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Invited Paper

Disentanglement in a two-qubit system subjected to dissipative environment: Exact analysis

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ABSTRACT

We investigate the time evolution of entanglement of various entangled states of two-qubit atomic system in vacuum environment using exact analysis. Compared to our earlier work under Markov approximation [M. Ikram, F.-L. Li, M.S. Zubairy, Phys. Rev. A 75 (2007) 062336] we show that disentanglement rate is slower and sudden death times are higher than the earlier study in each set of entangled state.

1. Introduction

The open system dynamics of composite systems, being initially in entangled states, is well explored in recent years. It is well known that the individual quantum systems obey half-life and decay exponentially. However, for a composite system–environment scenario, the coupling makes correlated dynamics quite complex. Yu and Eberly [1] were the first to study the behavior of a composite two-qubit mixed atomic system in the dissipative environment. They investigated that although local systems may decay asymptotically but, in contrast, the composite entangled systems may decay in finite times depending upon the mixing of doubly excited component. Their work was extended to a class of initially mixed and pure states for non-interacting [2–5] and interacting [6–8] qubits. All of this work is done exploiting the Markov approximation for weak system–environment coupling. This assumption ensures short memory in the sense that correlation time is very short and there is no feedback from the environment to the system. However, when the system–environment coupling is not weak, Markov approximation is no more valid [9]. In such a scenario, systems do have feedback from their environment and retain memory of interaction as implied by Jaynes–Cumming model. In such cases, memory effects are important and interesting from many points of views. During the time span when the memory effects are not negligible, the flow of energy and information from the system to the environment can be momentarily reversed. The reversal of these processes causes recoherence and restoration of previously lost superpositions [10]. These systems are treated as non-Markovian [11]. Non-Markovian systems appear in many branches of physics, such as quantum optics [12,13], solid sate physics [14], quantum chemistry [15] and quantum information processing [16]. Memory effects are usually characterized by a structured spectral density implying that the quantum system interacts more strongly with some modes of the reservoir than with others. Leaky optical cavities and photonic band-gap materials, for example, have such spectral densities [10,13].

The entanglement dynamics in strong coupling regime has been recently investigated under different theoretical models [17–20]. Particularly, the role of spectral width of system–environment coupling and mixing of the initial state is investigated for two-qubit systems [22]. In this paper, we investigate the entanglement dynamics of a two qubit system, with qubits as two-level atoms trapped in two leaky cavities, thus having structured vacuum reservoir inside the cavities. Due to structured reservoir–system interaction, Markov approximation cannot be applied here. Knowing that doubly excited component in the entangled state is the main source of disentanglement, we consider a set of atomic system having mixing of doubly excited component and study the entanglement evolution of these states in non-Markovian system–reservoir interaction. On contrary to the previous study [2], non-Markovian effect or the exact treatment not only suggests the

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postponement of the death of the entanglement but it exhibit an evident enhancement in the entanglement. The Wootters concurrence formula [21] is used as a quantitative measure of two-qubit entanglement.

The paper is organized as follows. In Section 2, we present the theoretical model employed to investigate the non-Markovian effects on the two-qubit entanglement dynamics. Further analytical and numerical results are presented for different cases of initial mixed states. Section 3 finally concludes the paper with a brief discussion.

2. Model

We consider the similar model as in [2], i.e., two two-level atoms representing a bipartite system trapped in two separate cavities containing structured vacuum acquired through the interaction of cavity fields with the outside vacuum as shown in Fig. 1. However, the correlation between the atoms depends only on the initial quantum entanglement between them. We also consider that the cavities are far apart with no direct cross-mutual interaction between the atoms or the cavity fields. The total Hamiltonian can be written as

$$H = H_o + H_I, \tag{1}$$

where H_o and H_I are the free and interaction parts, respectively, of the Hamiltonian, and are given by

$$H_o = \hbar\omega_o \sum_{i=1}^{2} \sigma_+^i \sigma_-^i + \hbar \sum_{k=1}^{N} \omega_k b_k^\dagger b_k, \tag{2}$$

$$H_I = \hbar \sum_{k=1}^{N} \sum_{i=1}^{2} \left(g_k \sigma_+^i b_k + g_k^* \sigma_-^i b_k^\dagger \right). \tag{3}$$

