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Coherent tunneling by adiabatic process in a four-waveguide optical coupler

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ABSTRACT

We numerically simulate Schrödinger-like paraxial wave equation of a four-waveguide system. The coherent tunneling by adiabatic passage in a four-waveguide optical coupler is analyzed by borrowing the dressed state theory of coherent atom system. We discuss the optical coupling mechanism and coupling efficiency of light energy in both intuitive and counterintuitive tunneling schemes and analyze the threshold condition from adiabatic to non-adiabatic regimes in intuitive scheme. The results show that this coupler can be used as power splitter under certain conditions.

1. Introduction

Although equations that describe the dynamics in quantum and classical physics are different, there are many similarities between quantum and classical physical phenomena [1]. These analogies have been exploited to mimic microscopic quantum effect at macroscopic level, including using engineered photonic structures to mimic Zener tunneling [2] and quantum effects such as Bloch oscillations [3]; using waveguide directional coupler to mimic the generation of discrete solitons [4], Aharonov–Bohm effect [5], Bloch oscillations [6], discrete Talbot effect [7], adiabatic mode conversion [8,9], Landau–Zener dynamics [10], Stimulated Raman adiabatic passage (STIRAP) [11,12], to name a few.

As an important tool which can manipulate the quantum structures, STIRAP is widely used in coherent atomic excitation [13,14], optical switching [15], quantum information processing [16], waveguide optics [12,17,18], generation of short terahertz pulses [19], etc. STIRAP is based on the dark state of the system. In a three-waveguide optical coupler, the coherent tunneling adiabatic passage is based on the existence of one dark state[11]; in a four-waveguide optical coupler, however, this adiabatic passage is base on the existence of two dark states, which is the linear superposition of the initial and target states. In 2012, Xin–Ding Zhang group discussed the non-Abelian geometric phase and the light energy adiabatic passage in a four-waveguide system which is composed of a central straight waveguide, a similar parabolic waveguide and two cosine-type waveguides [20]. In 2015, this group further discussed the Abelian geometric phase in a waveguide system which is composed of a central straight waveguide and three similar parabolic waveguides [21]. But they only discussed the structure in counterintuitive tunneling scheme, failing to elaborate on the adiabatic condition, the adiabatic conversion efficiency versus the distance between the upper and central waveguide, and the way to obtain adiabatic conversion condition in intuitive tunneling scheme.

In this paper we discuss a four-waveguide coupler which is composed of two straight waveguides and two similar parabolic waveguides. The central and upper waveguide are straight, the left and right waveguide are similar parabolic. By analogy to the STIRAP of the four level atoms, the coherent tunneling in a four-waveguide coupler is studied, progress in theoretical and experimental studies of multi waveguide array couplers can assure the rationality of the waveguide characteristic parameters selected in this paper [11,22]. When light is introduced from the left waveguide, the left waveguide is coupled with the center waveguide before the right one, therefore creating the intuitive tunneling scheme. On the other hand, when the right waveguide is coupled with the center waveguide before the left one, the counterintuitive tunneling scheme is created. By using the theory of dressed state we analyze the dynamic process of the energy adiabatic transfer in a four-waveguide coupler, discussing the energy coupling efficiency and adiabatic conditions in different tunneling rate scheme. The upper waveguide is straight, so such coupler is more easily to be manufactured than the structure in reference [21] and can be used as power splitter under certain conditions.

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2. Theoretical modeling

Waveguide structures are shown in Fig. 1(a). The left waveguide (L) and the right waveguide (R) are similar parabolic, which are located in the YZ plane with the central straight waveguide (C), the upper straight waveguide (U) and the central waveguide in the XZ plane. Light propagates along the z direction. Direct tunneling among the left, right and upper channels can be fully ignored because of their far distance. They can only be coupled with the intermediate waveguide. The refractive index profile $n_w(X, Y)$ of the waveguide is the same as in reference [11]. Fig. 1(b) is a section of the four waveguide coupler in the XY plane. Ω_s, Ω_p and Ω_q are the tunneling rate between the adjacent waveguide. The four set of contour lines is the distribution of the refractive index function in the XY plane of the four waveguides.

The monochromatic light propagating in the waveguide directional coupler along Z direction can be described by Schrödinger-like paraxial wave equation [11]

$$i\hbar\frac{\partial\psi}{\partial Z} = -\frac{\hbar^2}{2n_s}\nabla^2_{X,Y}\psi + V(X - X_0(Z), Y - Y_0(Z))\psi \tag{1}$$

In Eq. (1), $\hbar = \lambda/(2\pi)$ is reduced wavelength, $V(X, Y, Z)$ is spatial potential distribution function,

$$V(X, Y, Z) = [n_s^2 - n^2(X, Y, Z)]/(2n_s) \approx n_s - n(X, Y, Z),$$

n_s is substrate refractive index, $n(X, Y, Z)$ is the refractive distribution of four-waveguide directional coupler, $|\psi|^2$ denotes the beam intensity in the coupler.

