either the z-directed component of the total electric field for the s-polarization case, or the z-directed component of the total magnetic field for the p-polarization case [16]. Outside the corrugation region $\min g(x) \leqslant y \leqslant \max g(x)$, $f(x, y)$ is rigorously represented by the following Rayleigh expansions:

$$f(x,y) = \exp\left[i(\alpha_0 x - \beta_0^{(1)} y)\right] + \sum_{n=-\infty}^{+\infty} \rho_n \exp\left[i(\alpha_n x + \beta_n^{(1)} y)\right], \quad y > \max g(x), \tag{3}$$

$$f(x,y) = \sum_{n=-\infty}^{+\infty} \tau_n \exp\left[i(\alpha_n x - \beta_n^{(2)} y)\right], \quad y < \min g(x). \tag{4}$$

Here, $\{\rho_n\}_{n=-\infty}^{+\infty}$ and $\{\tau_n\}_{n=-\infty}^{+\infty}$ are scalar coefficients to be determined; and

$$\left.\begin{aligned} \alpha_0 &= \tfrac{\omega}{c}\sin\theta_0 \\ \alpha_n &= \alpha_0 + 2n\pi/d \\ \beta_n^{(1)} &= \sqrt{\tfrac{\omega^2}{c^2} - \alpha_n^2} \\ \beta_n^{(2)} &= \sqrt{\tfrac{\omega^2}{c^2}\epsilon_2\mu_2 - \alpha_n^2} \end{aligned}\right\}, \tag{5}$$

where c is the speed of light in vacuum. Note that $\beta_n^{(1)}$ is either purely real or purely imaginary; and the conditions

$$\left.\begin{aligned} \mathrm{Re}\left[\beta_n^{(1)}\right] \geqslant 0 \\ \mathrm{Im}\left[\beta_n^{(1)}\right] \geqslant 0 \end{aligned}\right\} \forall n \tag{6}$$

are appropriate for plane waves in the vacuous half-space $y > g(x)$. The refracting half-space $y < g(x)$ being filled by a material medium, $\epsilon_{2I} > 0$ and $\mu_{2I} > 0$ by virtue of causality constraints imposed on constitutive relations [17]. The refracted plane waves must attenuate as $y \to -\infty$, which requirement leads to the condition

$$\mathrm{Im}\left[\beta_n^{(2)}\right] > 0. \tag{7}$$

Fulfilment of this condition automatically fixes the sign of $\mathrm{Re}[\beta_n^{(2)}]$, regardless of the signs of ϵ_{2R} and μ_{2R}. We note here that the condition (7) ensures that the power density decays as $y \to -\infty$, but the enhancement of the amplitude of a field comprising an upgoing and a downgoing evanescent waves is still permitted in that direction [18].

2.2. Rayleigh hypothesis

In accordance with the Rayleigh hypothesis [10], expansions (3) and (4) can be used in the boundary conditions

$$\left.\begin{aligned} f(x,g(x)+) &= f(x,g(x)-) \\ \hat{n}\cdot\nabla f(x,g(x)+) &= \sigma^{-1}\hat{n}\cdot\nabla f(x,g(x)-) \end{aligned}\right\}, \tag{8}$$

where $\sigma = \mu_2$ for the s-polarization case and $\sigma = \epsilon_2$ for the p-polarization case, while $\hat{n}$ is a unit vector normal to the boundary. Invoking the Rayleigh hypothesis, and then projecting into the Rayleigh basis $\{\exp(\mathrm{i}\alpha_m x)\}_{m=-\infty}^{+\infty}$, we obtain a system of linear equations for $\{\rho_n\}_{n=-\infty}^{+\infty}$ and $\{\tau_n\}_{n=-\infty}^{+\infty}$. With slight changes, we repeated the steps sketched by Lester and Depine [14] to obtain the system of equations

$$\begin{aligned} &\sum_{n=-\infty}^{\infty} \frac{(1-\sigma)\left[\beta_n^{(1)}\beta_m^{(2)} + \alpha_n\alpha_m\right] - \frac{\omega^2}{c^2}[\mu_2\epsilon_2 - \sigma]}{\beta_m^{(2)} - \beta_n^{(1)}} \\ &\quad \times D_{mn}\left(\beta_n^{(1)} - \beta_m^{(2)}\right)\rho_n \\ &= \frac{(1-\sigma)\left[\beta_0^{(1)}\beta_m^{(2)} - \alpha_0\alpha_m\right] + \frac{\omega^2}{c^2}[\mu_2\epsilon_2 - \sigma]}{\beta_m^{(2)} + \beta_0^{(1)}} \\ &\quad \times D_{m0}\left(-\beta_0^{(1)} - \beta_m^{(2)}\right) \ \forall m, \end{aligned} \tag{9}$$

with

$$D_{mn}(u) = \frac{1}{d}\int_0^d \exp\left[-\mathrm{i}\frac{2\pi}{d}(m-n)x + \mathrm{i}ug(x)\right]\mathrm{d}x. \tag{10}$$

