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Polarization squeezing in polarized light

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ABSTRACT

It is shown that polarized light can be polarization squeezed only if it exhibits sub-Poissonian statistics with the Mandel's Q factor less than $-1/2$.

In classical optics, Stokes parameters are used to denote the polarization state [1,2]. For light beam traveling along the 3-direction, the Stokes parameters {S_{{0.1.2.3}}} are defined by

$$S_{0,1} = \left\langle \mathcal{E}_x^* \mathcal{E}_x \pm \mathcal{E}_y^* \mathcal{E}_y \right\rangle, S_2 + \mathrm{i}S_3 = 2\left\langle \mathcal{E}_x^* \mathcal{E}_y \right\rangle, \tag{1}$$

where $\mathcal{E} = \boldsymbol{e}_x \mathcal{E}_x + \boldsymbol{e}_y \mathcal{E}_y$ is the analytic signal [3] for electric field and conical brackets denote the average. For perfectly polarized light, $S_0^2 = |\mathbf{S}|^2 = S_1^2 + S_2^2 + S_3^2$, and the point with coordinate (S_1, S_2, S_3) lies on a sphere of radius S_0, called the Poincare's sphere [4], and the direction of $\mathbf{S} = (S_1,\ S_2,\ S_3)$ represents the polarization state. For unpolarized light [5–8], $\mathbf{S} = 0$ and for partially polarized light $|\mathbf{S}| < S_0$ and the point with coordinates $(S_1,\ S_2,\ S_3)$ lies inside the Poincare sphere. Since the Stokes parameters involve only the coherence functions [9] of order $(1, 1)$, they are not sufficient for describing polarization in the context of nonlinear interactions (also discussed in Ref. [6]). Quantum analogue of Stokes parameters are the Stokes operators defined by

$$\widehat{S}_{0,1} = \widehat{a}_x^\dagger \widehat{a}_x \pm \widehat{a}_y^\dagger \widehat{a}_y, \widehat{S}_2 + i\widehat{S}_3 = 2\widehat{a}_x^\dagger \widehat{a}_y, \tag{2}$$

where $\widehat{a}_{x,y}$ are the annihilation operators for the x and y linear polarization. Stokes operators satisfy the commutation relations,

$$\left[\widehat{S}_0, \widehat{S}_j\right] = 0, \left[\widehat{S}_j, \widehat{S}_k\right] = 2i \sum_l \varepsilon_{jkl} \widehat{S}_l \ \ (j, k, l = 1,\ 2 \text{ or } 3) \tag{3}$$

and lead to the uncertainty relations,

$$V_j V_k \geq \left\langle \widehat{S}_l \right\rangle^2, V_j \equiv \left\langle \widehat{S}_j^2 \right\rangle - \left\langle \widehat{S}_j \right\rangle^2, \quad j \neq k \neq l \neq j. \tag{4}$$

Here conical brackets denote the expectation values of the operators.

Relations for Stokes operators are very much similar to those for Dicke's collective atom hermitian operators [10] $\widehat{R}_{1,2,3}$ for two-level atoms (TLA's). If $|u\rangle_j$ and $|l\rangle_j$ are upper and lower states for the j^{th} TLA, for an assembly of N TLA's the Dicke's operators $\widehat{R}_{1,2,3}$ are defined by

$$\widehat{R}_1 + i\widehat{R}_2 = \sum_{j=1}^{N} \ |u\rangle_j \langle l|,\ \widehat{R}_3 = \sum_{j=1}^{N} \frac{1}{2}\left[|u\rangle_j \langle u| - |l\rangle_j \langle l|\right] \tag{5}$$

and satisfy

$$\left[\widehat{R}_j, \widehat{R}_k\right] = i \sum_l \varepsilon_{jkl} \widehat{R}_l,\ (j, k, l = 1, 2 \,\text{or}\, 3), \tag{6}$$

which is similar to those in Eq. (3), except for the factor of 2 on right hand side. These lead to uncertainty relations on the basis of which Walls and Zoller [11] defined atomic squeezing of $\widehat{R}_1$ if

$$\left\langle \widehat{R}_1^2 \right\rangle - \left\langle \widehat{R}_1 \right\rangle^2 < \frac{1}{2}\left|\left\langle \widehat{R}_2 \right\rangle\right| \text{ or } \frac{1}{2}\left|\left\langle \widehat{R}_3 \right\rangle\right|. \tag{7}$$