By introducing new variables, $x = X - X_0$, $y = Y - Y_0$, $z = Z$, after the gauge transformation:

$$\psi = \phi\,\exp\left\{ i(n_s/\hbar)[\dot X_0(Z)x + \dot Y_0(Z)y] + i(n_s/2\hbar)\int^Z d\xi\left[\dot X_0^2(\xi) + \dot Y_0^2(\xi)\right]\right\},$$

$\dot X_0$, $\dot Y_0$ indicates the derivative with respect to z, and substitute the new variables into Eq. (1), we can get:

$$i\hbar\frac{\partial\phi}{\partial z} = H\phi \tag{2}$$

The Hamiltonian $H = H_0 + H'$, where $H_0 = -\frac{\hbar^2}{2n_s}\nabla^2_{x,y} + V(x, y)$, $H' = n_s\left[\ddot X_0(z)\hat e_x + \ddot Y_0(z)\hat e_y\right]\cdot\vec r$,

if we expand the state vector $\phi(x, y, z)$ as the linear superposition of the basis vector $w_n(x, y)$.

$$\phi(x, y, z) = \sum_n C_n(z)\exp(-i\omega_n z)w_n(x, y) \tag{3}$$

and substitute Eq. (3) into Eq. (2), under the general hypotheses of nearest-neighbor approximation and weak waveguide coupling, we get:

$$\frac{d}{dz}C(z) = -iW(z)C(z) \tag{4}$$

where we have set

$$W(z) = \begin{pmatrix} 0 & \Omega_p(z) & 0 & 0 \\ \Omega_p^*(z) & 0 & \Omega_s(z) & \Omega_q(z) \\ 0 & \Omega_s^*(z) & 0 & 0 \\ 0 & \Omega_q^*(z) & 0 & 0 \end{pmatrix} \tag{5}$$

The state vector $C(z) = [C_L(z), C_C(z), C_R(z), C_U(z)]^T$ is complex amplitude of light field in four waveguides. $|C_n(z)|^2$ indicates the variation of beam intensity with the propagating distance z in the four waveguides. $\Omega_i(z)\,(i = p, s, q)$ is the tunneling rate between the adjacent waveguide.

Eq. (4) shows the propagation of light in a four-waveguide coupler system, which is similar to the STIRAP occurring in the interaction between the three pulsed laser fields and a four-state atomic system. The Rabi frequency of the pump pulse and Stokes pulse correspond to the tunneling rate between the adjacent waveguides, when light comes from the left waveguide in intuitive tunneling scheme ($\delta < 0$), or in counterintuitive tunneling scheme ($\delta > 0$). The eigenvalue of the tunneling rate matrix are: $\lambda_1(z) = \lambda_2(z) = 0$, $\lambda_3(z) = -\Omega_0(z)$ $\lambda_4(z) = +\Omega_0(z)$, $\Omega_0(z) = \sqrt{|\Omega_p(z)|^2 + |\Omega_s(z)|^2 + |\Omega_q(z)|^2}$. The two null-eigenvalues are degenerated, STIRAP occurs in these two dark states.

Defining the distance-dependent mixing angle $\theta(z)$ and $\varphi(z)$ as

$$\tan\theta(z) = \frac{|\Omega_p(z)|}{|\Omega_s(z)|} \tag{6}$$

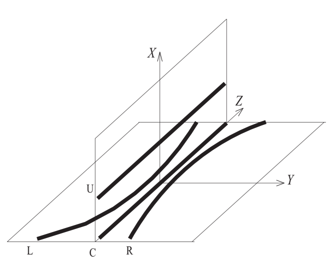
$$\tan\varphi(z) = \frac{|\Omega_q(z)|}{\sqrt{|\Omega_p(z)|^2 + |\Omega_s(z)|^2}} \tag{7}$$

The eigenvectors of the dressed state are

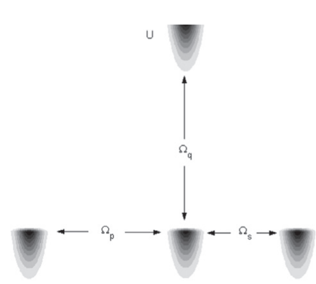
$$\Phi_1(z) = \begin{pmatrix} \cos\theta(z) \\ 0 \\ -\sin\theta(z) \\ 0 \end{pmatrix} \quad \Phi_2(z) = \begin{pmatrix} \sin\varphi(z)\sin\theta(z) \\ 0 \\ \sin\varphi(z)\cos\theta(z) \\ -\cos\varphi(z) \end{pmatrix} \tag{8}$$

$$\Phi_3(z) = \frac{1}{\sqrt{2}}\begin{pmatrix} \cos\varphi(z)\sin\theta(z) \\ -1 \\ \cos\varphi(z)\cos\theta(z) \\ \sin\varphi(z) \end{pmatrix} \quad \Phi_4(z) = \frac{1}{\sqrt{2}}\begin{pmatrix} \cos\varphi(z)\sin\theta(z) \\ 1 \\ \cos\varphi(z)\cos\theta(z) \\ \sin\varphi(z) \end{pmatrix} \tag{9}$$

In the adiabatic limit, $d\theta(z)/dz$ and $d\varphi(z)/dz$ are both small, the non-adiabatic coupling of dark states Φ_1 or Φ_2 to the dress states Φ_3 or Φ_4 can be ignored. If the system is initially in a dark state, it



(a). Schematic of the four-waveguide directional coupler.



