Optics Communications 332 (2014) 366–369

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Dicke and Fano effects in single photon transport

Maritza Ahumada [a], Natalia Cortés [a], M.L. Ladrón de Guevara [a], P.A. Orellana [b,*]

[a] Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile
[b] Departamento de Física, Universidad Técnica Federico Santa María, P.O. Box 110 V, Valparaíso, Chile

ARTICLE INFO

Article history:
Received 18 January 2012
Received in revised form
18 June 2014
Accepted 29 June 2014
Available online 10 July 2014

Keywords:
Single photon
Cavity
Waveguide

ABSTRACT

The single photon transport through a one dimensional array of cavities is studied theoretically. Analytic expressions of the reflection and transmission are given. The transmission shows an energy spectrum with forbidden and allowed bands that depend on the detuning parameter of the system. We show that the allowed mini band is formed due to the indirect coupling between the atoms in each cavity. In addition, the band edges can be controlled by the degree of detuning of the level of the atoms. We discuss the analogy between this phenomenon with the Fano and Dicke effects.

1. Introduction

In recent years, there has been great interest, both theoretically and experimentally, in the study of photon transport in quantum circuits [1,2]. The advantage of photons, in comparison with other kind of information carriers (e.g. electrons), is that they show high-speed transmission and low dissipation in nonlinear media (e.g. optical fibers). Photon transport in quantum circuits has many possible realizations, as for instance in superconductor line resonators [3–6] and semiconductor micro-cavities [7]. Additionally, many efforts have been devoted in order to implement single photon transport in one-dimensional wave-guides [8–18]. Besides applications on optical quantum computing and quantum information, the single photon transport has potential application in optical communication and quantum devices as photon transistors. The possibility of implementing photon transistors in analogy to the electron transistor is a great challenge. In order to do that, several kinds of wave-guides have been involved, such as metal nano-wires [19], silicon wires [20], and quantum resonator arrays [21,22]. It has been shown that the behavior of photons in quantum resonator arrays is in analogy to the movement of electrons in periodic potentials. The photon can feel the presence of the quantum resonators and it can be transmitted through allowed mini-bands and reflected by forbidden mini-bands [23]. The single photon scattering properties in the coupled resonator array wave-guide with individually embedded two-level or three-level atom in each resonator, have also been explored recently [23–25].

In the above context, recently Chang et al. [25] considered new quantum devices based on a quantum resonator array. In these works, the authors have proposed a set-up based on the coupled-resonator array with doped atoms, which is expected to exhibit perfect reflection within a wide spectrum of frequencies. Thus it can perfectly reflect an optical pulse, or namely a single photon wave-packet. They used a thick atomic mirror, which is made of an array of two-level atoms individually doped in some cavities arranged in a coordinate region of the one-dimensional coupled-cavity wave-guide.

Quantum interference effects as Fano effect can play a crucial role in single photon transport [26]. Very recently Li et al. [27] have experimentally observed Fano resonances in a single toroidal micro-resonator, in which two modes are excited simultaneously through a fibre taper. By adjusting the fibre-cavity coupling strength and the polarization of incident light, the Fano-like resonance line shape can be controlled.

In this work we report further progress along the lines indicated above. We study the single photon transport through a system of a 1D array cavity. We obtain analytic expressions of the reflection and transmission. We show that the transmission displays an energy spectrum with forbidden and allowed mini-bands that depends on the detuning parameter of the system. We show that by the indirect coupling between the levels of the atom in each cavity an allowed mini-band is formed. In addition, the band edges can be controlled by the degree of detuning of the level of the atoms. We discuss the analogy between this phenomenon with the Fano and Dicke effects.

2. Model

The system under consideration consists in a 1D coupled cavity array with an embedded N cavity-atom array, with one V-type three-level atom in each cavity as is shown schematically in Fig. 1.