This was generalized by Prakash and Kumar [12], who call the generalized component $\widehat{R}_{\boldsymbol{n}} \equiv \left(\boldsymbol{n} \cdot \widehat{\boldsymbol{R}}\right)$ of $\widehat{\boldsymbol{R}} = \left(\widehat{R}_1, \widehat{R}_2, \widehat{R}_3\right)$ along the unit vector $\mathbf{n}$ squeezed if

$$\left\langle \widehat{R}_{\boldsymbol{n}}^2 \right\rangle - \left\langle \widehat{R}_{\boldsymbol{n}} \right\rangle^2 \leq \frac{1}{2}\left[\left\langle \widehat{R}_{\boldsymbol{n}_{\perp 1}} \right\rangle^2 + \left\langle \widehat{R}_{\boldsymbol{n}_{\perp 2}} \right\rangle^2\right]^{1/2} = \frac{1}{2}\left[\left\langle \widehat{\boldsymbol{R}} \right\rangle^2 - \left\langle \widehat{R}_{\boldsymbol{n}} \right\rangle^2\right]^{1/2} \tag{8}$$

where $\mathbf{n}_{\perp 1}$ and $\mathbf{n}_{\perp 2}$ are any two unit vectors perpendicular to $\mathbf{n}$.

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For optical polarization, concept of polarization squeezing is introduced through Eq. (3) Chirkin et al. [13] gave the first definition in the form, $V_j < V_j(coh) = \langle \hat{S}_0 \rangle$, for $j = 1$, 2 or 3 where $V_j \equiv \langle \hat{S}_j^2 \rangle - \langle \hat{S}_j \rangle^2$ is the variance of the operator $\hat{S}_j$ and $V_j(coh)$ is variance for equally intense coherent state. Heersink et al. [14] used Eq. (3) and called operator $\hat{S}_j$ polarization squeezed if $V_j < |\langle \hat{S}_l \rangle| < V_k$, for $j \neq k \neq l \neq j$, which is similar to the definition Eq. (7) of Walls and Zoller [11] for atomic squeezing. This was generalized by Luis and Korolkova [15] who wrote criterion for squeezing of as

$$V_{\boldsymbol{n}} < \left| \left\langle \hat{S}_{\boldsymbol{n}_\perp} \right\rangle \right|, V_{\boldsymbol{n}} \equiv \left\langle \hat{S}_{\boldsymbol{n}}^2 \right\rangle - \left\langle \hat{S}_{\boldsymbol{n}} \right\rangle^2, \tag{9}$$

where $\mathbf{n}_\perp$ is a unit vector perpendicular to $\mathbf{n}$. This was written by Prakash and Shukla [16] in the form,

$$V_{\boldsymbol{n}} \equiv \left\langle \hat{S}_{\boldsymbol{n}}^2 \right\rangle - \left\langle \hat{S}_{\boldsymbol{n}} \right\rangle^2 < \left| \left\langle \hat{S}_{\boldsymbol{n}_\perp} \right\rangle \right|_{\max} = \left[\left| \left\langle \hat{S} \right\rangle \right|^2 - \left\langle \hat{S}_{\boldsymbol{n}} \right\rangle^2 \right]^{1/2}. \tag{10}$$

which is very much similar to definition Eq. (8) for atomic squeezing by Prakash and Kumar [12].

The basis states for study of polarization need not necessarily be the two linear polarizations and can in the most general case be the two general orthogonal elliptical polarizations represented by two orthogonal unit vectors, *say*, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_\perp$ which satisfy obviously $\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_\perp^* \cdot \boldsymbol{\varepsilon}_\perp = 1$ and $\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_\perp = 0$. Since we can write the expansion of vector potential $\hat{A}$ for a monochromatic unidirectional optical field in the form,

$$\hat{A} = \sqrt{\frac{2\pi}{\omega V}} \left[\left(\boldsymbol{\varepsilon} \hat{a}_{\boldsymbol{\varepsilon}} + \boldsymbol{\varepsilon}_\perp \hat{a}_{\boldsymbol{\varepsilon}_\perp} \right) e^{ikz} + h.c. \right] = \sqrt{\frac{2\pi}{\omega V}} \left[\left(\boldsymbol{e}_x \hat{a}_x + \boldsymbol{e}_y \hat{a}_y \right) e^{ikz} + h.c. \right] \tag{11}$$

in natural units, where *h.c.* stands for hermitian conjugate, it leads to [see, *e.g.*, Ref. [7] also]