(b). Refractive index profile $n_w(X,Y)$ of the waveguide and the tunneling pattern between different waveguide.

Fig. 1. (a) Schematic of the four-waveguide directional coupler. (b) Refractive index profile $n_w(X, Y)$ of the waveguide and the tunneling pattern between different waveguide.

always stays in the dark state during the adiabatic process. During the evolution, the states Φ_3 and Φ_4 have nothing to do with the adiabatic process. Distance-dependent state vector $\phi(z)$ can be expressed by degenerated adiabatic (dark) states Φ_1 and Φ_2. Let us suppose that the initial state vector $\phi(z)$ is either one of the dark state, $\phi_a(-\infty) = \Phi_a(-\infty)$, $a = 1$ or 2. At later distance we can expand the vector $\phi(z)$ as:

$$\phi_a(z) = \sum_b^N B_{ab}(z)\Phi_b(z)\ (a, b = 1, 2) \tag{10}$$

N is the number of the dark states. $B_{ab}(z) = \exp[i\varphi_{ab}(z)]$ are the elements of a 2×2 Abelian phase matrix. Substituting this equation into Schrödinger equation, taking the scalar product with dark states, because the dark states are orthonormal, we find immediately that $B_{ab}(z)$ obeys the equation

$$\frac{d}{dz}B_{ba}(z) = -\sum_c A_{bc}(z)B_{ca}(z)\ (a, b, c = 1, 2) \tag{11}$$

where $A_{bc}(z) = \left\langle \Phi_b(z)\middle|\frac{d}{dz}\middle|\Phi_c(z)\right\rangle$. Through direct integrating Eq. (11), we get the form, where $\mathcal{P}$ is the distance-ordered product. For a two-state system the simple formula can be get after the calculation

$$B(z) = \begin{bmatrix} \cos\gamma(z) & \sin\gamma(z) \\ -\sin\gamma(z) & \cos\gamma(z) \end{bmatrix} \tag{12}$$

where $\gamma(z) = \int_{-\infty}^{z} \frac{d\theta(z')}{dz'}\sin\varphi(z')dz'$ is Abelian geometric phase [21]. The output terminal of the four-waveguide optical coupler is

$$B(\infty) = \begin{bmatrix} \cos\gamma_f & \sin\gamma_f \\ -\sin\gamma_f & \cos\gamma_f \end{bmatrix} \tag{13}$$

$$\gamma_f = \int_C \frac{\Omega_q}{(\Omega_p^2 + \Omega_s^2)\sqrt{\Omega_q^2 + \Omega_p^2 + \Omega_s^2}}(\Omega_s d\Omega_p - \Omega_p d\Omega_s) \tag{14}$$

Considering dark state Φ_1 and Φ_2, if the initial state lies in the dark state Φ_1, the beam intensity can be only transferred between the left and right waveguide. And if the initial state lies in dark state Φ_2, the beam intensity can be transferred among the left, right and upper waveguide. In either case the distribution of light energy is small in the middle waveguide.

3. Coherent tunneling by adiabatic process

3.1. Counterintuitive tunneling scheme

If the incident light comes from the left waveguide in the counterintuitive tunneling scheme, Ω_s precedes Ω_p. Since the upper waveguide U is parallel to the central waveguide C, the tunneling rate $\Omega_q(z) = \Omega_q$ is constant. The tunneling rate $\Omega_p(z)$ and $\Omega_s(z)$ can be written as [20]:

$$\Omega_p(z) = n_s\eta\frac{\partial^2 d_L(z)}{\partial z^2}\exp[i(\omega_L - \omega_C)z];\ \Omega_s(z) = n_s\eta\frac{\partial^2 d_R(z)}{\partial z^2}\exp[i(\omega_R - \omega_C)z] \tag{15}$$

The left and right channel distance relative to the propagating distance z has the form:

$$d_L(z) = d_{\min} + \sum_{k=0}^{\infty}\frac{(-1)^k(z-\delta)^{2k+2}}{3900(2k+1)(2k+2)k!\eta L^{2k}};$$

$$d_R(z) = d_{\min} + \sum_{k=0}^{\infty}\frac{(-1)^k(z+\delta)^{2k+2}}{3900(2k+1)(2k+2)k!\eta L^{2k}}$$

$d_{\min} = 8.2\mu\text{m}$ is the nearest distance between the curving and straight waveguide. Substituting the expression $d(z)$ into Eq. (15) and taking into account the beam propagating in coupler with the relationship $\omega_C = \omega_R = \omega_L$, we get the Gaussian forms:

$$\Omega_p(z) = \Omega_p\exp[-(z-\delta)^2/L^2];\ \Omega_q(z) = \Omega_q;\ \Omega_s(z) = \Omega_s\exp[-(z+\delta)^2/L^2] \tag{16}$$

2δ is the distance along the z direction between the points which is the shortest distance from the left and right waveguides to the central one. L is the width of the tunneling rate $\Omega(z)$.