Here, in these equations, ω_o is the transition frequency of the two-level atom, σ_+^i (σ_-^i) is the raising (lowering) operator for the atom i and index k labels the different field modes of the reservoir with frequencies ω_k with $b_k(b_k^\dagger)$ being the field annihilation(creation) operator. Using the rotating-wave approximation, the interaction Hamiltonian between an atom and N-mode reservoir takes the form [23]

$$H_I = \hbar \sum_{k=1}^{N} \sum_{i=1}^{2} g_k^i (\sigma_-^i b_k^\dagger e^{-i(\omega_0-\omega_k)t} + \sigma_+^i b_k e^{i(\omega_0-\omega_k)t}), \tag{4}$$

where ${g_k}^i$ is the coupling constant between the atom i and the vacuum reservoir. We focus on the case for which the structured reservoir is the electromagnetic field inside the lossy cavity. It means that the discrete cavity modes can be effectively replaced with the spectral density function. We consider a case where the atom is interacting resonantly with the cavity field reservoir having Lorentzian spectral density that characterizes the coupling strength of the reservoir to the qubit as follows:

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_c - \omega)^2 + (\lambda)^2}. \tag{5}$$

This corresponds to a cavity supporting a single mode of frequency ω_c which can be leaked out through the non-ideal cavity walls with a probability proportional to λ^2, where λ is the spectral width of the field distribution inside the lossy cavity. It is connected to the reservoir correlation time τ_B by the relation $\tau_B = \lambda^{-1}$ and the time scale τ_R on which the state of the system changes is given by $\tau_R = \gamma_0^{-1}$. Here the parameter γ_0 is proportional to the strength of the atom–cavity coupling. Typically, in weak coupling regime ($\lambda > 2\gamma_0$), the qubit–reservoir system is Markovian and in strong coupling regime ($\lambda < 2\gamma_0$), non-Markovian dynamics occurs accompanied by a reversible decay.

In this paper, we are interested in two-qubit entanglement dynamics in strong coupling regime. To incorporate the parameters that control the atomic dynamics under strong coupling, we need to study the decay of a single two-level atom. We, therefore, consider a single two-level atom initially in excited state $|a\rangle$ trapped in a cavity containing vacuum modes, then time dependent wave function of the system and the environment can be written as

$$\left|\psi(t)\right\rangle = A(t)\left|a, 0\right\rangle + \sum_k B_k(t)\left|b, 1_k\right\rangle, \tag{6}$$

where $A(t)$ and $B_k(t)$ are the probability amplitudes of atom in excited state $|a\rangle$ with vacuum in cavity and atom in ground state $|b\rangle$ with cavity in single photon in kth mode $|1_k\rangle$, respectively. From Schrodinger equation we get the integro-differential equation

$$\dot{A}(t) = -\int_0^t d\acute{t} f\left(t - \acute{t}\right) A\left(\acute{t}\right), \tag{7}$$

where $f(t - \acute{t})$ is a correlation function defined in terms of continuous limits of the environment frequency as

$$f(t - \acute{t}) = \int_{-\infty}^{\infty} d\omega\, J(\omega) \exp\left[i(\omega_0 - \omega_k)\left(t - \acute{t}\right)\right]. \tag{8}$$

Considering the frequency distribution inside the cavity as defined in Eq. (5) for $\lambda > 0$ and $t - \acute{t}$ real, we get

$$f(t - \acute{t}) = \tfrac{1}{2}\gamma_0 \lambda e^{-\lambda(t-\acute{t})}, \tag{9}$$

where it is assumed that atomic transition frequency ω_0 is resonant with the cavity's central frequency mode ω_c. Now, we can solve the integro-differential equation (Eq. (7)) using initial condition $A(0) = 1$ i.e., atom is initially in excited state and vacuum in the cavity, as

$$A(t) = e^{-(1/2)t\lambda}\left(\cosh\left(\frac{dt}{2}\right) + \frac{\lambda}{d} \sinh\left(\frac{dt}{2}\right)\right), \tag{10}$$

where $d = \sqrt{\lambda^2 - 2\gamma_0\lambda}$ and is defined in the strong coupling regime $\gamma_0 > \lambda/2$ or $\tau_R < 2\tau_B$. The single atom dynamics exhibits an exponential decay by the oscillatory function $p(t) = \cosh(dt/2) + \frac{\lambda}{d}\sinh(dt/2)$. Thus we can easily calculate the modified decay rate using $\Gamma(t) = -2\,\mathrm{Re}[\dot{A}(t)/A(t)]$, as

Fig. 1. Two two-level atoms, initially prepared in an entangled state, trapped in two cavities having structured vacuum reservoir surrounded by vacuum environment.

