



Experimental performance evaluation of weak turbulence channel models for FSO links

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ABSTRACT

Outdoor wireless communication is subject to several outdoor environment conditions. Among them, atmospheric turbulence which happens on a daily basis, introduces signal fading and degrades the system performance. Many channel models have been proposed in the literature to model the turbulence fading in free space optic (FSO) communication systems. In this work, we aim to evaluate the performance of some key proposed channel models for FSO systems under weak turbulence condition. First, an outdoor experimental setup is utilized to measure the turbulence fading and build a dataset. The setup is all-optical, which gives us the advantage of isolating the solar radiation noise during measurements. Second, the obtained dataset is used to investigate the performance of five key FSO channel models claimed to perform well under weak turbulence conditions. The results show that for very weak turbulence with scintillation index (SI) $< 10^{-3}$, four models that claim to work well under such condition failed to fit the dataset. For weak turbulence with $SI > 10^{-3}$, all the models fit the dataset except *I-K* model. Third, we exploited the obtained dataset to propose a new empirical channel model that has good performance under weak turbulence conditions. Using this model, we studied the outage probability of FSO system under the joint effect of weak turbulence and fog attenuation.

1. Introduction

Free space optic (FSO) is proposed for high speed data rate communication links such as backhaul links in wireless networks. Nowadays, terrestrial traditional FSO systems support data rate up to 30 Gbps [1] while the experimental demonstrations show comparable speed to fiber thanks to using all-optical FSO systems [2]. In addition to the promised huge bandwidth, FSO technology has many other advantages including license-free, immunity to electromagnetic interference, and low power consumption.

In spite of these advantages, FSO is highly depending on the outdoor environment. Many atmospheric conditions can affect its performance and limit its applications. This include dust, fog, rain, snow, and turbulence. In clear weather conditions, sun heating of the air causes change in air refractive index. This change in turn causes signal fluctuations at the receiver (known as scintillation) which may fade the signal below the receiver threshold especially for long-distance links of several kilometers length. Under such conditions, the reliability of FSO system functionality can be deduced from the probability density function (PDF) of the irradiance signal.

In literature, many FSO channel models have been proposed for modeling turbulence conditions [3]. These models help in engineering the FSO link and improving its reliability and availability. Some of these models are dedicated for specific weak, moderate, or strong

turbulence conditions. Others claim suitability for all turbulence conditions. We aim in this work to study and compare the performance of different weak turbulence models using a dataset collected by an outdoor experimental setup. The setup is built over 100 m distance. The dataset contains measurements with different scintillation indices (SI). All the measurements are obtained under weak turbulence conditions, i.e. $SI < 1$. We consider five key channel models proposed in literature. Using the measurements, we test the goodness-of-fit (GoF) of each proposed model using two GoF metrics, R^2 and root mean square error (RMSE).

The results show that only log-normal distribution can model well the FSO channel under very weak turbulence conditions, i.e. $SI < 1 \times 10^{-3}$. Moreover, we found that *I-K* model is not suitable for modeling turbulence with $SI < 1$. For Gamma-Gamma model, the results indicate acceptable performance for $SI > 1 \times 10^{-2}$. Finally, using the dataset, we found that logistic distribution has similar performance to log-normal distribution and can be applied for modeling weak turbulence conditions. It has the advantage of producing analytical tractable expressions compared to the log-normal distribution found to have limited tractability for certain computations in FSO. Using this new model, we analyzed the performance of FSO communication system under the effect of weak turbulence and fog attenuation.

The remaining of the paper is organized as follows, in Section 2 we report the most key models for modeling the FSO channel under weak

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turbulence conditions. Section 3 illustrates the experimental setup and Section 4 reports the obtained results. FSO system performance analysis under the effect of weak turbulence and fog attenuation is reported in Section 5. Finally, we conclude in Section 6.

2. Weak turbulence channel models

Irradiance fluctuations in FSO links due to turbulence is described by SI which is given by [3]

$$SI = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1, \quad (1)$$

where I is the signal irradiance (intensity) and the angle brackets $\langle \cdot \rangle$ denote an ensemble average. SI is frequently used to quantify the amount of fading in FSO signal. For weak turbulence, $SI < 1$, while for strong turbulence, $SI > 1$. Different models have been reported in literature for modeling the FSO channel under weak turbulence. Here we briefly introduce them.