To determine the mixing angle $\theta(z)$ and $\varphi(z)$, we use the Eqs. (6) and (7). As the initial values $\theta(-\infty) \approx 0$ and $\varphi(-\infty) \approx \pi/2$, the dark states have the components $\Phi_1(-\infty) = (1, 0, 0, 0)^T$ and $\Phi_2(-\infty) = (0, 0, 1, 0)^T$. The state vector $\phi(-\infty) = \Phi_1$, With the increase of the coupling distance, at any position in the waveguide coupler, From Eqs. (10) and (12), we get $\phi(z) = (\cos\gamma\cos\theta + \sin\gamma\sin\varphi\sin\theta)C_L + (-\cos\gamma\sin\theta + \sin\gamma\sin\varphi\cos\theta)C_R - \sin\gamma\cos\varphi C_U$, after the interaction $\theta(+\infty) \approx \pi/2$, $\varphi(+\infty) \approx \pi/2$. $\phi(+\infty) = \sin\gamma_f C_L - \cos\gamma_f C_R$. According to Eq. (8), we can obtain the light intensity in four waveguides as $I_L(\infty) = \sin^2\gamma_f$, $I_C(\infty) = 0, I_R(\infty) = \cos^2\gamma_f$, $I_U(\infty) = 0$. If the upper waveguide is far from the central one, then Ω_q is less than Ω_p and Ω_s. The light power is transferred to the right channel via the adiabatic process. Through numerical calculation of the Eq. (2), we get the gray-scale plot of light transmission in coupler along the direction of light propagation in the counterintuitive tunneling scheme. It is shown in Fig. 2(a). In Eq. (2) the 2D index profile $n_w(x, y)$ is the form [11]

$$n_w(x, y) \approx n_s + \Delta n\left[g\left(x - \frac{a}{2}\right) + g\left(x + \frac{a}{2}\right)\right]f(y),$$

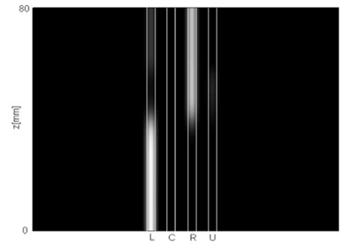
where the substrate refractive index $n_s = 1.5218$, the peck index change $\Delta n = 0.0148$,

$$g(x) = [erf((x+w)/D_x) - erf((x-w)/D_x)]/[2erf(w/D_x)],$$

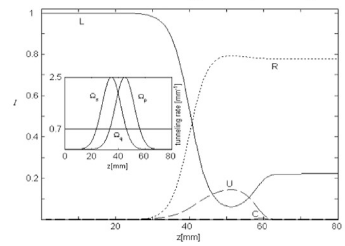
$f(y) = 1 - erf(-y/D_y)$, the waveguide width $2w$, D_x and D_y are diffusion lengths in x and y direction respectively. Eq. (2) shows that the similar parabolic waveguide appears to be straight by introducing new variables x and y [10]. Fig. 2(a) only shows the laws of light propagation in couplers, and does not represent the spatial structure of waveguides. Fig. 2(b) is the fractional beam intensity trapped in the four-waveguide directional coupler versus the propagation distance of light, and the small graph in Fig. 2(b) is the behavior of the tunneling rates along the directional coupler. Here Ω_s precedes Ω_p, but Ω_p precedes Ω_s in the intuitive tunneling scheme, when other parameters are the same, except for δ.

Because Ω_q is smaller than Ω_p and Ω_s, $\varphi(-\infty) \approx \pi/2$, $\varphi(+\infty) \approx \pi/2$. Through numerical simulations of the Eq. (14), we can get the accumulated Abelian geometric phase after light propagating along z direction, as shown in Fig. 3, it indicates that at the output planes the Abelian geometric phase $\gamma_f \approx 0.15\pi$, so $I_L(\infty) = \sin^2\gamma_f \approx 0.21, I_R(\infty) = \cos^2\gamma_f \approx 0.79$, as indicated in Fig. 2.

When the upper waveguide is far from the central one, $\Omega_q \approx 0$, Abelian geometric phase $\gamma \approx 0$, and $\phi(z) = \cos\theta C_L - \sin\theta C_R$. Then light transfers between the left and right channel. When the upper waveguide is close to the central one, Ω_q is bigger than Ω_p and Ω_s. Fig. 3 shows that under the case of $\gamma_f \approx \pi/2$, $I_L(\infty) = \sin^2\gamma_f = 1$, $I_R(\infty) = \cos^2\gamma_f = 0$, light energy will be located in the left channel.



(a). Grey-scale plot of beam intensity evolution in a four-waveguide directional coupler for counterintuitive tunneling rate, calculation parameters: $\Delta n = 0.0148$, $D_x = 4.6\mu m$, $D_y = 4.1\mu m$, $w = 2.5\mu m$, $\lambda = 980nm$.