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$$\Gamma(t) = \frac{2\gamma_0\lambda \sinh\left(\frac{dt}{2}\right)}{d \cosh\left(\frac{dt}{2}\right) + \lambda \sinh\left(\frac{dt}{2}\right)}. \tag{11}$$

To study the entanglement dynamics, a quantitative measure of entanglement is necessary. For any bipartite system, Wootters concurrence [21] is a convenient method. The concurrence can be calculated explicitly from the time dependent *X*-type density matrix $\rho(t)$ for two qubits [2] as

$$C(t) = \mathrm{Max}\left\{0, \Lambda_1(t), \Lambda_2(t)\right\}, \tag{12}$$

where $\Lambda_1(t) = 2(|\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)})$ and $\Lambda_2(t) = 2(|\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)})$.

3. Entanglement dynamics in structured reservoir

In this section, we consider the entanglement dynamics of the two qubit system in a structured vacuum reservoir. Using the general quantum reservoir theory, with the Hamiltonian (Eq. (4)), we can derive the following equation of motion for the reduced density matrix of the atoms interacting with the structured vacuum reservoir [12]:

$$\frac{d}{dt}\rho_S(t) = -\frac{1}{2}\sum_{i=1}^{2}\Gamma_i(t)\left[\sigma_+^i\sigma_-^i\rho_S(t) - 2\sigma_-^i\rho_S(t)\sigma_+^i + \rho_S(t)\sigma_+^i\sigma_-^i\right], \tag{13}$$

where $\Gamma_i(t)$ is the time-dependent decay rate of the ith atom. The exact decay rate for the two atoms trapped in two separate cavities containing structured vacuum reservoir is calculated and given in Eq. (11). This modification in the decay rate is due to the feedback from the cavity reservoir onto the atomic system. Henceforth, in deriving Eq. (13), we consider that the interaction between the atoms and reservoirs is non-Markovian and atomic transitions take place into integrated modes of the cavities. Since two identical atoms interact with almost the same environment in the cavities, so we may assume $\Gamma_1(t) = \Gamma_2(t) = \Gamma(t)$ (Eq. (11)).

The solution of time dependent master equation (13) depends upon the initial state of the atoms. We note that for a class of the initial states that will be considered below, the solution of Eq. (13) has the X-matrix form in the representation spanned by the basis $|1\rangle = |a_1, a_2\rangle$, $|2\rangle = |a_1, b_2\rangle$, $|3\rangle = |b_1, a_2\rangle$, $|4\rangle = |b_1, b_2\rangle$, and is given by

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix}. \tag{14}$$

The matrix elements $\rho_{ij}(t)$ are determined from the master equation (13) as [22]

$$\rho_{14}(t) = \rho_{14}(0)A^2(t), \tag{15}$$

$$\rho_{23}(t) = \rho_{23}(0)A^2(t), \tag{16}$$

$$\rho_{11}(t) = \rho_{11}(0)A^4(t), \tag{17}$$

$$\rho_{22}(t) = A^2(t)\left\{\left(1 - A^2(t)\right)\rho_{11}(0) + \rho_{22}(0)\right\}, \tag{18}$$

$$\rho_{33}(t) = A^2(t)\left\{\left(1 - A^2(t)\right)\rho_{11}(0) + \rho_{33}(0)\right\}, \tag{19}$$

$$\rho_{44}(t) = \left(1 - A^2(t)\right)^2\rho_{11}(0) + \left(1 - A^2(t)\right) \times \left(\rho_{22}(0) + \rho_{33}(0)\right) + \rho_{44}(0). \tag{20}$$

We can see that the decay of $\rho_{11}(t)$ is simply modified by the presence of $p^4(t)$ from the Markovian approximation. This term also imparts it with the oscillatory behavior and remarks the environment feedback. Therefore, entanglement dynamics now also depends on whether $\lambda < 2\gamma_0$ or $\lambda > 2\gamma_0$, the later is the weak coupling regime so we can readily consider the dynamics under Markov approximation in this case.