2.3. Perturbative approach

The integrals $D_{mn}(u)$ can be stated exactly as the power series

$$D_{mn}(u) = \sum_{j=0}^{\infty}\frac{(i)^j}{j!}u^j\tilde{g}^{(j)}(m-n), \tag{11}$$

where

$$\tilde{g}^{(j)}(m) = \frac{1}{d}\int_0^d [g(x)]^j \exp\left(-\mathrm{i}m\frac{2\pi}{d}x\right)\mathrm{d}x, \tag{12}$$

is the mth Fourier coefficient of the function $[g(x)]^j$. These coefficients can be obtained through the recurrence relation

$$\tilde{g}^{(j)}(m) = \sum_n \tilde{g}^{(j-1)}(m-n)\tilde{g}^{(1)}(n), \quad j \geqslant 1, \tag{13}$$

beginning with

$$\tilde{g}^{(0)}(m) = \delta_{m0}, \tag{14}$$

where δ_{mn} is the Kronecker delta.

Assuming the expansion [11–14]

$$\rho_n = \sum_{j=0}^{\infty}\frac{(-i)^j}{j!}\rho_n^{(j)}, \tag{15}$$

we arrive at an iterative scheme, whereby the coefficient $\rho_n^{(j)}$, $j \geqslant 1$, can be obtained in terms of all lower-order coefficients $\rho_n^{(j-1)}, \ldots, \rho_n^{(0)}$ as follows:

$$\begin{aligned} \rho_n^{(j)} = \frac{1}{M_{nn}}\Bigg\{ & N_n\left[\beta_0^{(1)} + \beta_n^{(2)}\right]^j\tilde{g}^{(j)}(n) \\ & - \sum_m\left[M_{nm}\sum_{p=1}^{j}\binom{j}{p}\left[\beta_n^{(2)} - \beta_m^{(1)}\right]^p\tilde{g}^{(p)}(n-m)\rho_m^{(j-p)}\right]\Bigg\}. \end{aligned} \tag{16}$$

This scheme commences with

$$\rho_n^{(0)} = \frac{\sigma\beta_n^{(1)} - \beta_n^{(2)}}{\sigma\beta_n^{(1)} + \beta_n^{(2)}}\delta_{n0}, \tag{17}$$

which is the planewave reflection coefficient for a perfectly flat boundary (i.e., $g(x) \equiv 0$) [19], and requires the computation of

$$N_n = \frac{(\beta_0^{(1)}\beta_n^{(2)} - \alpha_0\alpha_n)(1-\sigma) + \frac{\omega^2}{c^2}(\epsilon_2\mu_2 - \sigma)}{\beta_0^{(1)} + \beta_n^{(2)}}, \tag{18}$$

$$M_{nm} = \frac{(\beta_m^{(1)}\beta_n^{(2)} + \alpha_m\alpha_n)(1-\sigma) - \frac{\omega^2}{c^2}(\epsilon_2\mu_2 - \sigma)}{\beta_n^{(2)} - \beta_m^{(1)}}. \tag{19}$$

Provided the series (15) converges rapidly, the reflection coefficients ρ_n can be computed economically in order to obtain the diffraction efficiencies

$$e_n^r = \frac{\mathrm{Re}\left[\beta_n^{(1)}\right]}{\beta_0^{(1)}}\,|\rho_n|^2\,. \tag{20}$$

The principle of conservation of energy requires that

$$\sum_n e_n^r \leqslant 1. \tag{21}$$

3. Results and discussion

The perturbative scheme derived in the previous section was implemented for a variety of cases. Similarly to what occurs for PPV refracting mediums [12–14], our numerical studies showed that this scheme converges quickly for NPV refracting mediums as well – provided the boundary is weakly corrugated.

When the boundary is perfectly flat, the only non-zero reflection coefficient is ρ_0. The transformation $\{\epsilon_2 \to -\epsilon_2^*, \mu_2 \to -\mu_2^*\}$ – that is, the replacement of a NPV/PPV refracting medium by an analogous PPV/NPV refracting medium – changes the phase of ρ_0 but not its magnitude [8]; hence, the transformation does not affect e_0^r at all. For a weakly corrugated boundary, we therefore expect that the magnitude of the specular reflection coefficient would not be greatly affected by the transformation $\{\epsilon_2 \to -\epsilon_2^*, \mu_2 \to -\mu_2^*\}$, but the effect of the transformation should be unambiguously evidenced by the nonspecular diffracted orders.