(b). Fractional beam intensity evolution in a four-waveguide directional coupler for counterintuitive tunneling rate.

Fig. 2. (a) Grey-scale plot of beam intensity evolution in a four-waveguide directional coupler for counterintuitive tunneling rate, calculation parameters: $\Delta n = 0.0148, D_x = 4.6\mu m, D_y = 4.1\mu m, w = 2.5\mu m, \lambda = 980nm$. (b) Fractional beam intensity evolution in a four-waveguide directional coupler for counterintuitive tunneling rate.

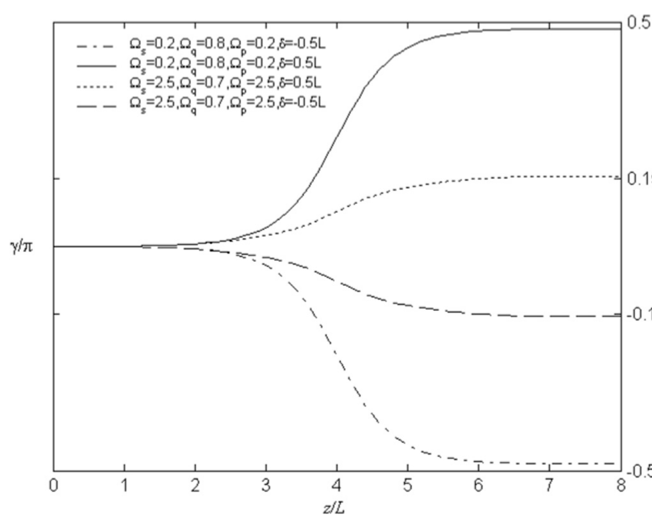


Fig. 3. Accumulated geometric phase γ/πversus z/L.

3.2. Intuitive tunneling scheme

For the intuitive tunneling scheme, Ω_p precedes Ω_s, the initial values at the input plane $\theta(-\infty) = \pi/2, \varphi(-\infty) = \pi/2$, the components of the dark state $\Phi_1(-\infty) = (0, 0, -1, 0)^T$, $\Phi_2(-\infty) = (1, 0, 0, 0)^T$, the state vector $\phi(-\infty) = \Phi_2$. With the increasing of the propagation distance, Ω_s and Ω_p increase at the beginning and then decreases. After the coupling interaction, $\theta(+\infty) \approx 0, \varphi(+\infty) \approx \pi/2$, the state is as: $\Phi_1(\infty) = (1, 0, 0, 0)^T$ and $\Phi_2(\infty) = (0, 0, 1, 0)^T$. From Eqs. (10) and (13), we get $\phi(\infty) = -\sin\gamma_f C_L + \cos\gamma_f C_R$. At the output terminal of the four-waveguide optical coupler, the light intensity in four channels are: $I_L(\infty) = \sin^2\gamma_f$, $I_C(\infty) = 0$, $I_R(\infty) = \cos^2\gamma_f$, $I_U(\infty) = 0$. If the upper waveguide is far from the central one, then Ω_q is smaller than Ω_p and Ω_s. Fig. 3 shows that the geometric phase $\gamma_f \approx -0.15\pi$, so $I_L(\infty) = \sin^2\gamma_f \approx 0.21$, $I_R(\infty) = \cos^2\gamma_f \approx 0.79$. The results are same to the counterintuitive tunneling scheme. Through numerical calculation of the Eq. (2), we get the gray-scale plot of light transfer in the coupler along the direction of propagation in the intuitive tunneling scheme. They show in Fig. 4(a), which other parameters of the waveguide are the same with counterintuitive tunneling scheme, except for $\delta = -5$mm. Fig. 4(b) is the fractional beam intensity evolution versus propagation distance.

When light is at any position in the waveguide coupler, we get

$$\phi(z) = (-\sin\gamma\cos\theta + \cos\gamma\sin\varphi\sin\theta)C_L + (\sin\gamma\sin\theta + \cos\gamma\sin\varphi\cos\theta)C_R - \cos\gamma\cos\varphi C_U,$$

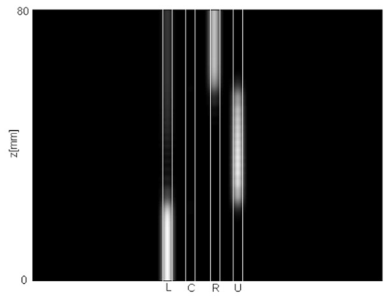
In the middle of the coupler, Ω_q is smaller than Ω_p and Ω_s, so φ_{middle} and γ_{middle} are both small, $I_U = \cos^2\gamma\cos^2\varphi$, as shown in Fig. 4.

In the intuitive tunneling scheme, when upper channel is close to central one, Ω_q is bigger than Ω_p and Ω_s. Fig. 3 shows $\gamma_f \approx -\pi/2$, at the output plane of the coupler $I_L(\infty) = \sin^2\gamma_f = 1$, $I_R(\infty) = \cos^2\gamma_f = 0$, and light intensity still lies in left channel, which has the same results with the counterintuitive tunneling scheme.