Let us first consider the entanglement dynamics of the Bell states which show maximum correlations in the two-qubit systems and has maximum concurrence which is 1, as follows:

$$\left|\Psi^\pm\right\rangle = \frac{1}{\sqrt{2}}(|a_1, b_2\rangle \pm |b_1, a_2\rangle), \tag{21a}$$

$$\left|\Phi^\pm\right\rangle = \frac{1}{\sqrt{2}}(|a_1, a_2\rangle \pm |b_1, b_2\rangle). \tag{21b}$$

The entanglement dynamics for these maximally entangled states under Markov approximation and through exact analysis are shown in Fig. 2. The entanglement dynamics is independent of the phases for these two states $(|\Phi^\pm\rangle, |\Psi^\pm\rangle)$ as evident from Eq. (12). The exact dynamics of these two states $|\Psi^\pm\rangle$ and $|\Phi^\pm\rangle$ is determined as $A^2(t)$ and $A^4(t)$, respectively, which clearly show that $|\Phi\rangle$ decays faster than $|\Psi\rangle$ mainly due the presence of doubly excited component in the entangled state $|\Phi\rangle$. Further the dynamics of these states is modified by factors of $p^2(t)$ and $p^4(t)$, respectively, for $|\Psi\rangle$ and $|\Phi\rangle$ from Markov approximation. It not only shows the higher amount of entanglement during evolution but it decay slower than the dynamics under Markov approximation.

It has been shown earlier [3,5] that Markovian dynamics of the state

$$|\chi\rangle = \beta_{aa}|a_1, a_2\rangle + \beta_{bb}|b_1, b_2\rangle, \tag{22}$$

with $|\beta_{aa}|^2 + |\beta_{bb}|^2 = 1$, shows asymptotic decay of entanglement when $|\beta_{aa}|^2 \leq |\beta_{bb}|^2$, and sudden death of entanglement (SDE) when $|\beta_{aa}|^2 > |\beta_{bb}|^2$. Thus, when the probability of doubly excited component is higher than 1/2, then instead of asymptotic decay of entanglement we observe finite time disentanglement with

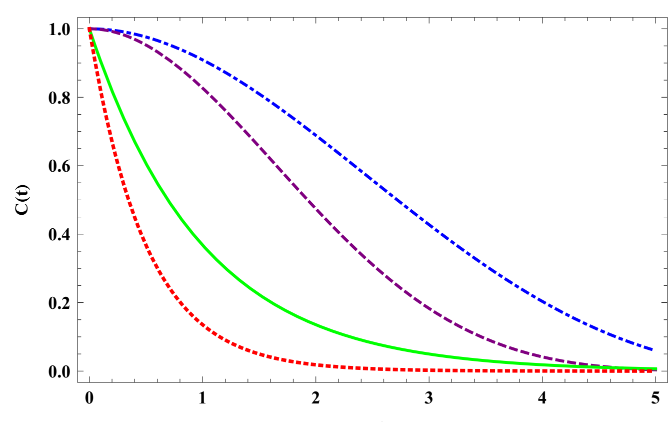

Fig. 2. Entanglement dynamics of Bell states $|\psi^\pm\rangle$ and $|\phi^\pm\rangle$. Green solid and blue dotted-dashed lines are for the evolution of the state $|\phi^\pm\rangle$ under Markov approximation and Exact analysis, respectively. Red dotted and purple dashed lines are for the evolution of the state $|\psi^\pm\rangle$ under Markov approximation and exact analysis, respectively. For exact treatment $\lambda = 0.2\gamma_0$ in all cases. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

sudden death time (SDT) under Markov approximation given as

$$t_d = \frac{1}{\gamma_0}\,\mathrm{Log}\left(\frac{\beta_{aa}}{\beta_{aa}-\beta_{bb}}\right), \tag{23}$$

where γ_0 is the spontaneous decay of a two-level atom. In the case of exact entanglement dynamics of the state $|\chi\rangle$, the concurrence comes out to be

$$C(t) = 2\beta_{aa}A^2(t)\left\{\beta_{bb}-\beta_{aa}\left(1-A^2(t)\right)\right\}. \tag{24}$$

The SDE, in this case, occurs when

$$e^{-\lambda t}p^2(t) = \left(\frac{\beta_{aa}-\beta_{bb}}{\beta_{aa}}\right), \tag{25}$$

where $p(t)$ is an oscillating function with discrete zeroes at

$$t_d = \frac{2}{d}\left(n\pi - \mathrm{Tan}^{-1}\left(\frac{d}{\lambda}\right)\right). \tag{26}$$

Hence, the decay probability $A^2(t)$ can only be analytical function for $t \in [0, t_0[$, where t_0 corresponds to the smallest possible zero of the decay dynamics.