Fig. 1 presents the diffraction efficiency e_0^r as a function of $\theta_0 \in (-\pi/2, \pi/2)$ when the boundary is sinusoidally corrugated as follows: $g(x) = \frac{h}{2}\cos(\frac{2\pi}{d}x)$. We chose $h/d = 0.07$ and $\omega d/c = 2\pi/2.1$, so that $e_n^r \equiv 0 \forall n \neq 0$. The refracting medium is of either the PPV ($\epsilon_2 = 5 + i0.01$, $\mu_2 = 1 + i0.01$) or the NPV ($\epsilon_2 = -5 + i0.01, \mu_2 = -1 + i0.01$) type. Calculations were made for both the s- and the p-polarization cases. Clearly, the transformation $\{\epsilon_2 \to -\epsilon_2^*, \mu_2 \to -\mu_2^*\}$ does not greatly affect e_0^r, except at low $|\theta_0|$.

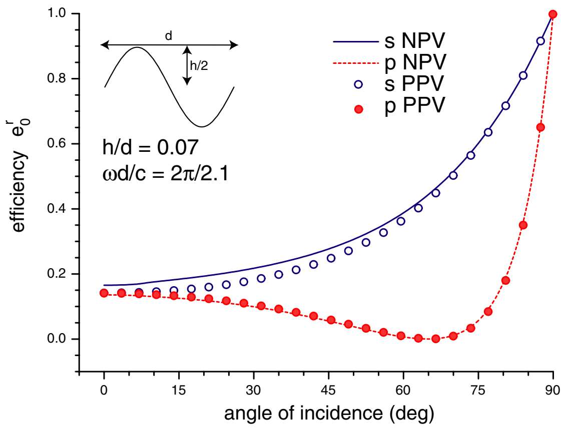

Fig. 1. Diffraction efficiency e_0^r as a function of the incidence angle θ_0, for a sinusoidally corrugated interface between vacuum and a linear homogeneous medium. The interface function $g(x) = 0.5h\cos(2\pi x/d)$, where $h/d = 0.07$ and $\omega d/c = 2\pi/2.1$. The refracting medium is of either the PPV ($\epsilon_2 = 5 + i0.01, \mu_2 = 1 + i0.01$) or the NPV ($\epsilon_2 = -5 + i0.01, \mu_2 = -1 + i0.01$) type. Calculations were made for both the s- and the p-polarization cases. Note that $e_0^r(\theta_0) = e_0^r(-\theta_0)$.

The situation for the specular diffraction efficiency does not change when $\omega d/c$ is increased to $2\pi/0.8$, as is clear from Fig. 2. In contrast, the same figure shows that the nonspecular diffraction efficiency e_{-1}^r, which is non-zero only for $\sin\theta_0 > -0.2$, is gravely affected by the type of the refracting medium. The diversity can be understood as follows: When the boundary is perfectly flat, the transformation $\{\epsilon_2 \to -\epsilon_2^*, \mu_2 \to -\mu_2^*\}$ leaves the magnitude of the reflection coefficient unchanged *only* for non-evanescent incident plane waves; but that is not a true statement for incident evanescent plane waves [20]. In the troughs of a corrugated boundary, the total field actually has both specular ($n = 0$) and nonspecular ($n \neq 0$) components. Most of the nonspecular components are like evanescent plane waves because they are characterized by $\mathrm{Re}[\beta_n^{(1)}] = 0$. Their presence ensures that the nonspecular diffraction efficiencies are considerably affected by the transformation of the refracting medium from NPV/PPV to PPV/NPV.

The foregoing understanding should hold for corrugations of other shapes as well. Therefore, we computed the diffraction efficiencies when the unit cell of $g(x)$ is an isosceles triangle. The counter-