2.1. Log-normal model

Log-normal distribution is widely accepted model for signal fading under weak turbulence conditions in literature. This model is given by

$$f(I) = \frac{1}{2I\sqrt{2\pi\sigma_I^2}} \exp \left(- \left(\ln \frac{I}{I_0} + \frac{\sigma_I^2}{2} \right)^2 / 2\sigma_I^2 \right), \quad (2)$$

where I_0 is the signal intensity with no turbulence and σ_I^2 is the scintillation index [4]. Although this distribution models well the weak turbulence conditions in FSO systems, the mathematical complexities inherent in the model limits its application for performance analysis. Therefore, other distributions are reported in the literature as alternatives that have better tractability.

2.2. I-K model

This distribution is a generalized one of the K -distribution that works under strong turbulence conditions only. I - K distribution is assumed to be applicable for all turbulence conditions including weak turbulence. This model is given by [5]

$$f(I) = \begin{cases} \frac{2\alpha}{b_0} \left(\frac{\sqrt{I}}{A} \right)^{(\alpha-1)} K_{\alpha-1} \left(2A\sqrt{\frac{\alpha}{b_0}} \right) \\ \times I_{\alpha-1} \left(2\sqrt{\frac{\alpha I}{b_0}} \right), & I < A^2. \\ \frac{2\alpha}{b_0} \left(\frac{\sqrt{I}}{A} \right)^{(\alpha-1)} I_{\alpha-1} \left(2A\sqrt{\frac{\alpha}{b_0}} \right) \\ \times K_{\alpha-1} \left(2\sqrt{\frac{\alpha I}{b_0}} \right), & I > A^2. \end{cases} \quad (3)$$

where α , and b_0 , are distribution parameters, and $I_n(\cdot)$ and $K_n(\cdot)$ are the modified Bessel functions of the first and second kind, respectively, with order n .

2.3. Gamma-Gamma model

Gamma-Gamma PDF is developed to model all turbulence conditions better than I - K model that showed shortcomings when compared to measurements [6]. This model is a two parameters model and assumes that the small-scale irradiance fluctuations of the wave are modulated by large-scale fluctuations, where both are modeled by Gamma distribution. This distribution is defined by [6]

$$f(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta I} \right), \quad (4)$$

where $\alpha > 0$ and $\beta > 0$ parameters represent the effective number of large-scale and small-scale cells of the scattering process, respectively, and $K_{\alpha-\beta}$ is the $\alpha - \beta$ order modified Bessel function of second

kind. Using numerical simulation data, the authors showed that the proposed model has a better fit than other models such as log-normal distribution, even for weak turbulence conditions.

2.4. Inverse Gaussian model

Because log-normal distribution is analytically intractable for some performance metrics in FSO, the authors in [7] proposed inverse Gaussian distribution as an alternative to model FSO channel under weak turbulence. This model is found to substitute log-normal distribution in RF systems for describing shadowing effects. The authors used Kolmogorov-Smirnov (KS) goodness to show that this model is a good alternative for the log-normal model and has comparable performance with less complexity. This model is given by [7]

$$f(I) = \sqrt{\frac{\lambda}{2\pi I^3}} \exp \left(- \frac{\lambda(I - \mu)^2}{2\mu^2 I} \right), \quad (5)$$

where $\mu > 0$ is the irradiance mean and $\lambda > 0$ is a scale parameter of the distribution.

2.5. Exponentiated Weibull model

In this work, the authors proposed a model and claimed its capability to work under all turbulence conditions. The performance of this model is compared with the log-normal and Gamma-Gamma models using simulation and experimental results. It is described by the following three parameters model [8]

$$f(I) = \frac{\alpha\beta}{\eta} \left(\frac{1}{\eta} \right)^{(\beta-1)} \exp \left[- \left(\frac{1}{\eta} \right)^\beta \right] \times \left(1 - \exp \left[- \left(\frac{1}{\eta} \right)^\beta \right] \right)^{\alpha-1}, \quad (6)$$

where $\alpha > 0$ and $\beta > 0$ parameters are shape parameters, and $\eta > 0$ is a scale parameter.