$$\hat{a}_{\boldsymbol{\varepsilon}} = \varepsilon_x^* \hat{a}_x + \varepsilon_y^* \hat{a}_y, \hat{a}_{\boldsymbol{\varepsilon}_\perp} = \varepsilon_{\perp x}^* \hat{a}_x + \varepsilon_{\perp y}^* \hat{a}_y \tag{12}$$

and also

$$\hat{a}_x = \varepsilon_x \hat{a}_{\boldsymbol{\varepsilon}} + \varepsilon_{\perp x} \hat{a}_{\boldsymbol{\varepsilon}_\perp}, \hat{a}_y = \varepsilon_y \hat{a}_{\boldsymbol{\varepsilon}} + \varepsilon_{\perp y} \hat{a}_{\boldsymbol{\varepsilon}_\perp}. \tag{13}$$

Operators $\hat{a}_{\boldsymbol{\varepsilon}}$ and $\hat{a}_{\boldsymbol{\varepsilon}\perp}$ are annihilation operator for two orthogonal modes having polarization represented by complex unit vector $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_\perp$. These help us define [17] state $|\psi\rangle$ of light polarized in the mode $\boldsymbol{\varepsilon}$ by

$$\hat{a}_{\boldsymbol{\varepsilon}_\perp} |\psi\rangle = 0, \langle \psi | \hat{a}_{\boldsymbol{\varepsilon}_\perp}^\dagger = 0 \tag{14}$$

To study polarization squeezing in this state, straight calculations using Eqs. (2) and (14) give,

$$\left\langle \hat{S}_0 \right\rangle = \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle, \left\langle \hat{S}_j \right\rangle = m_j \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle, \tag{15}$$

$$\left\langle S_j^2 \right\rangle = \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle + m_j^2 \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^{\dagger 2} \hat{a}_{\boldsymbol{\varepsilon}}^2 \right\rangle, \tag{16}$$

$$\left\langle \left\{ \hat{S}_j, \hat{S}_k \right\} \right\rangle = 2 m_j m_k \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^{\dagger 2} \hat{a}_{\boldsymbol{\varepsilon}}^2 \right\rangle, \qquad (j \neq k), \tag{17}$$

where

$$m_1 = |\varepsilon_x|^2 - \left|\varepsilon_y\right|^2, m_2 = \left(\varepsilon_x^* \varepsilon_y + \varepsilon_y^* \varepsilon_x \right), m_3 = \left(-i\varepsilon_x^* \varepsilon_y + i\varepsilon_y^* \varepsilon_x \right) \tag{18}$$

define a unit vector $\mathbf{m}$. If we write

$$\boldsymbol{\varepsilon} = \boldsymbol{e}_x \cos\frac{\theta_o}{2} + \boldsymbol{e}_y \sin\frac{\theta_o}{2} e^{i\varphi_o} \tag{19}$$

and let angles θ_o and φ_o define the polarization state, the polarization state will also be represented by unit vector,

$$m = \boldsymbol{e}_x \cos\theta_o + \left(\boldsymbol{e}_y \cos\varphi_o + \boldsymbol{e}_z \sin\varphi_o \right) \sin\theta_o. \tag{20}$$

We can write the unit vector $\mathbf{n}$, squeezing of components of S along which we are considering, in a similar form as

$$\boldsymbol{n} = \boldsymbol{e}_x \cos\theta + \left(\boldsymbol{e}_y \cos\varphi + \boldsymbol{e}_z \sin\varphi \right) \sin\theta. \tag{21}$$

Eqs. (15–17), then give

$$\left\langle \hat{S}_{\boldsymbol{n}} \right\rangle = \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle \cos\Phi, \left\langle \hat{S}_{\boldsymbol{n}}^2 \right\rangle = \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle + \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^{\dagger 2} \hat{a}_{\boldsymbol{\varepsilon}}^2 \right\rangle \cos^2\Phi, \tag{22}$$

where

$$\cos\Phi = (\boldsymbol{n} \cdot \boldsymbol{m}) = \cos\theta_o \cos\theta + \cos(\varphi_o - \varphi) \sin\theta_o \sin\theta \tag{23}$$