* Corresponding author.
E-mail address: pedro.orellana@usm.cl (P.A. Orellana).

http://dx.doi.org/10.1016/j.optcom.2014.06.068

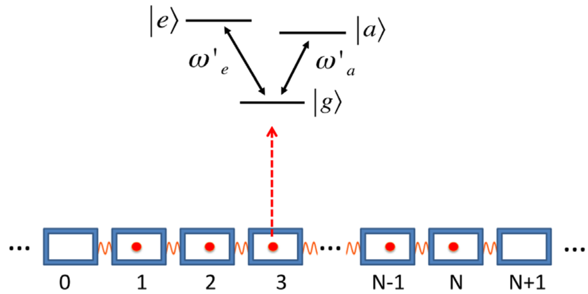

Fig. 1. 1D coupled cavity array with an embedded N cavity-atom array with one V-type three-level atom in each cavity.

We denote by $|g\rangle$ the ground level and by $|a\rangle$ and $|e\rangle$ the two excited metastable levels. The atom is coupled to the cavity mode of frequency ω through the transitions g–a and g–e. The total Hamiltonian is composed by the sum of a free Hamiltonian of the atom $\mathcal{H}_a$, which describes the propagation of a photon through the cavities $\mathcal{H}_p$, and the term which describes the atom–field interaction represented by a Jaynes–Cummings Hamiltonian under the rotating wave approximation $\mathcal{H}_I$:

$$\mathcal{H}=\mathcal{H}_p+\mathcal{H}_a+\mathcal{H}_I, \tag{1}$$

$$\mathcal{H}_p=\sum_{i=-\infty}^{\infty}\omega\hat{c}_i^\dagger\hat{c}_i+\sum_{i=-\infty}^{\infty}v(\hat{c}_i^\dagger\hat{c}_{i+1}+\hat{c}_{i+1}^\dagger\hat{c}_i), \tag{2}$$

$$\mathcal{H}_a=\sum_{j=1}^{N}(\,\omega'_a|a_j\rangle\langle a_j|+\omega'_e|e_j\rangle\langle e_j|), \tag{3}$$

$$\mathcal{H}_I=\sum_{j=1}^{N}[g_a(\hat{c}_j|a_j\rangle\langle g_j|+\hat{c}_j^\dagger|g_j\rangle\langle a_j|) +g_e(\hat{c}_j|e_j\rangle\langle g_j|+\hat{c}_j^\dagger|g_j\rangle\langle e_j|)], \tag{4}$$

where $\hat{c}_j^\dagger$ and $\hat{c}_j$ are the creation and annihilation operators of one photon in the j-th cavity, respectively, while v is the coupling between the cavities, $\omega'_{a,e}$ are the energy transitions and g_a (g_e) is the coupling constant between the field and transition $|g\rangle\to|a\rangle$ ($|g\rangle\to|e\rangle$). In what follows we assume $g_a=g_e\equiv\tilde{g}$.

The stationary state of the system can be written as

$$|E\rangle=\sum_{i=-\infty}^{\infty}u_i|\,1_i,g\,\rangle+\sum_{j=1}^{N}\left[d_{a,j}|\,0,a_j\,\rangle+d_{e,j}|\,0,e_j\,\rangle\right], \tag{5}$$

where u_i represents the amplitude to find a photon in the i-th cavity and the corresponding atom in the ground state, $d_{a,j}$ ($d_{e,j}$) the probability amplitude to find no photon in the cavity array and the atom in the j-th cavity in the excited state a (e), while the rest of the atoms keep in the ground state.

