



Hierarchical quantum-information splitting

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ABSTRACT

We present a scheme for asymmetric quantum-information splitting, where a sender distributes asymmetrically a quantum secret (quantum state) to distant partners in a network. The asymmetric distribution leads to that the partners have different powers to recover the sender's secret. In other words, their authorities for getting the secret are hierarchized. In the scheme, the partners do not need to make any nonlocal operation. The scheme can also be modified to implement threshold-controlled teleportation.

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The combination of information theory and quantum mechanics leads to the advent of quantum-information science [1]. Entanglement, one of the most striking features of quantum mechanics, is the center resource for quantum-information processing. The extensive applications of quantum entanglement should owe to its nonlocal correlations. One well-known example is the quantum teleportation [2,3], which utilizes the nonlocality of the quantum channel, i.e., a bipartite or multipartite entangled state, to transport an unknown quantum state from one site to another one. In the original teleportation protocol of Bennett et al. [2], the sender (Alice) and the receiver (Bob) initially share a maximally entangled state of two particles. Alice then performs a joint measurement on her particle of the entangled pair and the particle whose state is to be teleported. With the measurement outcome transmitted from Alice via a classical channel, Bob can recover the teleported state by appropriate local transformations.

Generally, the more particles that can be entangled, the more clearly nonlocal effects are exhibited [4], and the more useful the states are for quantum-information processing [5]. In addition, the usefulness of an entangled state is usually related to its entanglement properties [6,7]. Thus exploring and exploiting multipartite entangled states are very important tasks for the ones who are studying quantum-information theory. It has been attracting much interest that what classes of multipartite entangled states are competent for achieving a defined quantum-information-pro-

cessing task and what they can do. Greenberger–Horne–Zeilinger (GHZ) states [8] are typical multipartite entangled states. With the GHZ states, Hillery et al. [9] firstly introduced the concept of quantum-information splitting (QIS), where a quantum secret (quantum state) is distributed to two or more distant partners such that any one of them can recover the secret with the assistance of the others. QIS can be considered as a generalization of teleportation, and also called open-destination teleportation or quantum-state sharing in literature [10,11].

QIS has extensive applications in quantum-information science, such as creating joint checking accounts containing quantum money [12], secure distributed quantum computation [13], and so on. Since the outstanding work of Hillery et al., QIS has been attracting much attention [14–20], and a scheme has already been experimentally realized [10]. However, all of these schemes are focused on the symmetric case where every partner has the same status, i.e., the same authority for getting the sender's secret. In Ref. [21], Gottesman pointed out that a more general QIS scheme should involve the asymmetry between the powers of the different participants, and showed that it is possible to construct an access structure that some subsets of the shares can be combined to reconstruct the secret quantum state. This case was further studied later [22,23]. Their idea is based on the theory of quantum error-correcting codes, and thus nonlocal operations are required.

In this paper, we present a scheme for distributing a quantum secret to three distant partners asymmetrically. The asymmetric distribution leads to that the partners have different powers to recover the sender's secret. In other words, their authorities for

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getting the secret are hierarchized. In the scheme, the partners do not need to perform any nonlocal operation.

The quantum channel of our scheme is the four-qubit entangled state, recently proposed by Yeo and Chua [6],

$$|\chi_{ABCD}\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle|\varphi_{BCD}^0\rangle + |1_A\rangle|\varphi_{BCD}^1\rangle), \quad (1)$$

where

$$\begin{aligned} |\varphi_{BCD}^0\rangle &= \frac{1}{2}(|0_B0_C0_D\rangle \\ &\quad - |0_B1_C1_D\rangle - |1_B0_C1_D\rangle + |1_B1_C0_D\rangle), \\ |\varphi_{BCD}^1\rangle &= \frac{1}{2}(|0_B0_C1_D\rangle \\ &\quad + |0_B1_C0_D\rangle + |1_B0_C0_D\rangle + |1_B1_C1_D\rangle). \end{aligned} \quad (2)$$

The state $|\chi_{ABCD}\rangle$ has many interesting properties and exhibits more clear nonlocality in some aspects than the counterparts of the well-known GHZ and W states [6,24]. In addition, it can be easily verified that at least two single-qubit measurements are required in order to completely disentangle $|\chi_{ABCD}\rangle$. Thus such a state has higher persistency of entanglement than a GHZ state which can be completely disentangled by only one local measurement. This may lead to that our scheme is more robust against decoherence than the scheme of Ref. [9].