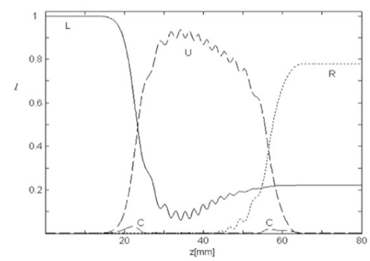
If there is no upper channel, the fractional light power transferred among left, central and right channels, finally the light is trapped in the central one [11]. But in our four-waveguide coupler scheme, the upper waveguide acts as an auxiliary waveguide. It is able to realize the adiabatic transferring of light energy. In the first step, the light transfers from the left channel to the upper one, in which the tunneling rate Ω_q is similar to Stokes pulses and the tunneling rate Ω_p is similar to pump pulses. Then in a second step, the light transfers from the upper channel to the right one, but this process is not STIRAP. For the four waveguide coupler system, regardless of the counterintuitive or intuitive scheme, using the parameters we choose, nearly 80% energy of light is located in the right channel and 20% energy of light is in the left channel ultimately. Completely energy transferred from left to right channel can take place in counterintuitive scheme while in intuitive scheme complete energy transferred can not occur. This result is different from the three waveguide coupler.

3.3. Coupling efficiency

In the counterintuitive tunneling scheme ($\delta = 5$mm), when the



(a). Grey-scale plot of beam intensity evolution in a four-waveguide directional coupler for intuitive tunneling rate.



(b). Fractional beam intensity evolution in a four-waveguide directional coupler for intuitive tunneling rate.

Fig. 4. (a) Grey-scale plot of beam intensity evolution in a four-waveguide directional coupler for intuitive tunneling rate. (b) Fractional beam intensity evolution in a four-waveguide directional coupler for intuitive tunneling rate.

distance between left, right and upper waveguide and the central one is changed from far to near, the tunneling rates among the waveguides increase from zero. Fig. 5 shows the variation of the beam intensity in right channel with the tunneling rate. And the eight curves from top to bottom indicate that the tunneling rate between the upper and central waveguides increases from zero. As the distance between the waveguides decreases, the tunneling rate increases, and vice versa. From Fig. 5 we can see that if the distance between the upper and central channel is fixed, and the distance of the left, right channel to the central one reduces, the

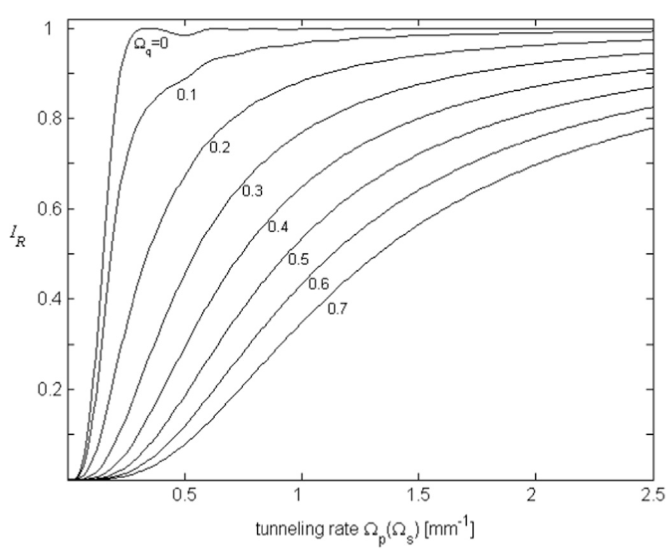


Fig. 5. Beam intensity in right waveguide versus tunneling rate for counterintuitive scheme. Different tunneling rate Ω_q represents the different distance between the upper and central waveguides, the abscissa indicates the tunneling rate between the left (right) and middle waveguides. As the distance between the waveguides decreases, the tunneling rate increases, and vice versa.

beam intensity in right channel enhances and its coupling efficiency increases. If the distance from the left, right channel to the central one is fixed, and the distance between the upper and central channel decreases, the beam intensity in right channel is weakened and its coupling efficiency decreases. In the case Ω_q=0, there is only one nondegenerate dark state, and the adiabatic process is proportional to $e^{-\Omega_q L}$, where $\Omega_q L$ is the effective tunneling rate area. Finally the light power in the left waveguide is completely transferred to the right one, in the case Ω_q>0, there are two degenerate dark states, the light power evolution is extremely different, when the upper channel is close to the central one, and the adiabatic process is not proportional to $e^{-\Omega_q L}$ anymore.

In the intuitive tunneling scheme ($\delta = -5$mm) and adiabatic limit, changing the distance between the channels in the same way, we find that the variety of the coupling efficiency is the same as in the counterintuitive tunneling scheme. In order to carry out comparative analysis, we plot the beam intensity in right waveguide versus tunneling rate for both counterintuitive and intuitive tunneling schemes in one picture, as shown in Fig. 6.