Concurrences of the initial entangled state equation (22), with and without Markov approximation, are compared in Fig. 3. It clearly demonstrates that the loss of entanglement as a result of decoherence under exact treatment is slower than the Markovian one. Thus it is obvious that the doubly excited component in the two qubit entangled state is the main cause of disentanglement. SDE occurs when $|\beta_{aa}|^2 > |\beta_{bb}|^2$ with disentanglement time being larger for the exact case than the one with Markov approximation.

In the following, we use the master equation (Eq. (13)) to investigate the exact dynamics of entanglement for different mixing of states with the Bell states that follow X-matrix (Eq. (14)) and compare with our earlier results under Markov approximation [2]. Fig. 4 compares the entanglement dynamics of various cases under Markov approximation in left column along with exact treatment in right column.

Case 1: Consider the initial state $\rho(0) = (1-a)|\Psi^+\rangle\langle\Psi^+| + a|a_1, a_2\rangle\langle a_1, a_2|$ where a is the mixing constant varying between 0 and 1. In this state, Bell state $|\Psi^+\rangle$ is mixed with the doubly excited state $|a_1, a_2\rangle$. The concurrence can be found from Eq. (12) as

$$C(t) = \mathrm{Max}\left\{0, \Lambda_1(t)\right\}, \tag{27}$$

where

$$\Lambda_1(t) = A^2(t)\left(1 - a - 2\sqrt{a\left(1-A^2(t)\right)\left(1-aA^2(t)\right)}\right). \tag{28}$$

The plots of concurrence against initial mixing a and time are shown in Figs. 4a and b under weak coupling (Markov approximation) and strong coupling (Exact) regimes, respectively. This state has maximum amount of entanglement when there is no mixing of doubly excited component i.e., $a=0$. We see that time of disentanglement or sudden death time (SDT) increases with increase in mixing parameter a. For this state, SDE is found for $a > 3-2\sqrt{2}$ considering Markov approximation, while it is much higher under exact treatment. Comparison of the two plots 4a and 4b clearly shows that the entanglement region including the amount of entanglement and duration is much higher than the one under the Markov approximation. From Eq. (28), we can see that as $a \to 1$, the initial state is separable and remains unentangled under Markov approximation while under exact treatment entanglement survives even with high mixing of doubly excited component. This happens due to the memory effect of the environment which is not there under the Markov approximation.

Case 2: Consider the initial state $\rho(0) = \frac{a}{2}|\Phi^+\rangle\langle\Phi^+| + 2 - a/2|a_1, a_2\rangle\langle a_1, a_2|$ $(0 < a \le 2)$. In this expression, Bell state $|\Phi\rangle$ is mixed with the doubly excited state $|a_1, a_2\rangle$. Here, unlike the state considered in the previous example, the maximally entangled state itself contains the double excitation component. For this initial state, the time evolution of concurrence can be calculated as

$$C(t) = \mathrm{Max}\left\{0, \Lambda_2(t)\right\}, \tag{29}$$

where

$$\Lambda_2(t) = \frac{A^2(t)}{2}\left\{a - (4-a)\left(1-A^2(t)\right)\right\}. \tag{30}$$

We can see that this state has finite SDT only when $A(t)$ becomes zero or when mixing $a = 41 - A^2(t)/2 - A^2(t)$. The plots of concurrence against initial mixing a and time are shown in Figs. 4c and d for the two regimes, Markovian and Non-Markovian, respectively. We see that it has maximum entanglement at $a=2$, which decays asymptotically, while with slight lower mixing $(a < 2)$, we always see SDE with time of disentanglement being higher under exact dynamics than the approximated one. Entanglement decreases as mixing parameter a decreases and it is separable at $a=0$. Entanglement dynamics at intermediate values of mixing a is slower than the approximated one. Region of entanglement is also larger than the one with Markov approximation case.