We consider that Alice, Bob, Charlie, and Diana possess particles A, B, C , and D , respectively. These particles are in the entangled state $|\chi_{ABCD}\rangle$. Alice has another particle S which is in the state

$$|\xi_S\rangle = \frac{1}{\sqrt{1+|\lambda|^2}}(|0_S\rangle + \lambda|1_S\rangle). \quad (3)$$

The state of the whole system is

$$\begin{aligned} |\xi_S\rangle|\chi_{ABCD}\rangle &= \frac{1}{\sqrt{2(1+|\lambda|^2)}}(|0_S0_A\rangle|\varphi_{BCD}^0\rangle + |0_S1_A\rangle|\varphi_{BCD}^1\rangle) \\ &\quad + \frac{\lambda}{\sqrt{2(1+|\lambda|^2)}}(|1_S0_A\rangle|\varphi_{BCD}^0\rangle + |1_S1_A\rangle|\varphi_{BCD}^1\rangle). \end{aligned} \quad (4)$$

The task is: Alice wants to distribute the state $|\xi_S\rangle$ to her three partners, i.e., Bob, Charlie, and Diana, such that any one of them can recover the secret state with the assistance of the other two. To this end, Alice performs a joint measurement on her two particles S and A in the Bell basis $\{|\Psi_{SA}^\pm\rangle, |\Phi_{SA}^\pm\rangle\}$, and then informs the outcome to them. The four Bell states are given by

$$\begin{aligned} |\Psi_{SA}^\pm\rangle &= \frac{1}{\sqrt{2}}(|0_S0_A\rangle \pm |1_S1_A\rangle), \\ |\Phi_{SA}^\pm\rangle &= \frac{1}{\sqrt{2}}(|0_S1_A\rangle \pm |1_S0_A\rangle). \end{aligned} \quad (5)$$

For Alice's four possible measurement outcomes, $|\Psi_{SA}^\pm\rangle$ or $|\Phi_{SA}^\pm\rangle$, the particles held by Bob, Charlie, and Diana collapse into the following corresponding entangled states:

$$\begin{aligned} |\psi_{BCD}^\pm\rangle &= \frac{1}{\sqrt{1+|\lambda|^2}}(|\varphi_{BCD}^0\rangle \pm \lambda|\varphi_{BCD}^1\rangle), \\ |\phi_{BCD}^\pm\rangle &= \frac{1}{\sqrt{1+|\lambda|^2}}(|\varphi_{BCD}^1\rangle \pm \lambda|\varphi_{BCD}^0\rangle). \end{aligned} \quad (6)$$

The non-cloning theorem [25] allows only one particle to be in the state $|\xi_S\rangle$, so that any one of Bob, Charlie, and Diana, but not all, can recover such a state.

In order to recover Alice's secret state, Bob, Charlie, and Diana need to cooperate. Before they come to an agreement, their single-particle state-density matrices are given by

$$\begin{aligned} \rho_{B(C)} &= \frac{1}{2}(|0_{B(C)}\rangle\langle 0_{B(C)}| + |1_{B(C)}\rangle\langle 1_{B(C)}|), \\ \rho_D^\pm &= \frac{1}{2}(|0_D\rangle\langle 0_D| + |1_D\rangle\langle 1_D|) \pm i \frac{\text{Im}(\lambda)}{(1+|\lambda|^2)}(|1_D\rangle\langle 0_D| - |0_D\rangle\langle 1_D|), \end{aligned} \quad (7)$$