From the eigenvalues equation $\mathcal{H}|E\rangle=E|E\rangle$ it is obtained the following linear system of difference equations:

$$(E-\omega)u_j=v(u_{j+1}+u_{j-1}), \tag{6}$$

for $j\le 0$ and $j>N$, and

$$(E-\omega)u_j=v(u_{j+1}+u_{j-1})+\tilde{g}(d_{e,j}+d_{a,j}), \tag{7a}$$

$$(E-\omega'_e)d_{e,j}=\tilde{g}u_j, \tag{7b}$$

$$(E-\omega'_a)d_{a,j}=\tilde{g}u_j, \tag{7c}$$

for $j=1,\dots,N$. From Eqs. (7b) and (7c) we obtain

$$d_{e,j}=\frac{\tilde{g}\,u_j}{E-\omega'_e},\quad d_{a,j}=\frac{\tilde{g}\,u_j}{E-\omega'_a}, \tag{8}$$

which inserted in Eq. (7a), it is reduced to the single equation:

$$(E-\omega-\tilde{\varepsilon})u_j=v(u_{j-1}+u_{j+1}),\quad j=1,\dots,N, \tag{9}$$

where

$$\tilde{\varepsilon}=\tilde{g}^2\left[\frac{2E-\omega'_a-\omega'_e}{(E-\omega'_a)(E-\omega'_e)}\right], \tag{10}$$

is the renormalized energy. Thus, the problem reduces to solve a linear chain of N sites with effective energies $\tilde{\varepsilon}$. In order to find the solutions of Eq. (9), we assume open boundary conditions. This means that the photon is described by a plane wave incident from the far left, with unitary amplitude and a reflection amplitude r while at the far right by a transmission amplitude t. That is,

$$u_j^{(k)}=e^{ikj}+re^{-ikj},\quad j<1,$$
$$u_j^{(k)}=te^{ikj},\quad j>N. \tag{11}$$

Inserting Eqs. (11) in Eq. (6) we obtain the following dispersion relation for the incident photon:

$$E=\omega+2v\cos(k). \tag{12}$$

From Eqs. (6), (9), and (11) we obtain an inhomogeneous system of equations for the probability amplitudes u_j ($j=1,\dots,N$), r and t, leading to the following expression for t:

$$t=\frac{2ie^{-ikN}}{\Delta}\sin k, \tag{13}$$

with Δ given by

$$\Delta=e^{-ik}\frac{\sin(N+1)q}{\sin q}+2\frac{\sin Nq}{\sin q}+e^{ik}\frac{\sin(N-1)q}{\sin q}, \tag{14a}$$

if $|(E-\omega-\tilde{\varepsilon})/2v|\le 1$ and

$$\Delta=e^{-ik}\frac{\sinh(N+1)\kappa}{\sinh\kappa}+2\frac{\sinh N\kappa}{\sinh\kappa}+e^{ik}\frac{\sinh(N-1)\kappa}{\sinh\kappa}, \tag{14b}$$

if $|(E-\omega-\tilde{\varepsilon})/2v|\ge 1$.

The reflection and transmission probabilities are $R=|r|^2$ and $T=|t|^2$. Then if $|(E-\omega-\tilde{\varepsilon})/2v|\le 1$ we have

$$R=\frac{\sin^2(Nq)(\cos q+\cos k)^2}{\sin^2(Nq)(\cos k\cos q+1)^2+[\sin k\sin q\cos(Nq)]^2},$$
$$T=\frac{1}{\cos^2(Nq)+[\sin(Nq)(1+\cos q\cos k)/(\sin q\sin k)]^2}, \tag{15a}$$

where $q=\cos^{-1}[-(E-\omega-\tilde{\varepsilon})/2v]$. We note that these probabilities oscillate as a function of both N and q. In this energy region, the system should show allowed mini-bands in the transmission.

On the other hand, if $|(E-\omega-\tilde{\varepsilon})/2v|\ge 1$ we get

$$R=\frac{\sinh^2(N\kappa)(\cosh\kappa+\cos k)^2}{\sinh^2(N\kappa)(\cos k\cosh\kappa+1)^2+[\sin k\sinh\kappa\cosh(N\kappa)]^2},$$
$$T=\frac{1}{\cosh^2(N\kappa)+[\sinh(N\kappa)(1+\cosh\kappa\cos k)/(\sinh\kappa\sin k)]^2}, \tag{15b}$$

where $\kappa=\cosh^{-1}[-(E-\omega-\tilde{\varepsilon})/2v]$. Then in this energy region $T\sim e^{-2N\kappa}$ tends to zero while R tends to unity, when N is large. This means that a mini-band gap in the transmission should be formed.