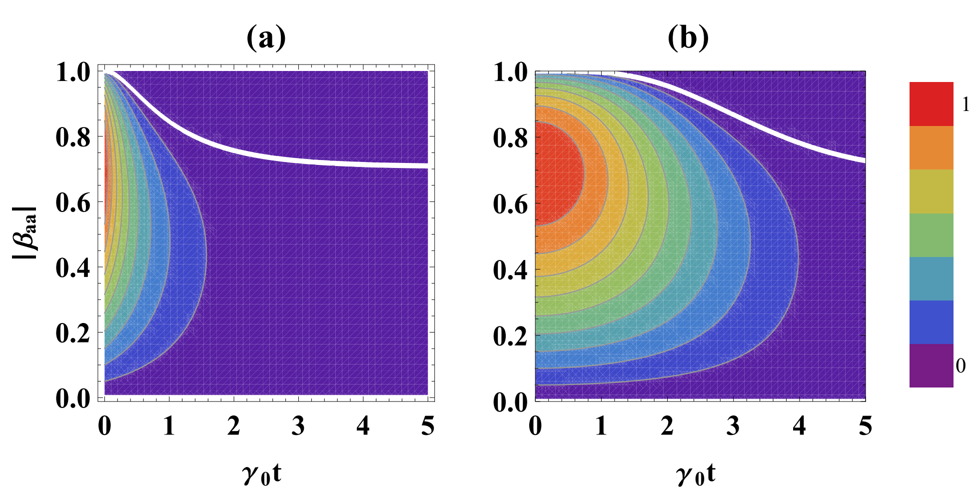

Fig. 3. Entanglement dynamics of the state $\beta_{aa}|a_1, a_2\rangle + \beta_{bb}|b_1, b_2\rangle$ with $|\beta_{aa}| + |\beta_{bb}| = 1$, in vacuum environment under (a) Markov approximation, (b) exact analysis with $\lambda = 0.2\gamma_0$. The white line in each plot shows sudden death of entanglement.

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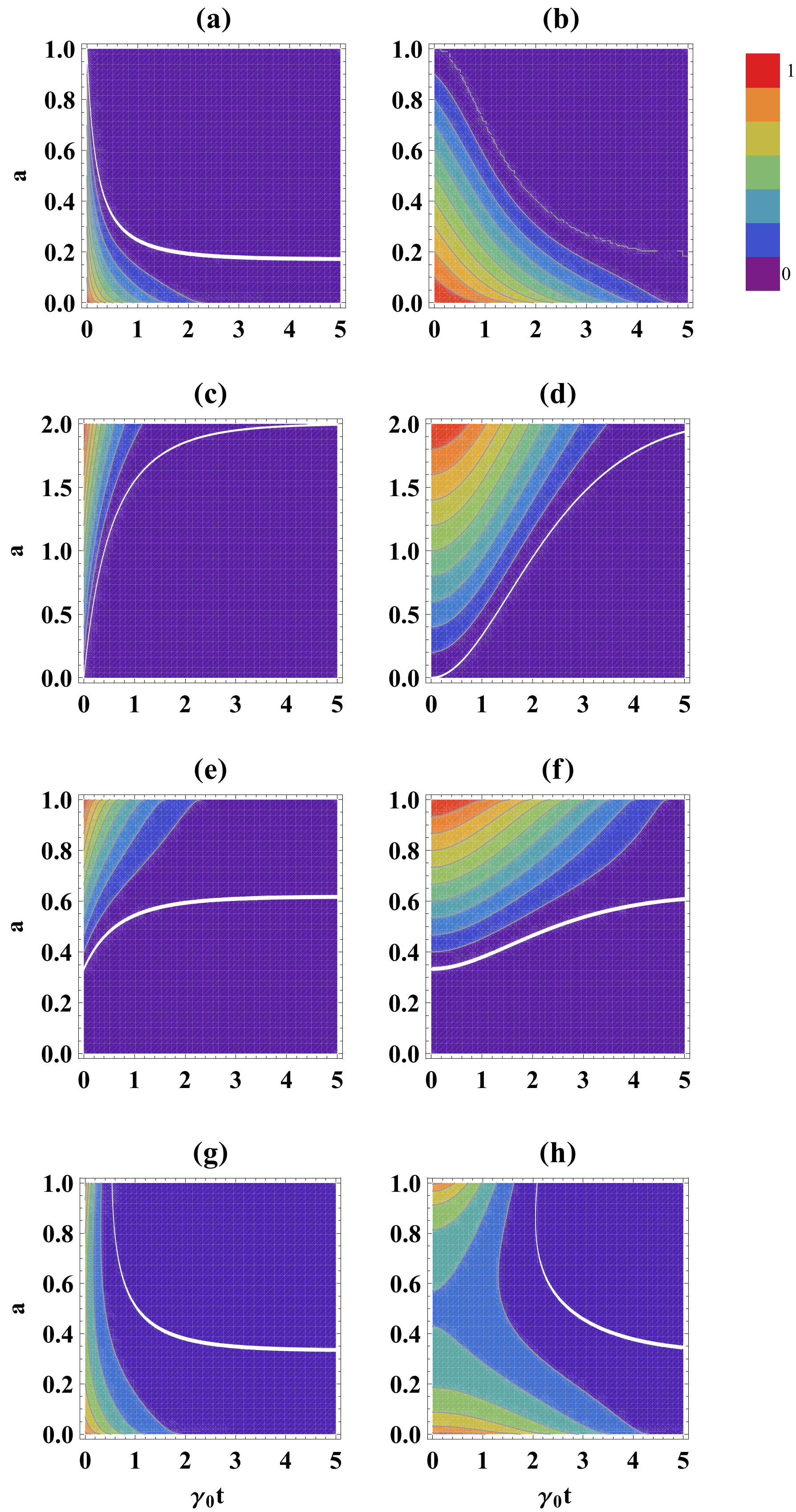