where ρ_D^\pm corresponds to Alice's measurement outcomes $|\Psi_{SA}^\pm\rangle$ and $|\Phi_{SA}^\pm\rangle$, and ρ_D^- corresponds to $|\Psi_{SA}^\pm\rangle$ and $|\Phi_{SA}^\pm\rangle$. It can be seen that Bob or Charlie knows nothing about the amplitude and phase of Alice's qubit S without the collaboration of the other two partners; Diana, however, has partial information about both the amplitude and phase of qubit S as long as receiving Alice's Bell-state measurement outcome. This case implies that Alice's quantum secret is distributed to Bob, Charlie, and Diana asymmetrically. We shall show that the asymmetric distribution leads to an interesting phenomenon: for recovering the secret, Bob (Charlie) needs the assistance of both of the other two partners, i.e., Diana and Charlie (Bob), while Diana only needs the help of any one of the other two, i.e., Bob or Charlie.

First, we assume that the three partners agree to let Diana possess the secret. We rewrite $|\psi_{BCD}^\pm\rangle$ and $|\phi_{BCD}^\pm\rangle$ as

$$\begin{aligned} |\psi_{BCD}^\pm\rangle &= \frac{1}{2\sqrt{1+|\lambda|^2}}[(|0_B0_C\rangle + |1_B1_C\rangle) \times (|0_D\rangle \pm \lambda|1_D\rangle) \\ &\quad - (|0_B1_C\rangle + |1_B0_C\rangle) \times (|1_D\rangle \mp \lambda|0_D\rangle)] \\ &= \frac{1}{\sqrt{2(1+|\lambda|^2)}}[|_{+B+C}\rangle(|_{-D}\rangle \pm \lambda|_{+D}\rangle) \\ &\quad + |_{-B-C}\rangle(|_{+D}\rangle \mp \lambda|_{-D}\rangle)], \\ |\phi_{BCD}^\pm\rangle &= \frac{1}{2\sqrt{1+|\lambda|^2}}[(|0_B0_C\rangle + |1_B1_C\rangle) \\ &\quad \times (|1_D\rangle \pm \lambda|0_D\rangle) + (|0_B1_C\rangle + |1_B0_C\rangle) \times (|0_D\rangle \mp \lambda|1_D\rangle)] \\ &= \frac{1}{\sqrt{2(1+|\lambda|^2)}}[|_{+B+C}\rangle(|_{+D}\rangle \pm \lambda|_{-D}\rangle) \\ &\quad - |_{-B-C}\rangle(|_{-D}\rangle \mp \lambda|_{+D}\rangle)], \end{aligned} \quad (8)$$

where $|\pm_j\rangle = (|0_j\rangle \pm |1_j\rangle)/\sqrt{2}$ ($j = B, C, D$). Obviously, if Alice and Bob measure their qubits in the basis $\{|+\rangle, |-\rangle\}$, their outcomes are always correlated, i.e., Bob's (Charlie's) outcome can be deduced by Charlie's (Bob's). This implies that the outcome $(|+\rangle)$ or $(|-\rangle)$ of any one of them is sufficient to help Diana recover the secret. Indeed, Diana can reconstruct the state $|\xi_S\rangle$ on qubit D by appropriate local operations based on the measurement outcomes of Alice and only one of Charlie and Bob. The local unitary transformations that Diana should perform (on qubit D) according to Alice's measurement outcomes (on qubits S and A) and Bob's or Charlie's measurement outcomes (on qubit B or C) are given in Table 1, where $\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$ and $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$ are the usual Pauli operators, and H is the Hadamard transformation functioning as $H|0\rangle = |+\rangle$ and $H|1\rangle = |-\rangle$. It can be seen from Eq. (8) that Bob and Charlie can also choose the measurement basis $\{|0\rangle, |1\rangle\}$, but then Diana needs the collaboration of both of them.