When $\Omega_q \geq 0.7$, whether in the intuitive tunneling scheme ($\delta = -5mm$) or the counterintuitive tunneling scheme ($\delta = 5$mm), the beam intensity in right waveguide and the coupling efficiency is the same. But when Ω_q<0.7, coherent tunneling by adiabatic process can take place in counterintuitive tunneling scheme, while in intuitive tunneling scheme, the adiabatic conditions are destroyed, so the evolution is not adiabatic.

In Fig. 6, the three intersecting points formed by the dashed line and three solid lines ($\Omega_q \geq 0.7$) indicate that the coupling efficiency of the right waveguide is 50%, since $\Omega_q \geq 0.7$ the tunneling process is adiabatic. As long as we select the value of the tunneling rate corresponding to the crossing points, for example $\Omega_p = \Omega_s = 1.9$, $\Omega_q = 1.0$. In the adiabatic limit, either the counterintuitive or intuitive tunneling scheme, the light energy in the left and right waveguides is half of the total energy, so such four-waveguide optical coupler can be used as a power splitter. Of

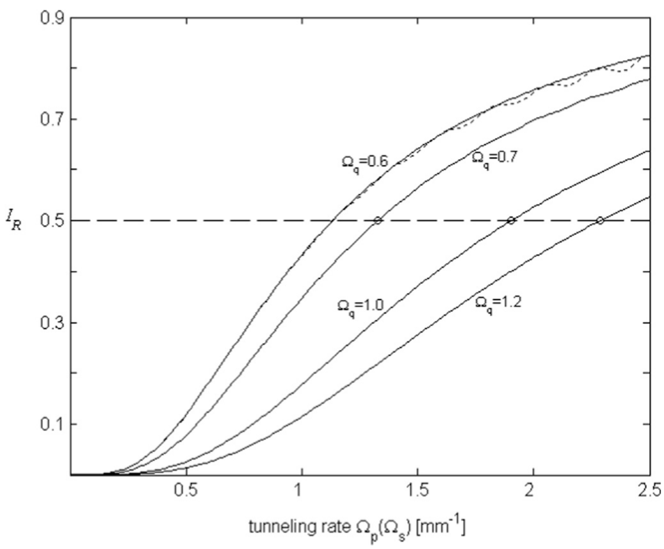


Fig. 6. Variation of the beam intensity in right waveguide with tunneling rate for counterintuitive and intuitive schemes. When $\Omega_q = 0.6$, the solid line corresponds to counterintuitive scheme, the dotted line refers to intuitive scheme. When $\Omega_q = 0.7$, 1.0 and 1.2, the solid and dotted curve coincide with each other, the two schemes are indistinguishable.

course, in addition to the above three intersecting points, there are a lot of potential intersection points can be selected if we change the value of the tunneling rate Ω_p, Ω_s and Ω_q, the counterintuitive coupling structure has more parameters to select compared with the intuitive scheme, for the counterintuitive scheme, when $0 \le \Omega_q < 0.7$, the adiabatic process can still occur.

3.4. Further discussion of adiabatic conditions

When the incident light is from the left waveguide and the tunneling rate $\Omega_q = 0$, then the coupler turned into a three-waveguide structure, the coherent tunneling process can occur in counterintuitive tunneling scheme while this is not the case in intuitive tunneling scheme. When $\Omega_q > 0$ and it is small, for counterintuitive tunneling scheme, the initial state vector $\phi(-\infty) = \Phi_1$, the energy transferring process is determined by $\theta'(z)$, but for intuitive tunneling scheme and the initial state vector $\phi(-\infty) = \Phi_2$, the energy transferring process is determined by $\theta'(z)$ and $\varphi'(z)$ together. If the tunneling rate is small, and $\varphi'(z)$ is big, the adiabatic conditions cannot be met. The light energy oscillates rapidly in the left, central and right channels, as the tunneling rate Ω_q gets bigger, $\varphi'(z)$ becomes smaller and the adiabatic condition is again satisfied. Then the adiabatic transfer process is carried out.

Under the adiabatic condition, according to the Eqs. (8), (10) and (12), In both the counterintuitive and intuitive tunneling schemes, we can get the same results: $I_L(\infty) = \sin^2 \gamma_f$, $I_C(\infty) = 0$, $I_R(\infty) = \cos^2 \gamma_f, I_U(\infty) = 0$, note that the light power in central waveguide (C) is small during the tunneling process, we should have the same light power in the left (L) and right (R) waveguides in both schemes, in the counterintuitive scheme adiabatic process can always occur owing to the small $\theta'(z)$, therefore when the light energy in right waveguide is different, the adiabatic condition is no longer valid for intuitive scheme, see Fig. 6, for $\Omega_q \ge 0.7$, the light power in right channel is same for two schemes, the solid and dotted curve coincide with each other, however, when $\Omega_q < 0.7$, with the decrease of the tunneling rate Ω_p, two cases give different results. difference of the light intensity in right waveguide gets bigger and bigger in two schemes, the solid line and the dotted line begin to grow apart, for the counterintuitive scheme, the solid line has a monotonic dependence of the light power in right waveguide on the tunneling rate, see Fig. 5, but for the intuitive scheme the dotted line has the oscillation characteristic, the smaller the tunneling rate Ω_q is, the more obvious the oscillation is, $\Omega_q = 0.7$ is the threshold from adiabatic to non-adiabatic domains. So for the four-waveguide directional coupler, the adiabatic conditions are different in the intuitive and counterintuitive tunneling scheme. This condition is stricter in the former, as besides $\theta'(z) \approx 0$, $\varphi'(z) \approx 0$, we need to add a constrained condition: $\Omega_q \ge 0.7$.