3. Experimental setup

Fig. 1 shows our experimental setup which includes a laser diode (LD) source in the transmitter side that transmits an optical signal with 16 dBm power at 1550 nm wavelength and a photodiode in the receiver side with -110 dBm sensitivity. The FSO link between the transmitter and the receiver is installed over 100 m distance. The installed link is all-optical with no electrical to optical or optical to electrical conversion. The FSO link includes two identical light collimators. The collimator in the transmitter side transmits the output beam of a single mode fiber (SMF) into free space with very low divergence while the collimator in the receiver side collects the light into SMF. Each collimator is equipped with a doublet lens with 7 mm aperture diameter and 0.016° (0.279 mrad) full angle beam divergence. The collimators are mounted on a post and mounting units are used to align the transmitter and receiver. Because the SMF has very narrow diameter $\sim 8 \mu\text{m}$, very precise alignment is required to reduce the misalignment loss. The theoretical geometrical loss due to beam spreading of a Gaussian beam is given by [9]

$$L_G = 10 \log \left(1 - \exp \left(-2 \left[\frac{r_{Rx}}{L \tan(\theta/2)} \right]^2 \right) \right), \quad (7)$$

where $L = 100$ m is the link length, $\theta = 0.279$ mrad is the full angle beam divergence, and $r_{Rx} = 7$ mm is the receiver aperture radius. The measured link power loss is 16 dB. This includes 9.3 dB geometrical loss according to (7), and 6.7 dB for link misalignment and power coupling loss. The collimators are connected to the laboratory using SMFs of 50 m length in the transmitter side and 150 m length in the receiver side. In the receiver side, the photodiode's output signal is sampled at 10 kHz sampling speed using an analog-to-digital converter

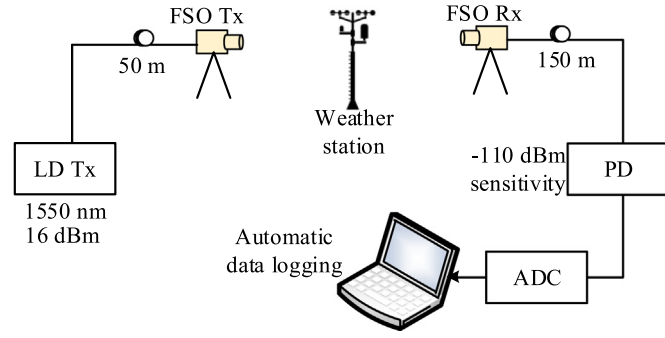


Fig. 1. Experimental setup of an all-optical FSO link for turbulence effect measurement. LD: laser diode, PD: photodiode, ADC: analog-to-digital converter.

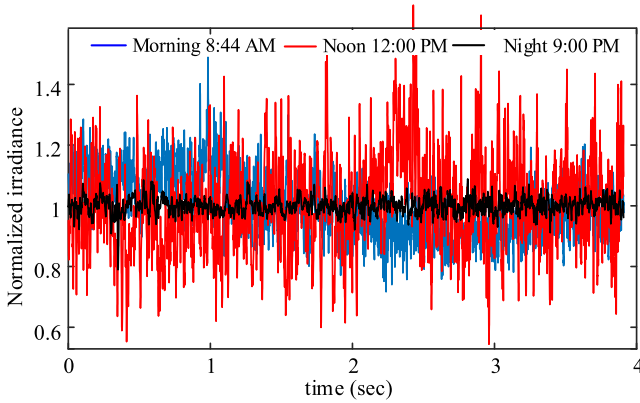


Fig. 2. Effect of turbulence fading on the transmitted signal at three different times.

and the output is connected to a PC to log the data automatically using Labview. Each measurement includes 40,000 samples. The irradiance measurements were carried out at three different times: morning at 8:44 AM, noon at 12:00 PM, and night at 9:00 PM. In addition, some weather parameters are collected using a weather station installed at the measurement location.

4. Results and discussions

Fig. 2 shows the normalized received signal at three different times. First, we notice from Fig. 2 that the turbulence fading increases from morning to noon and then decreases at night due to air temperature fluctuations that increases during day time to 27.6°C and decreases at night to 21.7°C according to the weather station measurements. Table 1 lists the scintillation index of each measurement. All the measurements have $SI < 1$, which define weak turbulence conditions.

Next, we investigate the effect of solar radiation on the taken measurements and on the whole all-optical FSO system. For this purpose, the transmitter is switched off and some measurements were taken at morning, noon and at night. The variance of the sampled signals at morning, noon, and night was 2.8×10^{-16} , 3.1×10^{-16} , and 3.1×10^{-16} , respectively. The three measurements have almost the same variance. This indicates that the noise measured at night (which is the receiver noise since there is no solar radiation) is similar to those measured at noon and morning. Hence, the solar radiation has no effect on the all-optical FSO system and also on the obtained measurements of the turbulence. The effect of solar noise is negligible because of using an optical collimator with a SMF in the receiver instead of a photodiode. This SMF limits the field of view (FOV) of the receiver and hence spatially filters the ambient light by sun [10]. This is not the case in traditional FSO links that use photodiode receiver which is sensitive to

Table 1

Scintillation index of the FSO signal at different times.