Now, we consider the case that they agree to let Bob possess Alice's secret, i.e., recover the state $|\xi_S\rangle$. We then rewrite $|\psi_{BCD}^\pm\rangle$ and $|\phi_{BCD}^\pm\rangle$ as

Table 1

The corresponding local operations that Diana should perform for recovering the state $|\xi\rangle$, according to Alice's Bell-state measurement outcomes and Bob's or Charlie's single-qubit measurement outcomes.

Alice's outcomes	Bob's or Charlie's outcomes	Diana's operations
$ \Psi^+\rangle$	$ +\rangle (-\rangle)$	$\sigma_x H (\sigma_z H)$
$ \Psi^-\rangle$	$ +\rangle (-\rangle)$	$\sigma_x \sigma_z H (H)$
$ \Phi^+\rangle$	$ +\rangle (-\rangle)$	$H (\sigma_x \sigma_z H)$
$ \Phi^-\rangle$	$ +\rangle (-\rangle)$	$\sigma_z H (\sigma_x H)$

$$\begin{aligned}
 |\psi_{BCD}^\pm\rangle &= \frac{1}{2\sqrt{1+|\lambda|^2}} [(|0_B\rangle \pm \lambda|1_B\rangle) |0_C 0_D\rangle \\
 &\quad - (|1_B\rangle \mp \lambda|0_B\rangle) |0_C 1_D\rangle + (|1_B\rangle \pm \lambda|0_B\rangle) \\
 &\quad \times |1_C 0_D\rangle - (|0_B\rangle \mp \lambda|1_B\rangle) |1_C 1_D\rangle], \\
 |\phi_{BCD}^\pm\rangle &= \frac{1}{2\sqrt{1+|\lambda|^2}} [(|1_B\rangle \pm \lambda|0_B\rangle) |0_C 0_D\rangle \\
 &\quad + (|0_B\rangle \mp \lambda|1_B\rangle) |0_C 1_D\rangle + (|0_B\rangle \pm \lambda|1_B\rangle) \\
 &\quad \times |1_C 0_D\rangle + (|1_B\rangle \mp \lambda|0_B\rangle) |1_C 1_D\rangle]. \tag{9}
 \end{aligned}$$

It can be seen that Bob can reconstruct the state $|\xi\rangle$ if and only if both Charlie and Diana measure their qubits in the basis $\{|0\rangle, |1\rangle\}$ and broadcast their outcomes. In other words, Bob needs the help of both of the other two partners, Charlie and Diana, for recovering the secret. The corresponding local operations that Bob should make according to Alice's Bell-state measurement outcomes and Charlie's and Diana's single-qubit measurement outcomes are given in Table 2. These results are also applicable to the case where Charlie is deputed to recover Alice's secret because $|\chi_{ABCD}\rangle$ is unchanged under the permutation of qubits B and C , which indicates that Bob and Charlie have the same status in the above QIS protocol.

In a word, for recovering the secret state $|\xi\rangle$, Diana only needs the assistance of any one of Bob and Charlie, while Bob (Charlie) needs the help of both Diana and Charlie (Bob). Thus, their authorities for getting the secret are hierarchized, and Diana is in a higher position relative to Bob and Charlie.

The above QIS protocol can also be modified to implement controlled teleportation [3] if we choose Diana as the receiver in advance. The scheme of Ref. [3] is a (2,2)-threshold controlling scheme where the achievement of teleportation is conditioned on the collaboration of both of the two supervisors, while the present one is a (1,2)-threshold controlling scheme where one of the two supervisors (Bob and Charlie) can help Diana successfully recover the teleported state. Very recently, several other types of (k, m) -threshold-controlled teleportation protocols have also been proposed [26–28].