Fig. 4. Time evolution of the concurrence of the initial atomic states for the cases 1–4 with $\lambda = 0.2\gamma_0$. The plots in the left column {(a), (c), (e), (g)} are the dynamics of the entangled states for four cases (1–4) under Markov approximation while plots in the right column {(b), (d), (f), (h)} are the dynamics of these states using exact analysis. The white lines in each plot show sudden death of entanglement. The amount of entanglement and sudden death of entanglement time in each case shows improvement in exact treatment than under Markov approximated ones.

Case 3: Consider the Werner state [24]

$$\rho(0) = a\left|\Psi^-\right\rangle\left\langle\Psi^-\right| + \frac{1-a}{4}(|a_1, a_2\rangle\langle a_1, a_2| + |b_1, b_2\rangle\langle b_1, b_2| + |a_1, b_2\rangle\langle a_1, b_2| + |b_1, a_2\rangle\langle b_1, a_2|). \quad (31)$$

In this expression, the maximally entangled state $|\Psi^-\rangle$ is mixed with the equally weighted four possible states of a two-qubit system. The concurrence for this state can be obtained as

$$C(t) = \mathrm{Max}\left\{0, \Lambda_3(t)\right\}, \quad (32)$$

where

$$\Lambda_3(t) = A^2(t)\left\{a - \sqrt{(1-a)(1 - A^2(t) + \frac{1-a}{4}A^4(t))}\right\}. \quad (33)$$

The plots of concurrence for Markov approximated dynamics and exact dynamics against initial mixing a and time are shown in Figs. 4e and f, respectively. When $a=0$, we have a state with four equally weighted basis and the concurrence is zero. Concurrence remains zero for $a < 1/3$ then it increases with increasing a until it becomes maximum at $a=1$. Entanglement dynamics for $a > 1/3$ shows SDE with disentanglement time as well as amount of entanglement being larger in exact analysis than that of Markov approximated dynamics.

Case 4: Finally we consider the initial state $\rho(0) = 2/3|\Psi^+\rangle\langle\Psi^+|) + a/3|a_1, a_2\rangle\langle a_1, a_2| + 1 - a/3|b_1, b_2\rangle\langle b_1, b_2|$. Unlike the initial state in case (1), this state contains both the double excited component and the double ground states components at the same time. This state was first considered by Yu and Eberly [1] and for this SDE is found for $a > \frac{1}{3}$ considering Markov approximation. The concurrence for this state can be obtained as

$$C(t) = \mathrm{Max}\{0, \Lambda_4(t)\}, \quad (34)$$

where

$$\Lambda_4(t) = \frac{2A^2(t)}{3}\left\{1 - \sqrt{a[3 - 2(1+a)A^2(t)] + aA^4(t)}\right\}. \quad (35)$$

The plots of concurrence against initial mixing a and time are compared with the Markov approximated case in Figs. 4g and h. The concurrence is maximum at extreme values, $a = 0$ and $a=1$, i.e., absence of doubly excited component $|a_1, a_2\rangle$ and absence of doubly grounded component $|b_1, b_2\rangle$, respectively. The concurrence decreases as a increases until $a = \frac{1}{2}$ when both mix states become equally weighted, then concurrence increases with increase in a until we get another maximally entangled state. The time dynamics of the state is asymptotic for $a < 1/3$. Entanglement dynamics for $a > 1/3$ shows SDE with disentanglement time and amount of entanglement larger than Markov approximated dynamics.

4. Conclusion

We consider the two-qubit entanglement dynamics of a class of initial mixed states for two atoms trapped in leaky cavities operating under comparatively strong coupling regime. In comparison to our earlier work related to weak coupling regime (using Markov approximation), our study demonstrated that exact system dynamics is slow and sudden death times are delayed in each case. The dynamics in strong coupling regime is modified and shows oscillatory behavior due to the feedback from the environment. The comparison of entanglement dynamics in exact treatment or strong coupling case with the weak coupling regime shows that entanglement under Markov approximation is under estimated. If the entangled atoms are trapped in the cavities containing vacuum then we have larger amount of entanglement and larger duration compared with the Markov approximated entanglement dynamics.