3. Results

To avoid the profusion of free parameters and for the sake of clarity we set $\Omega=E-\omega$, the energies of the atoms as, $\omega'_e=\omega_0-\Delta\omega$, and $\omega'_a=\omega_0+\Delta\omega$. In what follows, we express all energies in units of γ with $\gamma=g^2/2v$, hereafter.

In first place, we consider the degenerate case, $\Delta\omega=0$, for different values of N. Simple expressions for the transmission and reflection can be readily obtained for $N=1$. For $N=1$, the reflection

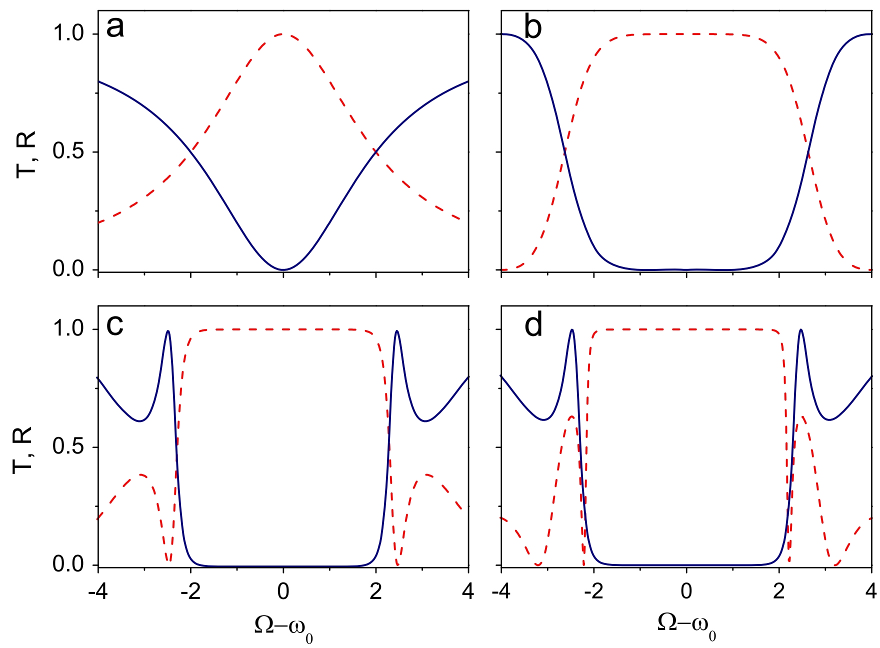

Fig. 2. Transmission (blue solid line) and reflection (red dash line) as a function of the detuning $\Omega-\omega_0$ for $\Delta\omega=0$ and (a) $N=1$, (b) $N=3$, (c) $N=5$ and (d) $N=7$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

reduces to a Breit–Wigner line shape of semi-width γ

$$R=\frac{\gamma^2}{(\Omega-\omega_0)^2+\gamma^2}, \tag{16}$$

and the transmission takes the form of a symmetrical Fano line shape

$$T=\frac{(\Omega-\omega_0)^2}{(\Omega-\omega_0)^2+\gamma^2}=\frac{(\epsilon+q)^2}{\epsilon^2+1}, \tag{17}$$

with $q=0$ and $\epsilon=(\Omega-\omega_0)/\gamma$. Notice that T vanishes just at $\Omega=\omega_0$. This result can be interpreted as follows. The photon has two possible paths when going from left to right, a direct and indirect one. In the latter the photon is absorbed and emitted by the atoms. The destructive interference between these two paths gives rise to the Fano effect.