As to the physical realization of the above QIS scheme, we mainly need to consider two points: the preparation of the entangled state $|\chi\rangle$ and the Bell-state measurement. Recently, several

methods for preparing the state $|\chi\rangle$ have been proposed [29–32]. In Ref. [29], one of our authors proposed a scheme for generating the state $|\chi\rangle$ with a well controllable ion-trap setup [33]. Ref. [30] showed that a four-photon $|\chi\rangle$ state can be effectively created even with conventional photon detectors. Ref. [32] presented a simple scheme for producing the $|\chi\rangle$ -type entanglement among four atoms separately trapped in four distant cavities [34,35], by emitted-photon interference. Based on the current techniques [33–36], these schemes are experimentally achievable. In addition, the Bell-state measurement has been well realized for both atomic and photonic qubits [37–39]. All these achievements may contribute to our QIS scheme in physical realization.

In conclusion, we have proposed a QIS protocol with a particular four-partite entangled state as the quantum channel, where the three partners, i.e., Bob, Charlie, and Diana, have different authorities for getting Alice's (the sender) quantum secret. Diana only needs the assistance of any one of the other two partners, Bob and Charlie, for recovering the secret; while Bob (Charlie) needs the help of both of the other two partners, Diana and Charlie (Bob). That is, Diana has a larger authority than Bob and Charlie to possess the final secret. In other words, their authorities are hierarchized, and Diana is in a higher position relative to Bob and Charlie. The security checking for the quantum channel is the same as that of Ref. [40]. The presented QIS scheme is experimentally achievable with state-of-the-art. Our scheme can also be modified to implement (1,2)-threshold-controlled teleportation.

The hierarchical QIS may be very interesting in view of the reliability of the partners in quantum communication and the access controlling in architecture of quantum computer, and should be more useful than the symmetric QIS in practice. Let us take a simple example that a dealer in Berlin wants to have an action taken on her behalf in Beijing. She has three partners who can carry it out for her, but she knows that some of them are dishonest and does not know whom they are. She cannot simply send a message to one of them, because the dishonest ones will try to sabotage the action, but she knows that if all of them carry it out together, the honest ones will keep the dishonest ones from doing any damage. Then she can encode the message in a quantum state (quantum secret) and distribute it among them through the generalized teleportation protocol discussed above. The partner who is the most reliable will be distributed a larger part of information. As a consequence, the most reliable partner can recover the secret with the cooperation of subset of the other ones, but the other ones cannot get the secret without the participation of the most reliable one.

It is worth pointing out that a more general hierarchical QIS scheme should involve more than three parties. However, it will be much more complicated and cannot be obtained by directly generalizing the above scheme. In fact, there are many types of hierarchies for the case where more than three parties are involved, and each one of them will relate to the construction of a special structure of multipartite entangled state. Let us take an example of the simplest case where the hierarchy only involves two ranks, i.e., only one of the parties is in a higher position and the other ones have the same status. Then we need to construct a multipartite entangled state in which at least one particle is different from the other ones, however, these ones are fully symmetrical and are equivalent to each other under permutation. As to a more general case where the hierarchy has a tree-like structure, the related entanglement channel will have a much more complicated configuration. The physical generation of the complicated many-party entangled states is also temporarily difficult. Thus one will need to make further efforts. In principle, however, these states could be constructed. Here, we just present a possible idea as follows. For clarity and simplicity, we also consider the simplest case where only one party has a larger authority than the other ones to possess the final secret. With loss of generality, we assume

Table 2

The corresponding local operations that Bob should perform for recovering the state $|\xi\rangle$, according to Alice's Bell-state measurement outcomes and Charlie's and Diana's single-qubit measurement outcomes.