	Morning	Noon	Night
SI	1.01×10^{-2}	2.44×10^{-2}	7.16×10^{-4}

ambient light and can highly limits its performance. In addition, the collimators are using coated lenses which behave as band pass filters.

Using the obtained dataset, we study the performance of some key FSO channel models, listed in Table 2, which are proposed in literature to work under weak turbulence conditions. These models are log-normal, Gamma-Gamma, Exponentiated Weibull, inverse Gaussian, and *I-K*. First, the unknown parameters of these models are estimated using non-linear least-squares regression method [11,12] toolbox in Matlab. This method is widely used in literature as a modeling method. Second, to test the GoF of the proposed channel models with the dataset, two well known and widely employed GoF metrics are used: R^2 and RMSE. The metric R^2 is defined mathematically as [13]

$$R^2 = 1 - \frac{SS_{reg}}{SS_{tot}}, \quad (8)$$

where $SS_{reg} = \sum_{i=1}^n (a_{m,i} - a_{p,i})^2$ is the square errors sum and $a_{m,i}$ and $a_{p,i}$ are the measured and predicted probability values, respectively, for the i th histogram bin of the received intensity. The term $SS_{tot} = \sum_{i=1}^n (a_{m,i} - \bar{a})^2$ is the sum of squares distances between the measured points mean and their distances. A model has good fit if R^2 has value closed to 1. The second metric is defined mathematically as [13]

$$RMSE = \sqrt{\frac{1}{M} \sum_{i=1}^M (a_{m,i} - a_{p,i})^2}, \quad (9)$$

where M is the number of measured samples. In terms of RMSE metric, the lower the value of RMSE, the better the model fitting to the measured data.

Fig. 3 and Table 2 show the fitting of the measured data with the different channel models. First, we notice that *I-K* channel model is not depicted in Fig. 3 because its performance in terms of R^2 and RMSE was bad over all measurements. Next, we can notice for noon measurements ($SI = 2.44 \times 10^{-2}$), all channel models (except *I-K*) showed high GoF with $R^2 > 0.98$ and $RMSE < 0.1$. For the morning measurement ($SI = 1.01 \times 10^{-2}$) which has a little bit less SI than noon measurement, we see that log-normal, exponentiated Weibull, and inverse-Gaussian showed high GoF with $R^2 > 0.95$ and $RMSE < 0.31$. For Gamma-Gamma model, the achieved R^2 and RMSE were a little bit worse, 0.77 and 0.67, respectively. When the turbulence fading becomes much less at night ($SI = 7.16 \times 10^{-4}$), the GoF test results show that only log-normal model achieved high GoF with $R^2 = 0.99$ and $RMSE = 0.41$. The other channel models achieved poor performance under very weak turbulence as illustrated in Fig. 3(c) and Table 2.

The results in Table 2 show that the log-normal distribution has the advantage of modeling very weak turbulence conditions. However, as mentioned before, the mathematical complexities of this model limit its application in FSO. Therefore, using the measured data of irradiance, we attempt to propose a new model for FSO channel under weak turbulence conditions capable of fitting well the dataset we obtained with better simplicity. For this purpose, we examined different types of continuous probability distributions. We found that logistic distribution has performance similar to log-normal model as listed in Table 2 and Fig. 4. This model is given by [14]

$$f(I) = \frac{\exp(-z)}{\sigma(1 + \exp(-z))^2}, \quad z = \frac{I - \mu}{s} \quad (10)$$

where s and μ are scale and location parameters of the distribution, respectively. The obtained R^2 and RMSE values for this model are very closed to that of log-normal model. Note that some of log-normal distribution statistics such as the moment generation function cannot be written in a closed form expression. This complicates analyzing

Table 2
Key weak turbulence channel models GoF.