Here Φ is angle between unit vectors $\mathbf{n}$ and $\mathbf{m}$ with $0 \le \Phi \le \pi$. Polarization squeezing therefore occurs if

$$\begin{aligned} \left\langle \hat{S}_{\boldsymbol{n}}^2 \right\rangle - \left\langle \hat{S}_{\boldsymbol{n}} \right\rangle^2 - \left[\left\langle \hat{\boldsymbol{S}} \right\rangle^2 - \left\langle \hat{S}_{\boldsymbol{n}} \right\rangle^2 \right]^{1/2} &= \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle (1 - \sin\Phi) \\ &+ \left[\left\langle \hat{a}_{\boldsymbol{\varepsilon}}^{\dagger 2} \hat{a}_{\boldsymbol{\varepsilon}}^2 \right\rangle - \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle^2 \right] \cos^2\Phi < 0 \end{aligned} \tag{24}$$

Mandel's Q factor is defined by [18]

$$Q = \left(\left\langle \hat{a}_{\boldsymbol{\varepsilon}}^{\dagger 2} \hat{a}_{\boldsymbol{\varepsilon}}^2 \right\rangle - \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle^2 \right) / \left\langle \hat{a}_{\boldsymbol{\varepsilon}}^\dagger \hat{a}_{\boldsymbol{\varepsilon}} \right\rangle. \tag{25}$$

For classical fields, $Q \ge 0$. $Q < 0$ gives the non-classical features of light, sub-Poissonian statistics. The criterion for polarization squeezing is then,

$$1 - \sin\Phi + Q\cos^2\Phi = (1 - \sin\Phi)(1 + Q[1 + \sin\Phi]) < 0. \tag{26}$$

This cannot be satisfied for $Q \ge 0$, i.e. for Poissonian or super-Poissonian statistics. For $Q < 0$ also, this cannot be satisfied if $Q \ge -1/2$. For $Q < -1/2$, however, this can be satisfied for values of Φ for which $1 > \sin\Phi > |Q|^{-1} - 1$, which is same as $0 < \cos\Phi < [2|Q|-1]^{1/2}/|Q|$. Polarized light in the state represented by $\mathbf{m}$ can thus be polarization squeezed in Stokes operator $\hat{S}_{\boldsymbol{n}}$ only if it exhibits sub-Poissonian statistics and $Q < -(1+\sin\Phi)^{-1}$. For a given value of Q which is less than $-1/2$, we thus get a cone of semi-vertical angle $\sin^{-1}(|Q|^{-1} - 1)$ about the unit vector $\mathbf{m}$ which describes the polarization state. If the line of $\mathbf{n}$ lies outside this cone and is not perpendicular to its axis then $\hat{S}_{\boldsymbol{n}}$ is squeezed.

It is also interesting to see that if $Q = -1$, the lowest possible value, which occurs for pure photon number state, the semi-vertical angle of cone is zero and hence all components $\hat{S}_{\boldsymbol{n}}$ are squeezed except those for which $\Phi = 0$ or $\pi/2$ [16].

If we use angles θ and φ, *i.e.*, unit vector $\mathbf{n}$, to define orthogonal complex unit vectors $\bar{\varepsilon}$ and $\bar{\varepsilon}_\perp$ by

$$\bar{\boldsymbol{\varepsilon}} = \cos\frac{\theta}{2}\, \boldsymbol{e}_x + \sin\frac{\theta}{2}\, e^{i\varphi} \boldsymbol{e}_y, \bar{\boldsymbol{\varepsilon}}_\perp = -\sin\frac{\theta}{2}\, \boldsymbol{e}_x + \cos\frac{\theta}{2} e^{i\varphi} \boldsymbol{e}_y, \tag{27}$$

it can be shown that

$$\hat{S}_{\boldsymbol{n}} = \left(\boldsymbol{n} \cdot \hat{\boldsymbol{S}} \right) = \hat{N}_{\bar{\boldsymbol{\varepsilon}}} - \hat{N}_{\bar{\boldsymbol{\varepsilon}}_\perp}, \left\langle \hat{S} \right\rangle^2 - \left\langle \hat{S}_{\boldsymbol{n}} \right\rangle^2 = 4 \left\langle \hat{N}_{\bar{\boldsymbol{\varepsilon}}} \right\rangle \left\langle \hat{N}_{\bar{\boldsymbol{\varepsilon}}_\perp} \right\rangle, \tag{28}$$

where $\widehat{N}_{\overline{\varepsilon}} = \widehat{a}^{\dagger}_{\overline{\varepsilon}} \widehat{a}_{\overline{\varepsilon}}$ and $\widehat{N}_{\overline{\varepsilon}_\perp} = \widehat{a}^{\dagger}_{\overline{\varepsilon}_\perp} \widehat{a}_{\overline{\varepsilon}_\perp}$ are photon number operators for light polarized along $\overline{\varepsilon}$ and $\varepsilon_\perp$. Eq. (28), helps us write