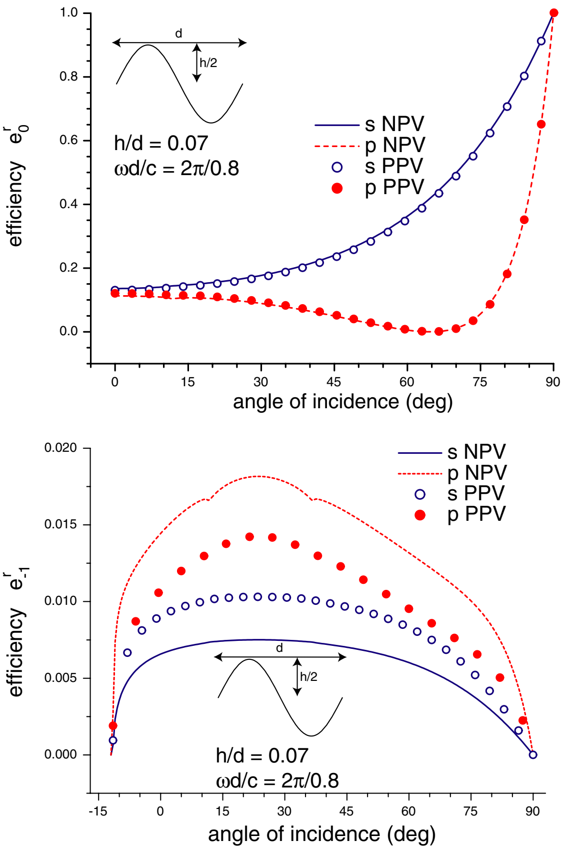

Fig. 2. Diffraction efficiencies e^r_0 and e^r_{-1} as functions of the incidence angle θ_0, for a sinusoidally corrugated interface between vacuum and a linear homogeneous medium. The interface function $g(x) = 0.5h\cos(2\pi x/d)$, where $h/d = 0.07$ and $\omega d/c = 2\pi/0.8$. The refracting medium is of either the PPV ($\epsilon_2 = 5 + i0.01, \mu_2 = 1 + i0.01$) or the NPV ($\epsilon_2 = -5 + i0.01$, $\mu_2 = -1 + i0.01$) type. Calculations were made for both the *s*- and the *p*-polarization cases. Note that $e^r_0(\theta_0) = e^r_0(-\theta_0)$.

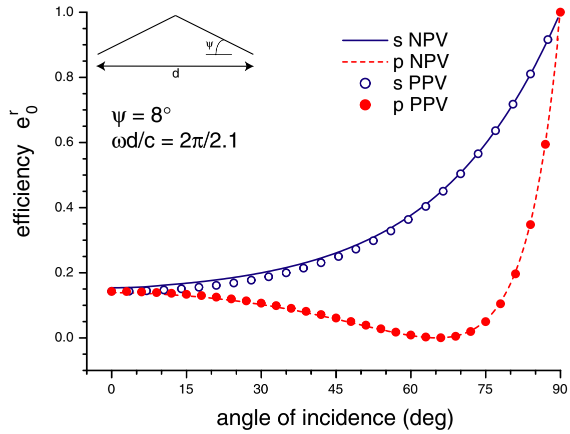

Fig. 3. Same as Fig. 1 except that the corrugations are shaped as isosceles triangles.

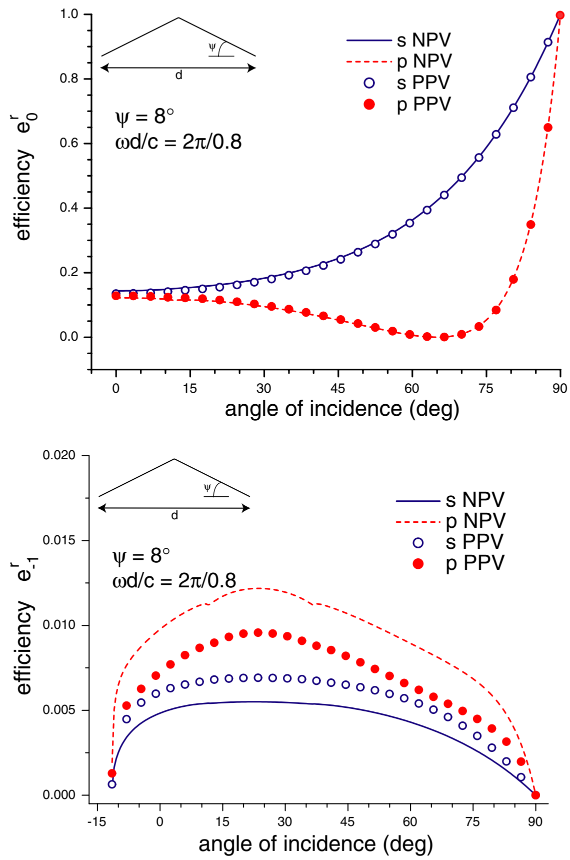

Fig. 4. Same as Fig. 2 except that the corrugations are shaped as isosceles triangles.

parts of Figs. 1 and 2 are Figs. 3 and 4, respectively. The latter two figures confirmed our understanding.

To conclude, we adapted a perturbative approach to study the changes that the transformation of a refracting medium from positive/negative phase-velocity to negative/positive phase-velocity has on the diffraction of a plane wave due to a periodically corrugated boundary. We concluded that nonspecular reflection, albeit weak for weakly corrugated boundaries, is highly affected by the type of refracting medium, in comparison with specular reflection – due to the involvement of evanescent waves in the troughs of the corrugated boundary.

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