Model	Turbulence condition	Meas. time	R^2	RMSE
Log-normal	Weak	Morning	0.9520	0.3118
		Noon	0.9888	0.0983
		Night	0.9934	0.4179
Gamma-Gamma	All conditions	Morning	0.7769	0.6721
		Noon	0.9895	0.0953
		Night	Not valid	Not valid
Exp. Weibull	All conditions	Morning	0.9770	0.2179
		Noon	0.9917	0.0853
		Night	Not valid	Not valid
Inverse Gaussian	Weak	Morning	0.9618	0.2796
		Noon	0.9889	0.0981
		Night	Not valid	Not valid
I - K	All conditions	Morning	Not valid	Not valid
		Noon	Not valid	Not valid
		Night	Not valid	Not valid
Logistic	Proposed	Morning	0.9329	0.3704
		Noon	0.9885	0.0998
		Night	0.9924	0.4513

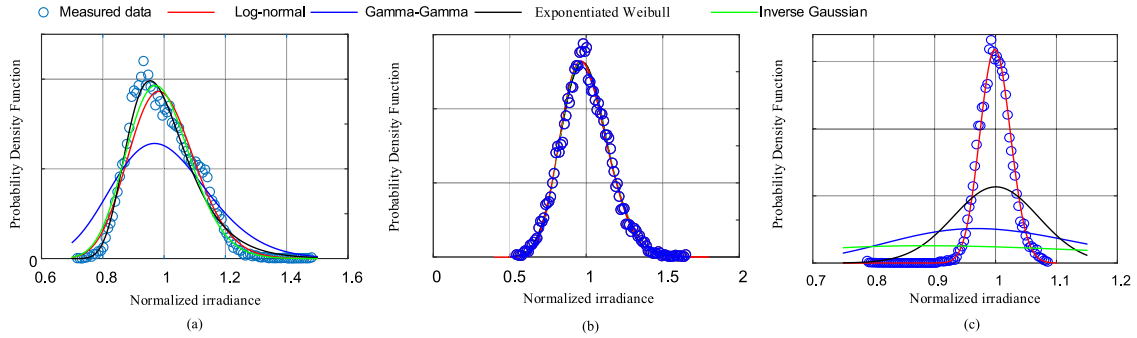


Fig. 3. Measured data fitting under weak turbulence and different channel models, (a) morning measurement, (b) noon measurement, and (c) night measurement. Note that I - K model is not indicated because it does not fit the data.

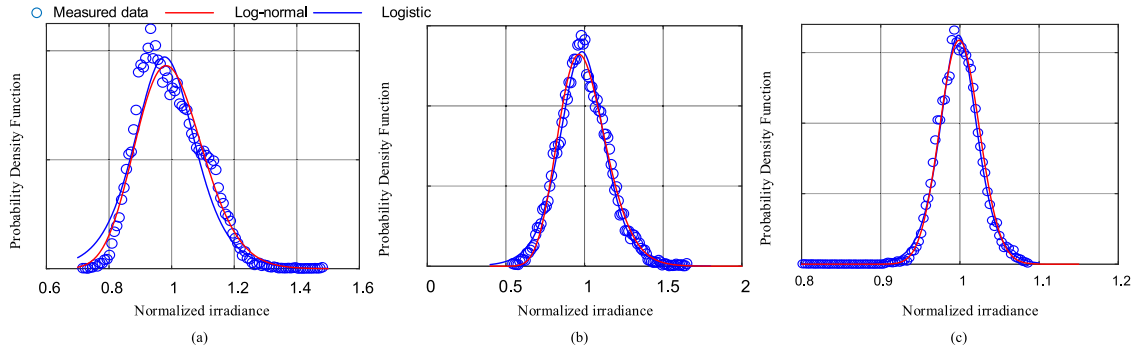


Fig. 4. Comparison of log-normal and logistic models fitting to the measured data, (a) morning measurement, (b) noon measurement, and (c) night measurement.

FSO systems performance and leads to intractable analytical expressions [7]. Therefore, logistic distribution could be a good alternative for log-normal distribution.

5. Performance analysis

In this section, we investigate the performance of FSO communication system in terms of outage probability under the joint effect of weak turbulence (I_a) represented by the proposed logistic channel model and atmospheric condition (I_l) represented by moderate to weak fog condition. The channel state is then given as

$$I = I_l I_a, \quad (11)$$

where I_a is a random variable defined by (10) and I_l is a deterministic model defined in [12] as

$$I_l = \exp(-22LV^{(0.2\lambda-1.04)}/4.343), \quad (12)$$

where V is the visibility range in km, L is the link length in km, and λ is the optical signal wavelength in micrometer. In addition, we consider intensity modulation/direct detection (IM/DD) communication system that uses ON/OFF keying (OOK) modulation format. The received signal is defined by [15]

$$y = IRx + n, \quad (13)$$

where R is the receiver responsivity, x is the transmitted signal intensity, and n is signal-independent additive white Gaussian noise with σ_n^2

Table 3
Simulation parameters.