Alice's outcomes	Charlie's and Diana's outcomes	Bob's operations
$ \Psi^+\rangle$	$ 0_C 0_D\rangle (0_C 1_D\rangle)$	$I (\sigma_x \sigma_z)$
$ \Psi^+\rangle$	$ 1_C 0_D\rangle (1_C 1_D\rangle)$	$\sigma_x (\sigma_z)$
$ \Psi^-\rangle$	$ 0_C 0_D\rangle (0_C 1_D\rangle)$	$\sigma_z (\sigma_x)$
$ \Psi^-\rangle$	$ 1_C 0_D\rangle (1_C 1_D\rangle)$	$\sigma_x \sigma_z (I)$
$ \Phi^+\rangle$	$ 0_C 0_D\rangle (0_C 1_D\rangle)$	$\sigma_x (\sigma_z)$
$ \Phi^+\rangle$	$ 1_C 0_D\rangle (1_C 1_D\rangle)$	$I (\sigma_x \sigma_z)$
$ \Phi^-\rangle$	$ 0_C 0_D\rangle (0_C 1_D\rangle)$	$\sigma_x \sigma_z (I)$
$ \Phi^-\rangle$	$ 1_C 0_D\rangle (1_C 1_D\rangle)$	$\sigma_z (\sigma_x)$

that Alice's $n + 1$ partners are {Bob, Charlie1, Charlie2, ..., Charlien}, and Bob has a larger power than charlies to recover Alice's quantum secret. Then the entanglement channel can be written as

$$|\psi\rangle_{ABC_1\dots C_n} = \frac{1}{\sqrt{2}} \left(|0\rangle_A |\phi^0\rangle_{BC_1\dots C_n} + |1\rangle_A |\phi^1\rangle_{BC_1\dots C_n} \right), \quad (10)$$

where $|\phi^0\rangle_{BC_1\dots C_n}$ and $|\phi^1\rangle_{BC_1\dots C_n}$ need satisfying the following conditions: (1) when Bob is chosen to possess Alice's secret state of Eq. (3),

$$\left. \begin{array}{l} \frac{1}{\sqrt{1+|\lambda|^2}} \left(|\phi^0\rangle_{BC_1\dots C_n} \pm \lambda |\phi^1\rangle_{BC_1\dots C_n} \right) \\ \frac{1}{\sqrt{1+|\lambda|^2}} \left(|\phi^1\rangle_{BC_1\dots C_n} \pm \lambda |\phi^0\rangle_{BC_1\dots C_n} \right) \end{array} \right\} \begin{array}{l} \text{local measurements are performed in subset of } \{C_j\} \\ \text{corresponding operations are made in } B \end{array} |\xi\rangle_B; \quad (11)$$

(2) when one of Charlies, e.g., Charlie1, is chosen to possess the final secret,

$$\left. \begin{array}{l} \frac{1}{\sqrt{1+|\lambda|^2}} \left(|\phi^0\rangle_{BC_1\dots C_n} \pm \lambda |\phi^1\rangle_{BC_1\dots C_n} \right) \\ \frac{1}{\sqrt{1+|\lambda|^2}} \left(|\phi^1\rangle_{BC_1\dots C_n} \pm \lambda |\phi^0\rangle_{BC_1\dots C_n} \right) \end{array} \right\} \begin{array}{l} \text{local measurements are performed in } \{B, C_j, j \neq 1\} \\ \text{corresponding operations are made in } C_1 \end{array} |\xi\rangle_{C_1},$$

$$\left. \begin{array}{l} \frac{1}{\sqrt{1+|\lambda|^2}} \left(|\phi^0\rangle_{BC_1\dots C_n} \pm \lambda |\phi^1\rangle_{BC_1\dots C_n} \right) \\ \frac{1}{\sqrt{1+|\lambda|^2}} \left(|\phi^1\rangle_{BC_1\dots C_n} \pm \lambda |\phi^0\rangle_{BC_1\dots C_n} \right) \end{array} \right\} \begin{array}{l} \text{local measurements are performed in } \{C_j, j \neq 1\} \\ \text{corresponding operations are made in } C_1 \end{array} |Error\rangle_{C_1}, \quad (12)$$

where $|Error\rangle_{C_1}$ denotes a wrong state differing from $|\xi\rangle_{C_1}$. In the same vein, the tree-structure many-party entangled states may also be possible to be constructed in theory. The multipartite entangled states involving more than four parties are under intensive research [26,41,42], and thus we believe that these states can be successfully constructed in the future.

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