4. Conclusion

In this paper, based on the analogue between the coherent tunneling theory in a four-waveguide optical coupler and the interaction theory between light and atoms, we analyze the coherent adiabatic process in the four-waveguide coupler using the theory of dressed state. The results show that the transfer of light between the waveguides depends on the Abelian geometric phase in parameter space and has nothing to do with the dynamics of the system, Under the adiabatic condition, by analyzing the numerical simulation results of the Schrödinger-like paraxial wave equation, we find that in both counterintuitive and intuitive tunneling schemes, if the tunneling rate parameters are identical, although the adiabatic transferring process of light energy is not the same, as shown in Figs. 2 and 4, the energy is eventually coupled to the right channel and the coupling efficiency is the same, the energy coupling efficiency of the adiabatic transferring process is independent of the energy coupling mechanism. In the adiabatic limit, if the coupling efficiency of the right waveguide is equal to 50%, in both of the schemes this coupler can be used as a beam splitter. As the upper waveguide is straight, such scheme is more feasible in designing quantum logic gates and optical switches,

The adiabatic condition is different in the two schemes, in counterintuitive tunneling scheme, adiabatic tunneling process can always be brought about, while in intuitive tunneling scheme, it can only be brought about under certain conditions.

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References

[1] D. Dragoman, M. Dragoman, Quantum Classical Analogies, Springer, Berlin, 2004.
[2] M. Ghulinyan, C.J. Oton, Z. Gaburro, L. Pavesi, C. Toninelli, D.S. Wiersma, Phys. Rev. Lett. 94 (2005) 127401.
[3] H. Trompeter, T. Pertsch, F. Lederer, D. Michaelis, U. Streppel, A. Bräuer, U. Peschel, Phys. Rev. Lett. 96 (2006) 023901.
[4] A. Szameit, D. Blomer, J. Burghoff, T. Schreiber, T. Pertsch, S. Nolte, A. Tünnermann, Opt. Express 13 (2005) 10552.
[5] K. Fang, Z. Yu, S. Fan, Phys. Rev. Lett. 108 (2012) 153901.
[6] N. Chiodo, G.D. Valle, R. Osellame, S. Longhi, G. Cerullo, R. Ramponi, P. Laporta, Opt. Lett. 31 (2006) 1651.
[7] R. Iwanow, D.A. May-Arrioja, D.N. Christodoulides, G.I. Stegeman, Y. Min, W. Sohler, Phys. Rev. Lett. 95 (2005) 053902.
[8] T.-Y. Lin, F.-C. Hsiao, Y.-W. Jhang, C. Hu, S.-Y. Tseng, Opt. Express 20 (2012) 24085.
[9] H. Zhou, X. Jiang, J. Yang, M. Wang, J. Mod. Opt. 54 (2007) 2769.
[10] S. Longhi, J. Opt. B 7 (2005) L9.
[11] S. Longhi, G.D. Valle, M. Ornigotti, P. Laporta, Phys. Rev. B 76 (2007) 201101, R.
[12] S. Longhi, Laser Photon. Rev. 3 (2009) 243.
[13] K. Bergmann, H. Theuer, B.W. Shore, Rev. Mod. Phys. 70 (1998) 1003.
[14] R.G. Unanyan, B.W. Shore, K. Bergmann, Phys. Rev. A 59 (1999) 2910.

[15] C.Y. Ye, V.A. Sautenkov, Y.V. Rostovtsev, M.O. Scully, Opt. Lett. 28 (2003) 2213.
[16] D. Møller, L.B. Madsen, K. Mølmer, Phys. Rev. A 75 (2007) 062302.
[17] S. Longhi, Phys. Rev. E 73 (2006) 026607.
[18] E. Paspalakis, Opt. Commun. 258 (2006) 30.
[19] N.G. Kalugin, Y.V. Rostovtsev, Opt. Lett. 31 (2006) 969.
[20] B.-L. Weng, D.-M. Lai, X.-D. Zhang, Phys. Rev. A 85 (2012) 053801.
[21] X.-D. Zhang, W.-J. Liu, G.-F. Yang, Quantum Inf. Process. 14 (2015) 921.
[22] T. Pertsch, U. Peschel, F. Lederer, J. Burghoff, M. Will, S. Nolte, A. Tünnermann, Opt. Lett. 29 (2004) 468.