$$V_{\boldsymbol{n}} - \left[\left\langle \widehat{\boldsymbol{S}}^2 \right\rangle - \left\langle \widehat{S}_{\boldsymbol{n}} \right\rangle^2\right]^{1/2} = \left\langle \widehat{N}^2_{\overline{\varepsilon}} \right\rangle + \left\langle \widehat{N}^2_{\overline{\varepsilon}_\perp} \right\rangle - 2\left\langle \widehat{N}_{\overline{\varepsilon}} \widehat{N}_{\overline{\varepsilon}_\perp} \right\rangle - \left(\left\langle \widehat{N}_{\overline{\varepsilon}} \right\rangle - \left\langle \widehat{N}_{\overline{\varepsilon}_\perp} \right\rangle\right)^2 - 2\left\langle \widehat{N}_{\overline{\varepsilon}} \right\rangle^{1/2} \left\langle \widehat{N}_{\overline{\varepsilon}_\perp} \right\rangle^{1/2} \tag{29}$$

This shows that to detect squeezing in $\widehat{S}_n$, therefore, only measurement of expectation values of $\widehat{N}_{\overline{\varepsilon}}$ and $\widehat{N}_{\overline{\varepsilon}_\perp}$ and their squares and product is required. This can be done easily by shifting phase of y-component by φ followed by rotating the beam by $-\theta/2$ about the direction of propagation and measurement in x and y linearly polarized components.

Also since Eq. (29), can be written as

$$V_{\boldsymbol{n}} - \left[\left\langle \widehat{\boldsymbol{S}} \right\rangle^2 - \left\langle \widehat{S}_{\boldsymbol{n}} \right\rangle^2\right]^{1/2} = \left\langle \widehat{a}^{\dagger 2}_{\overline{\varepsilon}} \widehat{a}^2_{\overline{\varepsilon}} \right\rangle + \left\langle \widehat{a}^{\dagger 2}_{\overline{\varepsilon}_\perp} \widehat{a}^2_{\overline{\varepsilon}_\perp} \right\rangle - 2\left\langle \widehat{a}^{\dagger}_{\overline{\varepsilon}} \widehat{a}^{\dagger}_{\overline{\varepsilon}_\perp} \widehat{a}_{\overline{\varepsilon}} \widehat{a}_{\overline{\varepsilon}_\perp} \right\rangle - \left[\left\langle \widehat{a}^{\dagger}_{\overline{\varepsilon}} \widehat{a}_{\overline{\varepsilon}} \right\rangle - \left\langle \widehat{a}^{\dagger 2}_{\overline{\varepsilon}_\perp} \widehat{a}^2_{\overline{\varepsilon}_\perp} \right\rangle\right]^2 + \left[\left\langle \widehat{a}^{\dagger}_{\overline{\varepsilon}} \widehat{a}_{\overline{\varepsilon}} \right\rangle^{1/2} - \left\langle \widehat{a}^{\dagger 2}_{\overline{\varepsilon}_\perp} \widehat{a}^2_{\overline{\varepsilon}_\perp} \right\rangle^{1/2}\right]^2, \tag{30}$$

if the density operator of light is written in the Sudarsan–Glauber diagonal representation [19] in the basis of coherent state $|\alpha, \beta\rangle_{\varepsilon, \varepsilon_\perp}$ in the form

$$\widehat{\rho} = \int d^2\alpha\, d^2\beta\, P(\alpha,\beta)\, |\alpha,\beta\rangle_{\varepsilon,\varepsilon_\perp} \langle \alpha,\beta|, \tag{31}$$

We have

$$V_n - [\left\langle \widehat{\boldsymbol{S}} \right\rangle^2 - \left\langle \widehat{S}_{\boldsymbol{n}} \right\rangle^2]^{1/2} = \int d^2a\, d^2\beta\, P(\alpha,\beta) \left[\left\{|\alpha|^2 - |\beta|^2 - \left(\left\langle |\alpha|^2 \right\rangle - \left\langle |\beta|^2 \right\rangle\right)\right\}^2 + (|\alpha| - |\beta|)^2\right]. \tag{32}$$

This shows that if polarization squeezing is exhibited, no non-negative $P(\alpha, \beta)$ can exist and therefore this is a purely non-classical feature.

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