Fig. 2 shows the transmission and reflection probabilities versus the detuning $\Omega-\omega_0$ for different values of N. As N grows (Fig. 2b–d) a forbidden mini-band (gap) is formed in T. It is apparent that R (T) tends to unity (zero) within a range $[-2\gamma, 2\gamma]$, and the system behaves as a quantum mirror within this interval of energies.

Now, let us consider the situation with $\Delta\omega\neq 0$. Fig. 3 displays the reflection and transmission vs detuning for different values of N. We note that an allowed mini-band develops at the center of the gap and the system becomes transparent. Moreover the transmission becomes always unity at the center of the allowed mini-band, independently of the value of N. This behavior is analogue of the electromagnetically induced transparency (EIT) which has recently been studied theoretically and experimentally for the structures based on micro-ring resonators and photonic crystal cavities [28,29].

Fig. 4 displays a zoom of the transmission at the center of the band for $\Delta\omega=0.5\gamma$, for two values of N, $N=1$ (upper panel) and $N=7$ (lower panel). We note the dramatic change of the width of the mini-band as $\Delta\omega$ decreases. In fact, for $N=1$, it is straightforward to show that for $\Delta\omega\ll\gamma$ the width of this allowed transmission band is $\Delta\omega^2/2\gamma$, the reflection can be written as a difference between two Breit–Wigner line-shapes with widths γ and δ, respectively, while the transmission can be written as a superposition of a symmetrical Fano line-shape and a Breit–Wigner line-shape:

$$R\approx\frac{\gamma^2}{(\Omega-\omega_0)^2+\gamma^2}-\frac{\delta^2}{(\Omega-\omega_0)^2+\delta^2}, \tag{18}$$

$$T\approx\frac{(\Omega-\omega_0)^2}{(\Omega-\omega_0)^2+\gamma^2}+\frac{\delta^2}{(\Omega-\omega_0)^2+\delta^2}, \tag{19}$$

with $\delta=\Delta\omega^2/2\gamma$.

The above phenomenon resembles the Dicke effect, which takes place in the spontaneous emission of a pair of atoms radiating a photon with a wavelength much larger than the separation between them [30,31]. In this effect, the luminescence spectrum is characterized by a narrow peak and a broad peak, associated with long and short-lived states, respectively. The former state, coupled weakly to the electromagnetic field, is called *subradiant*, and the latter, strongly coupled, *superadiant* state. In the present case this effect is due to indirect coupling between the atom states through the common cavity. The states strongly coupled to the continuum gives a forbidden mini-band with width 4γ, while the states weakly coupled to the continuum gives an allowed Dicke mini-band with width δ. This effect can be considered a special case the Fano–Feshbach resonances in the systems exhibiting more than one resonance [32].

A possible physical realization of this present set up may be made in a metal nano-wire coupled to quantum dots. The metal nano-wire plays the role of 1D continuum and the region where the quantum dots are coupled to the nano-wire.

4. Summary

In this work, we have studied the transport of a single photon through a system of a 1D array of cavities. We have obtained analytic expressions of the transmission and reflection. We have shown that the transmission displays an energy spectrum with forbidden and allowed bands that depend on the detuning parameter of the system. We have shown that the allowed mini-band