Parameter	Parameter	Value
Noise variance	σ_n^2	$10^{-14} \text{ A}^2 / \text{Hz}^2$
Receiver responsivity	R	0.5
SNR threshold	γ_{th}	6 dB
Transmission wavelength	λ	1550 nm
Transmission distance	L	1 km
Moderate fog visibility range	V	0.5 km
Light fog visibility range	V	1 km
Very light fog visibility range	V	10 km

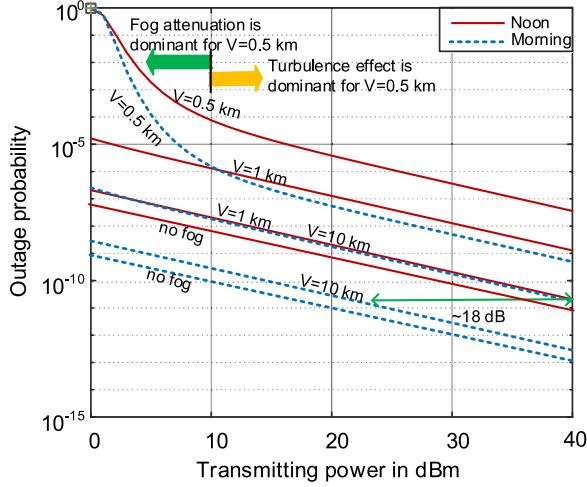


Fig. 5. Outage probability analysis under different fog and turbulence conditions.

variance. The received signal-to-noise ratio (SNR) is given as [15]

$$\gamma = \frac{2P_t^2 R^2 I^2}{\sigma_n^2} = \gamma_o I^2, \quad (14)$$

where P_t is the average transmitted power and $\gamma_o = 2P_t^2 R^2 I^2 / \sigma_n^2$. The outage probability that the SNR falls below a certain threshold γ_{th} is given as

$$P_{out} = P(\gamma < \gamma_{th}) = P(I_a < \sqrt{\gamma_{th}/\gamma_o}/I_l). \quad (15)$$

Substituting (10) in (14) and integrating yields

$$P_{out} = \frac{1}{2} \operatorname{sech} \frac{\mu}{s} \operatorname{sech} \frac{\mu - \sqrt{\gamma_{th}/\gamma_o}/I_l}{2s} \sinh \frac{\sqrt{\gamma_{th}/\gamma_o}/I_l}{2s}. \quad (16)$$

Using (16) and the simulation parameters in Table 3, we investigate the performance of the proposed system. Fig. 5 shows the system outage probability as a function of the average transmitted optical power. Two weak turbulence conditions are considered from the morning measurements and noon measurements as defined in Table 1. In addition, three fog conditions are considered; moderate ($V = 0.5$ km), light ($V = 1$ km), and very light ($V = 10$ km). First, we notice that the system performance at noon is worst than that at morning where for $P_t > 10$ dBm, the system performance at noon requires ~ 18 dB more power to achieve the same performance at morning. Second, we notice that when the transmitting power is low, $P_t < 10$ dBm and the visibility range is low (moderate fog), i.e. $V = 0.5$ km, fog attenuation is dominant resulting in the exponential decaying curve which agrees to its definition in (12). When the transmitting power increases or the visibility range improves, the effect of fog attenuation is minor and the turbulence effect is dominant. In Fig. 5, the isolated effect of turbulence without fog condition is indicated. The indicated curves represent the upper system performance's boundaries under a weak turbulence effect, where no better performance can be achieved.

6. Conclusion

Many turbulence models for FSO channel have been proposed in literature. We aimed to investigate the validity of such channel models which are dedicated for weak turbulence conditions. Using real measurements of weak turbulence fading, we tested five well known models in the literature. For a range of SI under weak turbulence, we found that log-normal is the best model. The other models show bad performance under very weak turbulence conditions. Hence, using non log-normal distribution for modeling very weak turbulence condition is not accurate. Moreover, the obtained data is exploited to propose a new model which showed good performance comparable to log-normal model. This model can overcome long-normal model by producing tractable analytical expressions for FSO links performance.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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