The enhancement of entanglement is basically due to the lessor number of modes in the cavities than the number of modes in Markov approximation having no memory effects and the information flow from the system to environment is completely lost. While in the exact analysis, due to less number of modes in the environment, information can flow from environment to the system. The photons which have been emitted by the atoms are reabsorbed at later time. The enhancement of entanglement also results in delay of sudden death of entanglement. Thus in exact analysis or in the non-Markovian regime entanglement and sudden death of entanglement are at higher values than the approximated ones in the Markovian dynamics.

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References

[1] T. Yu, J.H. Eberley, Phys. Rev. Lett. 93 (2004) 140404;
T. Yu, J.H. Eberley, Phys. Rev. Lett. 97 (2006) 140403.
[2] M. Ikram, F.-L. Li, M.S. Zubairy, Phys. Rev. A 75 (2007) 062336.
[3] M.F. Santos, P. Milman, L. Davidovich, N. Zagury, Phys. Rev. A 73 (2006) 040305(R).
[4] M.P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S.P. Walborn, P.H. Souto Ribeiro, L. Davidovich, Science 316 (2007) 579.
[5] R. Tahira, M. Ikram, T. Azim, M.S. Zubairy, J. Phys. B: At. Mol. Opt. Phys. 41 (2008) 205501.
[6] Z. Ficek, R. Tanas, Phys. Rev. A 74 (2006) 024304;
Z. Ficek, R. Tanas, Phys. Rev. A 77 (2008) 054301;
S. Chan, M.D. Reid, Z. Ficek, J. Phys. B: At. Mol. Opt. Phys. 42 (2009) 065507.
[7] R. Tahira, M. Ikram, M.S. Zubairy, Opt. Commun. 284 (2011) 3643.
[8] S. Das, G.S. Agarwal, J. Phys. B: At. Mol. Opt. Phys. 42 (2009) 141003;
S. Das, G.S. Agarwal, J. Phys. B: At. Mol. Opt. Phys. 42 (2009) 205502.
[9] F. Shibata, Y. Takahashi, N. Hashitsume, J. Stat. Phys. 17 (1977) 171.
[10] J. Piilo, et al., Phys. Rev. Lett. 100 (2008) 180402.
[11] J. Piilo, S. Maniscalco, K. Harkonen, K.-A. Suominen, Phys. Rev. A 79 (2009) 062112.
[12] H.-P. Breuer, F. Petruccione, The Theory of Open Quantum Systems, Oxford University Press, New York, 2002.
[13] P. Lambropoulos, G.M. Nikolopoulos, T.R. Nielsen, S. Bay, Rep. Prog. Phys. 63 (2000) 455.
[14] C.W. Lai, P. Maletinsky, A. Badolato, A. Imamoglu, Phys. Rev. Lett. 96 (2006) 167403.
[15] J. Shao, J. Chem. Phys. 120 (2004) 5053;
A. Pomyalov, D.J. Tannor, J. Chem. Phys. 123 (2005) 204111.
[16] D. Aharonov, A. Kitaev, J. Preskill, Phys. Rev. Lett. 96 (2006) 050504;
S. Maniscalo, S. Olivere, M.G.A. Paris, Phys. Rev. A 75 (2007) 062119.
[17] W. Cui, Z. Xi, Y. Pan, J. Phys. A: Math. Theor. 42 (2009) 155303.
[18] X.-F. Cao, H. Zheng, Eur. Phys. J. B 68 (2009) 209.
[19] Z.Y. Xu, M. Feng, Phys. Lett. A 373 (2009) 1906.
[20] X. Xiao, M.-F. Fang, Y.-L. Li, J. Phys. B: At. Mol. Opt. Phys. 43 (2010) 185505.
[21] W.K. Wootters, Phys. Rev. Lett. 80 (1998) 2245.
[22] B. Bellomo, R. Lo Franco, G. Compagno, Phys. Rev. Lett. 99 (2007) 160502;
B. Bellomo, R. Lo Franco, G. Compagno, Phys. Rev. A 99 (2008) 160502.
[23] M.O. Scully, M.S. Zubairy, Quantum Optics, Cambridge University Press, Cambridge, 1997.
[24] R.F. Werner, Phys. Rev. A 40 (1989) 4277.