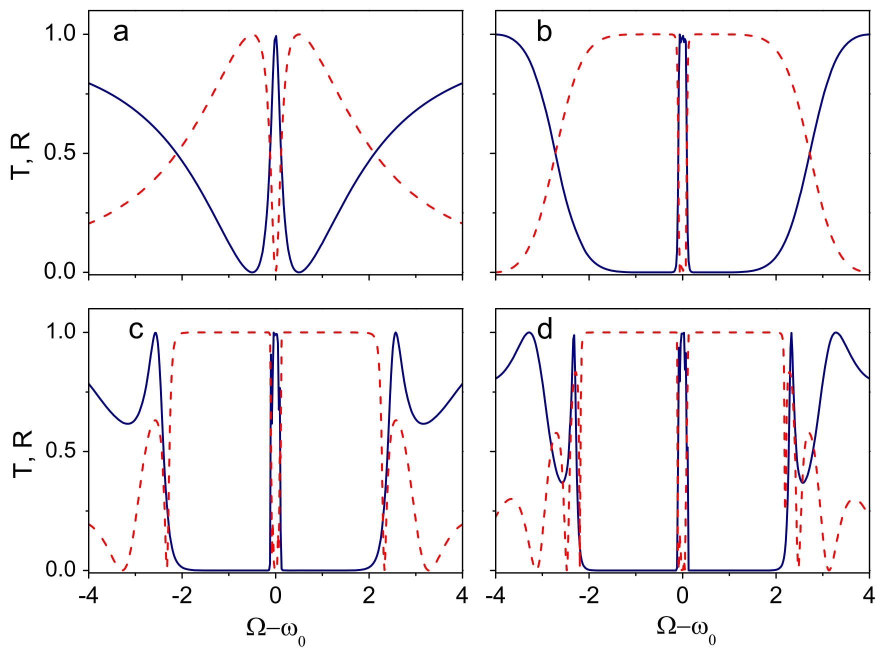

Fig. 3. Transmission (blue solid line) and reflection (red dash line) versus detuning for $\Delta=0.5$, for (a) $N=1$, (b) $N=3$, (c) $N=5$ and (d) $N=7$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

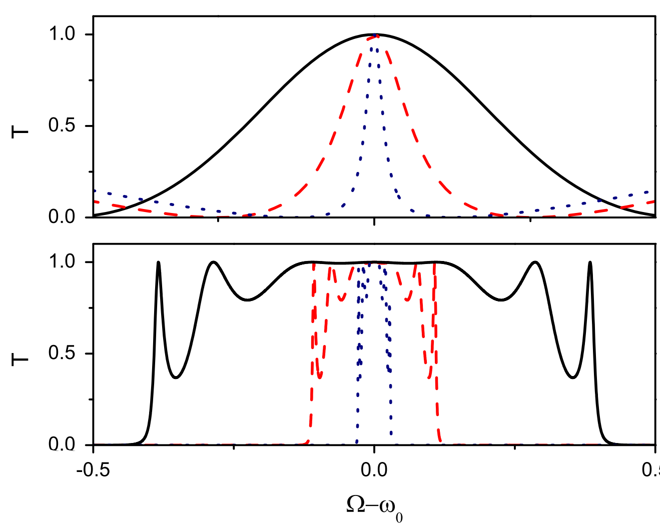

Fig. 4. Transmission versus Ω for $\Delta=1$ (black solid line) $\Delta=0.5$ (red dash line) and $\Delta=0.25$ (blue dotted line), for $N=1$ (upper panel) $N=7$ (lower panel). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

is formed by the indirect coupling between the levels of the atom in each cavity. In addition, the band edges can be controlled by the degree of detuning of the level of the atoms. We have discussed the analogy between this phenomenon with the Fano and Dicke effects. This set-up seems a suitable system to study these effects in experiments on single photon transport.

Acknowledgments

The authors would like to thank financial support from FONDECYT under Grants nos. 1080660 and 1140567, and CONICYT ACT 1204.

References

[1] I. Buluta, F. Nori, Science 326 (2009) 108.
[2] I. Buluta, S. Ashhab, F. Nori, Rep. Prog. Phys. 74 (2011) 104401.
[3] J.Q. You, F. Nori, Phys. Today 58 (11) (2005) 42.
[4] J.Q. You, F. Nori, Nature (London) 474 (2011) 589.
[5] P.D. Nation, J.R. Johansson, M.P. Blencowe, F. Nori, Rev. Mod. Phys. 84 (2012) 1.
[6] S.N. Shevchenko, S. Ashhab, F. Nori, Phys. Rep. 492 (2010) 1.
[7] K.J. Vahala, Nature (London) 424 (2003) 839.
[8] N.-C. Kim, J.-B. Li, Z. Yang, Z.-H. Hao, Q.-Q. Wang, Appl. Phys. Lett. 97 (2010) 061110.
[9] A. Wallraff, D.I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S.M. Girvin, R.J. Schoelkopf, Nature 436 (2005) 87.
[10] I.I. Smolyaninov, A.V. Zayats, A. Gungor, C. Davis, Phys. Rev. Lett. 88 (2002) 187402.
[11] A.L. Falk, F.H.L. Koppens, C.L. Yu, K. Kang, N. de Leon Snapp, A.V. Akimov, M.-H. Jo, M.D. Lukin, H. Park, Nat. Phys. 5 (2009) 807.
[12] K.M. Birnbaum, A. Boca, R. Miller, A.D. Boozer, T.E. Northup, H.J. Kimble, Nature 431 (2005) 163.
[13] D.E. Chang, A.S. Sorensen, E.A. Demler, M.D. Lukin, Nat. Phys. 3 (2007) 807.
[14] J. Kim, O. Benson, H. Kan, Y. Yamamoto, Nature 397 (1999) 500.
[15] B. Dayan, A.S. Parkins, T. Aoki, E.P. Ostby, K.J. Vahala, H.J. Kimble, Science 319 (2007) 1062.
[16] L. Zhou, H. Dong, Y.X. Liu, C.P. Sun, F. Nori, Phys. Rev. A 78 (2008) 063827.
[17] L. Zhou, S. Yang, Y.X. Liu, C.P. Sun, F. Nori, Phys. Rev. A 80 (2009) 062109.
[18] J.Q. Liao, Z.R. Gong, L. Zhou, Y.X. Liu, C.P. Sun, F. Nori, Phys. Rev. A 81 (2010) 042304.
[19] H. Ditlbacher, A. Hohenau, D. Wagner, U. Kreibig, M. Rogers, F. Hofer, F.R. Aussenegg, J.R. Krenn, Phys. Rev. Lett. 95 (2005) 257403.
[20] S. Zhu, Q. Fang, M.B. Yu, G.Q. Lo, D. L Kwong, Opt. Express 17 (2009) 20891.
[21] S. Mookherjea, J.S. Park, S.-H. Yang, P.R. Bandaru, Nat. Photon. 2 (2008) 90.
[22] L. Zhou, Z.R. Gong, Y.X. Liu, C.P. Sun, F. Nori, Phys. Rev. Lett. 101 (2008) 100501.
[23] Z.R. Gong, H. Han, L. Zhou, C.P. Sun, Phys. Rev. A 78 (2008) 053806.
[24] H. Ian, Y.X. Liu, F. Nori, Phys. Rev. A 81 (2010) 063823.
[25] Y. Chang, Z.R. Gong, C.P. Sun, Phys. Rev. A 83 (2011) 013825.
[26] U. Fano, Phys. Rev. 124 (1961) 1866.
[27] B.-B. Li, Y.-F. Xiao, C.-L. Zou, Y.-C. Liu, X.-F. Jiang, Y.-L. Chen, Y. Li, Q. Gong, Appl. Phys. Lett. 98 (2011) 021116.
[28] S.E. Harris, Phys. Today 50 (1997) 36.
[29] Q. Li, Z. Zhang, J. Wang, M. Qiu, Y. Su, Opt. Express 17 (2009) 933.
[30] R.H. Dicke, Phys. Rev. 89 (1953) 472.
[31] J. Pan, S. Sandhu, Y. Huo, N. Stuhrmann, M.L. Povinelli, J.S. Harris, M.M. Fejer, S. Fan, Phys. Rev. B 81 (1982) 041101 (R).
[32] P.A. Frasert, S.K. Burley, Eur. J. Phys. 3 (1982) 230.